Lec 12: Applications of 1D Photonic Crystal



Hello everyone, welcome to lecture 12 of the online course on Photonic Crystals Fundamentals and Applications.



- Applications of 1D PC:
 - Bragg Grating
 - Bragg Reflection
 - Bragg Grating A Simplified Theory
 - Stack of Partially Reflected Mirrors
 - Dielectric Bragg Grating
 - Omnidirectional Multilayer Mirrors
 - Periodic Dielectric Waveguides
 - Point Defects in Periodic Dielectric Waveguides
 - □ Periodic Dielectric Waveguides as Fiber Bragg Grating

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Source: B. E. Saleh and M. C. Teich, Fundamentals of photonics, John Wiley & Sons, 2019.

Today's lecture we will be discussing on the applications of 1D photonic crystals. So, here is the lecture outline, we will discuss about Bragg rating first. We will put forward the theory of Bragg reflection. Then how do you make Bragg grating using stack of partially reflected mirrors. Then we will also look for you know dielectric Bragg grating.

Then we'll discuss omnidirectional multilayer mirrors, periodic dielectric waveguides,

point defects in those dielectric waveguides. And then we'll look into the application of periodic dielectric waveguides as fiber bragg grating.

Lecture Outline	

Applications of 1D PC:

Bragg Grating

- Bragg Reflection
- Bragg Grating A Simplified Theory
- Stack of Partially Reflected Mirrors
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- Omnidirectional Multilayer Mirrors
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Applications of 1D PC: Bragg Grating

- The Bragg grating is introduced as a set of uniformly spaced parallel partially reflective planar mirrors.
- Such a structure has angular and frequency selectivity that is useful in many applications.
- Here, we generalize the definition of the Bragg grating to include a set of N uniformly spaced identical multilayer segments.
- Devices fabricated according to this prescription include distributed Bragg reflectors (DBRs) and fiber Bragg gratings (FBGs), which are often used in resonators and lasers.



Figure: Bragg grating comprising N identical mirrors.

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Source: B. E. Saleh and M. C. Teich, Fundamentals of photonics, John Wiley & Sons, 2019.

So let's focus on the first application that is bragg grating. So bragg grating is introduced as a set of uniformly spaced parallel partially reflective planar mirrors.

So this is how you can think of. So these are basically partially reflective mirrors which are uniformly spaced and they are parallel to each other. So there can be n number of identical mirrors. So such a structure has angular and frequency selectivity and that is useful for many applications. So, here we will generalize the definition of bragg gating to include a set of n uniformly spaced identical multi-layer segments.

So, N can be any practical number right. integer. So, devices fabricated according to this prescription are something like distributed Bragg reflectors which are DBRs in short form or you can think of fiber Bragg grating that is FBG. So, these are very often used in resonators, lasers you know filters.



So let us go into a little bit of more details of how exactly Bragg reflection takes place.

So let's consider light reflected at an angle θ okay from this M parallel refracting planes which are basically separated by a distance of Λ . So remember here the θ is basically measured from the you know plane of this reflecting planes okay. or from this reflecting planes you are measuring the θ . So, let us assume that only a small fraction of the light that is reflected from each of this plane okay are actually coming back and you can think of the amplitudes of this M reflected waves to be approximately equal okay. So, the reflected waves will have a phase difference of φ which is basically you know $k(2\Lambda sin\theta)$.

So, what is this θ ? Theta is basically this angle okay. And the phase difference is such that the intensity of the reflected light maximizes. So, you can think of you know $\sin\theta_B = \lambda/2\Lambda$. So, what is λ ? So, λ is basically the wavelength of light, capital lambda is this particular period you can say or the thickness between the two parallel refracting planes and $\sin\theta_B$ is basically your Bragg angle okay.

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So, here we understood that the maximum intensity of the reflected light can be achieved if θ_B satisfies this particular equation okay. So, such reflection are found in case when light encounters or light is reflected from a multilayer structure such as this. And as we mentioned, θ is basically defined with respect to this parallel planes.

Bragg Grating — A Simplified Theory

- The reflectance of the Bragg grating is determined under two assumptions:
 - > The mirrors are weakly reflective so that the incident wave is not depleted as it propagates.
 - > Secondary reflections (i.e., reflections of the reflected waves) are negligible.



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Source: B. E. Saleh and M. C. Teich, Fundamentals of photonics, John Wiley & Sons, 2019.

Now, the reflectance of bragg grating is determined under two assumptions. First is that all these mirrors are basically weakly reflective mirrors, okay, so that the incident wave is not depleted as it is propagating.

Means if these are strongly reflecting mirrors, then majority of the light will be reflected. So there will be very little light which will be able to get into and hit the second mirror or the third mirror and so on. So we assume that the mirrors are very weakly reflected. So most of the light basically goes through, penetrates inside and comes and hit the second one and so on. And finally, you actually get a transmission also out of this entire, you know, Bragg grating.

and second thing is we consider the secondary reflections which are basically the reflections of the reflected light okay those we consider to be negligible because if you consider those the calculations become much more messy so we are safe to assume that these secondary reflections are negligible because we consider the mirrors as very weakly reflective ones right.

Bragg Grating — A Simplified Theory

In this approximation, the reflectance R_N of an N-mirror grating is related to the reflectance R of a single mirror by the equation:

$$\mathcal{R}_N = \frac{\sin^2 N\varphi}{\sin^2 \varphi} \mathcal{R} \tag{L12.2}$$

The quantity φ denotes the phase between the successive phasors whereas here the phase is denoted by 2φ since it represents a round trip.



So, in this approximation you can actually calculate the reflectance of this entire structure to be Rn, ok. So, here n reminds you that you are talking about a n mirror grating, ok. So, how it is related to a reflectance R of the single mirror? You can actually look into this equation where R_N equals $(\sin^2 N\varphi)/(\sin^2 \varphi)$ R. So, this is how you correlate the overall reflectance of this grating to the reflectance of a single mirror.

So, here what is this φ ? The φ is basically denoting the phase between the successive phasors. Whereas, you know, here the phase is denoted by 2φ because it represents a round trip.

Bragg Grating – A Simplified Theory The factor sin²Nφ/sin²φ represents the intensity of the sum of N phasors of unit amplitude and phase difference 2φ. This function has a peak value of N² when the Bragg condition is satisfied, *i.e.*, when 2φ equals q2π, where q = 0, 1, 2, It drops away from these values sharply, with a width that is inversely proportional to N. In this simplified model, the intensity of the total reflected wave is, at most, a factor of N² greater than the intensity of the wave reflected from a single segment. Fure: Bragg grating comprising N identical mirrors.

So, if you think of this factor $(\sin^2 N\varphi)/(\sin^2 \varphi)$, okay, it represents the intensity of the sum of N phasors of unit amplitude and they have phase difference of 2φ , right. So, this function can have a peak value of N². okay when the Bragg grating condition is satisfied that means the round trip phase difference in each case will be integral multiple of 2φ .

So, you can write it as $q2\pi$ where q is nothing but 0 1 2 and dot dot dot all the integers right. So, what we understood that at the peak the value is very large it is equal to N square, but it drops away from this value sharply okay with a width that is inversely proportional to N okay. So, you can think of a very high Q kind of a reflection spectrum over here. So, in this simplified model the intensity of the total reflected wave is typically you know go up to a factor of N², then the intensity of the wave that is reflected from a single mirror. So, this is what is the contribution from this factor.

Bragg Grating — A Simplified Theory

- For a Bragg grating comprising partially reflective mirrors separated from each other by a distance Λ and a round-trip phase $2\varphi = 2k\Lambda\cos\theta$, where θ is the angle of incidence.
- Therefore, maximum reflection occurs when:



So, we understood that for a Bragg grating comprising partially reflective mirrors which are separated from each other by a distance of Λ . And if the round trip phase difference $2\varphi = 2k\Lambda\cos\theta$ considering that θ is now the angle of incidence not the angle from the surface of the mirrors ok. You can say that the maximum reflectance will occur when this round trip phase difference that is $2k\Lambda\cos\theta$ is equal to $2q\pi$ or you can write 2 q pi ok. So, from here you can write $\cos\theta$ is equal to q ($\lambda/2\Lambda$) you can write this in terms of frequency you can write it as q ($\omega_{\rm B}/\omega$) or you can write q ($\nu_{\rm B}/\nu$). So, what is this? This is the Bragg frequency ok and this is the frequency that you are applying ok.



So, what is the relation? As you can see that $v_{\rm B}$, the Bragg linear frequency is basically $=c/2\Lambda$ and the Bragg angular frequency will be 2π of this factor. So, it will be $\pi c/\Lambda$. You understand 2, 2 cancels out. So, you will be left with $\pi c/\Lambda$.

So, that is ω_B . Understood. So, only difference here is that the θ that we are considering is basically the incident angle. Okay. So, it is measured with respect to the normal.

Bragg Grating — A Simplified Theory

• Bragg Condition:
$$\cos \theta = q \frac{\lambda}{2\Lambda} = q \frac{\omega_B}{\omega} = q \frac{\nu_B}{\nu}$$
 (L15.3)

where
$$\nu_{\mathcal{B}} = \frac{c}{2\Lambda}$$
, $\omega_{\mathcal{B}} = \frac{\pi c}{\Lambda}$ Bragg Frequency (L15.4)

- At normal incidence ($\theta = 0^{\circ}$), peak reflectance occurs at frequencies that are integer multiples of the Bragg frequency, *i.e.*, $v_{B} = qv_{B}$.
- At frequencies such that $\nu < \nu_B$, the Bragg condition cannot be satisfied at any angle.
- At frequencies $\nu < \nu_B < 2\nu_B$, the Bragg condition is satisfied at one angle $\theta = \cos^{-1}(\lambda/2\Lambda) = \cos^{-1}(\nu_B/\nu)$.

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Source: B. E. Saleh and M. C. Teich, Fundamentals of photonics, John Wiley & Sons, 2019.
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Okay. So, if we consider normal incidence the peak reflectance basically occurs at frequencies which are integral multiple of the Bragg frequency which is v_B . So, you can consider v_B or you it will be basically v_B equals qv_B ok. So, all these frequencies so you can actually yeah that you can remove ok anyways. So, you read this as v_B equals v_B sorry v_B equals qv_B . So, at the frequencies where the frequency is less than the Bragg frequency, it is clear that Bragg condition cannot be satisfied at any angle.

And when you have frequencies, okay, which and when you have frequency, okay, which is in between Bragg frequency. So, I think this will be ν this will be ν_B and this will be $2\nu_B$ ok. So, when you have frequency in between ν_B and $2\nu_B$ that is where you know the Bragg condition can be satisfied at one angle and that angle can be found as θ equals $cos^{-1}(\lambda/2\Lambda)$ ok. So, that boils down to cos inverse of ν_B/ν . So, read this as you know ν_B less than ν less than $2\nu_B$ ok.

Bragg Grating — A Simplified Theory



- Locus of frequencies v and angles θ at which the Bragg condition is satisfied.
- For example, if $v = 1.5v_{\mathcal{B}}$ (dot-dash line), we have $\theta = 48.2^{\circ}$.
- This corresponds to a Bragg angle $\theta_B = (90 \theta) = 41.8^\circ$ (when measured from the plane of the grating.)

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Source: B. E. Saleh and M. C. Teich, Fundamentals of photonics, John Wiley & Sons, 2019.

Λ

θ

80° 70° 60° 50°

40° 30° 20°

10°

So, this is the you know simplified theory of break rating. So, remember here that we have considered break rating consisting of N identical mirrors and the separation between the mirrors is given by Λ And if you try to plot the locus of the frequencies and angle at which the Bragg grating is or the bragg condition is satisfied, you can obtain a figure like this. So, what does it tell you? So, if you consider ν equals 1.5 ν_B , okay. that is basically this point where ν by ν_B will be 1.5. So, if you go ahead and you will find out that θ comes out to be 48.2 degrees ok. So, this is the incident angle ok at which you will be able to get the break condition right. Now this corresponds to a Bragg angle so for θ equals 48.2 degree you can find θ_B that is 90- θ which is basically 41.8 degrees when you are measuring from the plane of the grating.

Stack of Partially Reflected Mirrors

- Bragg grating comprising N = 10 identical mirrors, each with a power reflectance $|r|^2 = 0.5$.
- Dependence of Φ on the inter-mirror phase delay $\varphi = nk_o\Lambda$.



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Source: B. E. Saleh and M. C. Teich, Fundamentals of photonics, John Wiley & Sons, 2019.

So if you consider a Bragg grating which has N equals 10 identical mirror and where the reflectance capital R or you can say modulus of modulus square of the reflection coefficient okay that is this one that is equal to 0.5 okay. So, in such case you can consider the dependence of φ on the inter-mirror phase delay. So, φ actually tells you about the inter mirror phase delay okay that is nk_o and what is the separation between the two mirrors that is Λ . What is n? n is basically the refractive index of this medium right and k_o is the vacuum wave number fine.

Stack of Partially Reflected Mirrors

- Within the shaded regions, Φ is complex and its imaginary part Φ_1 is represented by the dashed curves.
- Reflectance \mathcal{R} as a function of frequency (in units of the Bragg frequency $\nu_{\mathcal{B}} = c/2\Lambda$). Within the stop bands, the reflectance is approximately unity.



So if you think of this, so within the shaded regions, you can see that φ , the phase, is basically complex. And its imaginary part, that is φ_i , is represented by dash curves. And if you plot the reflectance, capital R, as a function of frequency, so the frequency is basically here. in terms of the break frequency v_B so you are taking in terms of say 0, v_B , $2v_B$ so at every integral multiple of v_B the break frequency you will see there is reflection so there is a stop band okay so the first one is here and then you have at v_B at 2 v_B , 3 v_B and so on So, what happens at the stop bands you can see that the reflectance is almost unity.

Dielectric Bragg Grating

- Power reflectance (\mathcal{R}) as a function of frequency for a dielectric Bragg grating comprising N = 10 segments, each of which has two layers of thickness $d_1 = d_2$ and refractive indices $n_1 = 1.5$ and $n_2 = 3.5$.
- The grating is placed in a medium with matching refractive index n₁.



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Source: B. E. Saleh and M. C. Teich, Fundamentals of photonics, John Wiley & Sons, 2019.

So, whatever is getting incident is basically getting reflected. So, now you can think of a dielectric bragg grating where you choose your you know grating materials. according to your need so let us calculate the power reflectance okay which is R as a function of the frequency for a dielectric bragg grating so here let us consider N equals 10 so you have got 10 segments and each of this segment is basically comprising of two layers So, one material is having refractive index of n_1 another one is got n_2 and n_1 we consider say 1.5 and n_2 is 3.5 and we have considered the thickness d_1 and d_2 to be equal in this case ok.

And another thing is you consider the grating to be placed in a medium that matches with the refractive index of n_1 . So, the outside media of this grating is also having refractive index of n_1 .

Dielectric Bragg Grating

• The reflectance is approximately unity within the stop bands centered about multiples of $v_B = c/2\Lambda$, where $c = c_o/\overline{n}$ and \overline{n} is the mean refractive index.





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Source: B. E. Saleh and M. C. Teich, Fundamentals of photonics, John Wiley & Sons, 2019.

So, when you do that ok the reflectance is approximately unity within the stop bands which are centered at v_B and this v_B is nothing but c/2 Λ where you have to consider c as you know c naught over n bar. So, what is c₀? c₀ is basically the speed of light in vacuum ok and \bar{n} is here the mean refractive index because the refractive index is changing over this period. So, you have to consider the mean refractive index right.

So, you can see that the stop bands behave like this for this kind of bragg grating.

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Omnidirectional Multilayer Mirrors

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Source: B. E. Saleh and M. C. Teich, Fundamentals of photonics, John Wiley & Sons, 2019.

Omnidirectional Multilayer Mirrors

> Off-Axis Propagation

00/2πc

Frequency

0.2

-0.5

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• So far, we have considered the modes of a one-dimensional photonic crystal which happen to have $k_{\parallel} = 0$; that is, modes that propagate only in the *z* direction.

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TM: x - polarized

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k,

In this section lets discuss off-axis modes.

0.5 Wave vector ka/2π



• Figure shows the band structure for modes with $k = k_y \hat{y}$ for the one-dimensional photonic crystal.

Source: B. E. Saleh and M. C. Teich, Fundamentals of photonics, John Wiley & Sons, 2019.

So, now let us move on to the next subtopic which is omnidirectional multilayer mirrors. So, when you think of omnidirectional multilayer mirrors, we are basically considering all the directions right. So, we have to consider off axis propagation. So, if you remember our discussion about this 1D photonic crystal, we were mainly focusing on the wave vector that lies along z axis.

But you know that has always given us the k_{\parallel} that is the wave vector along the plane to be 0 right because we only consider the modes to be propagating along the z direction. But now

let us consider the off axis modes as well. So, in this particular figure, you can see that there are some off axis modes shown for wave vector k equals $k_y \hat{y}$.

So, this is for the z case, but these are the bench structure which corresponds to off axis propagation. So the photonic band structure here which is plotted is basically for you know a multi-layer film which has got a lattice constant of a and it has got alternating layers of different widths.

So what has been considered epsilon equals 13 as one layer and the thickness for that layer is 0.2a. okay and the width of the other layer that is epsilon equals 1 that is air is considered to be 0.8 a. So, with that that particular band diagram okay is obtained.

- Band structure of a multilayer film
- The on-axis bands (0, 0, k_z) are shown on the left side, and an off-axis band structure (0, k_y, 0) is displayed on the right.
- On-axis, the bands overlap—they are degenerate.
- Along k_y, the bands split into two distinct polarizations.
- **Blue** indicates TM modes polarized so that the electric field points in the *x* direction.
- Red indicates TE modes polarized in the yz plane.



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Now here there are two interesting features. So, one you called as on axis bands. So, this ones So, they are represented as 0, 0, k_z that is on the left side and you also have off axis band structure that is this one which can be represented as 0, k_y , 0 right which is displayed on the right. So, here you see that the on axis the bands typically overlap. So, they are degenerate. However, when you go to ky that is the off axis case the bands basically split into two distinct polarization.

So, the blue one tells you about TM which is x polarized and the red one tells you about TE which is yz polarized okay. So, when I say yz polarized it means the polarization lies in yz plane right. So, the important difference between this on axis and off axis propagation is that there is no band gap in the case of off axis propagation that is possible for all the values of k_y. Okay, so this is always the case for a multilayer film because the off axis contains no periodic dielectric region to coherently scatter light and which can split open a gap. So you do not get a band gap which is so here you can see there is the band gap.

but here but here it actually merges ok. So, that is between n equals 1 and 2 ok. Here also you can see they are basically getting merged. So, there is no band gap that is wide open for entire value of k_y ok. So another important difference as we already discussed between on axis and off axis case is the degeneracy of the bands because here you can see the red and the blue are kind of overlapping right. So the electric field in the case of on axis are oriented at xy plane okay and we might choose the two basic polarizations as x and y directions.

• The most important difference between on-axis and off-axis propagation is that:

there are no band gaps for off-axis propagation when all possible k_y are included.

- This is always the case for a multilayer film, because the off-axis direction contains no periodic dielectric regions to coherently scatter the light and split open a gap.
- Another difference between the on-axis and off-axis cases involves the degeneracy of the bands.
- For the case of on-axis propagation, the electric field is oriented in the *xy* plane.
- We might choose the two basic polarizations as the x and y directions.



 Since those two modes differ only by a rotational symmetry which the crystal possesses, they must be degenerate.

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Source: B. E. Saleh and M. C. Teich, Fundamentals of photonics, John Wiley & Sons, 2019.

Now since those two modes differ only by a rotational symmetry which the crystal already proposes, possesses, so they are degenerate and rightly so you can see here that all these X and Y polarized, X polarized and YZ polarized are basically overlapping. Now, whenever a mode is propagating along any arbitrary direction of k, this symmetry is broken and the degeneracy is lifted and that is why you can actually see that all the bands are different in case of this ky, okay, of excess propagation. There are other symmetries for example, notice that the system is invariant under reflection through yz plane. So, whenever we are discussing about the system I hope all of you understand that this is the system we are discussing about right. So, they are saying that across yz plane it has got a reflection symmetry which is true ok.

- However, for a mode propagating with an arbitrary direction of k, this symmetry is broken: The degeneracy is lifted.
- There are other symmetries; for example, notice that the system is invariant under reflection through the *yz* plane.
- For the special case of propagation down the dielectric sheets, in the y direction, the possible polarizations are in the x direction (TM) or in the yz plane (TE).



 But there is no rotational symmetry relationship between these two bands, so they will generally have different frequencies as displayed in Figure.

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Source: B. E. Saleh and M. C. Teich, Fundamentals of photonics, John Wiley & Sons, 2019.

and the system is basically invariant under reflection through YZ plane yes because in YZ it is just you know along that plane this. So, we can see that the system is invariant under reflection through the YZ plane. So, for the special case of propagation down the dielectric sheets in the Y direction the possible polarizations are in the x direction that is you can call them as TM polarization or it is along the yz plane you can take it as TE polarization. but there is no rotational symmetry relationship between these two bands ok and they will generally have different frequencies as displayed in the figure.

- As described in the section Off-Axis Propagation, a multilayer film does not have a complete band gap, once one allows for a component of the wave vector that is parallel to the layers.
- Another way to state this is that for every choice of ω, there exist extended modes in the film for some wave vectors (k_{||}, k_z).
- Given this fact, it may seem paradoxical that a properly designed multilayer structure can still reflect light waves that are incident from *any* angle, with *any* polarization, if it has a frequency that is within a specified range.



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Source: B. E. Saleh and M. C. Teich, Fundamentals of photonics, John Wiley & Sons, 2019.

So, this is what you can see here. So, we have mentioned this again previously that for the case of off axis polarization a multi layer film does not have a complete band gap ok. Once one allows for a component of the wave vector that is parallel to the layers. So, another way to state that is that for every choice of ω there exist extended modes ok. In this in the film for some wave vector (k_{\parallel} , k_z) right. There is no band gap means for every frequency there is some kind of you know mode possible Okay and here the wave vectors will be (k_{\parallel} , k_z).

So given this fact it may seem paradoxical that a properly designed multilayer structure can still refract light that are incident from any angle with any polarization if the frequency lies within a certain specified range.

- Such a device, an omnidirectional mirror, relies on two physical properties.
 - First, *k* is conserved at any interface parallel to the layers, if the light source is far enough away that it does not interrupt the translational symmetry of the structure in the direction.
 - Second, light that is incident from air must have $\omega > c|\mathbf{k}|$, corresponding to the freely propagating modes above the light line.
- Modes that are below the light line are evanescent modes that cannot reach the mirror from a faraway source.
- Because of these two properties, the modes that the crystal harbors below the light line are irrelevant for the purpose of reflection.

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Source: B. E. Saleh and M. C. Teich, Fundamentals of photonics, John Wiley & Sons, 2019.

So, this kind of device is called an omnidirectional mirror and it relies basically on two physical properties. The first one is that the k vector is conserved at any interface parallel to the layers. If the light source is far away, okay that it does not interrupt with the translational symmetry of the structure in that direction. Second is light that is incident from air must have ω greater than c|k|.

okay. That means corresponding to freely propagating modes which are above the light line. So, ω equals c|k| is basically the light line. So, you consider you know In this case that light should which are incident from the air should have you know frequency more than ck. So, the modes that are below the light line will not be allowed. So, they are basically the evanescent modes that cannot reach the mirror from a far away source.

And because of these two properties, so there are two properties. So one is that the light source is far away and the second thing is the light which is incident from air must have ω greater than c|k|. So with these two, you can actually make the device work like a omnidirectional mirror. And because of these two properties the modes of the crystal harbours below the light line are irrelevant for the purpose of reflection.



So, to investigate this omnidirectional reflection let us plot this ω versus k_y.

So, again we plot in terms of normalized frequency and normalized wave factor. So, here you can see the modes okay which are the shaded regions for the off axis propagation and the propagation vectors are 0, k_z and k_y and what we have considered here we have considered a quarter wave stack with permittivity 13 and 2 ok. So, we have discussed about quarter wave stack where the thickness of each layer is basically quarter wavelength. So, they are not identical right because the permittivity is different for the two layers. Now the if you consider yz plane as a plane of incidence okay with y direction which is parallel to the layers and z direction is basically perpendicular to the layers right.

So, you always keep this in mind this particular schematic with this coordinate system okay. Now however, we must consider that both of the possible polarization states that is TM when the electric field is perpendicular to the plane of incidence ok and TE in which the electric field is within the plane of incidence ok. So, you can actually call what is the plane of incidence you can consider yz plane as your plane of incidence. So, when you have you know the field which is electric field in the plane of incidence you can call it also as p polarized light so you can say TE modes are basically p polarized and when it strikes out of the plane of incidence you can call it as s polarized light okay and that is why the TM modes can be called as s polarized light okay now here the example structure that you see okay, has got lot of interesting points, okay. Now, what are these different lines? So, you can see the green shaded regions correspond to the p-polarized ones and the blue shaded regions are basically corresponding to the s-polarized light, right.



And then there is a range of frequency here within which all of the modes of the multilayer are basically below the light line. Now where is the light line? This is the light line. So you can see a slope here. So in this particular frequency range, which is marked as yellow, you can see that the bands or you can see that all the modes of the multilayer film lies below the light line. So, it means that within this range any incident plane wave cannot couple to the extended states of the layers ok.

That means their field will decay exponentially within the quarter wave stick ok. And the transmission through such a mirror will drop exponentially with the number of layers. So, the light will be perfectly reflected if you consider that the material absorption is negligible. So, one more time let us clarify this particular band diagram which is very very important for omnidirectional scattering. So, here you can see couple of interesting points that is B which is basically a dashed white line which corresponds to the Brewster's angle.

ok. So, this gives rise to the crossing at B ok and as I mentioned this straight line the red straight line that you see here is basically the light line which is ω =ck_y ok above which the extended modes exist in air ok. So, these are air bands ok. Now if you consider the yellow shaded region okay. So, in this region you see the first frequency range of omnidirectional reflection.

So, L stands for the lower edge and U stands for the upper edge. And that is how you actually see this particular bend. So this is the case where no bend or mode is allowed. So that will be reflected. So, once again the blue region indicates TE or p-polarized and the sorry the green one the left side of the green indicates the modes with fields polarized in the yz incidence plane that means we are referring to TE or p-polarized modes. and on the right

side you have blue bands which corresponds to electric field polarized in the x direction that means we are talking about TM or s-polarized that is.



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So, what we understood that omnidirectional reflection is not a general property of 1D photonic crystal. So, there are basically two necessary condition first the dielectric constant contrast between the two mirror materials must be sufficiently large so that the point levelled U that is upper is lying above the point which is marked as L that is lower. Now if the bandgap is too narrow okay we will hit the top U of k_y equals 0 before the bottom of the gap okay, okay. where the gap has exited from the light cone that is exactly at this point L. So, it should not happen that this point is coming below this L point and that is why the dielectric contrast between the two materials should be large enough.

- The straight red line is the **light line** $\omega = ck_{y}$, above which extended modes exist in air.
- In yellow is shaded the first frequency range of omnidirectional reflection (with lower and upper edges at L and U, respectively).
- The dashed white line corresponds to Brewster's angle, which gives rise to the crossing at B.



the smaller dielectric constant that is ε_1 must be larger than the dielectric constant of the ambient medium by a critical amount ok. So, it should not be air that should be a you know material which has got slightly larger refractive index or dielectric constant than air and that is also required to satisfy this condition. So, this critical contrast with the ambient medium is reached when the p or TE modes are pulled down in frequency far enough that this B point in the figure where basically the first band and the second band intersect that does not fall above the light line, okay. So, you have to make sure that this point falls below the light line.

- Omnidirectional reflection is not a general property of one-dimensional photonic crystals.
- There are two necessary conditions:
 - First, the dielectric contrast between the two mirror materials must be sufficiently large so that the point labelled U (upper) is above the point L (lower).

If the band gap is too narrow, we will hit the top U of the $k_y = 0$ gap before the bottom of the gap has exited the light cone at L.

Second, the *smaller* of the two dielectric constants (ε₁) must be larger than the dielectric constant of the ambient medium (ε_a) by a critical amount.



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Source: B. E. Saleh and M. C. Teich, Fundamentals of photonics, John Wiley & Sons, 2019.



 This critical contrast with the ambient medium is reached when the p (TE) bands are pulled down in frequency far enough that point B in Figure (where the first and second p bands intersect) does not fall above the light line.



• This second criterion is why we chose $\varepsilon_1 = 2$ rather than 1 in the example.

Source: B. E. Saleh and M. C. Teich, Fundamentals of photonics, John Wiley & Sons, 2019.

So, the light line is this one. So, for that you need to have a critical contrast of this lower mid lower dielectric with the ambient medium. So, that is why you know we have chosen epsilon 1 equals 2 rather than taking that of air in this particular example. Now, the point B falls on a line that corresponds to the Brewster angle ok.



So, what happens you know at Brewster angle for p polarization light there will be no reflection ok at that $\varepsilon_1/\varepsilon_2$ interface and the lack of reflection is what permits the bands to intersect.

So, combining these two criteria okay. You can think of the size of the omnidirectional gap as a function of the ratio of $\sqrt{(\varepsilon_2/\varepsilon_1)}$ and $\sqrt{(\varepsilon_1/\varepsilon_a)}$ okay. for the case of quarter wave stack. So this is the interesting plot that tells you about this contrast of the two material and contrast of the lower dielectric material with ambient okay and this is the you know gap mid gap ratio.

okay. So, you can see that you know at 70 percent, 60 percent and so on. So, you can actually have very large gap mid gap ratio okay. Now here let us consider one particular system like silicon silica in air. So, epsilon 2, epsilon 1 and this is epsilon a okay. So, here also you can see that we are talking about quarter wavelength stack.

So, epsilon 2 thickness is much smaller as compared to epsilon 1. right. So, the figure here also shows that we are talking about you know light incident from an ambient medium which has got a dielectric constant of epsilon a. Now, our choice of material is such that epsilon 2 is larger than epsilon 1 and epsilon 1 is larger than epsilon a right. So, it does not matter which material forms the edge of the mirror. So, it can be epsilon 1 in the front meeting the epsilon a or other ways ok that does not actually make a difference.

- The gap-midgap ratios are labelled on the right side of each curve.
- The system is illustrated in the inset, in which light is incident from an ambient medium with dielectric constant ε_a.
- The two materials of the film have dielectric constants ε₁ and ε₂, with ε₁ < ε₂.
- It does not matter which material forms the edge of the mirror.
- The pink shaded area is the region in which there is a nonzero omnidirectional gap.
- Some common materials, such as the silicon/silica/air combination indicated by the arrow, fall within this region.



Source: B. E. Saleh and M. C. Teich, Fundamentals of photonics, John Wiley & Sons, 2019.

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So, here you see you have for this ranges you actually get this omnidirectional gap ok. So, if you choose this ratio to be say 3.5 okay and this ratio also to be 3.5 you can actually go up to very very wide band gap of something like more than 70 percent okay. So, if you choose common material like silicon, silica in air okay that falls typically in this region that is you can have a you know gap mid gap ratio within say 20 and 30 percent okay. So, strictly speaking a quarter wave stack does not maximize the size of the omnidirectional gap, but in practice it nearly does so.

So, people simply go with quarter wave stack ok and instead of this quarter wave stack if you would have done some optimization and used those optimized stacks you would have seen that the contours are displaced by less than 2 percent okay along either axis.



 In Figure, had we used the optimal layer spacings instead of the quarter-wave spacings, the contours would be displaced by less than about 2% along either axis.

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So, that is kind of you know not a very very productive exercise to do for further optimization of this stack layer you know thicknesses over the easier one which is the quarter wave stack okay because only improvement you can get is typically 2 percent okay. so a suitably designed multi-layer film can therefore function as a omnidirectional mirror but there are some things it cannot do its reflective properties depends on the translational symmetry of the interface and consequently it cannot confine a mode in three dimension in addition if the interface is not flat or if there is an object or a light source which is close to the surface then k is not conserved okay. In that case light will generally couple to the extended modes propagating in the mirror and they will be translated.

- A suitably designed multilayer film can therefore function as an omnidirectional mirror, but there are some things it cannot do.
- Its reflective property depends on the translational symmetry of the interface, and consequently it cannot confine a mode in three dimensions.
- In addition, if the interface is not flat, or if there is an object (or a light source) close to the surface, then k is not conserved.
- In that case, light will generally couple to extended modes propagating in the mirror and will be transmitted.

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Source: B. E. Saleh and M. C. Teich, Fundamentals of photonics, John Wiley & Sons, 2019.

So, you will not get a omnidirectional mirror. okay. So, this is an interesting exception. However, if the mirror is curved around a hollow sphere or cylinder, then the continuous rotational symmetry can substitute for this translational symmetry and light can be then localized within the core. As with the planar mirror, the leakage rate from the core to the exterior decreases exponentially with the number of layers. Here also, you can see the same thing. So you can actually make this kind of grating.

Omnidirectional Multilayer Mirrors	
 There is an interesting exception, however: if the mirror is curved around a hollow sphere or cylinder, then the <i>continuous rotational</i> symmetry can substitute for translational symmetry, and light can be localized within the core. 	Period Fiber Core Fiber Bragg Grating
 As with the planar mirror, the leakage rate from the core to the exterior decreases exponentially with the number of layers. 	
 The cylindrical case was called a Bragg fiber and the spherical case has been dubbed a Bragg onion. 	SiO: stem
 Here, one did not require omnidirectional mirrors to obtain localized modes, because a mode's rotational symmetry imposes restrictions on the angles that it can escape into at large radii. 	Bragg onion
Source: B. E. Saleh and M	. C. Teich, Fundamentals of photonics, John Wiley & Sons, 2019.

In a cylindrical case, you can call it Bragg fiber. and you can also have a core and this kind of you know silica silicon alternating layer cladding around the sphere and the spherical case is typically dubbed as Bragg-Onion. So, here one did not require omnidirectional mirrors

okay to obtain localized modes because the modes relational symmetry would impose the restriction on the angles that it can escape into a large radii. So, the localization of modes will be possible.

Lecture Outline	
 Applications of 1D PC: 	
Bragg Grating	
 Bragg Reflection 	
 Bragg Grating — A Simplified Theory 	
 Stack of Partially Reflected Mirrors 	
 Dielectric Bragg Grating 	
Omnidirectional Multilayer Mirrors	
Periodic Dielectric Waveguides	
Point Defects in Periodic Dielectric Waveguides	
Periodic Dielectric Waveguides as Fiber Bragg Grating	
Source: B. E. Saleh a	nd M. C. Teich, Fundamentals of photonics, John Wiley & Sons, 2019.
 Periodic Dielectric Waveguides Periodic dielectric waveguides, which have only a <i>one</i>-dimensionally direction of propagation, but have a finite thickness and a finite width. 	periodic pattern (or <i>grating</i>) along the
 Many periodic-waveguide structures are possible, such as those shown in Figure. 	(a) 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2
Figure: Examples of periodic dielectric waveguides.	(c)
IIT Guwahati MPTEL Source: J. D. Joannopoulos, S. 6 the Flow of Light", Princeton U	 Johnson, J. N. Winn & R. D. Meade, "Photonics Crystals: Molding niv. Press, 2008.

So, with that we move on to our next topic that is periodic dielectric waveguides. So, periodic dielectric waveguides which have only one dimensionally you know periodic pattern or grading along the direction of propagation, but the difference is that it has got finite thickness and finite width.

So, there are different types of structure which are possible one can be this you know in a slab thin slab you can have periodic 1D array of holes you can think of array of dielectric cylinders or you can have this kind of a structure.

It will turn out that, regardless of the geometry, all such structures exhibit common phenomena:

They have a form of photonic band gap along their periodic direction, and can confine light in the other directions by the principle of index guiding.



So, it will turn out that regardless of the geometry all these structures will exhibit a common phenomena they will form a typical photonic band gap along their periodic function along their periodic dimension and they can confine light in the other two directions by using the principle of index guiding ok. Because this will be like high index and outside there will be lower index material surrounding. So, in one direction they will confine light using the photonic band gap in the other directions they will be using the concept of index guiding ok.

- One-dimensional periodic pattern that will combine index guiding in one direction with a photonic band gap in the other direction.
- A dielectric waveguide ($\varepsilon = 12$) of width 0.4*a*.
- (a) y_{\downarrow} $\longrightarrow k$ (b) **Figure:** Dielectric waveguide.
- Periodic waveguide: a period-a sequence of $0.4a \times 0.4a$ dielectric squares. In both (a) and (b) there is a conserved wave vector k along the direction x of translational symmetry, resulting in guided modes.

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 Source: J. D. Joannopoulos,
 the Flow of Light". Princetor

Source: J. D. Joannopoulos, S. G. Johnson, J. N. Winn & R. D. Meade, "Photonics Crystals: Molding the Flow of Light", Princeton Univ. Press, 2008.

So, this is what we understood. Now, let us look into this particular dielectric waveguide and another waveguide which is basically a periodic you know 1D periodic structure of squares. So, one dimensional periodic pattern that will combine index waveguiding in one dimension and photonic bandgap in the other dimension. So, we are not thinking about the z direction here, we are just talking about a two dimensional figure okay. in this dielectric waveguide we are considering the material permittivity to be 12 and say it has got a width of 0.4a and a is basically the periodicity ok. So, what you have seen so 0.4a, 0.4a so we have got a square dielectric square and this is the direction of the propagation ok. And this is when you compare it with a uniform dielectric waveguide. So, what are the differences of these two structures in terms of their band gap? So, you can see the band diagrams of the waveguides.

- Band diagrams of waveguides, for TM-polarized in-plane $(k_z = 0)$ light only.
- Left: uniform waveguide. Right: periodic waveguide including twice the irreducible Brillouin zone.



So, we have considered only for TM waves that is for k_z equals 0 okay. So, in plain light. So, left one shows for uniform waveguide whereas, the right one shows for periodic waveguide ok. and the size of the bent diagram is typically twice the irreducible Brillouin zone. So, typically we take from minus we can keep the center across 0 and minus 0.5 to 0.5 or you can just put it you know 0 to 1. So, again these are normalized frequency and these are normalized wave vector. So, what is c? This blue shaded, so this is basically the light line. So, this is where you have modes propagating in air. So, these are the blue shaded region in the light cone.

- Blue shaded region is light cone (extended states propagating in air).
- Discrete guided bands are labelled even or odd according to the y=0 mirror symmetry plane.



So, that represents states or modes which are propagating in air. okay and then what you have you have discrete bands okay which are leveled as even and odd according to y equals 0 mirror symmetry plane. So, if you see here y equals 0 will go through the center of this particular waveguide okay same here also. So, depending on whether the electric field profile is symmetrical across this plane we can call it as even or odd ok. So, what you see here you have even band 1 even odd band 1 like that ok.

- The waveguide is symmetric under reflections through the plane y = 0 that bisects it. Consequently, all of the guided modes can be classified as *even* or *odd* with respect to mirror reflections in this plane.
- We see one even band and one odd band. The even band is the **fundamental mode**, for which the mode profile has the fewest nodes and the lowest frequency.



But for the periodic structure you have even band 1 and then you have even band 2. So, for even band 1 and 2 you have this much of band gap ok. So, the waveguide is symmetric under reflections through y equals 0 because y equals 0 basically bisects it. So, as I understood that all the guided modes can be classified as even or odd with respect to their mirror reflection from this particular plane. So, we see that there is one even band and one odd band and the even band is basically the fundamental mode because it has got lower frequency as compared to the odd bands.

Now, if you look into the patterns, so this is what your even band means. So, if you take the plane y equals 0, you will see it is symmetrical on up and bottom side ok. So, that way you can think of even band 1, this is even band 2 ok. So, here the modes or high low are splitting at the center of the square.



- E_z field patterns of the periodic waveguide at $k = \pi/a$, the Brillouin-zone edge.
- Left and middle panels correspond to the edges of the gap in the even guided modes, while the right panel has odd symmetry with respect to the y = 0 mirror plane. The dielectric squares are shown as dashed green lines.



So, this is a higher frequency mode. this is odd band 1. So, it is very similar to this just that you know upper side and the lower side these two are opposite to each other ok. So, that is how you can think of the electric field pattern. So, we are basically plotting E_z pattern ok at k equals π/a which is at the edge of the brilliant zone right. Now, if you think of periodic dielectric waveguide which is having a finite thickness that becomes a three dimensional dielectric strip.

So something like this suspended in air. So you have these holes. So the periodicity is considered to be a. So these are cylindrical air holes and you can plot it plot the band diagram only within the irreducible brilliance zone. So, here also you can see that you have this odd mode 1 and 2 giving you a band gap and so on ok.

- Band diagram for the waveguide: a three-dimensional dielectric strip, suspended in air, with a period-a sequence of cylindrical air holes. Only the irreducible Brillouin zone is shown.
- The discrete guided modes are labelled according to their symmetry as described in the text, with the fundamental E and M band gaps shaded light red and blue, respectively (the light cone is shaded darker blue, bounded by the light line in black).



So this is how the mode looks like. So this is the fundamental mode. This is the higher order mode. So this is 1 and 2. So the discrete guided modes are leveled according to their symmetry as described. And we have considered the fundamental E and M bandgaps that is the electric and the magnetic bandgaps shaded in red and blue ok. So, the light cone is shown here ok and here you can see this is one bandgap and this is another bandgap.

Lecture Outline

- Applications of 1D PC:
 - Bragg Grating
 - Bragg Reflection
 - Bragg Grating A Simplified Theory
 - Stack of Partially Reflected Mirrors
 - Dielectric Bragg Grating
 - Omnidirectional Multilayer Mirrors
 - Periodic Dielectric Waveguides
 - Point Defects in Periodic Dielectric Waveguides
 - Periodic Dielectric Waveguides as Fiber Bragg Grating

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Source: B. E. Saleh and M. C. Teich, Fundamentals of photonics, John Wiley & Sons, 2019.



- Now, let us consider a point defect.
- H_z field patterns of a localized resonant mode in a cavity formed by a defect in the periodic waveguide (suspended in air).
- The spacing between one pair of holes is increased from *a* to 1.4*a*.



So, now let us focus on the point defects in this periodic dielectric waveguides. So, what happens when you consider a point defect? So, you are basically thinking of a localized resonant mode in a cavity that is formed by this defect in the periodic waveguide and that is suspended in air. So, if you think of this geometry again let us go back here. So, when you take the cross section or electric field pattern along z equals 0 plane, that is along the xy plane, you will see like this, right? And when you take from y equals 0, that is from the side, you see this is how it looks like. So, whenever there is a, you know, say, how do you create a defect here? Say, one pair of holes, the spacing between them, you change from a to 1.4a and

that will become a, that particular one will become a defect. because normally otherwise that periodicity is a. So, you have changed one and made it 1.4a right.

G	Point Defects in Periodic Dielectric Waveguides
	The strong localization, exponentially decaying in the waveguide, is seen in the cross sections: the dielectric

The field decays only inversely with distance in the lateral directions, though, due to slow radiative leakage shown by the figure at right, which uses a saturated color scale to exaggerate small field values.

structure is shaded translucent yellow.



So, what you will see you will see strong localization and exponentially decaying fields in the waveguide as you can see from this cross section okay. And if you also see this particular you know saturated color scale you can see that the from the defect you know the field only decays inversely with the distance and it will slowly show some radiative leakage okay. And we are using you know saturated color scales to show this very small field values, but this is how the field will actually leak out. So, that mode will not propagate right.

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Periodic Dielectric Waveguides as Fiber Bragg Grating

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Source: B. E. Saleh and M. C. Teich, Fundamentals of photonics, John Wiley & Sons, 2019.

Periodic Dielectric Waveguides as Fiber Bragg Grating

• **Fiber Bragg grating**: A standard glass fiber, which guides light by index guiding, has been modified to include a weak periodic variation of the refractive index along the fiber axis.



So, we will see an application of this kind of periodic dielectric waveguide as fiber bragg grating. So, what is fiber bragg grating? So, you can take a fiber and then modulate the index of the fiber core by exposing it to a pattern of UV light exposure and you can create this high low high low kind of variation in the refractive index you can maintain the period to be capital lambda g. So, that is how you are able to create a periodic dielectric waveguide in the core of a optical fiber and that is very useful because you can take this standard glass fiber which is anyways guiding light by index guiding and what you are doing you are basically including a weak periodic variation of the refractive index along the fiber core. and from Bragg grating we understand that whichever wavelength will satisfy the condition of Bragg reflection will be basically reflected from this grating and remaining will be transferred.

So, you can think of a input signal which has got a wide frequency band okay. You can see only one particular band say the green yellow green wavelength or say the green wavelength is getting reflected. So, the transmitted light will look like this. So, this basically gives you a kind of notch or a band stop kind of a response okay. So these are the different applications of periodic dielectric waveguide as fiber break reading.



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So with that we conclude this lecture and in the next lecture we will discuss about the fundamentals of 2D photonic crystals. If you have got any query regarding this lecture you can write an email to me at this particular email address mentioning MOOC and photonic crystal in the subject line. Thank you.