

**Dr. Debabrata Sikdar**

Department of Electronics and Electrical Engineering  
Indian Institute of Technology Guwahati

Web: <https://www.iitg.ac.in/deb.sikdar>

Email: [deb.sikdar@iitg.ac.in](mailto:deb.sikdar@iitg.ac.in)



**NPTEL**  
NPTEL ONLINE CERTIFICATION COURSE  
AN INITIATIVE OF MoE, GOVT. OF INDIA

Lec 13: Fundamentals of 2D photonic crystals

Hello students, welcome to lecture 13 of the online courses on Photonic Crystals Fundamentals and Applications. Today's lecture will be on fundamentals of 2D photonic crystals

## Lecture Outline

### • Two-dimensional (2D) Photonic Crystal–

- Introduction
- A Square Lattice of Dielectric Columns
- A Square Lattice of Dielectric Veins
- A Complete Band Gap for All Polarizations
- Localization of Light by Point Defects
- Linear Defects and Waveguides



Eli Yablonovitch (born 1946) co-invented the concept of the photonic bandgap; he made the first photonic-bandgap crystal.



Sajeev John (born 1957) invoked the notion of photon localization and co-invented the photonic-bandgap concept.

Here is the lecture outline. We will have a quick introduction to the topic and then we will discuss about square lattice of dielectric columns. We will also consider a square lattice of dielectric vanes. Then we will see how we can achieve complete band gap for all polarizations.

We will further discuss the localization of light by point effects and application of linear defects and wave guides. So, when we discuss photonic crystal always remember these two gentlemen who are the pioneer of photonic crystal. So, this is the picture of Eli Avalonovich and this is Sajeev John. So, Eli Avalonovich co-invented the concept of photonic bandgap along with Sajeev John and he made the first photonic bandgap crystal.

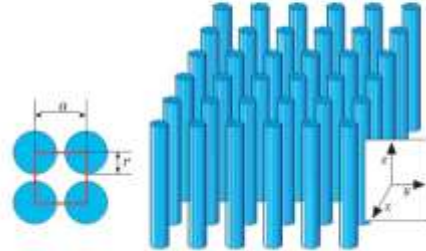
okay and Sajeev John as I mentioned he invented this concept of photonic band gap along with him and he also invoked the notion of photon localization



## Introduction

## Introduction

- No extended states are permitted (inside band gap), and incident light is reflected.
- Two-dimensional photonic crystal can prevent light from propagating in any direction within the plane.



**Figure:** A 2D photonic crystal with lattice constant  $a$  (periodic along  $x$  and  $y$ ). The left inset shows the square lattice from above, with unit cell in red frame.

So, let us have a quick introduction to photonic 2D photonic crystal. So, this is how a 2D photonic crystal will look like. So, it is basically a periodic structure along two lateral dimensions say  $x$  and  $y$  okay and it is homogeneous along the third axis that is  $z$ . So, a 2D photonic crystal will have this  $z$  axis infinitely long right.

## Introduction

- The system is homogeneous in the  $z$  direction, with no restrictions on the wave vector  $k_z$ .
- The system has discrete translational symmetry in the  $xy$  plane.
- Specifically,  $\epsilon(\mathbf{r}) = \epsilon(\mathbf{r} + \mathbf{R})$ , as long as  $\mathbf{R}$  is any linear combination of the primitive lattice vectors  $a\hat{x}$  and  $a\hat{y}$ .
- By applying Bloch's theorem, we can focus our attention on the values of  $\mathbf{k}$  that are in the Brillouin zone.

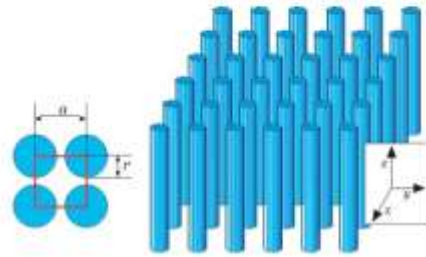


Figure: A 2D photonic crystal with lattice constant  $a$  (periodic along  $x$  and  $y$ ). The left inset shows the square lattice from above, with unit cell in red frame.

Now you can actually consider this typical specimen where you can think of a square lattice of dielectric columns which are having radius of  $r$  and they have dielectric permittivity of  $\epsilon$ . So, a mark here the lattice period okay. So, what we will see that for certain values of this  $a$  and radius  $r$ , you will be able to see a photonic bandgap in the  $xy$  plane. That means for those particular frequencies okay no propagation of electromagnetic wave or light is allowed through this crystal right. So, that will tell us that no extended modes are permitted inside the bandgap.

So, what will happen to the incident light that will simply get reflected. So, when we talk about a 2D photonic crystal, it can prevent light from propagating in any direction within that plane. So, always remember, so when we are talking about 2D photonic crystal. So, you can actually have light falling in any angle within this  $XY$  plane, but then you from all direction light should get reflected if the frequency of that incident light falls within the photonic band gap. Now, what happens along  $z$  direction? The system is homogeneous along  $z$  direction.

So, there are no restrictions on the wave factor  $k_z$  okay. So, that is how we come to this we can actually take help of the symmetries of the crystal to categorize its electromagnetic modes right. So, as I mentioned this system is basically homogeneous in  $z$  direction. So, the modes must be oscillatory in this particular direction with no restrictions on wave vector. In addition the system will have discrete translational symmetry along the  $xy$  plane where we have discussed this before that when you have periodicity along the dielectric constant you can express it as  $\epsilon(\mathbf{r}) = \epsilon(\mathbf{r} + \mathbf{R})$  where  $\mathbf{R}$  could be any linear combination of the primitive lattice vector  $a\hat{x}$  and  $a\hat{y}$ . So, you can think of a you know primitive lattice vector  $\mathbf{t}_1$  cap that will be equal  $a\hat{x}$  and  $\mathbf{t}_2$  cap that will be along equals to  $a\hat{y}$  right.

## Introduction

- Indexing the modes of the crystal by  $k_z$ ,  $\mathbf{k}_{\parallel}$ , and  $n$ , they take the now-familiar form of **Bloch states**

$$\mathbf{H}_{(n,k_z,\mathbf{k}_{\parallel})}(\mathbf{r}) = e^{i\mathbf{k}_{\parallel}\cdot\boldsymbol{\rho}} e^{ik_z z} \mathbf{u}_{(n,k_z,\mathbf{k}_{\parallel})}(\boldsymbol{\rho})$$

- In this equation,  $\boldsymbol{\rho}$  is the projection of  $\mathbf{r}$  in the  $xy$  plane and  $\mathbf{u}(\boldsymbol{\rho})$  is a periodic function,  $\mathbf{u}(\boldsymbol{\rho}) = \mathbf{u}(\boldsymbol{\rho} + \mathbf{R})$ , for all lattice vectors  $\mathbf{R}$ . Here,  $\mathbf{k}_{\parallel}$  is restricted to the Brillouin zone and  $k_z$  is unrestricted.
- Any modes with  $k_z = 0$  (i.e. that propagate strictly parallel to the  $xy$  plane) are invariant under reflections through the  $xy$  plane.
- Transverse-electric (TE) modes have  $\mathbf{H}$  normal to the plane,  $\mathbf{H} = H(\boldsymbol{\rho}) \hat{\mathbf{z}}$ , and  $\mathbf{E}$  in the plane,  $\mathbf{E}(\boldsymbol{\rho}) \cdot \hat{\mathbf{z}} = 0$ .
- Transverse-magnetic (TM) modes have just the reverse:  $\mathbf{E} = E(\boldsymbol{\rho}) \hat{\mathbf{z}}$  and  $\mathbf{H}(\boldsymbol{\rho}) \cdot \hat{\mathbf{z}} = 0$ .

So, you can actually think of capital R as any linear combination of these two primitive lattice vectors right. So, now if you apply block theorem you can actually focus our attention on those values of  $\mathbf{k}$  that are within the Brillouin zone right. So, by indexing the modes of the crystal by  $k_z$ ,  $\mathbf{k}_{\parallel}$  and  $n$ . So what is  $n$ ?  $n$  represents the band number. You can now write the familiar form of the block states.

So you can write  $h$ ,  $n$ ,  $k_z$ ,  $\mathbf{k}_{\parallel}$  as a function of  $\mathbf{r}$  can be written as  $\mathbf{H}_{(n,k_z,\mathbf{k}_{\parallel})}(\mathbf{r}) = e^{i\mathbf{k}_{\parallel}\cdot\boldsymbol{\rho}} e^{ik_z z} \mathbf{u}_{(n,k_z,\mathbf{k}_{\parallel})}(\boldsymbol{\rho})$  okay and then you have  $\mathbf{u}_{(n,k_z,\mathbf{k}_{\parallel})}(\boldsymbol{\rho})$  okay. So, in this particular equation what is  $\boldsymbol{\rho}$ ?  $\boldsymbol{\rho}$  is basically the projection of the position vector  $\mathbf{r}$  in the  $xy$  plane. So, once again keep this figure in mind that the crystal is periodic along  $x$  and  $y$  and it is homogeneous along  $z$ . okay. So, you can also think of that  $\mathbf{u}$  rho is basically a periodic function.

So, whenever you say periodic function it is like  $\mathbf{u}$  rho can be written as  $\mathbf{u}$  rho plus capital R right and that is true for all the lattice vectors R right. And any lattice vector R can be expressed as a linear combination of the primitive two lattice vectors right  $a\hat{x}$  and  $a\hat{y}$ . So when we talk about  $\mathbf{k}_{\parallel}$ ,  $\mathbf{k}_{\parallel}$  is the wave factor in  $xy$  plane and that should be restricted to the balloon zone, okay. However, your  $k_z$  is unrestricted, right. So any modes with  $k_z$  equals 0, that means if the mode propagates strictly parallel to the  $xy$  plane, okay, they are invariant under reflections through the  $xy$  plane.

okay and for T transverse electric mode okay they have magnetic field H normal to the plane. So, you can write  $\mathbf{H} = H(\boldsymbol{\rho})\hat{\mathbf{z}}$  okay whereas you have electric field which is basically in the plane. So, you can write  $\mathbf{E}(\boldsymbol{\rho}) \cdot \hat{\mathbf{z}}$  okay will be 0. On the other hand, if you look into the transverse magnetic case okay, so you can say that  $\mathbf{E} = E(\boldsymbol{\rho})\hat{\mathbf{z}}$ , where else you can write the magnetic field in that direction along the direction of you know Z cap will be 0, so the dot product is 0.



## A Square Lattice of Dielectric Columns

## A Square Lattice of Dielectric Columns

- The band structure for a crystal consisting of alumina ( $\epsilon = 8.9$ ) rods in air, with radius  $r/a = 0.2$ , is plotted here.
- Both the TE and the TM band structures are shown. The frequency is expressed as a dimensionless ratio  $\omega a/2\pi c$
- The horizontal axis shows the value of the in-plane wave vector  $k_{\parallel}$ .

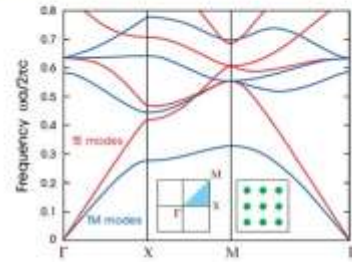


Figure: The photonic band structure for a square array of dielectric columns with  $r=0.2a$ .

Now let us come to some specific examples and discuss how we get band gap right and how do how can we obtain complete band gap.

When it is a complete band gap you need to obtain band gap for both the polarization T e and T m and also for all the possible k parallel values ok. So, let us consider light to propagate and you know in the x y plane. And we are considering basically a square array of dielectric which has got a lattice constant of A okay. So, what is this square array of dielectric cylinder made of? So, you can consider the cylinders to be made of alumina which is epsilon equals 9 okay.

## A Square Lattice of Dielectric Columns

- The square lattice array has a square Brillouin zone, which is illustrated in the inset panel of figure.
- The irreducible Brillouin zone is the triangular wedge in the upper-right corner; the rest of the Brillouin zone can be related to this wedge by rotational symmetry.
- The three special points  $\Gamma$ , X, and M correspond to
  - $\Gamma$ :  $k_{\parallel} = 0$ ,
  - X:  $k_{\parallel} = \pi/a\hat{x}$ , and
  - M:  $k_{\parallel} = \pi/a\hat{x} + \pi/a\hat{y}$

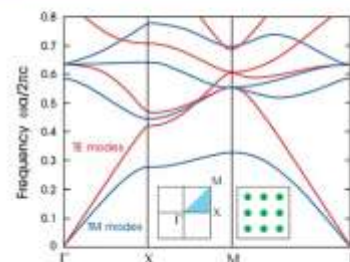


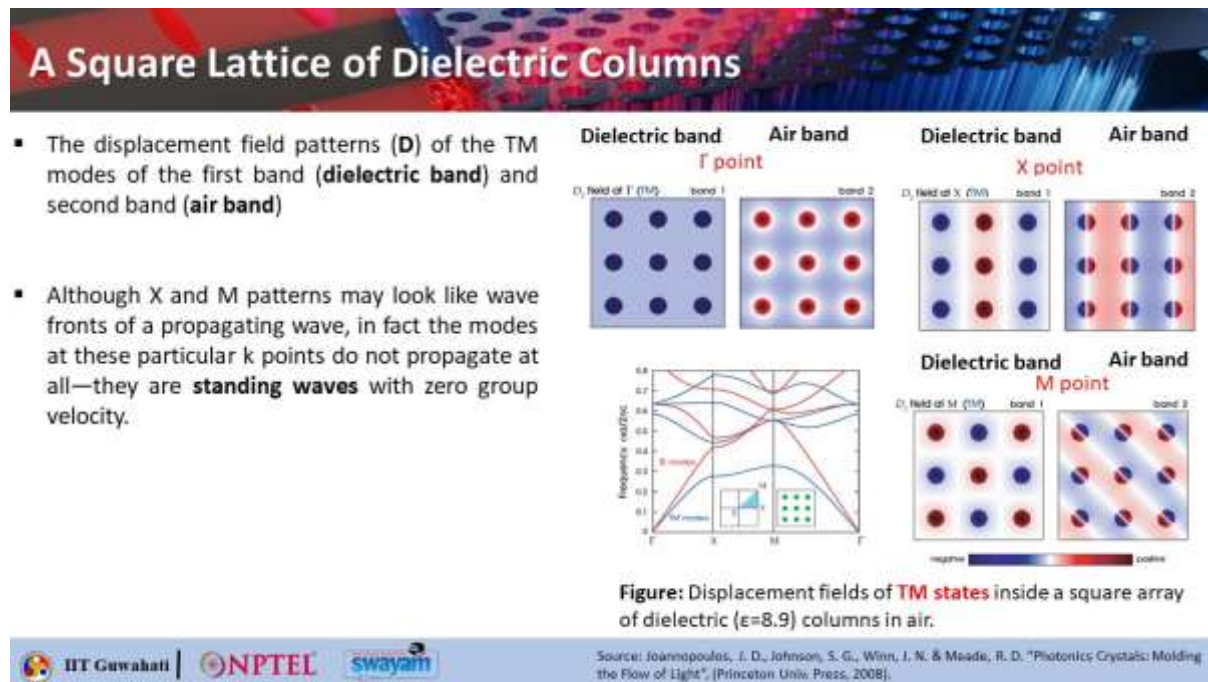
Figure: The photonic band structure for a square array of dielectric columns with  $r=0.2a$ .

So, these are those rods which are in air. okay and the radius to lattice constant ratio is 0.2. So, this is basically the photonic band structure of this square array of dielectric columns. Right? So once



again, what is there here? On the x-axis, you have marked the  $k$  parallel wave vectors, right, while traversing along this boundary of the irreducible Brillouin zone.

So important points of symmetry here is gamma, x, m, and gamma. So you actually travel from gamma to x to m to gamma, right? And then you actually obtain those frequencies for which there is a solution to the Maxwell's equation right.



Now in this particular diagram okay the band structure for both TE and TM modes are shown okay. So the TM modes are shown in blue and the TE modes are shown in red. okay and the frequency is basically dimensionless because you have normalized it you have written it as  $\omega a / 2\pi c$  which is also expressed as  $a / \lambda$  right.

So as I mentioned this is basically you know in plane wave vector which is  $k$  parallel. So, what is this square that is shown here? That is basically the Brillouin zone for this square lattice and if you see this blue shaded triangular region that represents the irreducible Brillouin zone. So, we have discussed this in the previous lectures and I hope this concept is clear to all of you. So, that is why if we only travel around this irreducible Brillouin zone that would give us the information about the entire Brillouin zone because this can be rotated and this other white triangle can be formed. So, you can actually get information about this quarter.

Once you have this quarter you can actually take a mirror symmetry and get this entire upper half once you have this entire upper half you can have this mirror symmetry horizontal mirror and then you can get you know the bottom half and the entire brilliant zone so only this particular blue shaded region contains independent values of  $k$  parallel for which you need to gather the information of which all frequencies are supported in this crystal So continuing this discussion you can also try to mark this important points of symmetry. So gamma can be written as  $k$  parallel equals 0 okay and the point x can be written as  $\pi / a \times \hat{x}$  okay and for m you have  $k$  parallel

which is  $\pi$  by  $a_x$  plus  $\pi$  by  $a_y$ . okay. So, that is how you can actually mark the different you know directions of the parallel you know wave vector. So, basically this gives you information of all the direction of light propagation or in light falling on this particular crystal along the xy plane right.

This is the reason why we have plotted  $k$  parallel only along the edge of the Brillouin zone is that the minima and the maxima of a given band which would determine the band gap eventually almost occurs at the zone edges and often at the corner. Right. So here also you can carefully see that if you consider the blue color lines only.

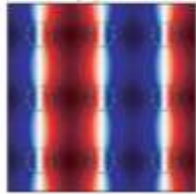
So this is TM mode. OK. So this will be band number one for TM mode. And this other blue line tells you it is band number two for TM mode. So if you consider TM mode you can see that there is a band gap. Right. For for all the values of  $K$ , you are actually able to find bandgap.

### A Square Lattice of Dielectric Columns

- The field patterns of the TE modes at the X point for the first and second bands are shown here.
- TE modes have  $\mathbf{D}$  (displacement mode) lying in the xy plane. The column positions are indicated by dashed green outlines, and the color indicates the amplitude of the magnetic field.
- Since  $\mathbf{D}$  is largest along nodal planes of  $\mathbf{H}$ , the white regions are where the displacement energy is concentrated.

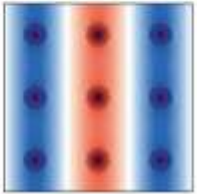
**Dielectric band**

$H_x$  field of X (TE) band 1



**Air band**

band 2



negative  positive

**Figure:** Magnetic fields of X-point TE states inside a square array of dielectric ( $\epsilon = 8.9$ ) columns in air.

So you need to find whether that gap exists for all the value of  $K$  or not. If not, then that is not useful. OK. How about TE modes? You consider the red lines.

So this is the fundamental one. So  $N$  equals 1 TE mode. And this one is  $N$  equals 2 TE mode. And you see that there is no gap between these two. Only gap is here but then this gap does not exist for all the values of  $K$ . It means T mode does not show a band gap in this particular structure.

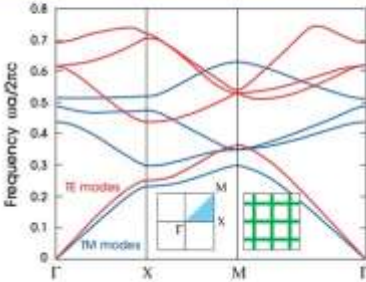
you can also you know analyze this further by considering the displacement field pattern that means if you plot  $d$  okay or the z component of  $d$  for this tm modes of the first band and the second band so we are considering only the blue lines here so this one is for the first band and this one is for the second band okay and we are plotting it at the gamma point okay so In one case it is here ok and the other case it is here ok. So, these are the two different frequencies that you can see ok. And if you look into the plot that the what is important here to see that the field pattern is exactly same at each unit cell ok. However, when you move to x point and to m point you see the field pattern okay are

not exactly same at each unit cell rather the field patterns actually alternate in each unit cell along the direction of the wave factor  $k_x$ . forming you know wave fronts which are basically parallel to  $y$  direction right in the case of  $x$  point clear.

so here you can see all blue that means negative values of  $dz$  then you have all red values that means it's a positive values of  $dz$  and then again negative here is like negative positive okay and so on and then here it alternates okay it's positive negative and then again it flips you get negative positive and so on Now, if you consider for  $M$  point, if you consider for  $M$  point, the sign of the field alternate in every unit cell. It is not like here. Here at least, you know, column wise they are same. Sorry, along a particular column, it is same. But here for every alternate cell, it is changing.

### A Square Lattice of Dielectric Columns

- **Another Example:** Square grid of dielectric veins (thickness  $0.165a$ ,  $\epsilon = 8.9$ ).
- This structure is complementary to the square lattice of dielectric columns, because it is a connected structure.
- The high- $\epsilon$  regions form a continuous path in the  $xy$  plane, instead of discrete spots.



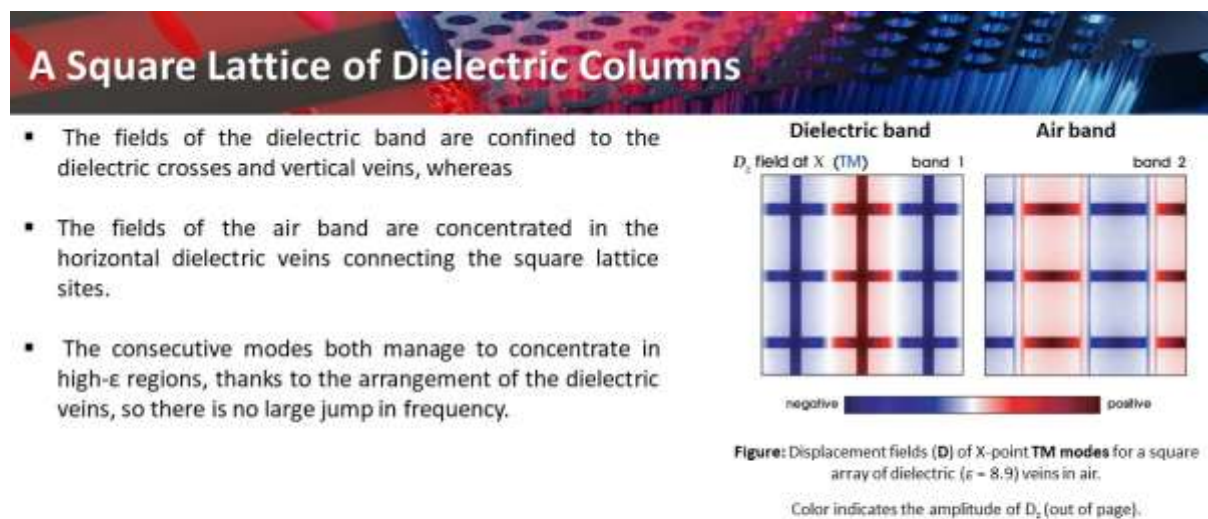
**Figure:** The photonic band structure for the lowest-frequency modes of a square array of dielectric ( $\epsilon = 8.9$ ) veins (thickness  $0.165a$ ) in air.

So, that forms a kind of checkerboard pattern and the lower bands always remember the lower band is called dielectric band and the upper band is called dielectric band. air band if you remember from our previous discussion. So, the band above the band gap is called air band the band below the band gap is called dielectric band. So, you can actually see that the left figure is for dielectric band the right one is for air band for each of the points that you have plot right. So, in this particular case the field of air band at  $M$  point okay are also looking like degenerate, one of a pair of degenerate states because you can see that you know they are similar kind of you know patterns that you see along this front okay.

So what is important here is to know that this is how you can analyze the displacement fields of the TM states in a square array of dielectric columns in air. So what it has given? It has given you TM bandgap. And we have also seen the modes at these important points, gamma,  $x$ , and  $m$ , how they look like. So gamma is basically here you have seen in the top. Then this is the  $x$  point, this is the  $m$  point.

And each set, only in the case of  $m$  point, we have seen that the air band actually shows degenerate

states. can further analyze that what happens to TE modes right so here you have seen the TE modes the red ones they do not actually show any band gap right so the fill patterns of the TE modes at x point so if you consider only the x point there is a small gap here okay so for the first band and the second band is shown in this case so what you can see say that this TE modes have this displacement field or displacement mode lying in the x y plane and the column positions are basically indicated by this dust green outlines if you are able to see here clearly okay and then the colors basically represent the strength of the magnetic field. So, we are not plotting dz here, we are basically plotting the Hz, the z component of the magnetic fields. So, that is the difference between the tm mode and here we are plotting dz where else here we are plotting hz, okay. So, this t modes they are having this d which is lying in the xy plane, right.



So, this d is called the displacement mode. fine. So, since D is largest along the nodal planes of H, so you can actually see that the white regions are where the displacement energy is concentrated and that is this point okay. Means where your H field is kind of minimal, you will have strongest D field okay or the displacement modes. So, you can see the patterns are different for dielectric band and air band okay and this tells you why you are actually seeing TM band gap in the case of square array of dielectric cylinders. Now let us consider another structure which is basically a square lattice of dielectric vanes.

So, what is a vane? Dielectric vanes will be like you know this is a square grid of connected wires kind of structure ok. So, you can consider the thickness to be  $0.165a$  and the material let us keep it to be same epsilon equals 8.9 as we have considered in the previous case.

Right. So here again, you can try to read the photonic band structure. You have this parallel wave vectors plotted here. Gamma X and gamma are nothing but, you know, the points along the edges of the irreducible Brillouin zone so here also you are having a square lattice so it's the same style of plotting the irreducible Brillouin zone and if you look carefully at this particular photonic band structure you see that for tm modes okay you are having a band gap okay Now, for TM modes,

which are basically blue in color, so they do not have any opening throughout. You see there is opening here, there is opening here. But if you try to find if there is opening throughout, it does not look like having a, you know, opening for all the frequencies.

Okay, let us try to draw it like this. Yeah, you see. they do not have anything like this, okay. But if you look carefully for the TE modes that is the red color curves. So, this curve and this curve this corresponds to  $n$  equals 1 and this one corresponds to  $n$  equals 2. So, clearly this time TE modes have got bandgap whereas TM modes do not support any bandgap.

### A Square Lattice of Dielectric Columns

- The veins provide high- $\epsilon$  roads for the fields to travel on, and for  $n = 1$  the fields stay almost entirely on them.
- Since the  $\mathbf{D}$  field will be largest along the nodal (white) regions of the  $\mathbf{H}$  field, the  $\mathbf{D}$  field of the lowest band is strongly localized in the vertical dielectric veins.
- The  $\mathbf{D}$  field of the next TE band ( $n = 2$ ) is forced to have a node passing through the vertical high- $\epsilon$  region, to make it orthogonal to the previous band.

**Dielectric band**

**Air band**

negative  positive

Figure: Magnetic fields of X-point TE modes for a square array of dielectric ( $\epsilon = 8.9$ ) veins in air.

So, this structure is basically complementary to the square lattice of dielectric columns because it is a connected structure and because of that you also see complementary nature in the photonic band gap. So, here you are seeing TE band gap whereas in the previous case we have seen TM band gap right. So, here the high permittivity regions form a continuous path in the xy plane instead of discrete spots ok. Now if you try to carefully analyze what happens to you know the x point ok. for the TM case that is for the blue curves you can see that you know the fields of the dielectric bands are mainly confined to this dielectric crosses and the vertical bends.

Whereas, the field in the air band if you consider the air band which is basically the band which is above the band gap though in this case we do not have proper band gap for all the frequencies, but at x point we do have a gap. So, if you consider the diagram of dz field or the distribution of dz field for air band You will see that the fields are concentrated in the horizontal dielectric veins which are connecting the square lattice sites. So the consecutive modes both manage to concentrate in high permittivity region and that is mainly because of the arrangement of dielectric vanes and that is why there is no large jump in frequency. If you remember that you can only see a jump in frequency if one mode is concentrated in the high permittivity region and the next mode is concentrated in the low permittivity region. In this case, both the modes are basically concentrated in high permittivity region, though their distribution is slightly different.

That is why there is a slight opening up here, but they do not give you a proper band gap for all the modes. you know, possible values of  $k$  parallel, right. So, with that analogy, you can also see one more time over here. So, what is happening in this case and why you do not have a band gap for T mode at x point, okay.

You can analyze this as well. now if you look into the d modes that is the red curves okay and let us consider the x point again so here you see you actually see a large jump in the frequencies right so what happens here if you consider if you consider the dielectric band that is  $n$  equal 1 or band 1. So, in this case you can see that this is how the magnetic field lines are concentrated ok. So, you again see the magnetic field null at the center of the vane ok. right and that is where the D field will be the largest and that is represented by this white line ok. And the D field of the lowest band is strongly localized at the vertical dielectric vein right.

so d field is strongly localized here in the on the veins itself but if you go to the next one you can actually see again the white line which represents the you know nodal regions of the magnetic field that means the electric fields or the D field will be concentrated here in the gaps where you know there is no vane available. That means they are mostly in a region where you know you do not have very high permittivity right. So, that way the D field of the  $n$  equals 2 band is forced to have you know nodes passing through this okay high permittivity region because it will have its so nodes means the minima okay. or the zero values. So, it will have its peak going through the middle of the two dielectric region and that is how you are actually able to get a kind of orthogonal kind of field pattern from this band to this band and that allows you to have a very large band gap.

okay and that is this will be possible because of a high or large jump in frequency and that is why you can see the frequencies of this mode and this mode is very different clear. So here again you can carefully see that there are some green dashed lines shown to mark the vanes for you okay and the blue and red are marking the negative and the positive magnetic fields. which are oriented in the z direction. So, that is why we are plotting HZ right. So, with that we understood that two different types of structures can give us two different types of band gap.

## A Complete Band Gap for all polarisations

- By combining our observations, we can design a photonic crystal that has band gaps for both polarizations.
- By adjusting the dimensions of the lattice, we can even arrange for the band gaps to overlap, resulting in a complete band gap for all polarizations.

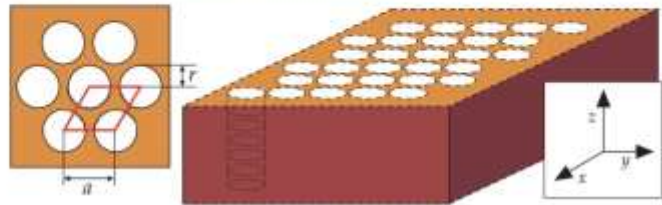


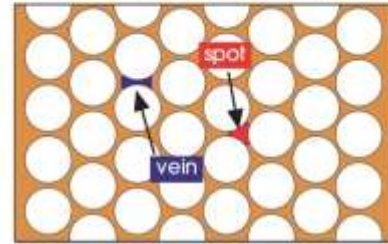
Figure: A two-dimensional photonic crystal of air columns in a dielectric substrate (which we imagine to extend indefinitely in the  $z$  direction). The columns have radius  $r$  and dielectric constant  $\epsilon = 1$ .

So, if you consider array of dielectric material something like dielectric rods you can get Tm band gap, but if you consider the inverse structure something like a connected square array of dielectric vanes that can give you TE band gap. Now, these are not a complete band gap because you want the band gap to be there for all polarization. It means for both TE and TM mode, the band gap should exist. And how do you do that? We have to use our understanding of the previous two cases and design a new structure that will support the feature of both those, dielectric air columns as well as those connected dielectric vanes. So, that you can actually have both Tm and Te band gap at the same time ok.

So, this is what we will do that by combining our observation of the previous two cases, we can now design photonic crystal that will have band gap in both directions. So, this is what we are looking for. So, we are actually having a two-dimensional photonic crystal of air columns.

## A Complete Band Gap for all polarisations

- The idea is to put a triangular lattice of low- $\epsilon$  columns inside a medium with high  $\epsilon$ .
- If the radius of the columns is large enough, the **spots** between columns look like localized regions of high- $\epsilon$  material.
- These are connected (through a narrow squeeze between columns) to adjacent spots.



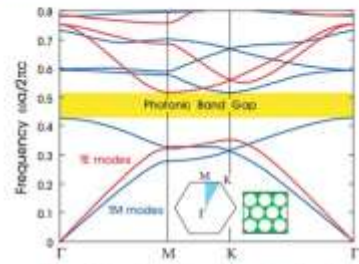
**Figure:** The spots and veins of a triangular lattice. Between the columns are narrow veins, connecting the spots surrounded by three columns.

So these are basically holes. You can say air columns. And you have made a hexagonal or triangular array in a dielectric substrate, which is imagined to be extended infinitely in the z direction. what are the parameters here you can consider the columns to have radius of  $r$  okay and they are definitely having these are air so dielectric constant of one and this is the period so you can actually adjust the dimensions of the lattice and this air gap in such a way that you are able to overlap the two band gaps in order to get complete band gap in both polarization right So how does this structure work? So you can see that the idea here is to put a triangular lattice of low permittivity column inside a medium of high permittivity. So if the radius of the columns is large enough, then the spot between the columns will look like localized regions of high permittivity material. So they will work as those kind of, you can imagine them as kind of dielectric columns. if these are very large, okay? And what are these vanes? Then these vanes are like, you know, narrow squeezed parts which are connecting the two of these parts.



## A Complete Band Gap for all polarisations

- The band structure for this lattice, shown here, has photonic band gaps for both the TE and TM polarizations.
- In fact, for the particular radius  $r/a = 0.48$  and dielectric constant  $\epsilon = 13$ , these gaps overlap, and we obtain an 18.6% complete photonic band gap.



**Figure:** The photonic band structure for the modes of a triangular array of air columns drilled in a dielectric substrate ( $\epsilon = 13$ ).

So they are looking like, you know, the vanes. So what is happening? You are basically trying to have both the two features like columns kind of structure as well as vanes together. right so once you have this kind of a structure and these are hexagonal lattice you will try to compute the photonic band structure and this is what you will obtain you will see that there is a photonic band gap present for both TE and TM polarization that means for all the polarization the band gap does exist okay you can see that for t modes for tm modes okay the first and the second you do not have a band gap but for second and third you have this as your band gap okay similarly for TE modes 1 and 2 you have a much wider band gap But you know you need to have the overlap of the two band gap so you will go with the minimum of the two or the more overlapping area of the two that is this yellow band that you that you can see here and that will be bandgap for all the directions. Now, this is called the complete photonic bandgap because this applies for all the values of  $k$ , this will work for all polarization of incident light okay and this is how you obtain a complete photonic bandgap. So, what are the specification of the structure? You have considered  $r/a$  to be 0.48, you have taken the dielectric constant of this base material to be epsilon equals 13 okays. And when you engineer after coming to this specification, you will see that the bandgaps overlap and you are actually able to obtain 18.6 percent complete photonic bandgap which is pretty decent one. So, with that we understood how complete photonic bandgap can be engineered. Now, let us look into couple of more interesting features like how do you localize light in a photonic crystal by introducing point defects.



## Localization of Light by Point Defects

## Localisation of light by point defects

- Perturbing just one site ruins the translational symmetry of the lattice
- Perturbing one column in the bulk of the crystal (yellow) might allow a defect state to be localized in both x and y.
- Perturbing one row in the bulk of the crystal (red) or truncating the crystal at a surface (green) might allow a state to be localized in one direction (x).

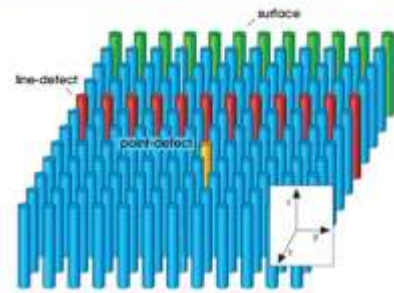


Figure: Schematic illustration of possible sites of point, line, and surface defects.

So, here we shall discuss the effect of point defects which is created in a photonic crystal array. So, how do you create a point defect? So, in a 2D photonic crystal you can think of removing a single column this yellow one you just remove it or you replace it by a different size or different kind of material or different shape. thing so that this looks different than rest of the lot. So, this will be treated as a point defect right. So, in short you can say perturbing just one site okay in a photonic crystal 2D photonic crystal you can introduce a point defect and that ruins the translational symmetry of the lattice.

Now if you think of you know that this is how you introduce a point defect and if you part of one particular row part of means either. You remove that particular column or you replace those columns with a different kind of material or different size and shape so that they look different than rest of the lot. So that can actually work as a line defect. You can also truncate your photonic crystal and like this and that will also allow you to you know localize in one particular direction.

## Localisation of light by point defects

- Perturbing a single lattice site causes a defect along a line in the z direction.
- We refer to this perturbation as a **point defect**.
- Removing one column may introduce a peak into the crystal's density of states within the photonic band gap.
- If this happens, then the defect-induced state must be evanescent.

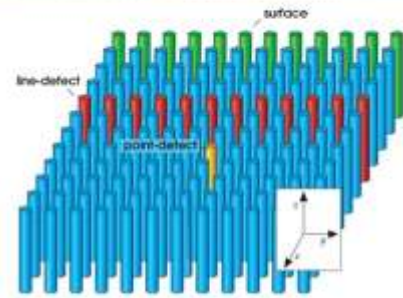


Figure: Schematic illustration of possible sites of point, line, and surface defects.

Here it is in x direction right. So, this one doing it only at a particular point we can introduce point defect right. Now, when you remove a column or a line and introduce defect you call it line defect. Now what happens for this now defects what are the effect of these defects on the photonic bandgap. So, you may think that you know the defect induced state must be evanescent.

## Localisation of light by point defects

- The defect mode cannot penetrate into the rest of the crystal, since it has a frequency in the band gap.
- Any defect modes decay exponentially away from the defect. They are localized in the xy plane, but extend in the z direction
- By removing a rod from the lattice, we create a **cavity** that is effectively surrounded by reflecting walls.
- If the cavity has the proper size to support a mode in the band gap, then light cannot escape, and we can pin the mode to the defect

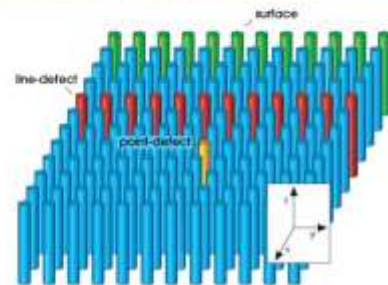


Figure: Schematic illustration of possible sites of point, line, and surface defects.

So the defect mode will not be able to penetrate into rest of the crystal. So if you put a frequency of light in this defect, which is falling within the band gap of this photonic crystal, then that light will get localized there or it will get trapped there because the light cannot escape into the crystal because the crystal is not supporting the propagation of that frequency because the frequency falls within the band gap of that crystal. So, what happens any defect mode will decay exponentially away

from the defect, but then they are localized in the xy plane and, but they extend in the z direction right. So, when you remove this you are actually forming a cavity okay and you can think of this cavity being surrounded by reflecting walls for frequency which lies in the band gap. So, if you you need to choose the cavity size such that you support a mode in the band gap then light cannot escape and you can actually pin that particular mode into the defect so this is how you can you know have light localization in the photonic band gap



## Linear Defects and Waveguides

### Linear Defects and Waveguides

- We can use point defects in photonic crystals to trap light.
- By using linear defects, we can also guide light from one location to another.
- Light that propagates in the waveguide with a frequency within the band gap of the crystal is confined to the defect, and can be directed along the defect.

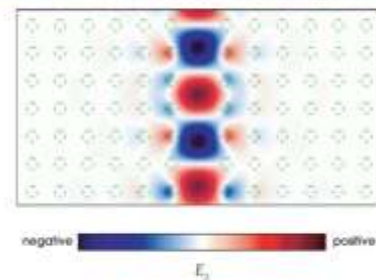


Figure: Electric-field ( $E_z$ ) pattern associated with a linear defect formed by removing a column of rods from an otherwise-perfect square lattice of rods in air.

now let us look into how we can have linear defects and create wave guides so you can use point defects to trap light you can also create linear defects by removing one row or one column or

replacing the conventional unit cells with the different size ones and you can actually create a kind of you know waveguide. So, here again what happens light will not be able to escape into the you know crystal because the frequency of the light falls within the band gap of the crystal.

So, that way you can actually whatever you know shape of the waveguide you require you can create a line defect like that and you will be able to guide light along that particular path. So that is something amazing because you can actually bend light to any degrees and then take it through the crystal. So here what is the important factor? The important thing is that you need to curve a waveguide out of an otherwise perfect photonic crystal. okay and how you do that you do that by modifying a linear sequence of unit cell okay.

So, here you can see how it is done. So, in this particular figure you can see that for the wave factor  $k_y$  equals  $0.32\pi/a$  along the defect, there is a propagation. It means this particular waveguide mode falls within the bandgap. And this dashed circular things tells you about the position of the dielectric rods which are forming this particular photonic crystal. So light that propagates in the waveguide should have frequency within the band gap of the crystal so that it can remain confined by those reflecting walls of the photonic crystal and then it can be guided along the defect.



So, with that we will stop here that is all for this lecture if you have got any query. So, you can write an email to me at this particular email address mentioning MOOC and photonic crystal in the subject line. Thank you.