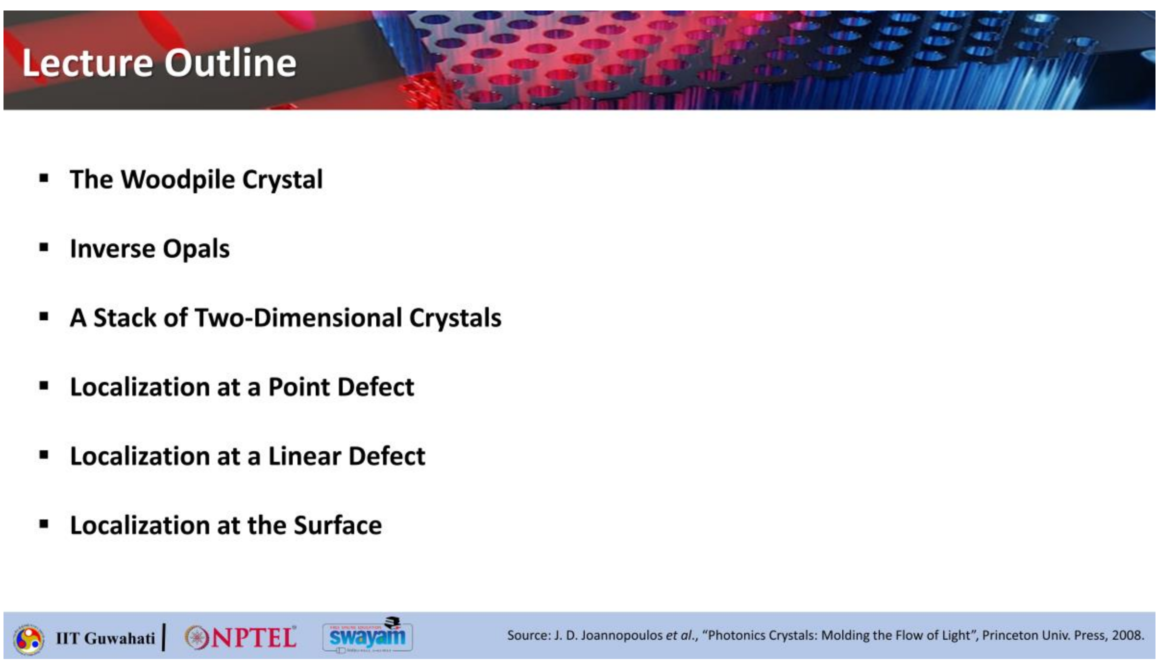
Lec 17: Crystals with complete bandgap

Hello students, welcome to lecture 17 of the online course on Photonic Crystals Fundamentals and Applications.



Today's lecture will be on crystals with complete band gap. So, this is basically in continuation of the discussion that we had in the last lecture. So, in this lecture we will have a look at the wood pile structure in more details. the inverse opal structure and also the stack of two-dimensional crystals that also we have seen briefly in the previous lecture. And then we will discuss about localization at a point defect, at linear defect and at surface.

The Woodpile Crystal

- The first three-dimensional photonic crystal with a complete gap to be fabricated on *micron* scales, for light at *infrared* wavelengths, was the crystal shown in **Figure**.
- This structure was proposed independently by K. M. Ho group (1994) and Sözüer and Dowling group (1994), and was dubbed a ***woodpile*** structure by the latter authors.
- *The woodpile crystal is formed by a stack of dielectric "logs" (generally rectangular) with alternating orthogonal orientations*.

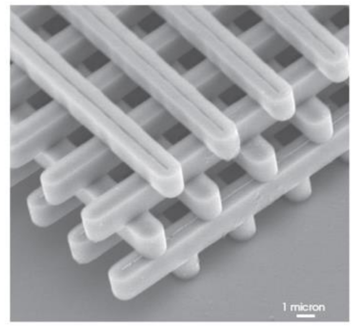


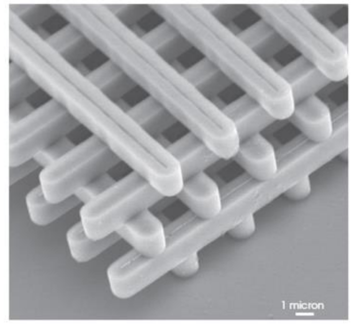
Figure: Electron-microscope image of a "woodpile" photonic crystal.

IIT Guwahati | NPTEL swayam Source: J. D. Joannopoulos *et al.*, "Photonics Crystals: Molding the Flow of Light", Princeton Univ. Press, 2008.

so the wood pile structure as i mentioned earlier that this is an electron microscope image of that wood pile photonic crystal and this is a beautiful 3D photonic crystal okay and this was the first three-dimensional photonic crystal with a complete band gap to be fabricated on micron scale okay and it was done for light at infrared wavelengths. The structure was proposed independently by K M Ho group in 1994 and Sergiver and Dowling group in 1994 as well ok and was dubbed as wood pile structure by later authors ok by this. So, why the name is wood pile because it is basically a stack of dielectric logs which are typically rectangular. with alternating orthogonal orientation.

The Woodpile Crystal

- The main advantage of the woodpile:
  - *the woodpile can be fabricated as a sequence of layers deposited and patterned by lithographic techniques developed for the semiconductor electronics industry*.
- Using just such a process, the woodpile structure was fabricated out of silicon ($\varepsilon \approx 12$) logs by Lin *et al.* (1998) and a band gap was measured around a wavelength of 12 μm.
- Subsequently, Lin and Fleming (1999) were able to reduce the size by nearly a factor of eight, resulting in a band gap around a wavelength of 1.6 μm.



The dielectric "logs" form an fcc lattice stacked in the [001] direction.

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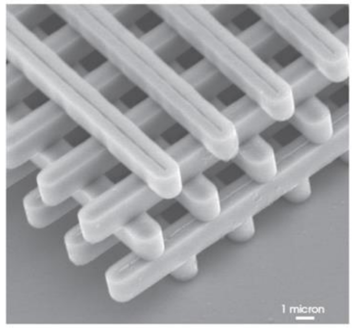
Source: J. D. Joannopoulos *et al.*, "Photonics Crystals: Molding the Flow of Light", Princeton Univ. Press, 2008.

So you can see how they are stepped. So what are the main advantages of this particular structure? So wood pile structure can be fabricated as a sequence of layers deposited and patterned by lithographic technique, which are developed for semiconductor electronics industry. So you can actually take help of the semiconductor foundries to fabricate your structures. okay and using just such a process the wood pile structure was fabricated out of silicon which is which has got permittivity around 12 okay silicon logs okay that was done by Lin et al. in 1998 and they could measure a band gap at wavelength around 12 micrometer okay So, subsequently Lin and Fleming in 1999 they were able to reduce the size of the wood pile structure they fabricated by a factor of 8 and they could bring it down to near infrared range and they could get band gap at wavelength of 1.6 micrometer. right.

The Woodpile Crystal

- The simplest woodpile-like stack would be an *ABAB* · · · sequence, where *A* denotes one log orientation and *B* denotes the orthogonal orientation, but this sequence does not produce a significant gap.
- Instead, it turns out that a four-layer *ABCDABCD* · · · sequence is better, in which *C* and *D* are layers with the same orientation as *A* and *B*, but are *offset* by half of the horizontal spacing, as illustrated.



Source: J. D. Joannopoulos *et al.*, "Photonics Crystals: Molding the Flow of Light", Princeton Univ. Press, 2008.

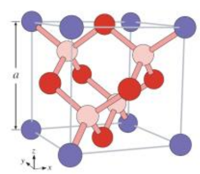
So, if you think of simplest structure wood pile structure it could be simply ABAB.

..... sequence where A is one log orientation and B is the log in orthogonal orientation ok. And the problem is that if you do that only ok. So, you have one log ok and then at the below that log you have another orthogonal log and so on. They were not able to see any significant band gap, but then if you try to make it a four layer something like you know ABCDABCD.

...... ok. So, where you know this layer is basically a shifted version of the top layer ok. So, it is like A, B, C and D ok. So, A and C are basically a shifted version by half the horizontal spacing ok and this also like that this B is basically shifted from D ok by again half the horizontal spacing and this structure could give you a significant band gap which is shown here ok.

The Woodpile Crystal

- The band diagram of this structure, for a dielectric contrast of 13: 1, is shown in **Figure**.
- It exhibits a 19.5% complete photonic band gap between the second and third bands.
- This structure has the periodicity of an fcc lattice, in which each fcc "atom" is replaced by a pair of orthogonal logs in the $\hat{x}$ + $\hat{y}$ [110] and $\hat{x}$ - $\hat{y}$ [$1\bar{1}0$] directions.



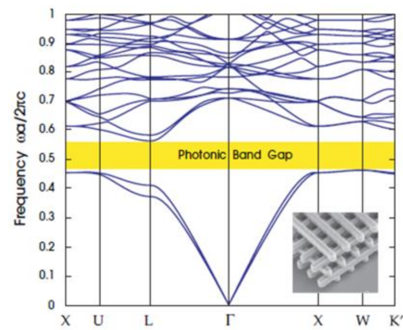


Figure 7: The photonic band structure for the lowest bands of the woodpile structure with $\varepsilon = 13$ logs in air.

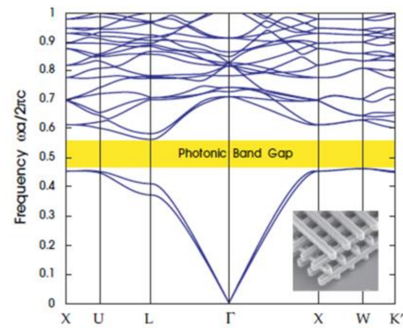
Source: J. D. Joannopoulos *et al.*, "Photonics Crystals: Molding the Flow of Light", Princeton Univ. Press, 2008.

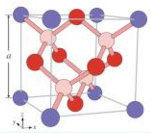
So, if you consider in simulation that you are considering dielectric contrast of 13 is to 1 and then you run your simulation you can see that this particular structure can give you a complete photonic band gap.

And, it exhibits around 19.5 percent complete photonic band gap between the second and the third bands. So, this structure has the periodicity of FCC lattice. So, if you think of each FCC atom can is here replaced by a pair of orthogonal logs ok in x cap plus y cap that is basically 1 1 0 direction and also along x cap minus y cap which is 1, 1 but 0 direction, right. So, you can actually take this and think of these atoms being replaced by those logs, okay.

The Woodpile Crystal

- *The irreducible Brillouin zone is larger than that of the fcc lattice, because of reduced symmetry—only a portion is shown, including the edges of the complete photonic band gap.*
- In fact, the woodpile crystal, like Yablonovite, can also be understood as a distorted form of the diamond lattice (with lower symmetry).
- If we imagine taking the diamond (in below figure) and *flattening* the bonds so that they lie parallel to the *xy* plane, then we obtain a woodpile stacking.





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Source: J. D. Joannopoulos *et al.*, "Photonics Crystals: Molding the Flow of Light", Princeton Univ. Press, 2008.

It is a pair of orthogonal logs. So, one is like lying along x, another could be lying along y. So, what you see here the irreducible Brillouin zone is basically larger than that of the FCC lattice because of the reduced symmetry and only a portion is shown here including the edge of the complete photonic band gap okay. If you look into the wood pile structure, something like the Yablonovite, okay, it is basically a distorted form of the diamond lattice which has got a slightly reduced symmetry. However, this reduction in symmetry has not hampered much on the complete photonic band gap because you are still able to get the complete photonic band gap.

If you imagine taking the diamond which is this particular figure and you know flattening out. So, this is basically x and y. So, this is x axis this y axis. So, if you take this diamond structure and try to flatten out all the bands. So, that they will lie parallel to the xy plane you basically get this wood pile stacking ok.

So, it is a FCC structure with reduced symmetry and that is why you will see slightly reduced complete band gap or the band gap is narrower in this case.

Inverse Opals

- An fcc lattice of dielectric spheres does *not* have a complete band gap.
- Nevertheless, it still has interesting optical properties that are responsible for the brilliant appearance of natural opals.
- Moreover, it turns out that the *inverse* structure, fcc air holes in high dielectric, *can* have a complete photonic band gap.
- Using electron microscopy, Sander's group found that precious opal mineraloids are formed of close-packed arrangements of submicron-diameter silica spheres in a silica–water matrix, with a relatively low dielectric contrast.

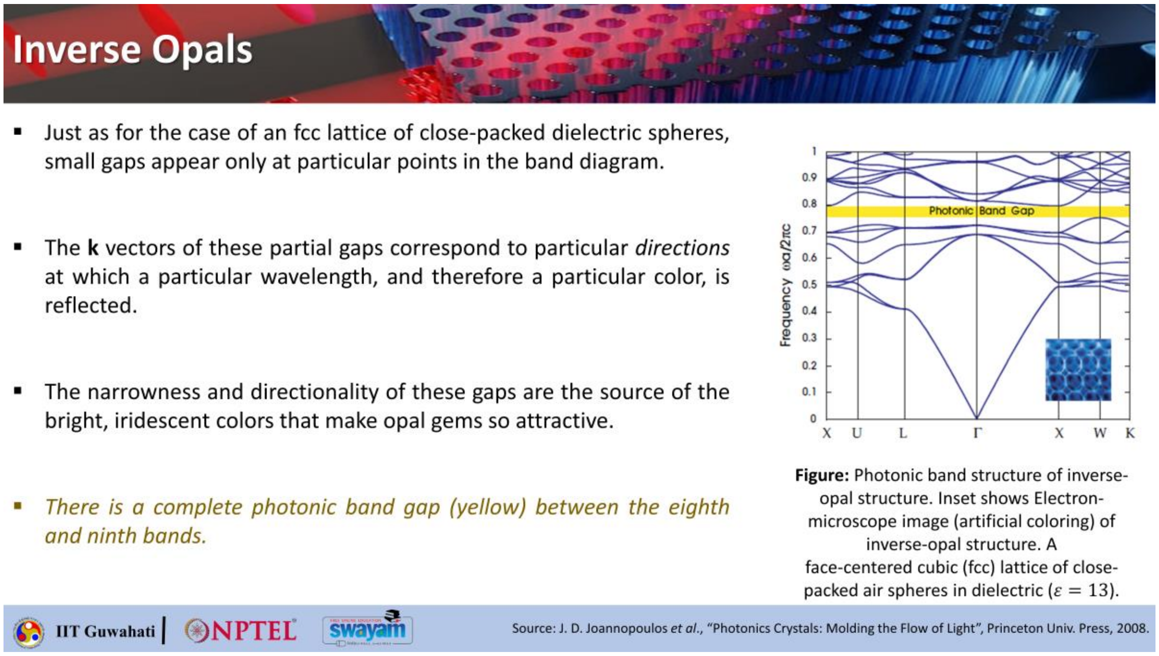
Figure: Electron-microscope image (artificial coloring) of inverse-opal structure.

Source: J. D. Joannopoulos *et al.*, "Photonics Crystals: Molding the Flow of Light", Princeton Univ. Press, 2008.

Now, let us move on to another interesting structure that could provide three-dimensional complete photonic band gap and it is inverse opal. So, opal is basically like a 3D stone, it is a gemstone spherical one okay. So, you can think of inverse opal means you are basically making a spherical hole So, you can consider first an FCC lattice of dielectric spheres that you have seen before that they are all fused together, but that could not give you a complete band gap rather it did not give you a proper band gap right which can be for all the directions of K. So, it was obviously interesting that you know the optical properties which are responsible for the beautiful appearance of natural opal.

It means natural opals have this effect naturally coming, okay. So, you actually have those band gaps in natural opal. But however, if you try to replicate that in your simulations, you are not able to reproduce any band gap. So, you can try the inverse structure and that is what has happened. So, when you tried the inverse structure like you actually have FCC air holes made in high dielectric, you can get a you know complete photonic band gap.

So, this is an electron microscope image which was done by Sander's group. They found that you know that the Opal mineraloids, they are formed of close packed arrangements of this kind of submicron diameter silica spheres in a silica water matrix and they have relatively low dielectric contrast, okay.



So, based on that, this kind of inverse opal structure was designed. So, inverse opal as you can see here these are basically you know holes or you can say air spheres repeated in a dielectric substrate. So, just as for the case of this FCC lattice of closed packed dielectric spheres.

Small gaps appear at particular points in the band diagram, as you can see here. In this particular figure, we are actually showing the photonic band gap of inverse opal structure. So this inset shows the electron microscope imaging. Of course, the coloring is artificial. You don't see any color in SEM images, okay.

And this FCC lattice, so these are the points of high symmetry in the irreducible brilliance zone. So, this air spheres are closely packed in a dielectric substrate with permittivity epsilon equals 13. And you can obtain this narrow band gap, though it is a complete photonic band gap, right. So, the k vectors of the spatial gaps okay correspond to particular directions at which so you can think of you know here there is slightly larger bandgap here there is bit narrower one okay So this is the overall one but then for some directions you have you know slightly larger partial gaps which correspond to particular directions okay at which a particular wavelength or a color will get reflected and the narrowness and function directionality of these gaps are the source of those bright beautiful colors that make these opal gems very attractive.so I will not be repeating it. So these are basically the points on your FCC Brillouin zone.

And this structure is particularly considered to be an FCC lattice of air sphere that are just touching one another. So, you have considered r to be $a/\sqrt{8}$ ok and the dielectric medium is considered to have permittivity of 13. So, the gap in this case is not between the second and the third band as it was for the other diamond based structure which we discussed previously. Here you could get only the band between 8th and 9th band and this is a bit narrow because you can see it is only 6 percent gap mid gap frequency ratio. So this is just another structure that gives you 3D photonic bandgap, but this is not very useful one because of its narrow bandgap.

But as you can see, because it goes for higher frequencies, so you can actually think of using this if you want something to be reflected from it at higher frequencies.

Right. So here, when you discuss about the inverse opal structure, you can see that there is a complete photonic band gap between the eighth and the ninth band. OK, so the wave vector here varies across the irreducible Brillouin zone . as leveled between the different high symmetry points starting from X U L Γ X W K. We have seen this couple of times The next one is a stack of two-dimensional crystals.

A stack of two-dimensional crystals

- The three-dimensional crystal formed of a stacked sequence of layers, like the woodpile.
- It has a somewhat larger gap than the woodpile, and it repeats every three layers (*ABCABC* · · ·) rather than every four.
- It is particularly simple to visualize and understand this structure: it consists of a stack of finite-thickness "two-dimensional" rod and hole crystals.

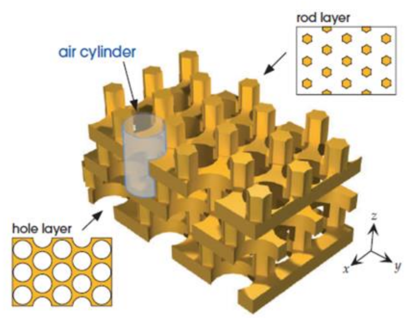


Figure: Three-dimensional photonic crystal formed by a stack of layers with two-dimensional cross sections: triangular lattices of dielectric rods in air (upper-right inset) and holes in dielectric (lower-left inset).

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Source: J. D. Joannopoulos *et al.*, "Photonics Crystals: Molding the Flow of Light", Princeton Univ. Press, 2008.

So you can think of a two-dimensional crystal which gives us beautiful band gap and start making a stack of that, something like the wood pile. So here is the three-dimensional photonic crystal, which is formed by a stack of layers with the two-dimensional cross-sections. So you can actually see what is this.

triangular lattice of dielectric rods in air okay as you can see here okay and this are this is how the holes are also you know forming a triangular lattice right So this is a kind of stack. So what do you have done here? So it has somewhat larger gap than the wood pile, and it repeats every three layers. So you have the gap here, then slightly shifted one, and then again another one.

So, it repeats like ABCABC..... something like that ok other than 4. So, in case of wood pile if you remember it is basically ABCD and then that was basically repeated here you are repeating it as ABCABC.

A stack of two-dimensional crystals

- These layers are paired in bilayers with an *ABCABC* · · · stacking, and are formed from a face-centered cubic (fcc) lattice of overlapping air cylinders in dielectric (one is highlighted) oriented along the [111] direction.
- The structure is shown in figure along with its horizontal cross sections, which fall into two categories:

  Rod layers: triangular lattice of high dielectric rods in air

  Hole layers: triangular lattice of cylindrical air holes in high dielectric

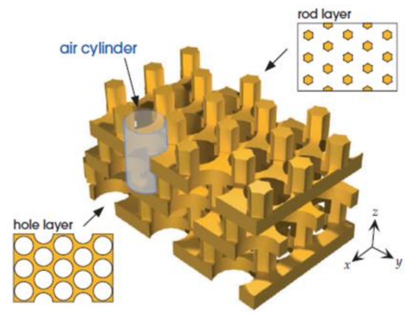

Figure: Three-dimensional photonic crystal formed by a stack of layers with two-dimensional cross sections.

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Source: J. D. Joannopoulos *et al.*, "Photonics Crystals: Molding the Flow of Light", Princeton Univ. Press, 2008.

it is basically a stack of finite thickness two-dimensional rod and hole crystals that we mastered in 2D photonic crystals. So these layers will be also paired in bilayers with an ABC-ABC kind of stacking. and they will form FCC lattice of overlapping air cylinders in dielectric, something like this, oriented along the 111 direction. So the structure shown here, along with the horizontal cross-sections, it will give us, it basically falls into two categories.

One is the rod layer with which is giving you triangular lattice of high dielectric rods in air and then also you have this whole layers which is basically triangular lattice of cylindrical air holes in high permittivity dielectric.

A stack of two-dimensional crystals

- For a dielectric contrast of 12:1 (similar to common semiconductor materials at infrared wavelengths), the structure has a 21% complete photonic band gap, as shown in the band diagram.
- Again, this structure turns out to be a distorted diamond lattice that is closely related to Yablonovite, and hence the gap is between the second and third bands.
- To see that it is diamond-like, think of the spot between three holes in a hole layer as an "atom," with four "bonds" formed by the three in-plane veins to neighboring spots and the one rod either under or over the spot.

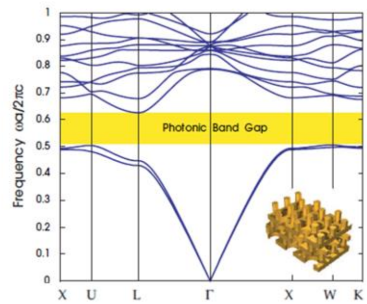

Figure: The photonic band structure for the lowest bands of the layered structure.

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Source: J. D. Joannopoulos *et al*., "Photonics Crystals: Molding the Flow of Light", Princeton Univ. Press, 2008.

So when you calculate the photonic band structure, considering a dielectric contrast of 12 is to 1 for this particular structure. So 12 is to 1, it is simple that you are taking air and silicon. So that gives you a 21% complete photonic band gap, which is pretty good. as compared to wood pile and the inverse opal structure that we have just seen.

Again, if you look into the structure carefully, this is another you know distorted diamond lattice that can closely relate to the Yablonovite and that is why you can also see the gap is here between the second and the third band. Okay, and to see that it is diamond like you can think of the spots between the three holes in a whole layer as atoms with four bonds formed by the three in-plane veins of the neighboring spots and the one rod either under or over that particular spot.

A stack of two-dimensional crystals

- **Top:** The structure is fabricated by depositing a sequence of dielectric layers with thickness $d = a/\sqrt{3}$, where a is the fcc lattice constant, into each of which is drilled a lattice of holes offset as shown.
- Rod/hole sublayer cross sections are formed by regions indicated, where adjacent holes do/don't overlap, respectively.
- **Middle:** horizontal cross section of a hole sublayer A, showing triangular lattice of holes and offsets of holes in subsequent layers.
- The horizontal lattice constant is $\tilde{a} = a/\sqrt{2}$.
- **Bottom:** top view of silicon structure fabricated for a gap around 1.3 microns, with three layers of holes visible (artificial coloring).

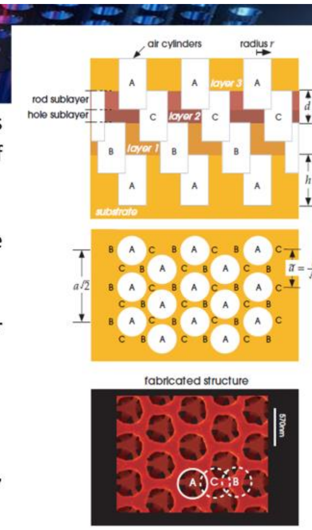


Figure: Vertical cross section of layered structure showing air cylinders in an fcc lattice forming an *ABC* stacking along [111] dir.

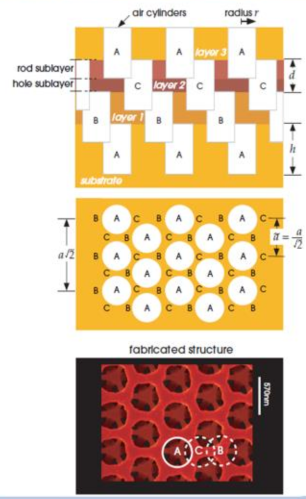
IIT Guwahati | NPTEL swayam Source: J. D. Joannopoulos *et al.*, "Photonics Crystals: Molding the Flow of Light", Princeton Univ. Press, 2008.

So you can carefully look into how this structure is made and you can actually think of how you can imagine this like a diamond like structure. ok. So, this also can show you that if you consider the structure to be fabricated by depositing a sequence of dielectric layers with thickness of D equals A by √3, where A is basically the axis the lattice constant ok into each of which is drilled into each of which it is a drilled hole is there ok with an offset.

So, you can think of like this you can see air cylinders ok then then you have this rod sub layer you have the next one next hole being drilled and so on ok. rod hole sublayer cross sections are formed as you can see here where adjacent holes do or in some case do not overlap okay so in rod layer they do overlap but in whole layer they do not overlap okay here you can see the horizontal cross-section of a whole sublayer which shows basically a triangular lattice of holes and offset of holes in the subsequent layers. So you can see that they are having some offset and the horizontal lattice constant is considered to be $a/\sqrt{2}$. And the bottom one actually shows you the top view of the silicon structure which is fabricated with a gap of 1.3 micron with three layers of holes visible through artificial coloring. So you can see that the three holes are basically offset.

A stack of two-dimensional crystals

- The rods have a strange-looking shape because they are formed from the *remnant* left over from six overlapping cylindrical air holes.
- The entire structure is an fcc lattice of overlapping air cylinders (with height $h = 0.93a$ and radius $r = 0.293a$) in a dielectric medium with $\varepsilon = 12$.
- The *ABCABC* · · · stacking of these cylinders is shown in figure, in which the stacking direction is the body diagonal [111] direction of the fcc lattice.
- In this way, a rod and hole bilayer is formed from each layer of cylindrical air holes.



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Source: J. D. Joannopoulos *et al.*, "Photonics Crystals: Molding the Flow of Light", Princeton Univ. Press, 2008.

So this is the vertical cross-section of layered structure. okay that that shows not this this one okay this is the top view. So, this is basically the vertical cross section of the layered structure that shows you the air cylinders in FCC lattice which form this ABC kind of stack along 111 direction.

So the rods have a strange looking shape because they are formed from the remnants which are basically left over from six overlapping cylindrical holes obviously. And then the entire structure you can think of an excess lattice of overlapping air cylinders which have got height say h equals 0.93a and the radius is optimized to be 0.293a. And you consider the dielectric medium to be silicon that is epsilon equals 12.

So, with this kind of ABC ABC kind of stacking of the cylinders as you can see in the figure the stacking direction is basically the body diagonal which is 1 1 and direction 1 1 1 for the FCC lattice. So, in this way what you can do you can actually see that a rod and a hole bilayer is formed from each layer of the cylindrical holes.

Localization at a Point Defect

- Defects in photonic crystals can localize light modes.
- In one dimension, this meant we could confine light to a single plane.
- In two dimensions, we could localize light to a single line, which can also be considered a single point in the *xy* plane.
- In three dimensions, we can perturb a single lattice site, and thereby localize light to a single point in the crystal.
- It is trapped in all three dimensions.
- The point defect pulls a state from the continuum above or below the gap *into* the gap itself, resulting in a localized mode.

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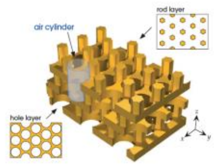
Source: J. D. Joannopoulos *et al.*, "Photonics Crystals: Molding the Flow of Light", Princeton Univ. Press, 2008.

So, with that we understood how this you know three dimensional photonic crystals are fabricated and as you can understand that it is not very easy to make this kind of structures with you know homogeneous or you know with all the dimensions perfectly matched in all the sides. So, this is a bit of challenging task to get 3D photonic bandgap. But why people still try to make that because that is very close to the natural crystals.

And you can take help of the localization at point defect, line defect, and surface defect on these photonic crystals, which can be used for many different applications. So now let us focus on localization at a point defect. so when you talk about localization at a point defect we are basically talking of defects in photonic crystals okay which can localize light modes so we have seen that in one dimension this means we can confine light to a single plane when you consider now two dimension when you say we could localize light It is basically localizing along a single line, okay, which can also be considered as a single point in XY plane. And when we see, you know, in three dimension, when we perturb of a single lattice point, we can actually localize light to a single point in the crystal. So, in this case, light will be trapped in all three dimensions.

Localization at a Point Defect

- Two simple ways to perturb a single lattice site are to add extra dielectric material where it does not belong or to remove some of the dielectric material that should be there.
- Consider the first case as a **dielectric defect** and the second case as an **air defect**.
- A single rod in a rod layer is removed to create an air defect (left).
- The radius of rod have increased its radius to create a dielectric defect (right).



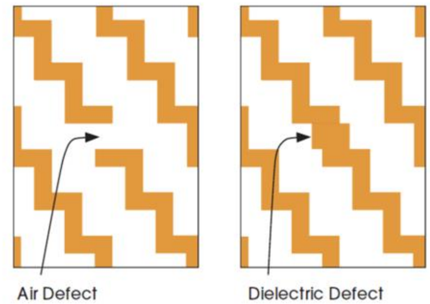


Figure: Vertical cross-section of the layered structure from a stack of two-dimensional crystals.

Source: J. D. Joannopoulos *et al.*, "Photonics Crystals: Molding the Flow of Light", Princeton Univ. Press, 2008.

So the point defect basically pulls the state from the continuum above or below the band gap into the gap itself, resulting into a localized mode. And that is what is the effect of point defect. So two simple ways to perturb a single lattice site in a three-dimensional photonic crystal is to add an extra dielectric material that does not belong okay or you can simply remove some of the dielectric material that should be there okay that is how you introduce a defect so it you can actually create a air defect by removing this particular you know dielectric material or you can add this extra dielectric and create a dielectric defect so these are the two ways you can basically you know create the defect in this particular three-dimensional photonic crystal. So, these are basically showing you the vertical cross section of the layered structured from the stack of two-dimensional crystals right.

So, consider the first case. So, this is basically a dielectric okay this is basically the dielectric defect and this is the air defect as you can see. So, a single rod in a rod layer can be removed ok like this to create an air defect and an a radius of the rod can be rather increased ok that can help you create a dielectric defect which is shown here.

Localization at a Point Defect

- The defect is like a cavity with perfectly reflecting walls.
- If light with a frequency within the band gap somehow winds up near the defect, it cannot leave, because the crystal does not allow extended states at that frequency.
- Therefore if the defect allows a mode to be excited with a frequency within the band gap, that mode is forever trapped.
- The defects create localized modes within the photonic band gap.
- The air defect introduces a single, nondegenerate state into the photonic band gap, which crosses from the dielectric to the air band as the defect frequency is raised.

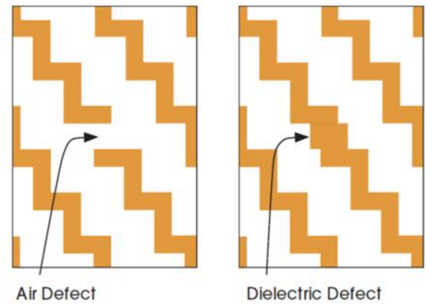


Figure: Vertical cross-section of the layered structure from a stack of two-dimensional crystals.

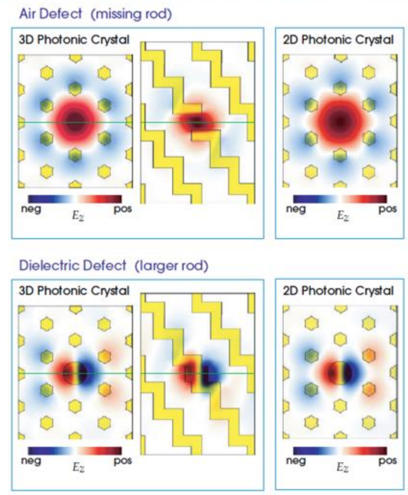
Source: J. D. Joannopoulos *et al.*, "Photonics Crystals: Molding the Flow of Light", Princeton Univ. Press, 2008.

So, the defect is like a cavity which has got, you know, reflecting walls in all dimensions, direction around it. So if light with the frequency within the band gap somehow, you know, winds up near the defect, it cannot leave that cavity, right? Because the crystal where that mode has to escape, that crystal does not support that particular mode because that mode falls within the band gap of the crystal, right? Therefore, if the defect allows the mode to be excited with the frequency within the band gap, it gets trapped in that particular cavity.

That mode gets trapped in that cavity forever. The defects could create localized modes within this photonic band gap. So here you can see the air defect introduces a single non-degenerate state into the photonic band gap which crosses from the dielectric to air band with you know defect frequency as the defect frequency is raised.

Localization at a Point Defect

- Horizontal and vertical cross sections (intersecting at green lines) of E_z field patterns of point defects in the layered structure (left), formed by modifying a single rod.
- The dielectric material is shown in yellow.
- *Top:* nondegenerate **monopole** state trapped by completely removing a rod.
- *Bottom:* dielectric defect formed by replacing a rod with an enlarged dielectric cylinder of radius 0.35 $\tilde{a}$ (where $\tilde{a}$ is the in-plane lattice constant).
- This is a doubly degenerate **dipole** state, where the degenerate partner is *roughly* its 90° rotation.



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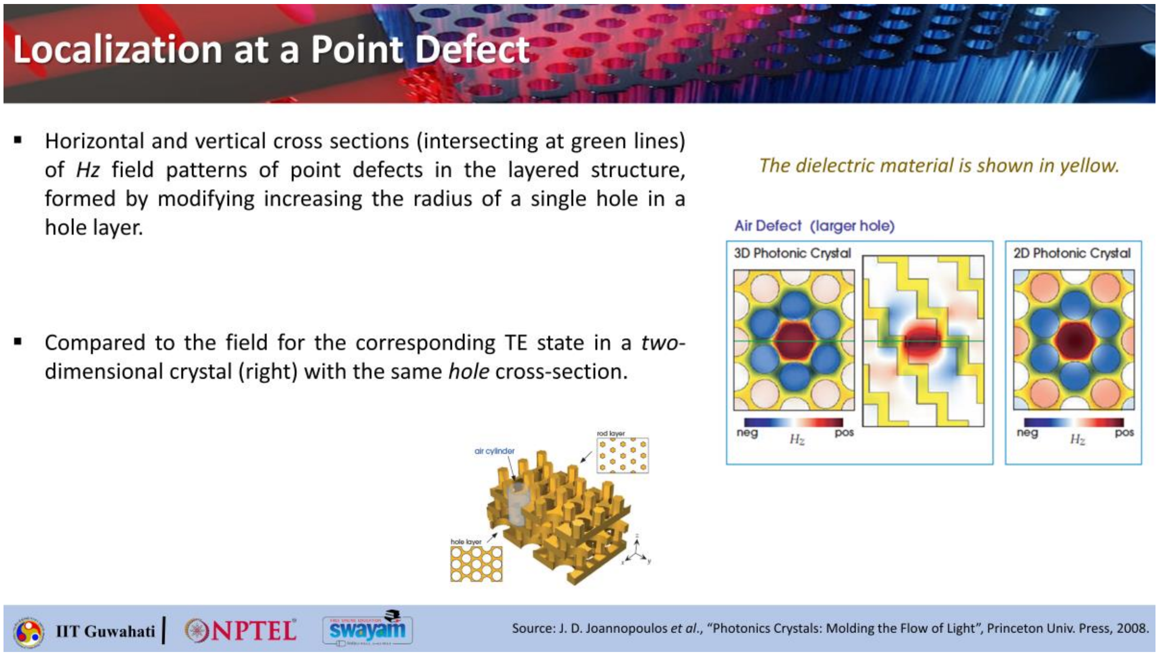
Source: J. D. Joannopoulos *et al.*, "Photonics Crystals: Molding the Flow of Light", Princeton Univ. Press, 2008.

so here are the horizontal and vertical cross section okay so you can actually see the intersecting you know green lines okay so this is where exactly you can locate the air defect that is basically the missing rod okay so this is the E_z profile Okay, this is for comparison we have kept it for that 2D photonic crystal. So here it happens in this plane, but here it happens in three dimensions. So this is the vertical cross section and here you can see it is basically trapped in a cavity where it is surrounded by in three dimension.

Okay. So the dielectric materials are shown in yellow that you can see here. So what you see here you are actually seeing a non-degenerate monopole state trapped by completely removing this particular rod. But when you see here this is basically a dielectric defect introduced by increasing the radius of a particular rod and that gives you okay. So here what they have done they have created the defect by replacing a rod with an enlarged dielectric cylinder of radius 0.35a okay. So, bigger one is put here okay. So, this actually gives you a doubly degenerate dipole state right. And where the degenerate partner is basically 90 degree rotation, right?

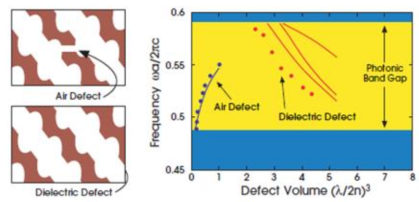


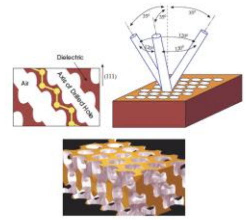
So that way you can see that the two cases basically gives you two different types of non, one gives you non-degenerate mode monopole mode and the other one gives you a doubly degenerate dipole mode right and for comparison you can see this is what happens in the 2D photonic crystal right. So, here we can see the horizontal and vertical cross section again. intersecting at the green lines, okay. And here we are seeing the magnetic field pattern HZ pattern of the point defects in the layered structure.

So, again if you think of air defect that has been introduced as a larger hole, okay. So, if you so this happens in the whole layer, right. In the whole layer if you increase one particular hole radius that gives you a hole defect. So, you can compare this with 2D case yeah this and this top view looks very similar, but then vertically it is also trapped ok in all three dimensions. So, compared to the field for the corresponding TE state in 2 dimensional it you have very similar kind of images or field distribution pattern for same whole cross section right.

Localization at a Point Defect

- Plotted are the frequencies of the localized modes of Yablonovite **(bottom figure)** as the defect size varies.
- The dots indicate measured values, the lines indicate computed values, and the yellow region is the photonic band gap.
- The modes on the blue line (leftmost) result from an air defect (top left), while the modes on the red lines (rightmost) result from a dielectric defect (bottom left). **(Top Figure)**
- The strength of the defect is expressed in terms of the volume of dielectric added or removed, in units of $(\lambda/2n)^3$, where λ is the midgap vacuum wavelength and $n = \sqrt{\varepsilon}$ is the refractive index.





Source: J. D. Joannopoulos *et al.*, "Photonics Crystals: Molding the Flow of Light", Princeton Univ. Press, 2008.

So, these are you know for the Yablonovite crystal. So, you can see that the different kind of defects here air defect is introduced here dielectric defect is introduced ok and this particular figure shows the frequencies ok for which air defect will show up this is the photonic band gap and this is where the dielectric defects will appear within the photonic band gap. So this basically tells you about the frequencies of the localized modes of the Yablonovite crystal for different kind of defect sizes. So this is the defect volume as you can see here and these lines indicate the computed values within the photonic bandgap. right. So, the strength of the defect is basically expressed in terms of the volume of the dielectric which is added or removed.

So, you can express this in terms of $(\lambda/2n)^3$ where λ is basically the mid gap vacuum wavelength and n is square root of epsilon. So, that way you can actually see the effect of you know the size of the defect and where it will end up within the photonic band gap.

Localization at the Surface

- Suppose we terminate a three-dimensional photonic crystal in the *z-direction*.
- By doing this, we destroy the translational symmetry in that direction, and we can no longer classify the states of the crystal with a definite k_z.
- However, the crystal still has translational symmetry parallel to the surface, and the electromagnetic modes *do* have a definite $\mathbf{k}_{||}$.
- We must project the full three-dimensional band structure onto the surface Brillouin zone.

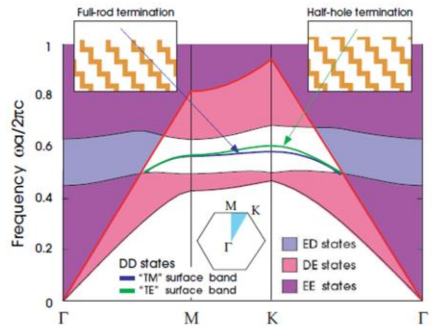


Figure: The band structure of the (111) surface of the layered structure, along special directions in the surface irreducible Brillouin zone (center inset).

Next we look into localization at a linear defect. So, we have seen the effect of linear defect in 2D photonic crystal which can give you beautiful waveguides for waveguiding lights in a particular direction okay. So, that the same thing can be done in case of 3D photonic crystal as well, but then here you know it will be confined also in the third dimension right.

So, here again this is useful for forming waveguide. So, you can remove a row of rods from a single rod layer of the three-dimensional crystal. So, what will happen? So, you will actually result in a waveguide state which is like this and which is pretty much similar to what you see in 2D photonic crystal just that this vertical cross section tells you that it is actually confined within that particular layer.

Localization at the Surface

- The shading denotes regions in which light is transmitted (purple, *EE*), internally reflected (pink, *DE*), and externally reflected (blue, *ED*).
- The lines in the gap correspond to *DD* surface bands in which light is localized at the surface, for two terminations.
- The green line (upper) is the "TE-like" band that results when the surface is terminated with half of a hole layer (top right inset).
- The blue line (lower) is the "TM-like" band that results when the surface is terminated with a full rod layer (top left inset).

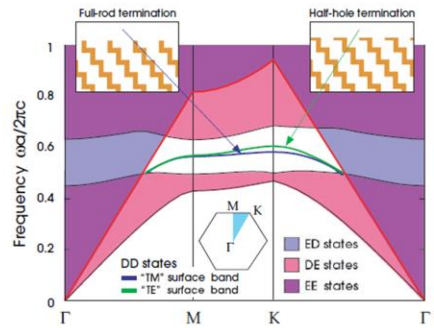


Figure: The band structure of the (111) surface of the layered structure, along special directions in the surface irreducible Brillouin zone (center inset).

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Source: J. D. Joannopoulos *et al.*, "Photonics Crystals: Molding the Flow of Light", Princeton Univ. Press, 2008.

So, it is mostly TM polarized okay E_z okay in the mid plane of the rods and it is confined primarily in the air of the defect row like this. Here however, the mode is also confined into vertical dimension and exponentially decaying away from the waveguide and this is the difference in the case of 3D and 2D photonic crystal. So, here you can see the projected band diagram of the lined effect formed by a missing row of rods in the case of a single layer you know 3D photonic crystal.

So, this is what happens in 3D photonic crystal and this is what happens in a 2D photonic crystal ok. So, this is just for comparison they look very similar ok. So, the linear defects are basically analogous to metallic waveguides where light will be trapped within perfectly reflecting walls. So, here unlike or you can say just like hollow metallic tubes, the waveguide mode will depict some interesting feature. okay so there is a point at the edge of the bend where the slope will go to 0 and okay that is where here you can see the slope goes to 0 and it basically becomes a slow light waveguide so in which the velocity of the light can be arbitrarily slowed in principle So, the red line in the guided band is showing the complete photonic band gap ok and here it is for comparison you can also see the same feature is available for your 2 dimensional Photonic crystal also for the same kind of rod cross sections if you remove one rod, okay.

Localization at the Surface

- The bands for two possible surface terminations: one in which *half* of a hole layer is left on top of the crystal, and one in which a *full* rod layer is left on top.
- The Brillouin zone of **k** is the same as that of the triangular lattice, with the K point corresponding to the nearest-neighbor direction.
- The fields for these two terminations at the K point is shown are shown, and exhibit the expected localization at the surface, in either the surface rod or hole layer.
- *Left*: E_z for "TM-like" band when the surface is terminated with a full rod layer.
- *Right*: H_z for "TE-like" band when the surface is terminated with half of a hole layer.

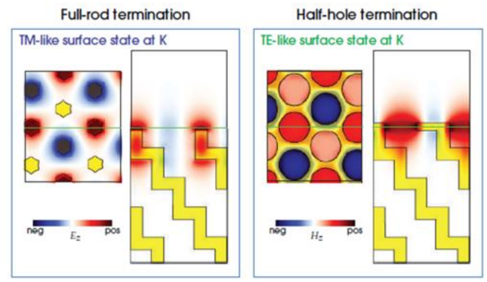


Figure : Horizontal and vertical cross sections (intersecting at green lines) of field patterns for the two crystal terminations at the K point.

Source: J. D. Joannopoulos *et al.*, "Photonics Crystals: Molding the Flow of Light", Princeton Univ. Press, 2008.

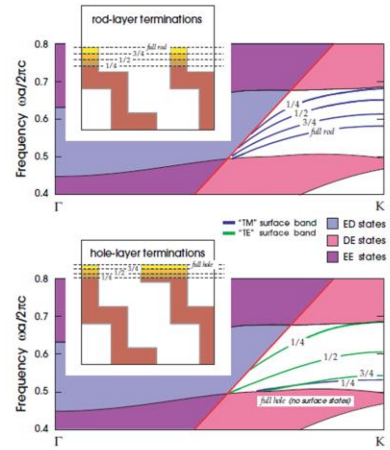
So, you can actually get this kind of thing, but only thing is that the vertical confinement will be missing. And then lastly we will look into another effect which is photonic. localization at the surface. So, suppose here we terminate the three-dimensional photonic crystal in z direction okay. So, by doing this we will basically destroy the translational symmetry in that direction and we can no longer classify the states of the crystals with the definite k_z.

However, the crystal will still retain its translational symmetry parallel to the surface. That means the electromagnetic modes will have definite k parallel, right? So we must project full three-dimensional band structure onto the surface brilliant zone and this is the band structure of 111 surface of the layered structure okay so this shows full rod termination and here it shows half hole you know half the hole is there so it is half hole termination okay So, the shading regions ok. So, here you can see there are two colors being used. So, one is for TM surface band, the other one is for TE surface band ok. This one is for TE, this one is for TM ok and this tells you the different colors tell you the ED, DE, and EE states.

So, what are these basically? The shading regions denote specific areas or specific regions in which light is transmitted. So, where it is transmitted it is basically purple E, where it is internally reflected we represent by pink D and where it is externally reflected we represent by blue that is ED. And the lines in the gap that basically tells you about the DD states okay. in which the light is localized at the surface for the two terminations which are shown here. One is for full rod termination like this and this is for half hole termination, right?

Localization at the Surface

- The Γ–K band structures of the (111) surface of the layered structure, for various terminations ending at a rod layer (top) or a hole layer (bottom).
- The figure zooms in on the gap region, using the same classification scheme *EE* etc.
- The rod-layer terminations (top) all support a "TM-like" surface state localized in the topmost rods (blue lines).
- The hole-layer terminations (bottom) localize a "TE-like" state in the topmost hole layer (green lines); leaving 1/4 of a hole layer also supports a "TM-like" surface band in the adjacent rods (bottom-most blue line).
- As the surface termination increases in the vertical direction, dielectric material is added and the corresponding bands are pulled *down* in frequency.



The insets indicate the corresponding surface terminations.

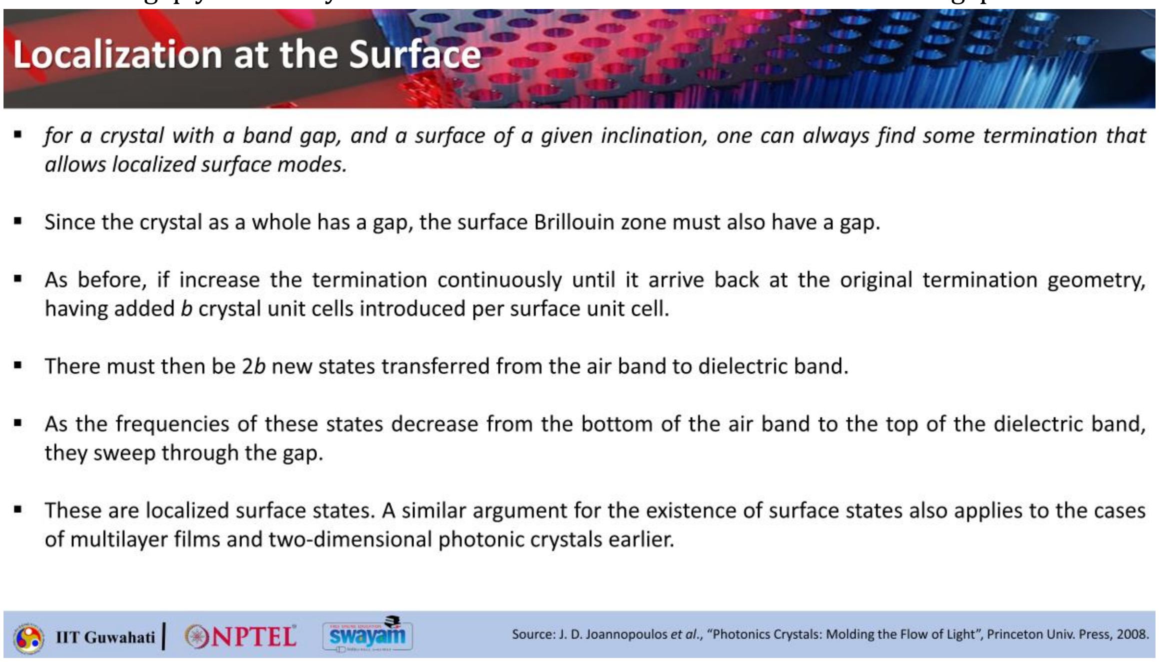
Source: J. D. Joannopoulos *et al.*, "Photonics Crystals: Molding the Flow of Light", Princeton Univ. Press, 2008.

So here the green line, the upper one is like the TE-like band that results when the surface is terminated with half of a whole layer.

which is shown here and the bottom one shows the blue line which is like TM band where the surface is terminated with a full rod layer. Now, the bands for two possible surface termination, so one in which half of a whole layer is left on top of the crystal and in another case you know a full rod layer is left on top of the crystal. So, this is for full rod termination, this is for half rod termination you can see here how they look like. So, the Brillouin zone of k will be the same as that of the triangular lattice with the K point corresponding to the nearest neighbors.

And the fields for these two terminations at the K points are shown here ok. So, you can see the E_z plot the red shows you positive and blue tells you the negative one ok. So, you can see the terminations over here. So, these are the horizontal and vertical cross section and this is where they are matched ok this particular intersecting blue lines ok at the k point. So, the both so this is like TM like surface state and this is TE like surface state. So, one is for half rod sorry full rod termination and this is for half hole termination clear.

So, this is what we just discussed. So, TM like band is seen here for full rod termination and TE like band or you can see H_z over here which shows up for half hole termination. So, the Γ–K band structure of this 111 surface of the layered structure for various termination rod layer termination and whole layer termination are shown here okay. So, this figures basically zooms in on the gap region using the same classification scheme EE okay. And here also you can see that the rod layer termination all supports the TM like surface modes or you can say TM like surface state which are localized on top of the you know rods something like this okay. And the whole layer termination here localized TE like states in the topmost whole layer, leaving 1 by 4 of a whole layer which supports TM like surface bands. in the adjacent rod. So, you can see the bottom most blue line that basically corresponds to the TM like surface bands which is coming from the one fourth of the whole layer ok. So, as the surface termination increases in the vertical direction dielectric material is added and the corresponding bands are pulled down in frequency. So, you can see from here to here ok. So, what we understood For a crystal with a band gap and a surface of a given inclination one can always some find some kind of termination that allows localized surface modes. So, since the crystal has a hole or you can say now since the crystal as a hole has a band gap you can say that the surface Brillouin zone must also have a gap.



And as before if increase the termination continuously until it arrive back at the original transmission termination geometry having added b crystal unit cells introduced per surface unit cell ok. If that happens then there must be you know 2b new states transferred from air band to dielectric band. So, let me repeat this. So, if you increase the termination continuously until it arrive back at the original you know termination geometry having added b crystal unit cells ok.

per surface unit cell ok. Then there should be like 2b new states transferred from air band to the dielectric band. As the frequencies of these states decrease from the bottom of the air band to the top of the dielectric band, we basically sweep through the gap right. So, that has to go through the band gap. So, these are the localized surface states and a similar argument for the existence of surface states also applies to the cases in for multi-layer films and two-dimensional photonic crystals which we have discussed earlier.

So, with that we will stop here for this particular lecture. So, we will start discussing about applications of 3D photonic crystals in the next lecture. If you have got any query or doubt regarding this lecture, you can drop an email to this email address mentioning MOOC and photonic crystal on the subject line. Thank you.

Thank You