Lec 19: Overview & Modelling of Periodic Dielectric Waveguides

Hello students, welcome to lecture 19 of the online courses on Photonic Crystals, Fundamentals and Applications. Today's lecture will be on an overview and modeling of periodic dielectric waveguides.

Lecture Outline

- Introduction
- Overview
- A Two-dimensional Model
- Periodic Dielectric Waveguides in Three Dimensions
- Modelling of the periodic dielectric waveguide

So, here is the lecture outline. We will briefly introduce and provide overview of the topic. We will show a two-dimensional model of this and then we will also consider periodic dielectric waveguides in 3D. and we will see how we do modelling in COMSOL for this periodic dielectric waveguide and that gives us the results which are reported in the literature.

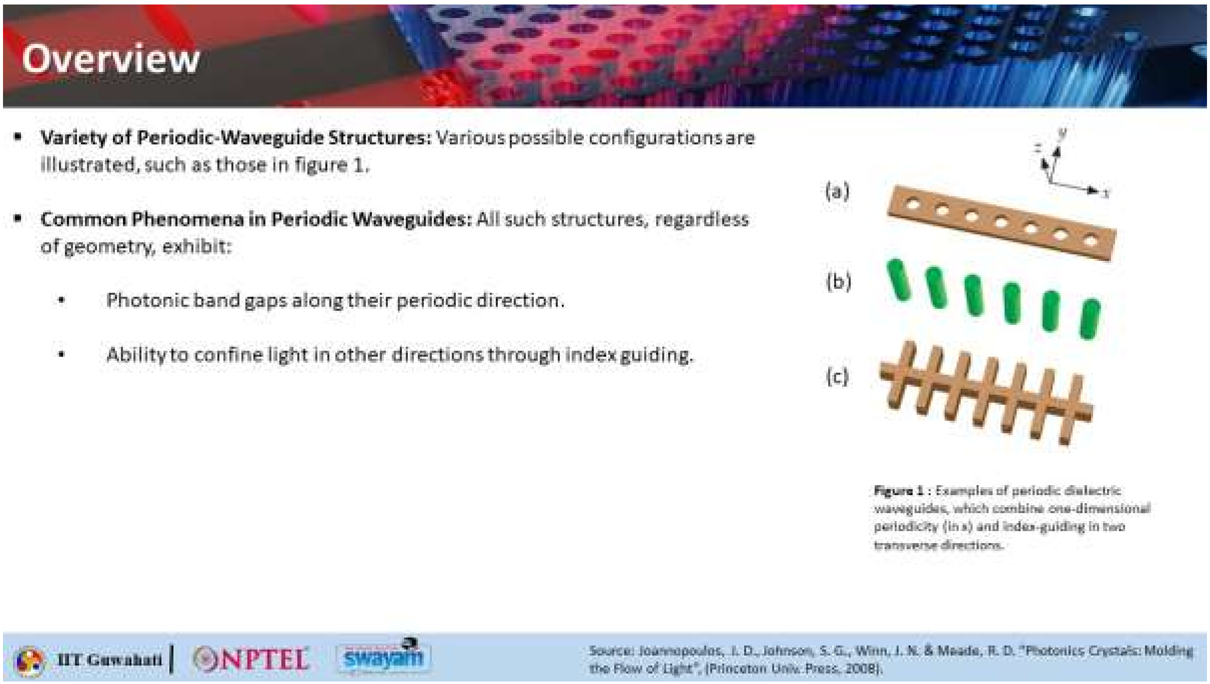
Introduction

- **Three-Dimensional Photonic Crystals:** These structures can confine light in all three dimensions.
- **Engineering Materials:** These materials can localize light at a single point (optical cavity), direct it along a specific path (waveguide), or confine it on a two-dimensional surface.
- **Fabrication Challenges:** Creating a structure that is periodic in all three dimensions is technically challenging.

  Alternative Approach: The discussion shifts to simpler structures such as periodic dielectric waveguides.
- **Characteristics of Periodic Dielectric Waveguides:** These have a one-dimensional periodic pattern or grating along the direction of propagation, and possess finite thickness and width.

So, as we saw in the previous lecture that about the analysis of 3D photonic crystals that 3D photonic crystals are those which are able to you know confine light in all three dimensions. Now, these materials can localize either at a single point that is like an optical cavity, or it can direct light along a specific path that will work as a wave guide, or it can confine into a two-dimensional surface. So all these three types of defects, this will be called as point defect, this will be a line defect, and this is a surface defect. All these possibilities are there in a 3D photonic crystal.

Now, what are the fabrication challenges? As we have seen that it is not very simple to fabricate those 3D photonic crystals, right? The fabrication looks very challenging. And if you compare this with the 2D photonic crystals, you will see that the 2D photonic crystals are much more, easier to adapt, okay, to the current technological advances and it is easy to fabricate. So, lot of people actually try to restrict their applications to 2D photonic crystals. So, we will try to now shift this discussion to simpler structures such as periodic dielectric waveguides and we will see how they are useful. So, when you talk about periodic dielectric waveguides, what are their characteristics so these are basically one-dimensional periodic pattern or grating which is along the direction of propagation and this waveguide possesses finite thickness and width so these are some examples of periodic dielectric waveguide which basically has one-dimensional periodicity.



Here it is along x direction and basically you have index guiding along the other two directions that is y and z. So regardless of the geometry, these three different periodic waveguides, they have something in common. It is that there is a photonic band gap along their propagation direction that is along x direction and in the other two directions they are able to you know confine light through index guiding. So, these are the similarities in this kind of periodic waveguide structure. Now, in the next two lectures we will explore different forms of hybrid system that will be able to combine the periodicity with other mechanism to confine light in three dimensions.

We will also discuss about you know the periodic planar waveguides which are known as photonic crystal slabs. So, which utilize two dimensional periodicity along with vertical index guiding. We will also examine photonic crystal fibers. These are basically special types of waveguides where the periodicity is basically transfers to the direction of propagation.

Overview

- **Hybrid Systems:** Different forms of hybrid systems that combine periodicity with other mechanisms to confine light in three dimensions.
- **Photonic-Crystal Slabs:** Discussion on periodic planar waveguides known as photonic-crystal slabs, which utilize two-dimensional periodicity along with vertical index-guiding.
- **Photonic-Crystal Fibers:** Special Waveguides where the periodicity is transverse to the direction of propagation.

A Two-dimensional Model

- **Strip Material Orientation:** The strip extends in the x direction and confines light in the y direction through index guiding, while remaining uniform in the z direction.
- **Light Propagation:** As seen in lecture 13, we focus on light that propagates in the *xy* plane ($k_z = 0$) and specifically restrict our analysis to TM polarization (E_z only).
- **Introduction of Periodic Interruptions:** Periodic interruptions are added along the x direction of the strip, creating a pattern of dielectric squares.
- **Specifications of the Material and Design:**

  Dielectric constant of the material is $\varepsilon = 12$.
  The spatial period of the squares is denoted by a.
  Each square measures $0.4a \times 0.4a$

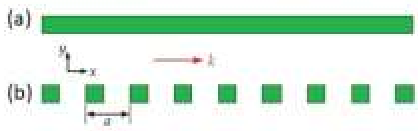


Figure 2: (a) Two-dimensional dielectric waveguide (ε=12) of width 0.4a. (b) Periodic waveguide: a period-a sequence of 0.4a ×0.4a dielectric squares.

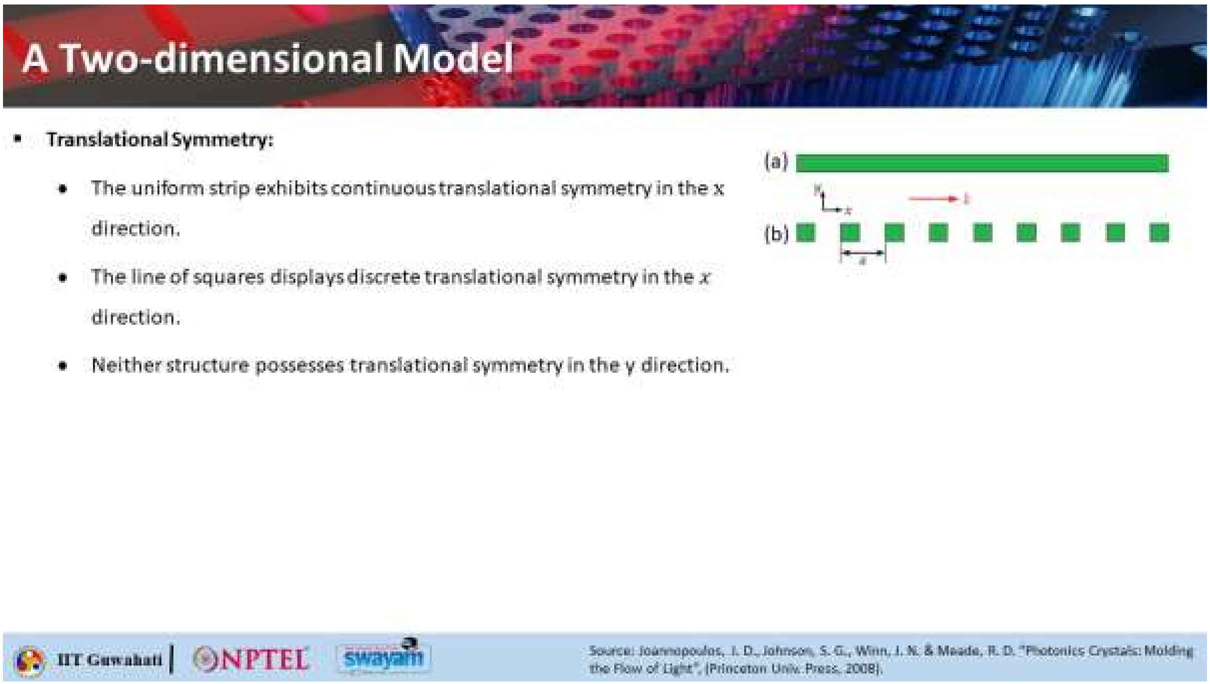
Source: Joannopoulos, J. D., Johnson, S. G., Winn, J. N. & Meade, R. D. "Photonics Crystals: Molding the Flow of Light", (Princeton Univ. Press, 2008).

We are starting with a simple model that is a two-dimensional model.

Although the real motivation for this lecture is to confine light in three dimensions, we will begin our discussion with a very simple two-dimensional model that will be able to showcase the essential physics that is involved. We will combine index guiding in one direction with the photonic band gap in other direction. So, let us first think of a strip that is a material strip that extends in x direction and it confines light in y direction through index guiding right and there and it remains uniform along the z direction which is basically out of this screen ok or into the screen whichever way you want to imagine. Now if you consider the light propagation as seen in lecture 13, we will focus on light that propagates in the xy plane. That means you can actually take kz equals 0 okay and specifically restrict our discussion or analysis to only Tm polarization.

That means you can only calculate the ez component or z component of the electric field. Right. So this is the uniform strip. Now you can also try to introduce some kind of periodic interruption in this strip. So you can actually think of adding periodic interruption along the x direction, which will create a pattern like this kind of dielectric squares.

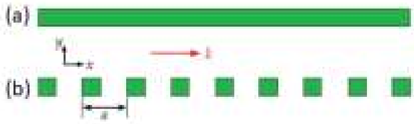
So we can think of a periodic waveguide, which is having a period of A. and each dielectric square can be thought of having dimension of 0.4a by 0.4a and the material that is involved here has a dielectric constant of 12. So, with that we can start our discussion with a two-dimensional period oh sorry a periodic waveguide right.



So it is having the periodicity only along one direction okay along y and z it is uniform. So how about the translational symmetry? You can see that the uniform strip exhibits continuous translational symmetry that is along the x direction and the line of squares okay which is having discrete translational symmetry and that also along the x direction. We have discussed about continuous and discrete translational symmetry in our previous lectures. So, neither structure possesses translational symmetry in the y direction, isn't it? So, how do you talk about you know the conservation and simplification of web numbers? So, here as the periodicity is along x, you can say kx is conserved due to the symmetry in the x direction. k y is not conserved reason is it reflects the lack of symmetry along y direction ok.

A Two-dimensional Model

- **Conservation and Simplification of Wave Numbers:**
  - k_x (referred to as k) is conserved due to the symmetry in the x direction.
  - k_y is not conserved, reflecting the lack of symmetry in the y direction.
- **Analysis of Band Structure:**
  - As discussed in lecture 13, it is beneficial to compute the projected band structure $\omega_n(k)$.
  - The mode frequencies are plotted as a function of k, although they technically depend on the full vector k for modes far from the waveguide.



Source: Joannopoulos, J. D., Johnson, S. G., Winn, J. N. & Meade, R. D. "Photonics Crystals: Molding the Flow of Light", (Princeton Univ. Press, 2008).

A Two-dimensional Model

- **Uniform Strip Band Diagram:**
  - The projected band diagram for the uniform strip is shown in the left panel of figure 3.
  - The diagram displays the range $0 < k < \frac{2\pi}{a}$, although k is technically unrestricted due to continuous translational symmetry.
- **Light Cone Definition:**
  - The region where $\omega \geq ck$ is referred to as the light cone.
  - In this region, there exist extended states that propagate in the air.
- **Beneath the Light Cone:**
  - The higher index of the waveguide beneath the light cone pulls down discrete guided modes.
  - These modes are localized due to total internal reflection.

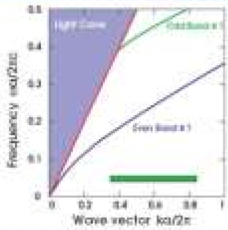


Figure 3: Band diagram of **Uniform Strip** waveguide, for TM-polarized in-plane (k_z =0) light only.

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Source: Joannopoulos, J. D., Johnson, S. G., Winn, J. N. & Meade, R. D. "Photonics Crystals: Molding the Flow of Light", (Princeton Univ. Press, 2008).

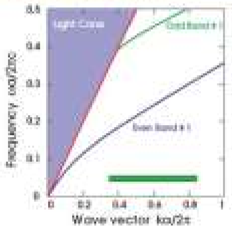
So, if you consider this as the uniform strip which you have seen before. So, this is the band diagram for this particular uniform strip waveguide. okay and this is computed for Tm polarized okay in plane light way only. So, when you say Tm polarized in plane light you can say kz equals 0. So, this is basically only kx component okay.

Again these are normalized frequencies so it is omega a by 2 pi c and this is normalized wave vector so it is k a by 2 pi. So, this is the band diagram and what you can see here that the diagram actually displays the range of k starting from 0 to 2 pi by a. Although k is technically unrestricted due to this you know continuous translational symmetry. But you actually plot it from 0 to 2 pi by e. So, what is the definition of the light cone? We have discussed earlier that you know the region which lies above the light line that is omega equals ck.

So, anything above this that is giving you the light cone. So, the lower boundary of the light cone gives you the light line okay and, in this region, there are extended states that could propagate in air okay. So, this blue shaded region actually tells you about the light cone and what they contain they basically contain extended states which are allowed to propagate in air. Now, what are these two lines? These are basically discrete guided bands that are labeled as even band and odd band. And there is a band numbering that you can see.

A Two-dimensional Model

- **Symmetry of the Waveguide:**
  - The waveguide is symmetric under reflections through the plane $y = 0$, which bisects the waveguide.
- **Classification of Guided Modes:**
  - All guided modes can be classified as either even or odd with respect to mirror reflections in the $y = 0$ plane.
  - Symmetries that might seem present in other planes perpendicular to the waveguide axis are actually broken when $k \neq 0$.
- **Mode Bands:**
  - The diagram shows one even band and one odd band.
  - The even band represents the fundamental mode, characterized by having lowest frequency in the mode profile.



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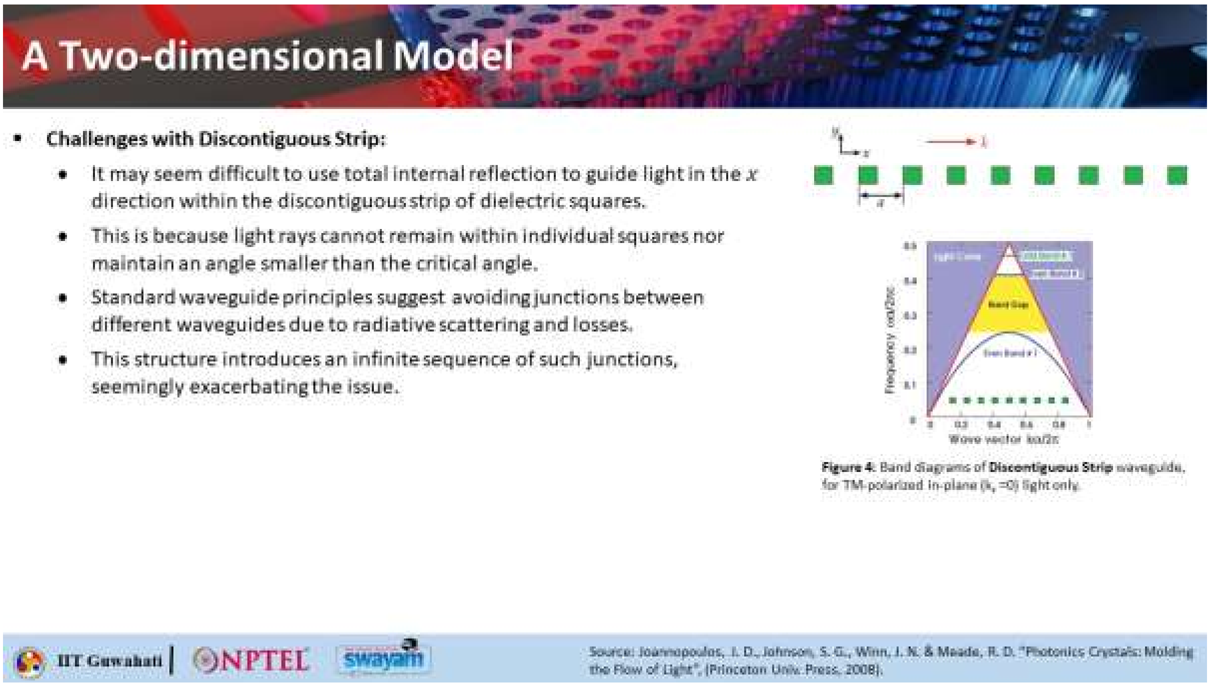
Source: Joannopoulos, J. D., Johnson, S. G., Winn, J. N. & Meade, R. D. "Photonics Crystals: Molding the Flow of Light", (Princeton Univ. Press, 2008).

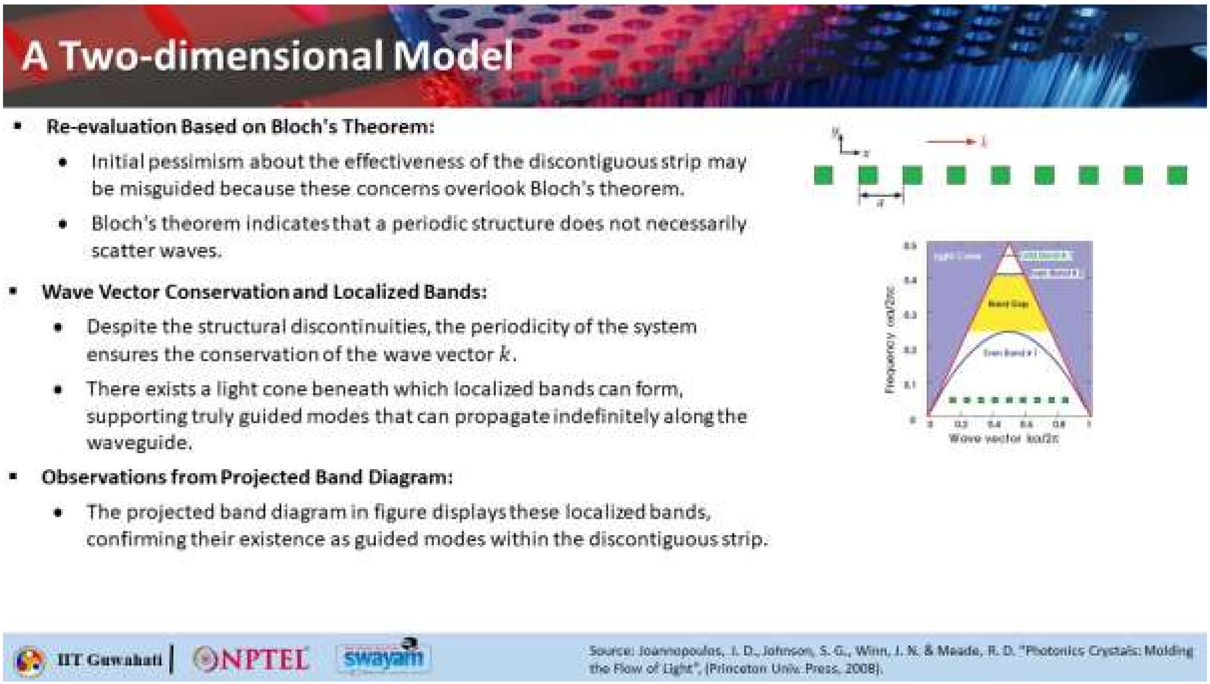
And even and odd are basically decided based on y equals 0 mirror symmetry plane. So if you take a horizontal plane that bisects this particular strip, okay that is basically y equals 0 plane and depending on the field profile whether it is symmetrical across the this mirror symmetry plane the odd and even modes are basically decided. So that is what happens beneath the light cone. So the higher index of the waveguide beneath the light cone pulls down these discrete guided modes. And these modes are localized due to total internal reflection.

So, how does you know the symmetry of the waveguide help? So, if you look into this waveguide, this waveguide has a symmetry plane at y equals 0 that basically bisects the waveguide. You can think of a horizontal plane as I told you. So why you are using that? That can be used for classification of the guided modes. So all guided modes can be classified as either even or odd with respect to the mirror reflections in this y equals 0 plane. And symmetries that might seem present in other planes perpendicular to the waveguide axes are actually broken when k is not equal 0.

So, you are mainly focused about this symmetry across the mirror reflection plane. Now, as I mentioned that this diagram shows one even band and one odd band. So, the even band basically contains the lowest energy. So, that corresponds to the fundamental mode and then odd band will have some higher order thing higher order modes. Now when you move from the continuous strip to this discontinuous strip that is means you know this discretized strip



What are the challenges? First thing is that it may seem difficult to use total internal reflection to guide light along X within this broken piece of strip. So why that happens because you know light rays cannot remain within the individual square or you know nor maintain an angle that is smaller than the critical angle. So this discretization actually poses a great challenge. So, that is why you know when you design standard waveguides the principle suggests that you should avoid junctions. Because whenever there is a junction between two different waveguide that will give rise to radiative losses or radiative scattering losses and here it is like full of you know small small scatterers.



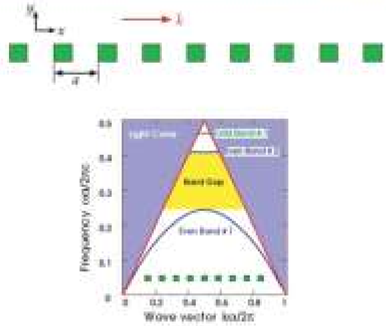
So, that way this make this ideally should make a very very bad waveguide, but is that the case. Is it that, you know, this kind of infinite sequence of these junctions which is repeating periodically, okay, is further worsening your scattering loss or there is some magic involved. So, the thing is you need to analyze this periodic medium using, you know, block theorem. So, the initial pessimism that we had regarding the effectiveness of this particular discontinuous strip. okay may be misguided because these concerns overlook the block theorem okay and the block theorem indicates that you know a periodic structure does not necessarily scatter waves okay.

Despite all these structural discontinuities, the periodicity of the system ensures the conservation of the wave vector k. That means there exists a light cone beneath which you will have you know the localized bands which can form and they can support truly guided modes which will propagate indefinitely along that waveguide okay. So this is the beauty of the block theorem that describes the propagation of guided modes in a discontinuous medium or periodic medium. So here as we mentioned that this is the strip waveguide for which the band structure has been calculated or the band diagram is calculated and it is shown here same. y axis normalized frequency a by lambda naught or you can write it as 2 pi by omega a by 2 pi c and the x axis is nothing but your normalized wave factor that is k a by 2 pi.

What you see here you see that you know light cone is basically this is basically symmetric ok and this part the right part is basically folded ok. So, along this line you can see a symmetry ok. So, what happens here you can see there is an even band 1, you have even band 2 and then you get odd band 1. So, from the now peak of this even band 1 to the bottom of the even band 2 you can actually see a band gap ok.

A Two-dimensional Model

- **Finite Brillouin Zone for Discontiguous Strip:**
  - The discontiguous strip features a finite Brillouin zone, unlike the uniform strip, leading to unique wave behavior.
  - The range $\pi/a < k < 2\pi/a$ is equivalent to the range $-\pi/a < k < 0$, which corresponds to the reverse of the irreducible Brillouin zone $0 < k < \pi/a$.
- **Light Cone Repetition:**
  - A light cone must exist within each of these zones.
  - The tip of the original light cone, located at $k = 0$ in the strip, repeats periodically at $k = 2\pi/a, 4\pi/a$, and so forth.

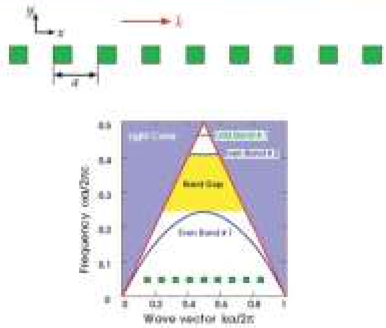


We will come into that details. So, what is important here to notice that the first Brillouin zone for this discontinuous strip. So, first thing is that this strip, this fragmented strip features a finite Brillouin zone, which is not seen in that uniform strip waveguide. So, whenever it has got a brilliant zone it leads to a unique behavior right. So, you can think of a range something like pi by a to 2 pi by a the wave vector anything in between is nothing but equivalent to minus pi by a to 0. okay or you can actually think of this as a reverse of the irreducible Brillouin zone which lies from 0 to pi by a.

So, you can think of a mirror symmetry at the half point which is at pi by a okay this one 0.5. So when k by 2 pi is equal 0.5, that is the midpoint. So that is why you will see that this is repeating or you can say it's a mirror image.

A Two-dimensional Model

- **Behavior of the Lowest Band:**
  - The lowest band starts at zero frequency at $k = 0$, flattens at $k = \pi/a$, and then bends downwards, returning to zero frequency at $k = 2\pi/a$.
  - This bending causes a band gap to open between the first two guided modes, similar to the behavior observed in one-dimensional crystals discussed earlier.
- **Nature of the Band Gap:**
  - The band gap is considered incomplete because it excludes only the guided modes.
  - Radiating modes (those within the light cone) can still exist for any frequency ω, not affected by the gap.



So a light cone exists within each of these zones. So you can see in this zone, a light cone is there. In this zone also, a light cone is there. And the tip of the original light cone, which is located at k equals 0, in the strip will also repeat periodically when k becomes 2 pi by a. So, when k becomes 2 pi by a, k a by 2 pi will become 1.

So, it will repeat here again it will repeat when this will become 2 and so on ok. So, you can only study this part and you can talk about the band features. You can actually only take half of this and then you can understand what is going on in this particular discontinuous periodic waveguide. So you can also observe that the lowest band starts at zero frequency right at k equals zero and it flattens at this point which is basically k equals pi by a okay or you can say k by 2 pi equals 0.5 and then it starts bending downwards and returns to zero frequency again at k equals 2 pi by a or you can say k by 2 pi equals 1.

So, this band actually this banding actually causing a band gap to open between the two first two guided modes which are both even bands 1 and 2. And this is something that you have seen already in one dimensional crystals that you have discussed earlier. So, now let us understand what is the nature of this band gap. The bandgap is considered incomplete here because it excludes only the guided modes. It means whatever is happening here are basically radiating modes because those are within the a light cone, but those can still exist for any frequency omega and they are not affected by this gap.

So you can see that the light cone exist for all the frequencies. And that is why this kind of bandgap is basically an incomplete bandgap. If you try to remember or recollect the complete bandgap we have seen, that was there for all the values of k. So it was going from this left to right boundary.

But here it is not like that.

A Two-dimensional Model

- **Field Profiles of Guided Modes:**
  - The field profiles for the three guided modes at $k = \pi/a$ are depicted in figure 5 .
- **Even-Symmetry Modes:**
  - Two modes exhibit even symmetry, similar to those discussed in lecture 10 .
  - The lower band mode is peaked within the dielectric, while the next-higher band features a node in the dielectric.
- **Odd-Symmetry Mode:**
  - The odd band mode contains a nodal line along the x-axis within the dielectric, which raises its frequency.
  - This mode is less tightly confined to the waveguide compared to the even modes because it is closer to the light cone.

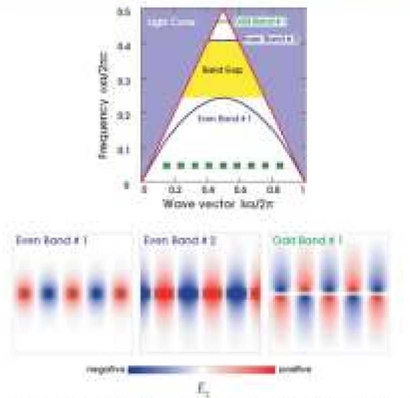
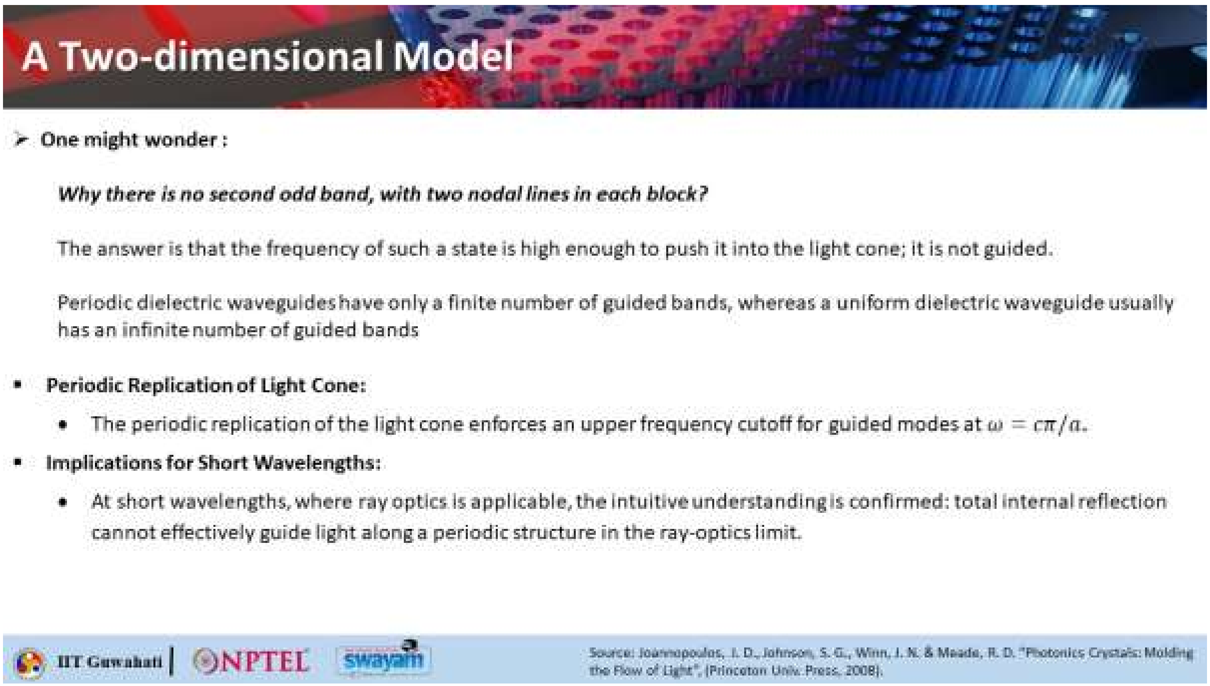


Figure 5: E_z field patterns of the discontiguous periodic waveguide at k=π/a, the Brillouin-zone edge.

Source: Joannopoulos, J. D., Johnson, S. G., Winn, J. N. & Meade, R. D. "Photonics Crystals: Molding the Flow of Light", (Princeton Univ. Press, 2008).

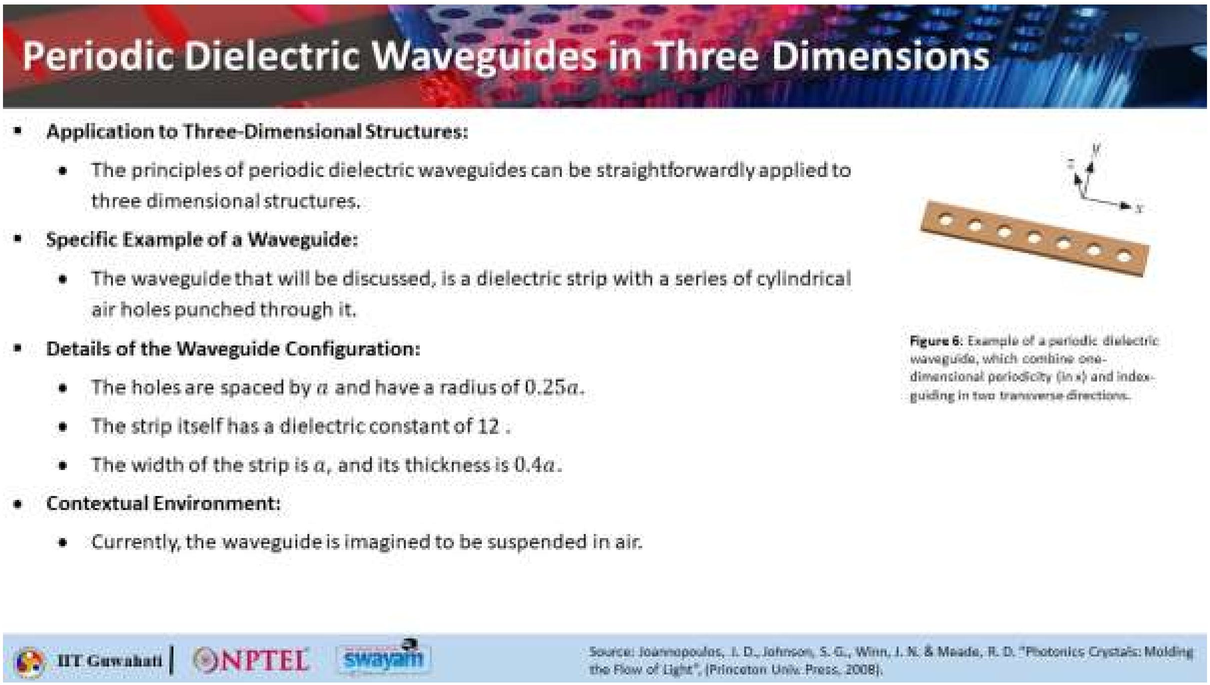
So now let us evaluate the field profiles of the guided modes. and they are being plotted here so this is the same band diagram we have seen in the last slide and this is the field profile which actually plots the magnitude of ez which is the z component of the electric field okay red showing the positive and blue the negative part and you can see the dielectric square is basically marked here So, this is how the even band 1 looks like. So, why they are called even? So, you can actually take y equals 0 plane which is going through the middle of this band. Let me try to draw it. So it is like this and you can see on the top part and the bottom part top part and bottom part it is basically symmetrical.

So it is called a even band. Same this one you can draw the same kind of plane over here okay and you will see it is also the top and bottom has got symmetrical. However in this case it is not symmetrical okay and that is why it is called a odd band. right so the lower band has basically picked within the dielectric as you can see here and if you go for the next higher band that is even band number two it has got a white line in between the dielectric that tells you that there is a node means zero uh electric field point okay in the middle of the dielectric okay And for the odd symmetry, you can also see that the odd band basically contains a nodal line along the x-axis within the dielectric. And because of this, it is having further higher energy as compared to your even band number 2. And this mode is less tightly confined to the waveguide as compared to the even modes the reason being that it is also closer to the light cone okay and you can also see that the mode extends more into air and it is less into the dielectric So, what I mean to emphasis here is that the electric field distribution or the field profiles of the mode actually gives you the understanding of why their energies are low and how the field is localized inside this particular dielectric strip waveguide.



Now, one might wonder why there is no second odd band. Okay, that is something like having two nodal lines in each block. The answer is that the frequency of such a state would be high enough to get pushed into the light cone. So if you see here, this one is already very close to the light cone and when you think of 2 nodal lines that is the odd band number 2 that would be already in the light cone ok.

So, it is not basically a guided mode ok. So, the periodic dielectric waveguides have only a finite number of guided bands Whereas, if you compare this with the uniform dielectric waveguide, they can have infinite number of guided bands. So, this is one important difference between the uniform waveguides and this periodic dielectric waveguides. Now the periodic replication of light cone in the case of periodic dielectric waveguide also enforces an upper frequency cutoff for the guided mode, which is basically omega equals C pi by A. So you can see that no band can actually have higher frequency than this, right? So that way it is also putting a higher cutoff. So does that have any implication at short wavelength? The answer is yes at short wavelength where you know you can apply ray optics the intuitive the intuitive understanding gets confirmed that means you know the total internal reflection cannot effectively guide light along a periodic medium in the ray optics limit and that is what happens here as well.

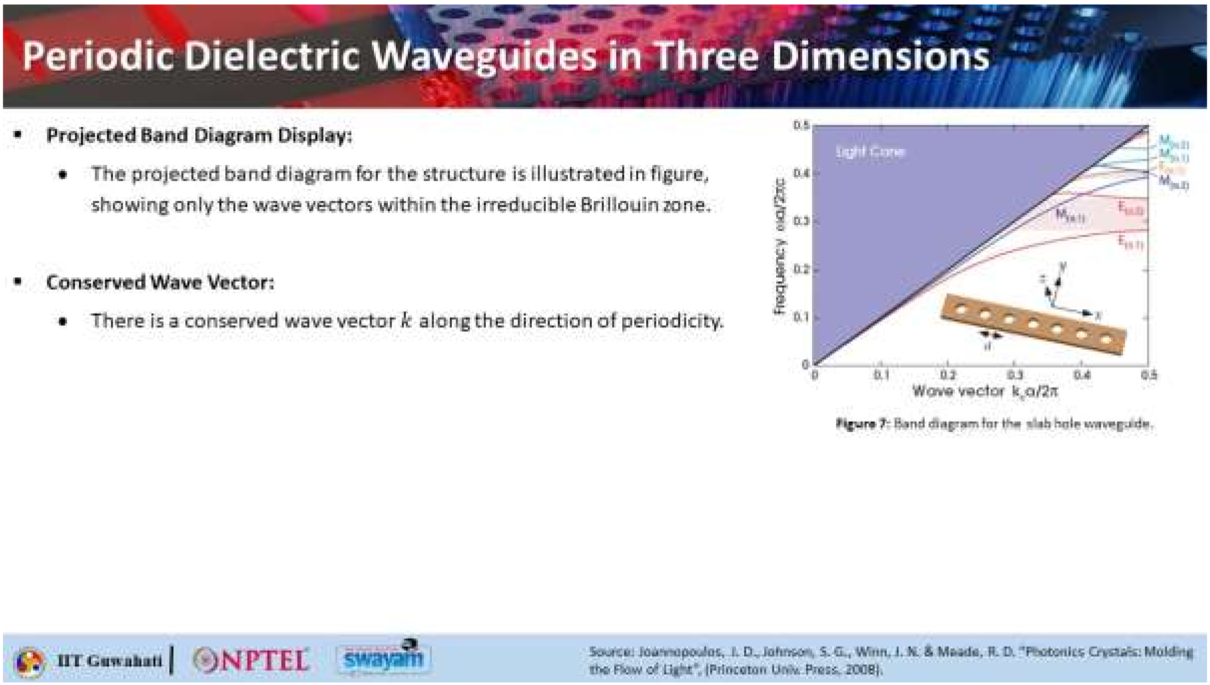


Now let us focus on periodic dielectric waveguides in three dimension. So the previous study was all about dielectric waveguides using a simplified two-dimensional model. Now let us move on to a more realistic one which is a three-dimensional model for the periodic dielectric waveguides. So the principles that we have understood so far from the periodic dielectric waveguides can be straightforwardly applied to this kind of three-dimensional structure. So, here we have taken example of an periodic dielectric waveguide which basically has one-dimensional periodicity of air holes drilled into a dielectric medium ok along the x direction.

That means there is index guiding in the other two transverse direction along y and z. So this one is particularly easy to fabricate and that is why it is a popular choice for examples being discussed because this dielectric strip has a series of cylindrical air holes punched through.

So looks pretty... simple to fabricate this kind of structure. So, the holes are considered to be spaced or separated by A. So, you can consider center to center to be A and they have a radius of 0.25 A. The strip is considered to have a dielectric constant epsilon equals 12.

And you consider the width of the strip to be A and its thickness to be 0.4a. So currently the waveguide is imagined to be suspended in air.

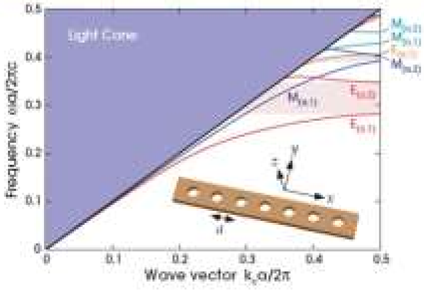


So for the analysis purpose, so when you calculate the band diagram for this particular waveguide, this is what you will obtain. You will see that this particular line which has a slope of 1 marks the light line because there is a normalized frequency and normalized wave factor.

So anything above the light line is having the light cone. So, this is the dispersion relation or band diagram for this slab hole waveguide ok and here as you can see only the irreducible Brillouin zone is shown. So, we are not showing the now extended version of the Brillouin zone where this would have been you know mirror imaged ok. So, we are only showing the irreducible Brillouin zone. And what you can see here that a different type of levelling is used for the bands.

Periodic Dielectric Waveguides in Three Dimensions

- **Light Cone and Guided Bands:**
  - The diagram includes a light cone, where $\omega \geq ck$, representing the extended states in air.
  - Below the light line, there are discrete guided bands.
- **Complexity of Guided Bands:**
  - The guided bands are more numerous and complex compared to previous examples.
  - This increased complexity is due to the inclusion of all modes, whereas previous diagrams considered only the TM polarization.



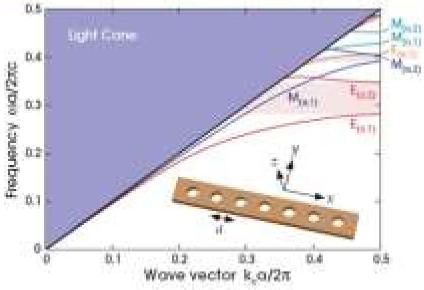
Source: Joannopoulos, J. D., Johnson, S. G., Winn, J. N. & Meade, R. D. "Photonics Crystals: Molding the Flow of Light", (Princeton Univ. Press, 2008).

So we will come into those. And different colors are also used. However, light cone is basically shaded in darker blue. And it is bounded by this black light line. So what is important here? The question is, is there any conserved wave vector? Yes, there is a conserved wave vector k that is along the direction of periodicity. So, you can actually take kx here, okay. How about the light cone? As we discussed that this diagram includes the light cone which marks the area for frequency greater than equals ck, okay.

That represents the extended states in air and below this light line are the discrete guided bands. Now, you can see that there are many guided bands and things are complex in this case. So, the guided bands are more numerous and complex as you compare with the previous example, which was a simple two-dimensional model. Now, this increased complexity comes from the inclusion of all modes, okay, because in the previous case, we only considered for TM polarization.

Periodic Dielectric Waveguides in Three Dimensions

- **Mirror-Symmetry Planes of the Waveguide:**
  - The three-dimensional waveguide features two mirror-symmetry planes: $z = 0$ (perpendicular to the hole axes) and $y = 0$ (parallel to the hole axes).
- **Classification of Modes:**
  - All modes can be classified as either even or odd with respect to reflections in the z and y planes.



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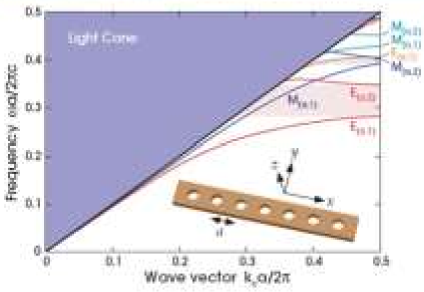
Source: Joannopoulos, J. D., Johnson, S. G., Winn, J. N. & Meade, R. D. "Photonics Crystals: Molding the Flow of Light", (Princeton Univ. Press, 2008).

. Okay, now how about the mirror symmetry plane? Okay, so the three-dimensional waveguide actually features two mirror symmetry plane, isn't it? So you can actually have one plane z equals 0 that will be lying in the xy plane.

Okay, so you can call it a z equals 0 plane which is perpendicular to the whole axis. Okay, and there can be y equals 0 plane, which is basically parallel to the whole axis. So you can think of the cylinder being drilled inside. So the axis of the cylinder is lying along z. Okay, so y equals 0 is basically xz plane and that tells you that you are basically parallel to the whole axis.

Periodic Dielectric Waveguides in Three Dimensions

- **Labeling of Modes:**
  - z-even modes are referred to as "E" modes, which are TE-like.
  - z-odd modes are labeled "M," indicating they are TM-like.
- **Further Specification of Modes:**
  - A subscript "e" (even) or "o" (odd) is added to indicate the mode's symmetry under y reflection.
  - An additional subscript, n, is used to identify the band number.
- **Example of Mode Labeling:**
  - The second band of modes that are z-even and y-odd is labeled $E_{(o,2)}$.



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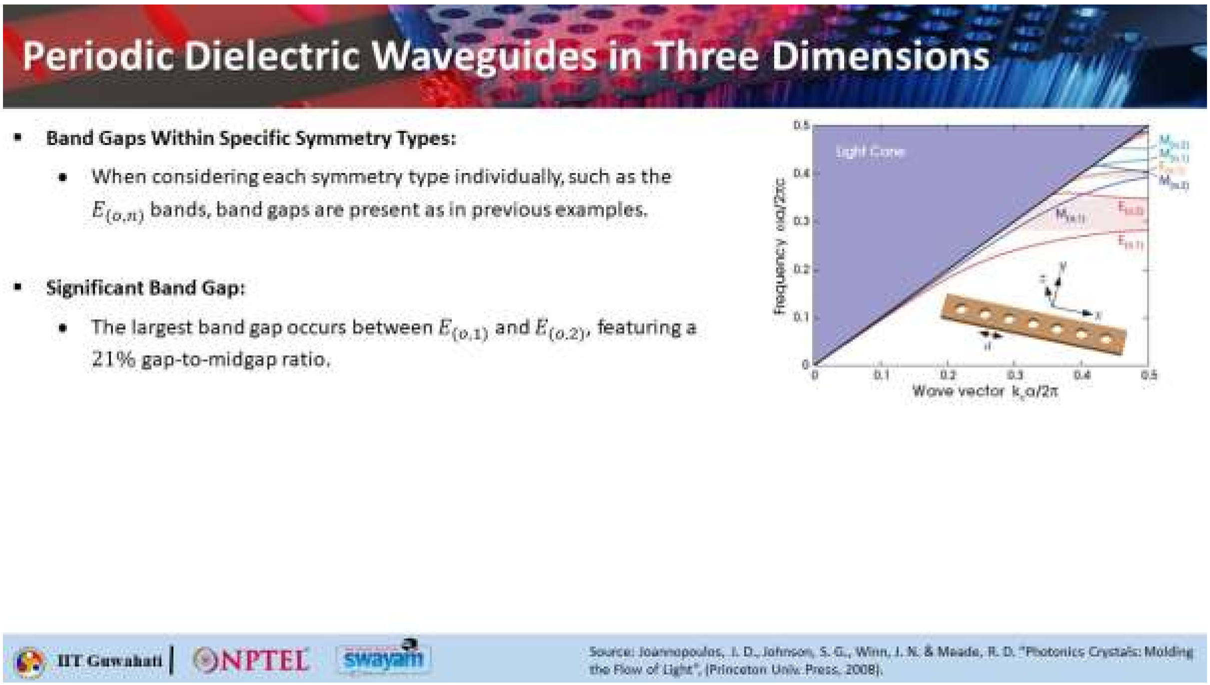
So there are two mirror symmetry plane and based on those you can classify the modes. So all the modes can be classified as either even or odd with respect to the reflections in the z and y planes. So let us look into the example. So, the Z even modes are referred to as E modes because they are mostly like TE modes and Z odd modes are labeled as M because they are more like TM modes.

So, that is why you see E and M being used here. So, this E and M are basically depending on the Z equals 0 plane symmetry. If it is even, you use E. If it is odd, you use M. Now how about the subscript? You see O and E written in different places. So here even and odd indicate the mode symmetry under Y reflection which we have also seen earlier.

So based on that you can identify whether the mode is even or odd okay based on Y reflection. The first one E and M are decided based on Z reflection. and small e and small o that is even and odd are decided based on y reflection. Now, what is this number 1, 2, 3 something like that these are basically additional subscript which identify the band number ok.

So, here now you can see it is e o 1. So, in z reflection it is even in Why reflection? It is odd, but it is the lowest energy band and it is band number one. And then you have this ME1, then you have EO2 and so on. OK, so for the events, you number them as 1, 2 and so on. OK, for the odd, then for even, E even you again start from 1, 2, 3 and so on.

So each type will have its own band numbers 1, 2, 3. So the second band of modes that are Z even So Z even means it will have capital E. It is Y-odd, so it will be O. And it is second band, so it will be numbered as 2. So this is how you can read this particular diagram. Now, our job here will be to identify the band gap in this particular band diagram.



So when considering each symmetry type individually, such as, let us try to only focus on E, capital E, O, and N bands, okay? So you can see that between these two bands, you are actually able to see the band gap being present. And this is something very similar to the previous example, right? And the largest band gap here occurs between these two bands. And if you measure them, you can see that they are almost 21% gap to mid-gap ratio. So that's pretty large band gap. So now, let us further analyze the modes by studying their electric field distribution.

Periodic Dielectric Waveguides in Three Dimensions

- **Localization of Modes:**
  - The modes within these bands are strongly localized, which will be further discussed in the next lecture.
- **Magnetic Field Component:**
  - The dominant component of the magnetic field in these modes is H_z, which is depicted in figure.

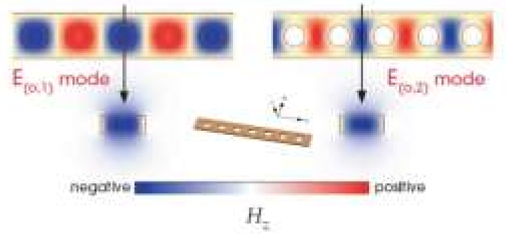


Figure 8: Cross sections of H_z field for lowest-order TE-like modes of the slab hole waveguide at the Brillouin-zone edge, k=π/a.

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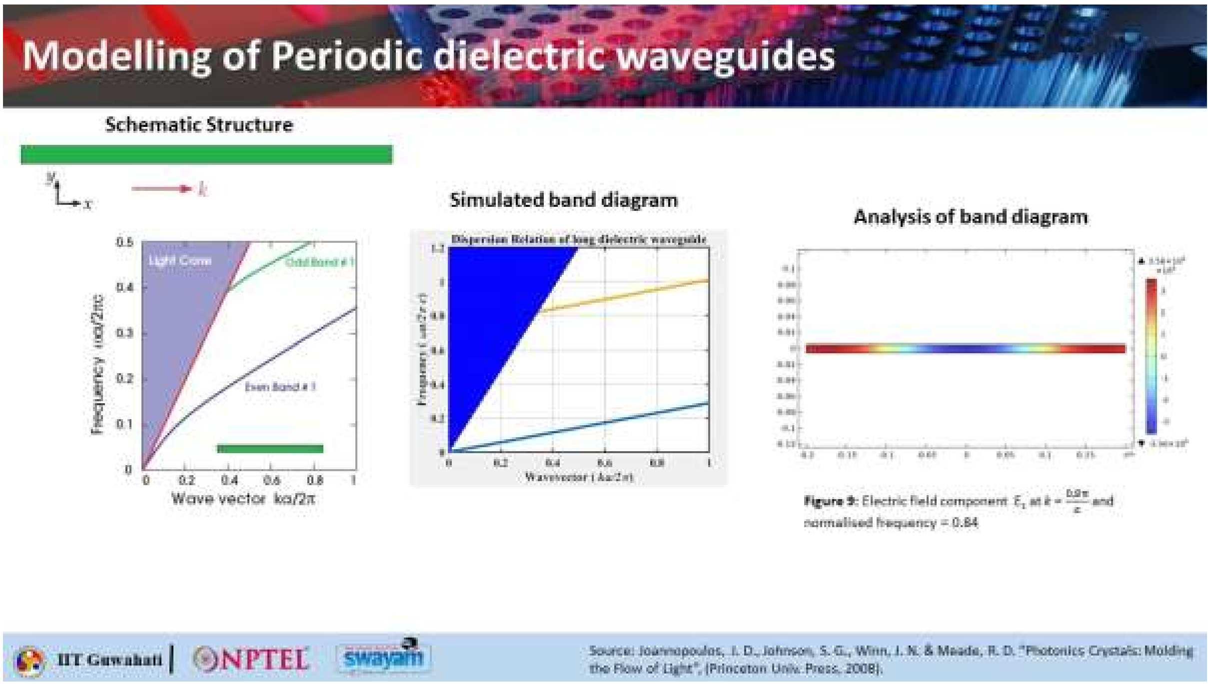
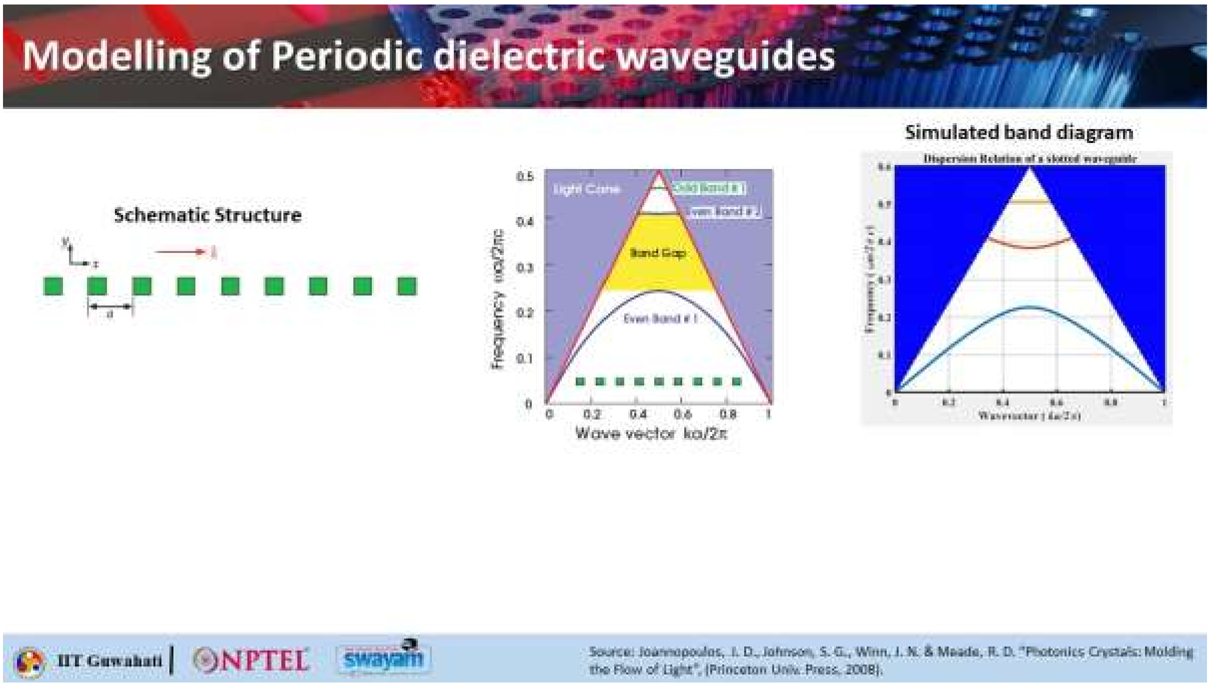
So that will help us. towards the localization of the modes. So, this is again you are plotting the z component of the magnetic field so red denotes positive blue denotes negative so this is what you have okay you have E(o,1) mode which looks like this so what are these circles that you see here these are basically those air holes right so you can see that the first band is mostly you know localized within the dielectric holes. And the second band is mostly within the gap between the two holes. So this is something interesting to see that difference between the two modes. So here you can actually see that strongest field is within the air holes here it is in the dielectric fine. Now this was a schematic structure and this is the one that is reported in the literature and based on the Comsol modeling okay that we already discussed in few lectures back.

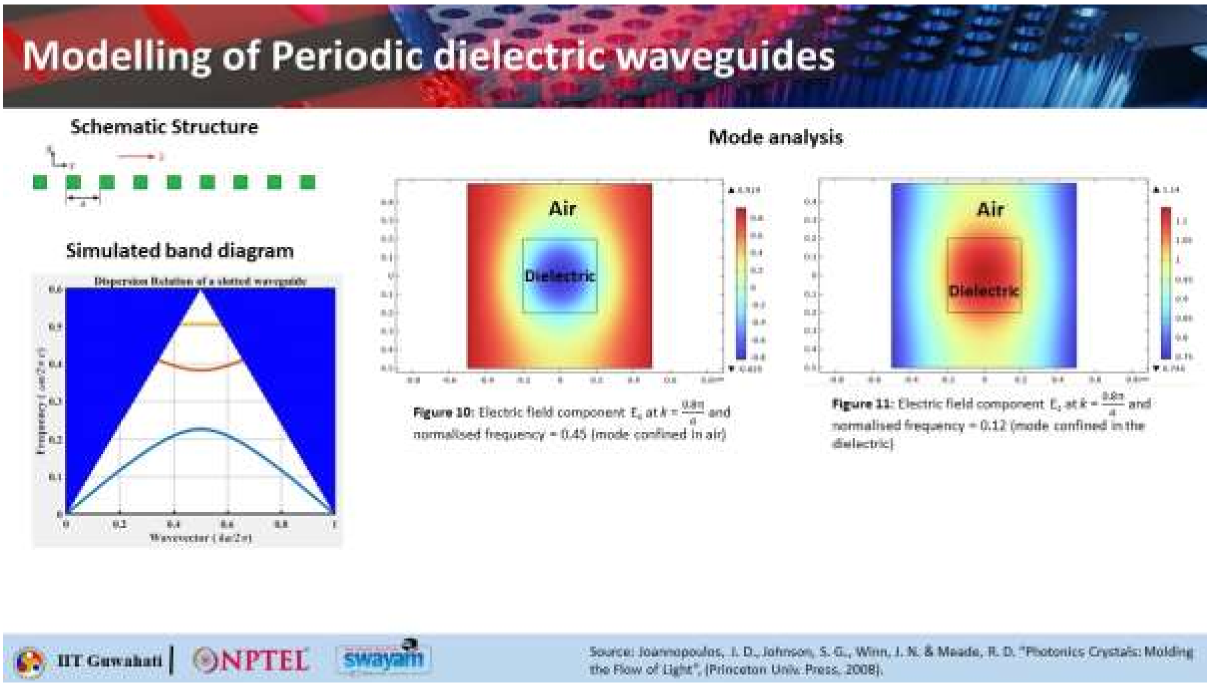


The TA in this course he has reproduced this particular band diagram and you can see that you are actually able to reproduce the band features more or less similar to the one reported in the literature. Right. So the accuracy of these two depends on a couple of factors like the amount of meshing and all those things.

But more or less you can see that the information is kind of reproducible. Right. So, this is done using this commercially available software COMSOL you can use other softwares also ok to solve for this one. And what we have simulated we basically simulated this structure ok where we took a rough guess on A, but then you know this is basically normalized to A. So, it should not matter. So, here the blue zone actually is the one that is showing the light cone okay and that looks that that shows us the radiating or the extended modes in air and these are those discrete guided bands okay.

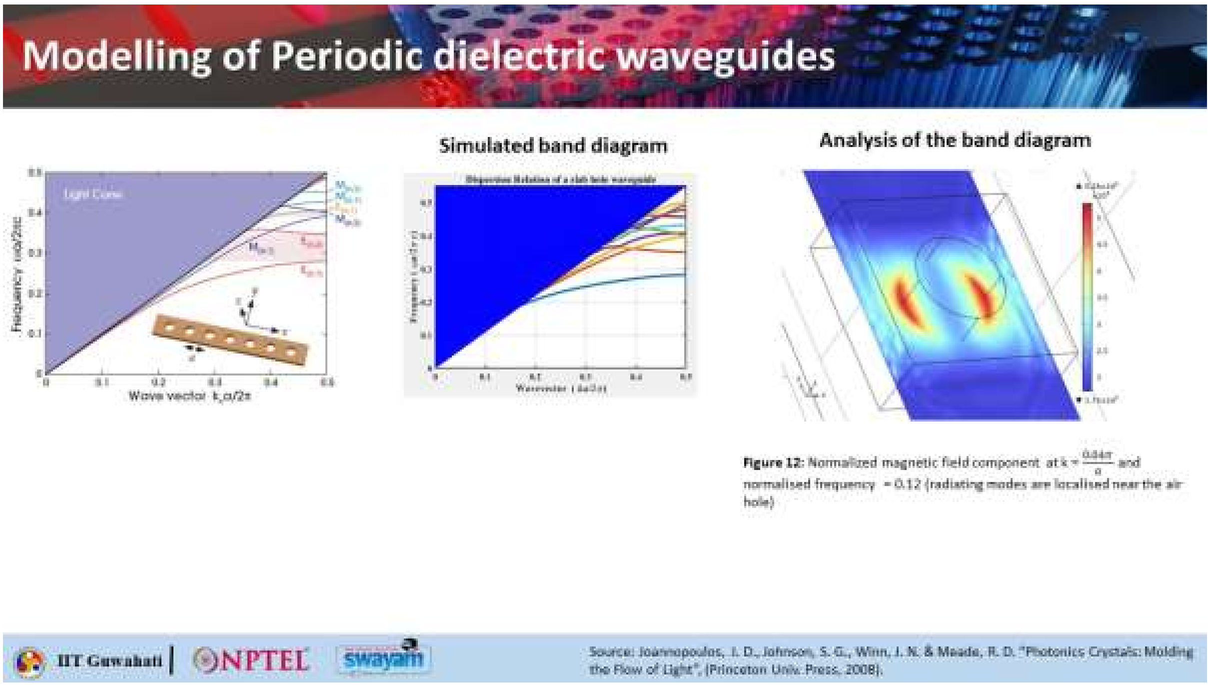
 So, this is for this particular structure. You can also do the mode analysis where you can see that the You know, the first band is mostly concentrated in, you know, in the dielectric, okay, whereas you will see that the second band. So, what is the difference here? As you can see that this corresponds to the same wave factor that is K equals 0.8 pi by A. So, you can find out what is what will be your k by 2 pi so k by 2 pi will be 0.4 somewhere here okay and you are talking about normalized frequency of 0.45 so you are somewhere here so that is basically a air mode okay so this mode is basically in air So, this is how the mode will look like because the strong fields are mostly in the air. But if you consider the same wave factor, so you are here 0. 4 and the normalized frequency is 0.12. So, you are somewhere on this. So, this is basically a guided mode and this is how the electric field distribution works. will look like okay. So what we understand from here the model analysis that the modes correspond to the region which lies above the light cone are basically radiating in nature and that is why the fields are mostly concentrated outside the dielectric structure. Whereas for the modes which are basically guided modes in this dielectric structure they fall below the light cone you will see that the fields are mostly concentrated you know within the structure itself. So, this is also another modelling of the dielectric slab waveguide and you can see that simulation can also reproduce.





So, I am showing this that to give you the confidence that you can achieve exactly same results from your own simulation model also okay. So, that is why I am showing this that here you can also find out what is the light cone which are the guided mode what will be the band gap and all this information. So, you can simulate and try and reproduce this results which are mentioned in the literature right. So, here again you can do the same kind of you know analysis model analysis of the band diagram you can look for a particular k value or the wave vector value and the normalized frequency value ok.

So, if you consider again this kind of k equals 0.04 pi by a ok. So, it is very close to this one ok and normalized frequency of 0.12. So, you are basically looking for radiating modes ok. So, you can actually see that from the diagram itself. the field diagram itself and if you want to locate something in the guided modes you have to choose that value k and the normalised frequency and plot this diagram again you will be able to see the field localization in the case of guided modes.



So, with that we come to an end to this lecture. And, if you have any queries regarding this, you can always drop an email to me mentioning MOOC and photonic crystal on the subject line.