Lec 20: Point Defects in Periodic Dielectric Waveguides & Q-factors of Lossy Cavities

Hello students, welcome to lecture 20 of the online courses on Photonic Crystals Fundamentals and Applications. Today's lecture will be on point defects in periodic dielectric waveguides and the quality factors of the lossy cavities.

Lecture Outline

- Symmetry and Polarisation
- Point Defects in Periodic Dielectric Waveguides
- Quality Factors of Lossy Cavities

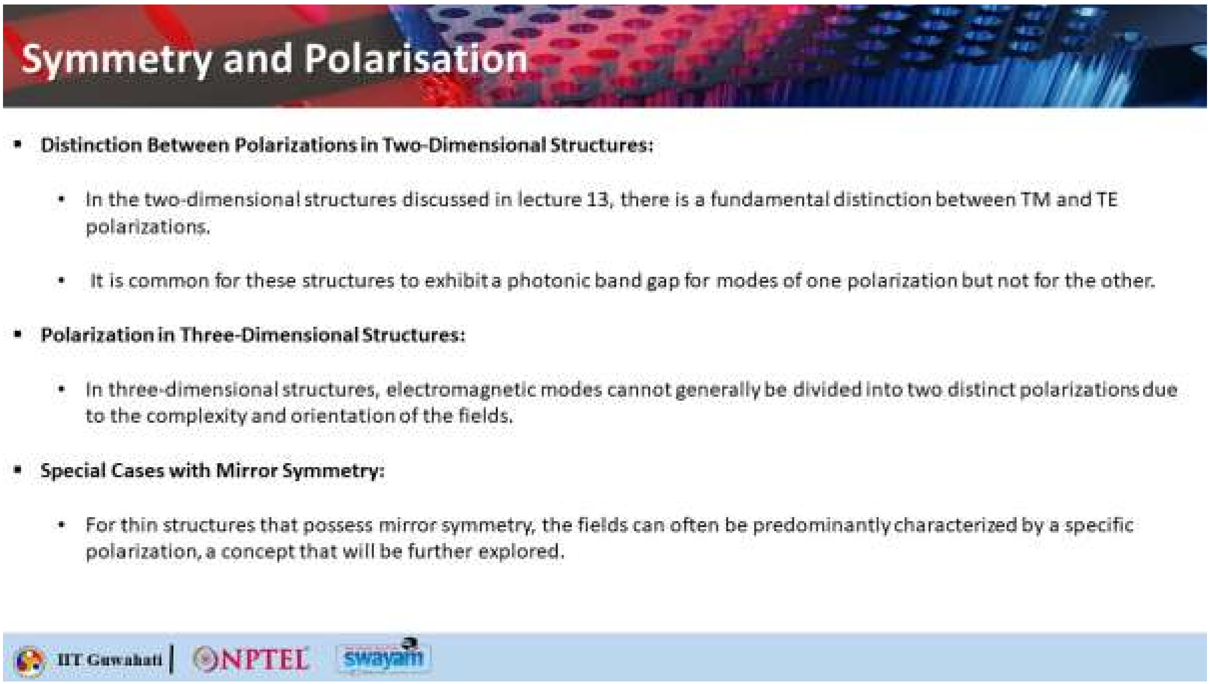
So, here is the lecture outline. We will have a discussion on the symmetry and polarization. Then we will discuss about the point defects in periodic dielectric waveguides. and how to calculate the quality factors of lossy cavities.

Symmetry and Polarization

So, let us focus on the first topic of the day that symmetry and polarization. So, if you think about the distinction between polarization in two-dimensional structures okay that we discussed in lecture 13 okay. We could see that the band structure looks different when you choose TM or TE polarization, isn't it? So, it is common for these two-dimensional structures to exhibit a photonic band gap for modes of one polarization and not for the other. Right.

So usually they may have a different band gap and you basically try to overlap those band gaps to maximize your complete band gap. So when you talk about complete band gap, we want the band gap to be for all the directions that all that means all the values of k. Okay. In the irreducible brilliance zone and also for both the polarizations. right and when you talk about polarization in three-dimensional structures so you can see that in three-dimensional structures the electromagnetic modes cannot generally be divided into two distinct polarizations because of their complexity and the orientation of the field.

So, you will typically call them like TE like or TM like modes. So, you can actually think of the special cases where mirror symmetry can be applied. So, these are applicable for thin structures that possess mirror symmetry and in that case the fields are often characterized by specific polarization and this concept will be explored further in this lecture.



. So, here you can see the electric fields profile of a mode. So, the figure on the right it shows the schematic depiction of electric field lines which are the E field okay for a thin dielectric structure that is shown in this grey shading okay and it is considering z equal to 0 as a mirror symmetry plane.

So this is your z axis this is your y axis so this is z equals 0 axis okay or z equals 0 plane. So the modes that are even with respect to this mirror plane okay So, they are considered to be TE like okay. So, as you can see that the electric field is mostly parallel to the mirror plane okay and they are exactly parallel in the case when it is Z equals 0 okay. And for the odd modes which are shown here, so these are like even modes or like TE like modes okay. You can also think of the odd modes which are more like TM.

So, there the electric field is almost perpendicular to this mirror symmetry plane that is z equals 0, okay. But then it is exactly perpendicular at z equals 0, isn't it? So that is how you can see that you can you know differentiate the mode profiles into even which is TE like or odd modes which is TM like okay. So what we observed here that away from the mirror plane the fields tend to maintain their you know TM and TE like characteristics and this tendency holds as long as you know the waveguide thickness remains smaller than the wavelength of the light

Symmetry and Polarisation

- **Electric Field Profiles of Modes:**
  - The electric field profiles of modes that are either even or odd with respect to z reflections are schematically shown in figure 1.
- **Behavior in the Symmetry Plane:**
  - Within the symmetry plane ($z = 0$), the even modes are purely TE polarized, and the odd modes are purely TM polarized.
- **Field Characteristics Away from the Mirror Plane:**
  - Away from the mirror plane, the fields tend to maintain their respective TE-like and TM-like characteristics.
  - This tendency holds as long as the waveguide thickness remains smaller than the wavelength.

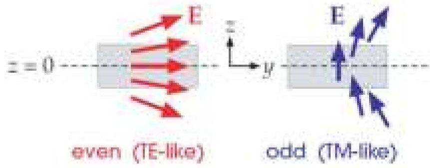


Figure 1: Schematic depiction of electric field lines (**E**) for a thin dielectric structure (gray shading) with a z=0 mirror symmetry plane.

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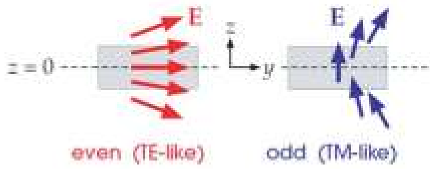
Source: Joannopoulos, J. D., Johnson, S. G., Winn, J. N. & Meade, R. D. "Photonics Crystals: Molding the Flow of Light", (Princeton Univ. Press, 2008).

Now we can further classify you know the most polarized states. So we have seen that you know most TE like states are those which are even with respect to Z symmetry or Z equals 0 plane and odd with respect to Y that is like you know Y equals 0.

So those can be called as E. odd n kind of states. Conversely, you can think of the other other possibility where you have you know TM like states. So, these are basically those which are even with respect to both z and y. So, you can actually call them as M, E, N.

Symmetry and Polarisation

- **Additional Mirror-Symmetry Plane ($y = 0$) :**
  - The presence of another mirror-symmetry plane at $y = 0$ affects the polarization characteristics of the modes, with symmetries being reversed compared to those along $z = 0$.
- **Polarization Characteristics Relative to $y = 0$:**
  - A TE-like mode is odd with respect to y reflections, while a TM-like mode is even with respect to y.



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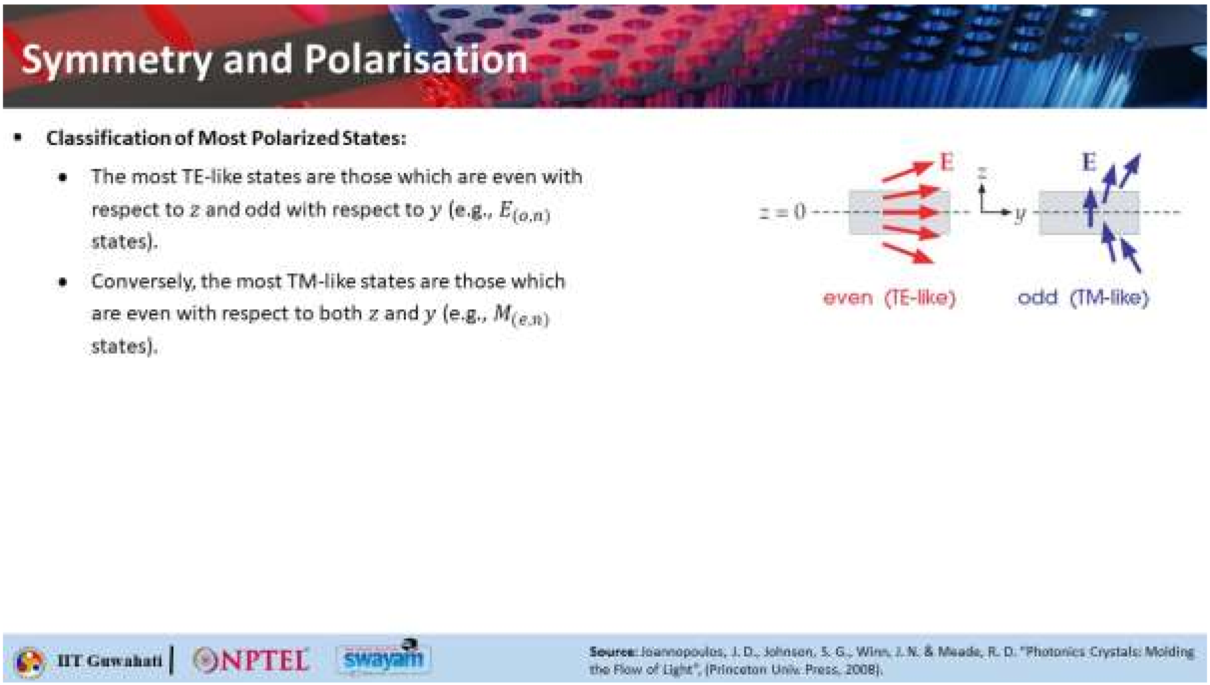
Source: Joannopoulos, J. D., Johnson, S. G., Winn, J. N. & Meade, R. D. "Photonics Crystals: Molding the Flow of Light", (Princeton Univ. Press, 2008).

So, what is N? N is basically telling you the band number. So, let us consider two TE like modes. So, when I say TE like modes, we will be talking about capital E and then they are odd with respect to y equals 0. That is why you are having this odd representation O ok and then 1 and 2 are basically your numbers ok. So, what we see here given their TE like nature it is basically appropriate to focus only on the H z component that is basically the z component of the magnetic field ok as there is only H field component that will be present on z equals 0 plane ok.



So z equals 0 plane you can see from here you are basically considering the xy plane right. So in this particular figure you can consider EO1 and EO2 which are basically these two states right and they both are TE like and that is why we choose to plot now HZ as you can see here, okay. And if you look into this band diagram which you have seen in the previous lecture, we can also see that the first band which is the lower edge of the gap, okay. And the second one is basically the upper edge of the band gap, right. So, that marks your 1 and 2, okay.

when you see the fill plots you can actually see that you know the lower energy one or the lower frequency one is basically having the fills concentrated on the air holes okay and for the higher energy the field is mostly concentrated in the dielectric between the air holes right. And for easy understanding you can see that the dielectric material is basically marked with this translucent yellow shading. So, this is something very similar to what we have seen in the previous lecture okay. So, what is important here is to understand the relationship between the symmetry and the polarization okay. Focus on the symmetry of the HZ plane so the HZ plot that you see here so red tells you about positive magnetic field okay and blue is the negative one okay.

Symmetry and Polarisation

- **TE-like States $E_{(o,1)}$ and $E_{(o,2)}$:**
  - Given their TE-like nature, it is appropriate to focus on the H_z field component for these states, as this is the only H field component present in the $z = 0$ plane.
- **Field Component Plotting:**
  - The H_z field component is plotted as demonstrated in figure 4, providing insights into the relationship between symmetry and polarization.

Figure 2 : Example of a periodic dielectric waveguide, which combine one-dimensional periodicity (in x) and index-guiding in two transverse directions.

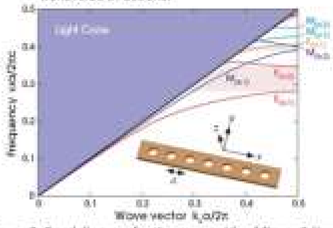


Figure 3: Band diagram for the waveguide of figure 2 (inset): a three-dimensional dielectric strip, suspended in air, with a period-a sequence of cylindrical air holes.

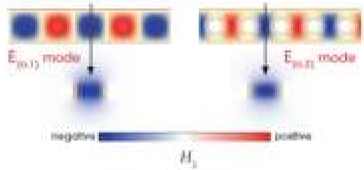


Figure 4: Cross sections of H_z field for lowest-order TE-like modes from figure 3 at the Brillouin-zone edge, k=π/a.

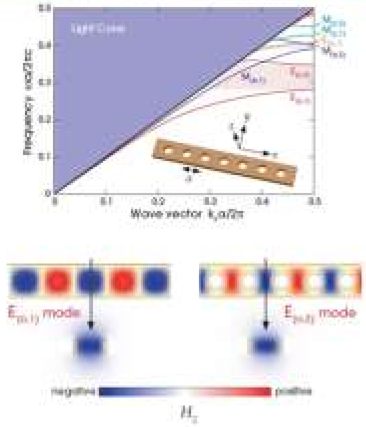
Source: Joannopoulos, J. D., Johnson, S. G., Winn, J. N. & Meade, R. D. "Photonics Crystals: Molding the Flow of Light", (Princeton Univ. Press, 2008).

So HZ plot appears to exhibit even symmetry with respect to y equals 0 okay and that is like you know with respect to this and odd symmetry with respect to z equals 0, fine. So, if you consider the subtlety of this symmetry and the field nature, you will see that despite appearances there is no actual difference in symmetry. The subtlety lies in the nature of H as a pseudo vector which incurs an additional factor of minus 1 under mirror reflection. So, sometimes you know in reflection you will see that you know there is an additional factor of minus 1. So, your field lines actually get you know reversed.

So, positive becomes negative, negative becomes positive and so on. So, typically when discussions simplify this kind of simplicity by focusing solely on the symmetry of the E field avoiding the subtlety that is associated with the H field ok. So, that is why sometimes you need to you know you should try to focus on electric field and magnetic field distribution both sometimes one helps in understanding or explaining a particular feature more than the other right. And we have understood that if you are able to compute one field that is E field H field is also related. So, it is also possible to get the magnetic field information as well.

Symmetry and Polarisation

- **Symmetry of H_z Field:**
  - The H_z plot appears to exhibit even symmetry with respect to $y = 0$ and odd symmetry with respect to $z = 0$.
- **Subtlety in Symmetry and Field Nature:**
  - Despite appearances, there is no actual difference in symmetry. The subtlety lies in the nature of H as a pseudovector, which incurs an additional factor of -1 under mirror reflections.
  - Typically, discussions simplify this complexity by focusing solely on the symmetry of the E field, avoiding the subtlety associated with H.



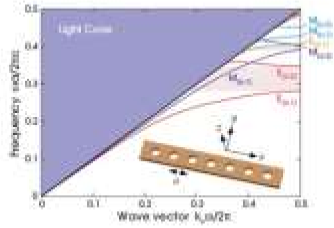
Source: Joannopoulos, J. D., Johnson, S. G., Winn, J. N. & Meade, R. D. "Photonics Crystals: Molding the Flow of Light", (Princeton Univ. Press, 2008).

Now, what are the advantages of dual mirror symmetries in waveguide? So, having two different mirror symmetries in a waveguide is beneficial because it allows a TETM like band gap to persist even if one symmetry is disrupted. Okay, so we have seen this particular thing and if you remember that this was considered for an array of cylindrical holes in a slab of finite thickness, but this was considered to be in air. Okay, now as soon as you cannot make a waveguide that is in air right in practice. So, when you try to make this waveguide on a low index substrate, okay. So, you do not want to keep it in you know floating in air because that is not feasible.

So, when you put it on top of a substrate what will happen you know it breaks the z equals 0 mirror symmetry. So, one symmetry is kind of lost. So, only you have the y symmetry. So, despite the loss of Z symmetry, you will see that the Y symmetry remains intact. And what would be the effect of that on the bandgaps? Because of the preservation of the Y symmetry, you can ensure that the large gap between the Y odd modes, which is depicted in the band diagram, they still continue to exist.

Symmetry and Polarisation

- **Advantages of Dual Mirror Symmetries in Waveguides:**
  - Having two different mirror symmetries in a waveguide is beneficial because it allows a TE/TM-like band gap to persist even if one symmetry is disrupted.
- **Impact of Substrate on Symmetry:**
  - Placing the waveguide on a low-index substrate instead of suspending it in air breaks the z = 0 mirror symmetry.
- **Preservation of y Symmetry:**
  - Despite the loss of z symmetry, the y symmetry remains intact.
- **Effect on Band Gaps:**
  - The preservation of y symmetry ensures that the large gap between the y-odd modes (depicted in band diagram figure) continues to exist.



Source: Joannopoulos, J. D., Johnson, S. G., Winn, J. N. & Meade, R. D. "Photonics Crystals: Molding the Flow of Light", (Princeton Univ. Press, 2008).

So this is what will happen when you do the simulation by placing this particular waveguide on a substrate. You will see that although you have compromised on the Z symmetries or Z mirror symmetry, you can still get this bandgap with, you know, Y symmetry being preserved. Make sense? So let us further look into uh like if you take some specific examples something like you know what is uh We will come to that later. Now, first we need to also analyze that what is the importance of the dielectric constant difference. So, remember that it is crucial for the substrate to have much smaller dielectric constant than the waveguide itself.

Symmetry and Polarisation

- **Importance of Dielectric Constant Difference:**
  - It's crucial for the substrate to have a much smaller dielectric constant than the waveguide to ensure that the guided modes and the band gap remain well below the light cone.
- **Common Material Pairing:**
  - A typical material pairing at infrared wavelengths is silicon (Si) on silica (SiO_2), with a dielectric contrast of approximately 12:2.
- **Role of the Substrate:**
  - The substrate lowers the frequencies of both the light cone and the guided modes, as the modes slightly penetrate into the substrate.
- **Additional Effects of the Substrate:**
  - The substrate also influences point defects, which will be discussed in more detail in the next section.

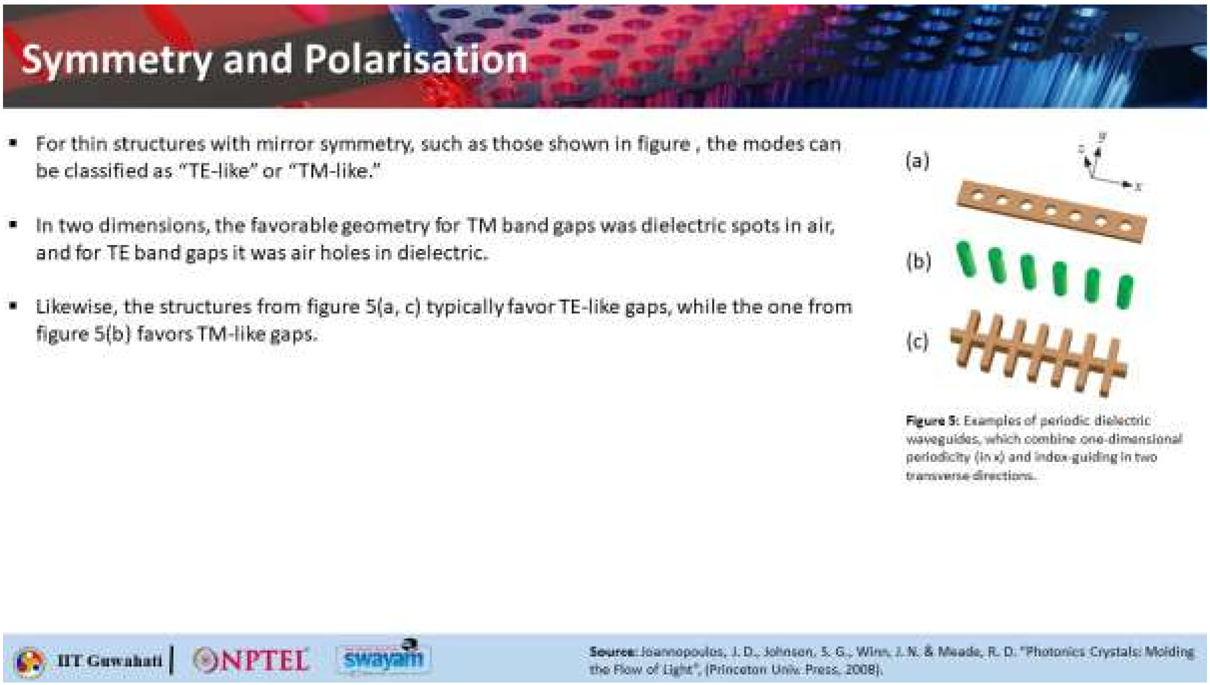
Source: Palik, Edward D., ed. Handbook of optical constants of solids. Vol. 3. Academic press, 1998.

So, that will ensure that the waveguide modes and the band gap remain well below the light cone. So what is the problem if it goes or close to the light cone, the guided modes will start leaking out.

So you don't want that. So you want your substrate to have a much lower or smaller dielectric constant as compared to the material that you have used to make your waveguide. So, what are the common material pairing? So, if you think of common material pairing that is done at infrared wavelengths that also cater to your telecom wavelength of 1550 nanometer right.

So, typically it is silicon on insulator or silica. So, it is silicon on silica platform. So, silica, what is the dielectric constant? It is typically 2. Silicon, it is typically 12. So, you can see the amount of contrast present.

So, what is the role of the substrate here? The substrate basically lowers the frequency of both light cone and the guided modes. And that happens because the modes slightly penetrate into the substrate. So, what are the additional effects of the substrate? The substrate also influences point defects and we will take this discussion in more details in the next section.



So, what do you understood from this discussion on symmetry and polarization? So, we understood that for thin structures with mirror symmetry such as those shown in this figure which are the examples of periodic dielectric waveguides which combine one dimensional periodicity in x and typically they have index guiding helping them in the two other orthogonal directions. right so the modes in this kind of waveguides can be classified as te like or tm like okay then in two dimension the favorable geometry for tm band gaps as we have seen are basically the dielectric spots in air and sorry dielectric spots like this, okay.

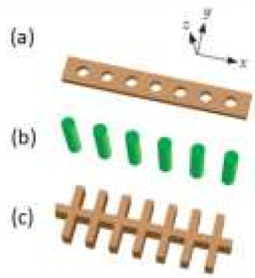
And for T, it was basically air holes in dielectric. So, this kind of structure basically gives you T bandgap and this kind of structure that is dielectric spots in air typically give you TM bandgap. right. So, you can actually try to combine this structure as we have seen earlier to get both TE and TM band gap overlapped and you can get complete band gap for the structure that you want to make.

right. So, here you can see that structure A and C okay they basically favor TE like gaps and the one in the middle this one it favors Tm like gaps. Let us now move on to the next topic which is point defects in periodic dielectric waveguides. We will analyze this in more details. So, one respect in which periodic dielectric waveguides differ from the systems considered in the previous lectures is that the defect states are not perfectly localized. So, periodic dielectric waveguides in those if you want to create point defects.

Point Defects in Periodic Dielectric Waveguides

Point Defects in Periodic Dielectric Waveguides

- **Difference in Defect States:**
  - Periodic dielectric waveguides differ from systems in previous lectures because defect states are not perfectly localized.
- **Creating Point Defects:**
  - Various methods exist to create point defects, such as altering the radius of a hole within the waveguide structure.
- **Specific Method for Creating a Defect:**
  - In this instance, the spacing between one pair of holes is changed from a to $1.4a$, adding extra dielectric material.



Source: Joannopoulos, J. D., Johnson, S. G., Winn, J. N. & Meade, R. D. "Photonics Crystals: Molding the Flow of Light", (Princeton Univ. Press, 2008).

So, what are the methods? So, the various methods exist and one easy way if you consider the structure in this case A would be to alter the radius of the hole within the waveguide structure. or another way would be to you know alter the spacing. So, here the holes are spaced at a distance A that is the light spacing from each other. So, if you change one pair of holes ok not following that. So, instead of A if you change it to 1.4 A ok by adding some extra dielectric material that also gives rise to defect ok. So, this you can do. So, you can actually increase the whole spacing and because of that what will happen you know it pulls down a mode from the upper EO2 band into the band gap okay and that is how it creates a defect state. So, you can actually look that in simulation okay and here this shows Z equals 0. plane this is y equals 0 plane okay or this actually tells you that this particular one is the one where you know the defect is created.

Point Defects in Periodic Dielectric Waveguides

- **Effect of Increased Spacing:**
  - Increasing the hole spacing pulls down a mode from the upper $[E_{(0,2)}]$ band into the gap, creating a defect state.
- **Characteristics of the Defect State:**
  - The resulting defect state is TE-like, with its frequency shown in the left panel of figure 6.
  - The frequency of this defect state is approximately $\omega_0 \approx 0.308(2\pi c/a)$.

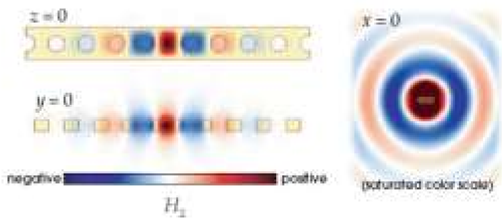


Figure 6: H_z field patterns of a localized resonant mode in a cavity formed by a defect in the periodic waveguide (suspended in air) of figure 5(a)

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Source: Joannopoulos, J. D., Johnson, S. G., Winn, J. N. & Meade, R. D. "Photonics Crystals: Molding the Flow of Light", (Princeton Univ. Press, 2008).

Okay, so what is the state of this web guide? It is basically suspended in air. So we take the simplified structure for simulation purpose. So only difference here is that the spacing between one pair of holes have been increased from A to 1.4 A. So because of that strong localization, okay, and exponential decay of field can be seen inside the waveguide and you can also see here that the field decays only inversely with distance in the lateral dimensions.

So, they quickly you know diminishes And if you look from x equals 0 plane you will be able to see that you know exactly at that particular point defect the field is trying to leak out through radiative leakage and this saturated color scheme has been used here to exaggerate the small field values for better visualization. So the resulting defect state is basically a TE type and its frequency is basically shown in the left panel of figure 6 that you can see here. And the frequency of this defect state is basically omega naught which is 0.308 times 2 pi C by A. So, you can find out the exact frequency if you know what is the LED is facing.

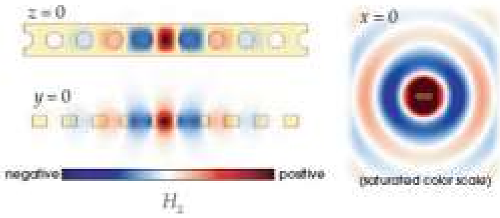
Point Defects in Periodic Dielectric Waveguides

- **Incompleteness of the Band Gap:**
  - Unlike defect states in previous chapters, the band gap here is incomplete, meaning light-cone modes exist for any frequency.
- **Nature of the Defect State:**
  - The defect state is a leaky mode or resonance due to the incomplete band gap, forming a resonant cavity at the point defect.
- **Breaking of Translational Symmetry:**
  - The introduction of the defect breaks translational symmetry, leading to non-conservation of **k**, the wave vector.

Now, unlike the defect states in the previous chapter, here you can see that the band gap is incomplete. That means you know light cone modes exist for any frequency. So, the band gap is only for a specific range, is not it? you can go back and quickly have a look at the bank diagram so that we are referring to here yeah so you can see the light cone modes basically exist for all frequencies and the this is only for some certain you know frequency and also for a specific direction this part works right and you basically have pulled down this mode So, this is the mode which is above the band gap, but something has been pulled down and within the gap you have localized that particular mode into the defect state. So, what is the nature of the defect state? The defect state is basically a leaky mode or resonance due to incomplete band gap and it forms a resonant cavity at the point of defect. So introduction of the defect basically breaks the translational symmetry and that would lead to non-conservation of the wave vector K.

Point Defects in Periodic Dielectric Waveguides

- **Coupling of Defect Mode to Light-Cone Modes:**
  - The defect mode can couple with light-cone modes that share the same frequency (ω), resulting in intrinsic radiative loss.
- **Visualization of the Defect Mode:**
  - The right panel of figure displays the defect mode profile, using a color scheme designed to emphasize the small-amplitude, outward-radiating wave.



Source: Joannopoulos, J. D., Johnson, S. G., Winn, J. N. & Meade, R. D. "Photonics Crystals: Molding the Flow of Light", (Princeton Univ. Press, 2008).

So if you have a closer look at the defect, you can understand that the main loss here is the radiative loss. So the defect mode can couple with the light mode. that share the same frequency omega because there are light modes existing at those frequencies. So, that way they can leak out easily through radiation.

So, that is the intrinsic radiative loss. Now, you can visualize this you know outward radiating wave by using this kind of color scheme which is a saturated color scheme which are used to you know emphasize small amplitude waves which are leaking out from this defect.

Point Defects in Periodic Dielectric Waveguides

- **Main Disadvantage of Incomplete-Gap Systems:**
  - The intrinsic radiative loss of point defects is a significant drawback compared to the complete photonic band gaps
- **Challenges Presented by Radiative Loss:**
  - Quantification of Loss: It is crucial to accurately measure the extent of the loss, which will be elaborated on in the following section.
  - Tolerance Assessment for Applications: Determining how much loss can be tolerated for specific applications is essential and will be explored in more detail during lectures 31-36.

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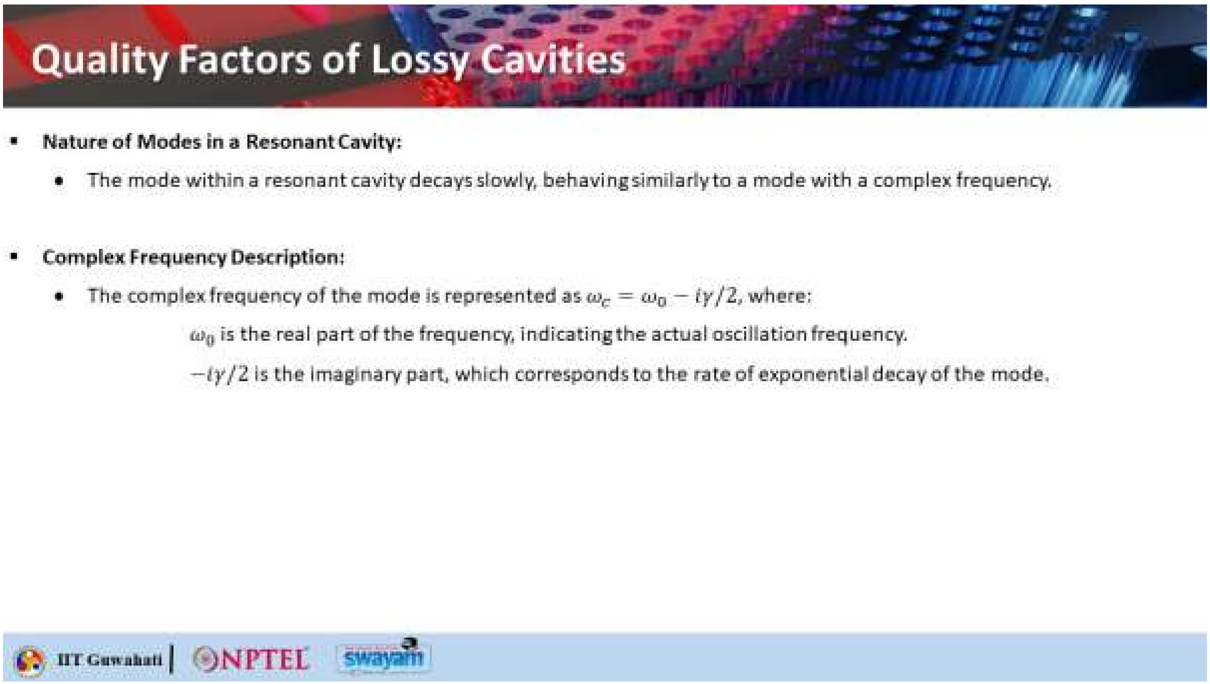
So, the main disadvantage of the incomplete bandgap system is that you know the intrinsic radiative loss of point defect remains as a significant drawback here as compared to complete photonic band gap. In complete photonic band gap what happens for those bands no you know light modes exist that means you know there would not be any radiative loss, but here there are radiative losses ok. so there are challenges presented by this radiative loss something like you know the first thing is the quantification of the loss so it is very important to accurately measure the extent of loss and that needs to be elaborated and also the tolerance assessment for application so That means you know how do you determine that how much loss can be tolerated for some specific application ok and that is very important and we will go into the details of that towards the end of this course ok. So right now we just keep this in mind that you know tolerance assessment for point defects ok in this kind of periodic dielectric waveguides which are popularly used is very important ok.



So now let us focus on the next subtopic which is quality factors of lossy cavities. So the nature of mode in a resonant cavity okay is basically the one it is kind of trapped inside the cavity but it decays slowly okay. because it decays you can think of a mode which has got a complex frequency. So, you can use this kind of complex frequency description for the mode that is within a cavity and you can consider $\omega_c = \omega_0 - i\gamma/2$. where ω_0 represents the real part of the frequency indicating that that is the actual oscillation frequency.

And the other part that is $-i\gamma/2$, okay, is the imaginary part and that corresponds to the rate of exponential decay of the mold, okay. So, you actually can cover for both the factors. Now, the decay of the field and energy, can be estimated. So, you can think that the field within the cavity decays according to e to the power minus gamma t by 2 leading to the energy within the cavity to decay as you know field is decaying like this.

So, the energy will be like square of it. So, $e^{-\gamma t/2}$.

Quality Factors of Lossy Cavities

- **Decay of Field and Energy:**
  - The field within the cavity decays according to $e^{-\gamma t/2}$, leading to the energy within the cavity decaying as $e^{-\gamma t}$.
- **Characterization of Loss Rate:**
  - While the loss rate can be characterized by γ, the scale-invariance of the Maxwell equations makes the dimensionless quantity $Q = \omega_0/\gamma$ more suitable.

Quality Factors of Lossy Cavities

- **Quality Factor (Q):**
  - The quality factor, Q, is a crucial concept in discussions of resonance.
  - First, $1/Q$ serves as a dimensionless decay rate, mathematically expressed as $\frac{1}{Q} = \frac{P}{\omega_0 U}$, where P is the power lost and U is the stored energy.
  - Second, Q is a dimensionless lifetime, the number of optical periods that elapse before the energy decays by $e^{-2\pi}$.
  - Third, 1/Q is the fractional bandwidth of the resonance.

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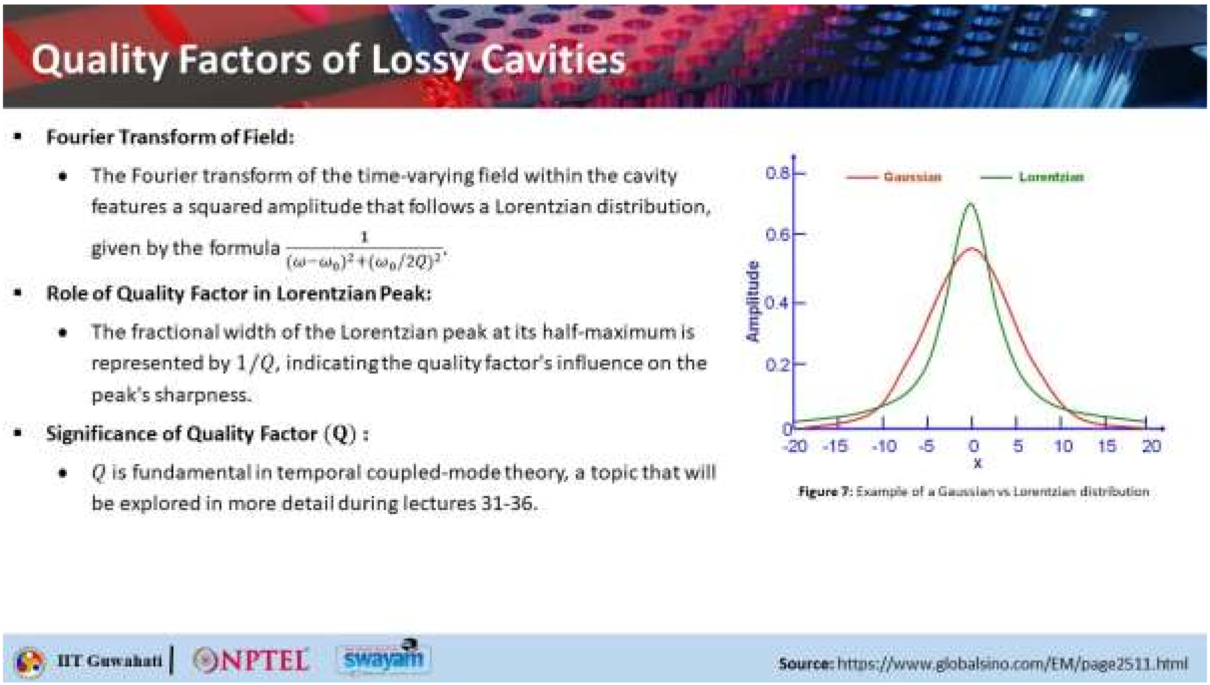
So, how do you characterize the loss rate? So, the loss rate can be characterized by gamma The scale variance of Maxwell's equation makes the dimensionless quantity Q which is defined as omega naught by gamma to be more suitable for characterization of loss rate. Now, this Q is a very interesting parameter. It is called the quality factor of the lossy cavities. So, Q the quality factor is a very important concept when you discuss resonance, isn't it? Firstly, you know 1 by Q serves as a dimensionless decay rate.

So, mathematically it can be expressed as $\frac{1}{Q} = \frac{P}{\omega_0 U}$. What is P? P is the power lost and U is the stored energy. Secondly, Q is also a dimensionless lifetime that means the number of optical period that elapse before the energy decays by $e^{-2\pi}$ that factor. And third that 1 by Q is basically the fractional bandwidth of the resonance. So, you can see the quality factor Q can actually convey lot of information right.



Now the Fourier transform of the time varying field that is within the cavity features a squared amplitude that follows a Lorentzian distribution which can be written using this formula 1 over omega minus omega naught square plus omega naught over 2 q whole square. So, what is the role of Q factor in Laurentian peak? The fractional bandwidth of Laurentian peak at its half maximum is basically represented by 1 by Q. it indicates that the quality factor has got influence on the peaks sharpness. So, here there is an example of Gaussian. So, this is the Gaussian one and this is the Laurentian one, ok.

So, there is a difference between the two distribution, ok. And Q is fundamental in the coupled mode theory which will also be explored in more details towards the end of the course okay in say lecture number 31 to 36 okay

Quality Factors of Lossy Cavities

- **Multiple Decay Mechanisms in Resonance:**
  - When a resonance has multiple decay mechanisms, it is beneficial to characterize each mechanism with its own quality factor (Q).
- **Example of Point-Defect Cavity in Air Bridge Waveguide:**
  - The net Q of the resonant mode in the air bridge waveguide varies with the number N of holes on either side of the cavity, as depicted in the top (blue) curve of figure 8 .
- **Q Increase and Saturation:**
  - In photonic crystals with a complete gap, Q would typically increase exponentially with N. However, in the described case, Q increases with N but saturates as N becomes large.

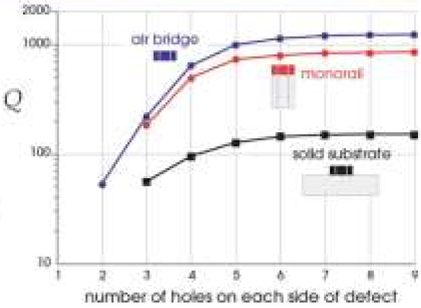


Figure 8: Total Q of the point defect state from figure 6, as a function of the number N of holes on either side of the defect.

Source: Joannopoulos, J. D., Johnson, S. G., Winn, J. N. & Meade, R. D. "Photonics Crystals: Molding the Flow of Light", (Princeton Univ. Press, 2008).

So, if a resonance has a multiple decay mechanism how do you actually analyze the or how do you characterize each mechanism with its own quality factor Q. So take an example of a point defect cavity in air bridge waveguide. So when I say air bridge waveguide, it is basically that periodic dielectric waveguide in air. So the net Q of the resonant mode in that air bridge waveguide varies with the number n of holes on either side of the cavity that is clear right one particular hole is the point defect and on either side of the cavity you will have more number of holes. So, with the number of holes the quality factor actually increases ok as you can see here. Now, what is interesting is that Q does increase ok and it also saturates. So, in photonic crystals with a complete gap Q typically increase exponentially with n. However, in this particular case because it is not a photonic crystal with perfect complete gap in this case Q increases with n, but when n becomes large it tends to saturate ok.

Quality Factors of Lossy Cavities

- **Decay Mechanisms:**
  - The mode in this cavity can decay through two primary mechanisms: it can decay into the uniform dielectric strip on either side of the holes or it can radiate into the surrounding air.
- **Net Dimensionless Decay Rate:**
  - The net dimensionless decay rate $1/Q$ can be expressed as the sum of the waveguide decay rate ($1/Q_w$) and the radiative decay rate ($1/Q_r$), with the formula $1/Q = 1/Q_w + 1/Q_r$.
- **Behavior of Individual Q-values:**
  - When the Q-values are large, they can generally be treated as independent:
  - Q_w increases exponentially with N due to the band gap effect.
  - Q_r remains roughly constant regardless of N.

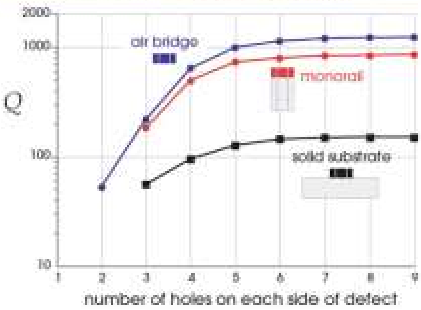
So, this particular one shows the Q factor of the point defect state from this particular figure we have discussed before as a function of n that is the number of holes on either side of the defect. So, there are three cases being considered one is air bridge that means the waveguide is basically in air you have monorail kind of arrangement where And this substrate basically mirrors the cross section of the waveguide including the holes which are also shown here was vertical lines. And this is the another case of solid substrate okay. So, this one tells you the substrate is large and it is not having any holes on it. So, the quality factor actually degrades for this kind of substrate support and for solid substrate it is this way below what we expected from the air bridge ok.

And this happens because the mode in this cavity can basically decay through two primary mechanism ok. So, first is it can decay into uniform dielectric strip on either side of the hole ok. or it can radiate into the surrounding air. right. So, how do you estimate the net dimensionless decay rate? You can consider the net dimensionless decay rate as 1 by q and you can express it as a sum of the waveguide decay rate that is 1 over q w and the radiative decay rate that can be written as 1 over q r.

So, 1 over q is basically 1 over q w plus 1 plus 1 over Q R. So, what are what is the behavior of this individual Q values? So, when the Q values are large they can generally be treated as independent ok. So, you can see that Q w increases exponentially with n due to the band gap effect. So, Q w is basically the decay into the waveguide ok and Q r remains roughly constant regardless of n. So coming back to this figure where you start seeing saturation with increase in the number of holes. So this is happening because QW increases while QR is remaining constant.

Quality Factors of Lossy Cavities

- **Explanation for Saturation:**
  - The saturation observed in figure is due to Q_w increasing while Q_r remains constant.
  - As N approaches infinity, Q converges to Q_r, which is approximately 1200 in this example.



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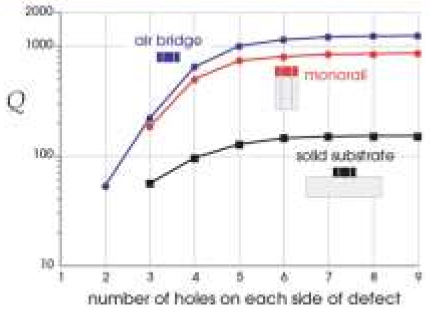
Source: Joannopoulos, J. D., Johnson, S. G., Winn, J. N. & Meade, R. D. "Photonics Crystals: Molding the Flow of Light", (Princeton Univ. Press, 2008).

So the rate actually slows down. So when n approaches infinity, Q actually converges to QR. okay and that is typically 1200 in this particular example okay. So, that is where the Q value gets saturated.

Quality Factors of Lossy Cavities

- **Radiated Power as Loss in Device Applications:**
  - In many device applications, radiated power is considered a loss, and to minimize this loss, it is generally preferred for Q_r (radiative quality factor) to be greater than Q_w (waveguide quality factor).
- **Substrate Effects Illustrated :**
  - The black and red lines in figure show how Q varies with N (number of holes on either side of the cavity) for two different types of substrates.



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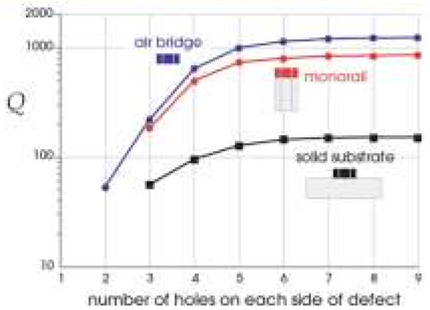
Source: Joannopoulos, J. D., Johnson, S. G., Winn, J. N. & Meade, R. D. "Photonics Crystals: Molding the Flow of Light", (Princeton Univ. Press, 2008).

. So, what would be the radiative power or radiated power in this kind of device? So, the radiated power is basically loss okay. So, in many device application the radiated power is considered as a loss and you have to minimize this loss.

So, it is generally preferred for qr which is the radiative quality factor to be greater than qw which is the waveguide quality factor. Because if qr, if you remember this formula, so you can actually think of the decay rate. So, if this is large as discussed here. So, it is generally preferred that you know QR to be larger than that of QW right.

Quality Factors of Lossy Cavities

- **Common Properties of Substrates and Waveguide:**
  - In both substrate scenarios, the waveguide has a dielectric constant (ϵ) of 12 , while the substrate has a dielectric constant of 2.25 .
- **Two Substrate Types:**
  - The first case (red line) involves a monorail substrate, which mirrors the cross-section of the waveguide, including the holes.
  - The second case (black line) uses a solid substrate without any holes.



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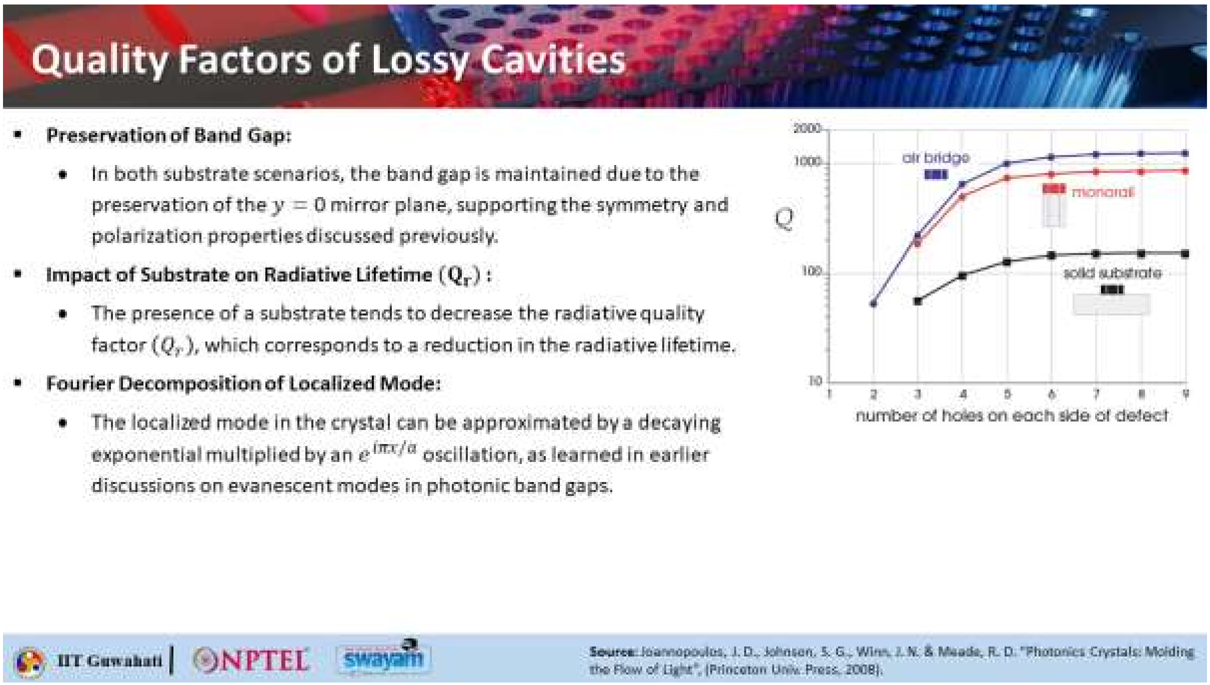
Source: Joannopoulos, J. D., Johnson, S. G., Winn, J. N. & Meade, R. D. "Photonics Crystals: Molding the Flow of Light", (Princeton Univ. Press, 2008).

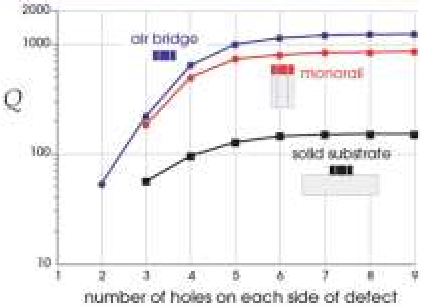
So, here you can also see the effect of substrate. that the black and the red lines will show you how quality factor varies with n that is the number of holes on either side of the cavity. And only thing that is changing here is the two substrate. So, one is a identical substrate of the pattern that you have made here that is called a monorail substrate and the other one is a solid substrate. So, in both cases what the material has been used is same. So, the waveguide is made of material which is a dielectric constant of 12, but the substrate in both case have a permittivity of 2.25. So, only difference is that this thing is a monorail substrate which basically mirrors the cross section of the waveguide itself. So, the substrate also has a hole drilled in it. However, this is a solid substrate without any holes. So, in both substrate scenarios, the bandgap is maintained due to the preservation of y equals 0 mirror plane and that supports the symmetry and the polarization properties that we have discussed today in the lecture. And the last thing is that what is the impact of substrate on radiative lifetime that is QR.



So, you can see that you know the substrate blocks one part ok, it breaks the Z equals 0 plane symmetry. So, the presence of substrate tends to decrease the radiative quality factor that is QR and this corresponds to a reduction in the radiative lifetime. So, the localized mode that you see in the crystal can be approximated by a decaying exponential multiplied by an $e^{i\pi x/a}$ oscillation as we have understood and learned from the earlier discussions on the evanescent modes in photonic bandgap. So, finally we can conclude by comparing the two substrate is that the monorail type of substrate which consists mainly of air has a weaker effect on the light cone as compared to the solid substrate. So, with this you can actually get the features very close to the air bridge kind of structure that you have used for your simulation for simplification ok.

Quality Factors of Lossy Cavities

- **Comparative Impact of Substrate Types:**
  - The monorail substrate, consisting mainly of air, has a weaker effect on the light cone compared to the solid substrate.
- **Effect on Radiative Quality Factor (Q_r) :**
  - The monorail substrate reduces Q_r by approximately 30%.
  - The solid substrate reduces Q_r by almost a factor of ten, indicating a more significant impact on the radiative losses.



So monorail substrate basically reduces the QR by approximately 30%. So from 1200 it will come down to say 900 or something. But if you use the solid substrate like this, Qr the quality factor, radiative quality factor comes down by almost by a factor of 10. That means you have significant you know impact on the radiative loss because of this solid substrate. So, thank you all. So, that will be all for this particular lecture on point defect in periodic dielectric waveguides and analysis of quality factors for lossy cavities.

So, if you have any query on this particular lecture, you can always drop an email to this particular email address dev.shikdar at itj.ac.in, but do not forget to mention MOOC.