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Lec 23: Different types of defects in Photonic Crystal Slabs

Hello students, welcome to lecture 23 of the online course on photonic crystals, fundamentals and applications. Today's lecture we will be discussing about different types of defects in the photonic crystal slabs.



- Linear defects in slabs
- Point defects

So, here is the lecture outline, we will be discussing linear defects in slabs in details and then we will just touch upon point defects in this lecture and in the next lecture we will have more discussion about point defects. So, we have briefly discussed already we know how point defects look like in a photonic crystal and with that you can create waveguides and other kind of devices.



Linear defects in slabs

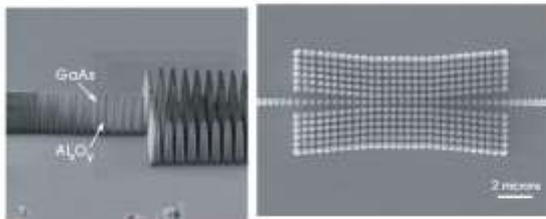


Figure 1: Two views of a reduced-radius waveguide fabricated in a rod slab.

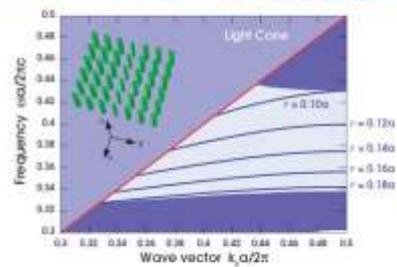


Figure 2: Projected band diagram of TM-like (z -odd) states in a linear waveguide in the rod slab of figure 2(left), formed by reducing the radius of a row of rods (inset), as a function of the wave vector k_x along the defect.

- **Modification of Rod Radii:** Specifically, the radius of all the rods in a particular row is shrunk, as demonstrated in the fabricated example shown in figure 1.
- **Band Diagram Analysis:** In figure 2, the projected band diagram is plotted for various shrunken radii sizes, focusing specifically on TM-like modes.

Today we will go into more details of it. So, I hope you remember from the previous lecture where we saw in the video tutorial that you know when you create a line defect that can change the band diagram right.

So, it basically introduces a guided mode in the band and all the frequencies in the guided mode can propagate along the defect thereby creating a waveguide. So, however, the localization of waveguide modes relies on both the bandgap within the plane of the periodicity and also on index guiding in the vertical dimension and this will restrict the kind of modes that you know we can guide through this kind of waveguide. So, in our discussion of 2 and 3 dimensional crystals we have formed a waveguide by removing a you know row of rods as we have seen in the previous lecture and in

this case right in this particular slide we will show you that you know we will remove the row gradually okay and that is by shrinking the radius of the rods okay and we will show you how the defect mode basically forms so look here on the slide on the first figure that is figure one it gives you the views of reduced radius waveguide fabricated in a rod slab so you can see the You know, this one, this particular waveguide is formed by having rods of reduced radius and these are the material, okay. And you can see they are also having bit of tapered shape for all of the nanorods, okay.

They are not proper cylinders. So, the structure was basically designed to couple the reduce radius waveguide via adiabatic taper to dielectric strip waveguides at the ends. So, both ends you can see there is dielectric strip waveguides and there is adiabatic taper that actually helps you couple this waveguide to those dielectric strips. And now this is the fabricated sample and the bar here the white bar shows the dimension that is 2 microns for this much length of the sample. Now look at figure 2, figure 2 basically is showing in the projected band diagram of Tm like that is Z odd states in a linear waveguide in the rod slab of the figure that is shown here you can see and that is formed by reducing the radius of the row at the centre.

Linear defects in slabs

- **Mode Localization and Band Gap:**

The extended modes of the crystal are highlighted in the dark blue region of the diagram.

The band gap is represented in light blue, where a single guided band for each radius choice is observed.

The TM-like waveguide mode, localized to the line defect within the band gap, is depicted in figure 3.
- **Coupling Characteristics:**

The localized mode cannot couple to the extended modes within the crystal due to its placement within the band gap.

It also cannot couple to extended modes in the air because its frequency is below the light line.

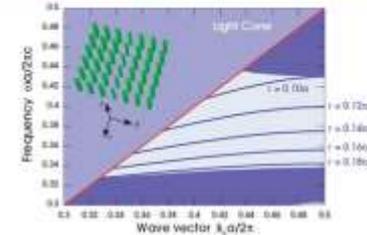
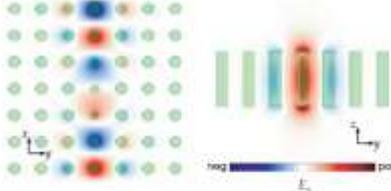



Figure 3: E_z field cross sections in reduced-radius line-defect waveguide from figure 2, for a defect rod radius of $r=0.14a$ at a wave vector $ka/2\pi=0.42$.





Source: J. D. Joannopoulos et al., "Photonic Crystals: Molding the Flow of Light", Princeton Univ. Press, 2008.

So, the inset shows you that this particular row has a lower or smaller radius than the other rods and here also you can see the plot ok. So this is a band diagram. So you actually have frequency and wave factor and it is for the finite height or thickness. So you have got this you know light cone at a slope of 1 okay. So it is 0.5 here 0.5 you can see this is actually this line tells you the boundary of the light cone okay and the blue dark blue shaded region which are below the light line shows you the extended modes which are supported by the crystal and the rods in the bulk crystal they have radius of $0.2a$. And what we have done here we have seen that when so usually this is the band gap ok. So, between these two dark blue regions you will have band gap if the entire crystal is made of $0.2A$ radius rods, but now if you introduce a defect by changing the centre row to different different row radius okay. So, you will see that localized waveguide bands will be introduced within the gap. So, they will actually allow propagation okay for certain frequencies. So, that is how you can see that when the rod radius is usually r equals $0.2a$, from that you slightly reduce, you can get a mode over

here, you further reduce, you get here, you further reduce, you get here and so on.

So, slowly what will happen, you will get the modes over here, right. So, those are the extended modes in air, fine. So this is what is shown. So the radius of all the rods in a particular row is shrunk and this is the band diagram for different shrunken radii sizes. So, let us look into the mode localization and the bandgap in further details.

So, as I mentioned the extended modes of the crystals are basically highlighted in this dark blue region and the bandgap is shown in this light blue region where you can see this kind of single guided band for each of those reduced radius that we can choose. So, either you choose r equals $1a$ or r equals $0.12a$, you will get one single guided band. So, this modes are actually showing TM like profile. So, you can actually see that from the field distribution.

So, this is the Field distribution, so you can see look into this slab from the top or the side view, okay. And here we have considered the defect rod radius to be $0.14a$, that is we are basically talking about this one, okay. And we consider the wave vector of 0.42 , that is we are basically at this particular point where the cursor is currently placed.



- **Propagation Properties:**

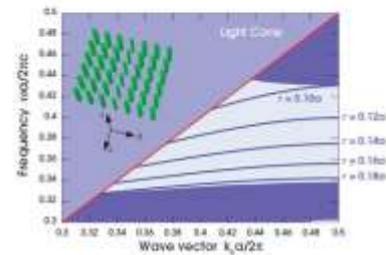
Despite the band gap being incomplete, the waveguide mode persists, supported by the conservation of k_x (wavevector component along the propagation direction).

This waveguide mode will propagate indefinitely in a perfectly periodic system.

So, at this point this is what is the field distribution looking like. Now the localized mode that you see cannot couple to the extended modes within the crystal due to its placement within the bandgap. So it also cannot couple to the extended modes in air, this one, because the frequency is below the light line. So it cannot actually couple to other modes or the air modes and cannot leak out. So despite the bandgap being incomplete, the waveguide mode will exist and it is supported by the conservation of k_x that is the wave vector along the propagation direction.

Linear defects in slabs

- **Presence of Light Cone:** The band diagram of a photonic crystal slab, unlike that of a truly two-dimensional photonic crystal, features a light cone, which adds a significant constraint on the behavior and existence of waveguide modes.
- **Limitation on Rod Removal:** Completely removing a row of rods is not feasible if one intends to support a waveguide mode, as the necessary conditions for mode guidance and confinement are not met.
- **Reduction of Rod Radius and Mode Behavior:**
As depicted in figure, reducing the rod radius influences the waveguide mode's proximity to the light cone.



When the rod radius is reduced to $0.10a$, the mode approaches and eventually hugs the light cone at the top of the gap, indicating a critical limit for mode confinement.

So, here you can see x is the direction of propagation. and this waveguide mode will propagate in this waveguide mode will propagate indefinitely in a perfectly periodic system. So, what does this presence of light cone tell you? So, the band diagram in of this photonic crystal slab okay will have this light cone right. So, this is not present in a perfectly two dimensional photonic crystal right. So, when there is this finite height or thickness of the photonic crystal slab, you will have this light cone and this adds a significant constraint on the behavior and the existence of the waveguide modes.

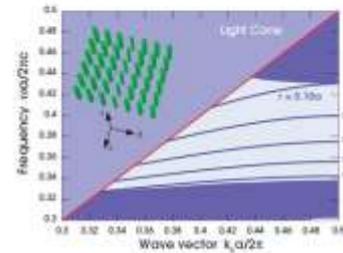
So, what are the limitations of rod removal? As you can see, if you completely remove the rod that is not going to be possible because in that case you will not be able to support a waveguide mode. So, you need to have a mode within the band gap to support propagation of light through it, but if you make it further you know smaller it will actually go into this region. So, here you can see that the mode radius basically influences the waveguide modes proximity and it you keep on reducing and it is getting closer and closer to the light cone. So when the rod radius is $0.1a$, the mode already approaches and eventually hugs the light cone at the top of the gap.

Linear defects in slabs

- **Guidance and Confinement Challenges:**

Further reducing the rod radius beyond $0.10a$ causes the mode to lose its guided characteristics, effectively ceasing to be a waveguide mode.

Attempting to guide light primarily through the air gaps between the rods fails due to insufficient vertical confinement from index guiding.



You see here indicating this is the critical limit for the mode confinement. So what are the challenges in guidance and confinement? So further reducing the rod radius below $0.1a$ would cause the mode to lose its guided characteristics, especially it will not be a waveguide mode anymore. Attempting to guide light primarily within the air gaps okay between the rods also fail due to you know there will be no vertical confinement due to index guiding. So, both possibilities are not there and you will not be able to guide light through the gap between the air gap between the rods and you cannot make the rod size very small or completely get a missing row that will not work.

Linear defects in slabs

- **Common Features:** All three structures share the same two-dimensional periodicity and feature a triangular lattice of rods, with one row removed.

They all have identical cross-sections at $z=0$

- **Structure Descriptions:**

First Structure: This is a two-dimensional photonic crystal where the rods extend infinitely in the vertical direction and exhibit a complete TM band gap.

Second Structure: This is a three-dimensional photonic crystal, also with a complete band gap, (detailed in the lecture 17). A row of rods missing from a single layer of the 3D photonic structure.

Third Structure: This is a photonic-crystal slab.

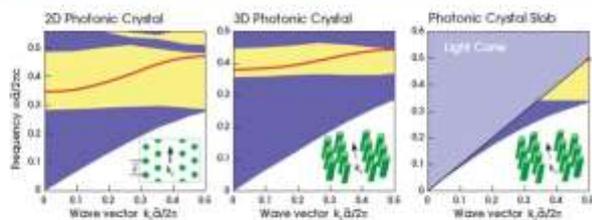
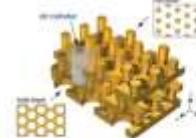


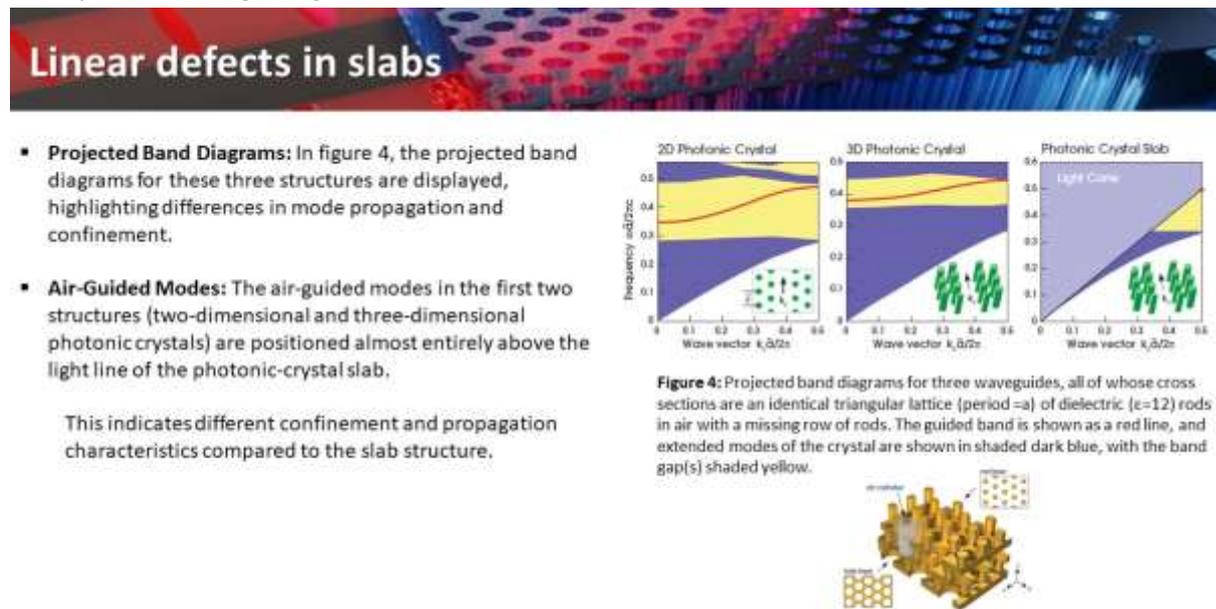
Figure 4: Projected band diagrams for three waveguides, all of whose cross sections are an identical triangular lattice (period $=a$) of dielectric ($\epsilon=12$) rods in air with a missing row of rods. The guided band is shown as a red line, and extended modes of the crystal are shown in shaded dark blue, with the band gap(s) shaded yellow.



So, to draw out the comparison further we will consider three different structures that share the same kind of periodicity in two dimension, but they basically differ in the third or the vertical

dimension. So, look at the figure here, here you know all the structures are basically two dimension, at you know Z equals 0 cross section, they look identical. So, this structure is basically this one, you are just considering one plane. So, that is why it is a part of 3D photonic crystal. And this is a photonic crystal slab.

So, these are basically rods of finite height. What is this? This is basically a two dimensional photonic crystal, that means the rod would extend infinitely in the vertical dimension and that is why it is able to support a complete TM band gap. Now if you look into the second structure as I mentioned this is a three-dimensional photonic crystal. It has got a complete band gap, but as you can see the gap is slightly reduced as compared to this one. And here what we are considering we are basically looking at you know one of the row of rod is basically missing from the single layer of this you know 3D photonic crystal structure okay. And the third one we have been discussing a lot about this photon crystal slab. This is basically a finite height two-dimensional periodic photonic crystal. So, what we understand here? So, all these band diagrams are plotted for a dielectric constant of epsilon equals 12. So, that is the permittivity of the rod material in air and all of them have got one row missing. Because of this row missing thing, you are basically having a, in this case, in the two-dimensional case, you are having this guided band which is shown as the red line.



And the extended modes of the crystal are shaded in dark blue as also you can see. Here also you can see the same thing, just that the band gap got narrowed. this is the guided band and the yellow color shows you the band gap. So, things are not as exciting for photon crystal slab with a missing row as we discussed previously. So, here I will come to this later that only one particular point shows you some hope in feasibility.

So, this is what has been displayed in this particular figure. So, the project, these are the band diagram of these three particular cases, 2D photonic crystal, 3D photonic crystal and photonic crystal slabs. And this clearly shows you the difference in mode propagation and confinement for the three cases. What about the air-guided modes? The air-guided mode in the first two structures that is this two-dimensional and three-dimensional photon crystals are positioned almost entirely above the

light line of the photonic crystal slab.

So, this is the light line. So, the air guided modes are all on top of that in the case of you know 2D and 3D photonic crystal slab. So, this basically indicates different confinement and propagation characteristics of this two crystal as compared to the slab structure. So, one thing we possibly missed. So, it was basically that here you can see that there is a red dot shown and this is basically showing a very weakly guided state right at the top of the bandgap edge and this is the only possible guided mode in this particular case because you have completely removed the row of rods.



Removed holes

- **Defect Creation:** The defect in the photonic structure was created by altering the average dielectric constant along a specific line, impacting the guided band's position within the band gap.

- **Manipulating Dielectric Constant:**

Decrease in Dielectric Constant: Reducing the dielectric constant pushed a guided band upward from the lower edge of the band gap.

Increase in Dielectric Constant: Conversely, increasing the dielectric constant pulls bands downward from the upper edge of the band gap.

Illustrative Example: This dynamic is demonstrated in figure 5, which shows the projected band diagram for a hole slab where a row of holes has been filled in, effectively increasing the average dielectric constant.

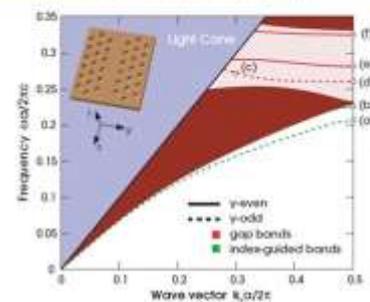


Figure 5: Projected band diagram of TE-like (z -even) states for a "W1" defect in the hole slab, formed by a missing row of nearest-neighbor holes along the x direction. Dark-red shaded regions indicate extended TE-like modes of the crystal.

Now we can discuss about the you know opposite structure that is like if you have a slab of holes, how removing holes will affect the band gap.

So, that is also possible that is also type of defect. So, the defect in the photonic structure was created by altering the average dielectric constant along a specific line impacting the guided bands position within the band gap. So, what you can do you can either fill a complete row of holes with the same material. So, that can be also called as a row missing row of holes or you can change the size of the holes that will also bring you know changes or that will also be considered as defects. So, these are the methods of decreasing dielectric constants or say manipulating dielectric constant.

when you say manipulating dielectric constant, you can decrease the dielectric constant and that is possible and reducing the dielectric constant would push a guided band upward from the lower edge of the band gap. So, what happens in the other case, if you increase the dielectric constant, So, opposite thing will happen. So, increasing the dielectric constant pulls down you know the bands from the upper edge of the band gap. And these things will be shown here in this particular figure which shows the projected band diagram for a hole slab where a row of holes have been filled. So, that is basically increasing the average dielectric constant.

So, what do you see? This is the projected band diagram for TE-like states. So, they are also called Z-

even states we discussed earlier. For a W1 defect in the whole slab. So, what is this W1 defect? Now, it is a general form you can think of W_n defect which will involve removal of n rows.

So, here only one row is removed. So, you call this kind of defect as W1. So, how it happened you basically you created a missing row. along the x direction. So, this is x direction. So, here you can see that the dark shaded regions indicate the extended T like modes of the crystal.

And you can also see that there is a guided mode which are introduced in the, so there are basically two guided modes which are introduced in the gap. So, they are shown as these red bands in this pink shaded region and below all of the extended modes of the crystal. So, which are basically the green bands below the red shaded regions and the guided modes are classified as Y even or you can say solid lines and Y odd which are basically the dashed lines that you can see over here. So, these are the modes possible and y odd and y even these are basically decided based on y equals 0 mirror symmetry that also we have discussed briefly earlier. This is a practical application of that kind of system of holes missing.

Linear defects in slabs

Removed holes

- **Practical Application:** A fabricated example of this type of waveguide, specifically a suspended membrane, is displayed in figure 6.

This shows how the theoretical concepts are applied in actual photonic device structures.

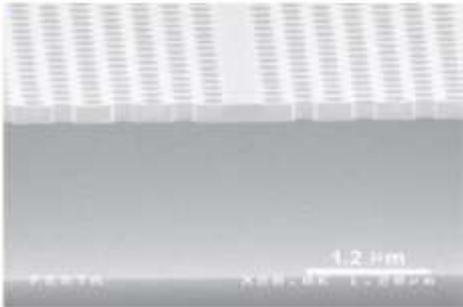


Figure 6: SEM image of a waveguide formed by a missing row of holes in a suspended-membrane hole slab

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Source: J. D. Joannopoulos et al., "Photonic Crystals: Molding the Flow of Light", Princeton Univ. Press, 2008.

So, a fabricated example of this kind of waveguide is shown here. So, this is a suspended membrane means there is no substrate below its air and this shows how you know theoretical concepts are applied in actual photonic device structures. What are the characteristics of the guided modes in this case? So, the waveguide supports a series of guided modes that are confined horizontally by the bandgap and they are confined vertically by index guiding due to difference in the refractive index between the waveguide material and surrounding air.

Linear defects in slabs

Removed holes

- **Guided Modes Characteristics:** The waveguide supports a series of guided modes that are confined:

Horizontally: By the band gap.

Vertically: By index guiding due to the difference in the refractive index between the waveguide material and the surrounding air.

- **Second Category of Guided Modes:**

Cause: These additional guided modes arise due to the waveguide having a higher average dielectric constant compared to the surrounding air.

Guiding Mechanism: These modes are index-guided in all directions and lie below the extended modes of the crystal

The second category of guided modes are also there they arise due to waveguide having a higher average dielectric constant as compared to the surrounding air. So, here the guiding mechanism you can guess it is basically they are all index guided in all the directions and they lie below the extended modes of the crystal.

Linear defects in slabs

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- **Mode Type and Visualization:**

Mode Type: All of the guided modes are TE-like and fundamental in the z-direction, meaning they have no nodes along this axis.

Visualization: These modes can be visualized by plotting the magnetic field component H_z in the $z=0$ plane, which helps illustrate their spatial field distribution.

- **Field Patterns and Identification:**

Display: Figure 7 displays the field patterns for the five guided modes.

Reference: These modes are identified with letters in figure 5, linking their spatial characteristics with their representation in the band diagram.

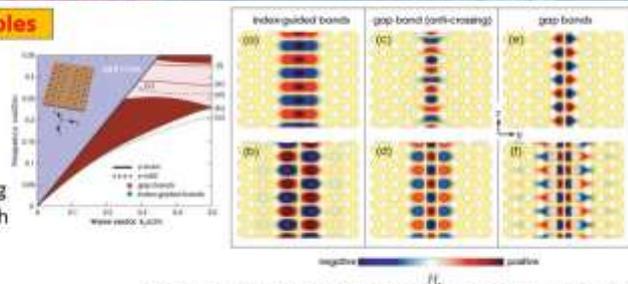


Figure 7: H_z field cross section for the missing-hole waveguide, with (a-f) corresponding to the labelled points of figure 5. Dielectric material is shown as translucent yellow.

So, what are the different mode types? So, all of the guided modes are TE like and fundamental in the z direction, meaning they have no nodes along this axis. So, you can visualize this by plotting the magnetic field component H_z in the $z=0$ plane which will help to illustrate their spatial field distribution. So, figure 7 here displays this H_z field cross section for the 5 guided modes. So, which are also numbered over here. So, these modes are basically identified with these letters that you see here and their spatial characteristics are shown here and you can map them with their position in

the band diagram.

So, what do you see carefully? If you look then the left side is telling you about y odd on the top and you have y at the bottom and this is the y axis, this is the coordinate system that has been marked over here. So, these are the index guided modes at k_x equals π/a which have lower frequencies than any extended mode of the crystal or air at that k_x . So, and because H is a pseudo vector, the even modes look odd and vice versa. In the middle you can see they correspond to C and D points.

So, you have C here, you have D here. So, they all corresponds to those things. So, they are both in the band gap. So, these points in the same wired gap guided bands and they are taken at k_x equals for the top one it is 0.3 times $2\pi/a$ and the bottom one is taken as k_x equals π/a and the drastic field change that you can see corresponds to an anti-crossing and that is why you see this kind of difference.

On the right, you basically correspond to E and F. So, you can see this one is E, this one is F. So, they are basically showing you the guided bands. So, these are two higher order Y even guided bands. So, a pseudo vector also known as an axial vector would behave differently under coordinate system transformations compared to a regular vector or a polar vector. So, under reflection pseudo vectors basically reverse directions which contrast with the scalar fields or polar vectors that would retain their orientations under the same transformation.



Linear defects in slabs

- A pseudovector (also known as an axial vector) behaves differently under coordinate system transformations compared to a regular vector (polar vector).
- Under reflection, pseudovectors reverse direction, which contrasts with scalar fields or polar vectors that retain their orientations under the same transformation. Common examples of pseudovectors include magnetic fields and angular momentum.
- **Implications in Photonic Crystals**
In photonic crystals, especially those with line defects or other forms of structural asymmetry, the behavior of electromagnetic modes (both electric E and magnetic H fields) can be categorized based on their symmetry properties.

The symmetry classifications are often referred to as even or odd, based on how the field patterns reflect across a symmetry plane (e.g., $y=0$).

1. Even Modes: Typically, for scalar fields, an even mode would mean that the field pattern is symmetric about the plane of reflection — if you flip it across the plane, it looks the same.
2. Odd Modes: An odd mode would mean the field pattern is antisymmetric — flipping it across the plane would result in the field pattern looking like its negative (it inverts).

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Source: J. D. Joannopoulos et al., "Photonic Crystals: Molding the Flow of Light", Princeton Univ. Press, 2008.

So, common examples of pseudo vectors would include like magnetic fields and angular momentum and that is why we have mentioned it here that because you know H magnetic field is a pseudo vector, the even modes will look odd and vice versa, okay, because we are talking about y equals 0 mirror symmetry, okay. So, what are the implications in photonic crystals? In photonic crystals, especially those with line defects or other forms of structural asymmetry, the behavior of the electromagnetic modes, both electric and magnetic fields can be categorized based on their

symmetric properties. The symmetric classifications are often referred to as odd or even based on how the field patterns reflect across the symmetric plane. So you can consider $y = 0$ plane. So even modes, so typically for scalar fields and even mode would mean that the field pattern is symmetric about the plane of reflection.

So now if you flip it across the plane, it will look the same. and odd modes would say that you know the field pattern is asymmetric. So, flipping it across the plane would result in a field pattern that would look like is inverted version or negative version. So, we understood that you know magnetic field vector is basically magnetic field is a pseudo vector and its symmetric properties under reflection are basically counterintuitive. So, the even modes would appear odd upon reflection and even mode of the magnetic field across a symmetry plane.



- **Magnetic Field as a Pseudovector**

Since the magnetic field H is a pseudovector, its symmetry properties under reflection are counterintuitive:

Even Modes Appear Odd: When reflecting an even mode of the magnetic field across a symmetry plane, the direction of the magnetic field reverses (because it's a pseudovector), making an even mode appear as if it were odd.

Odd Modes Appear Even: Conversely, an odd mode will appear to maintain symmetry under reflection, akin to an even mode, because the inversion inherent in the pseudovector nature of H counteracts the expected inversion from the mode's odd symmetry.

We will do this again. So, even modes would appear odd. When reflecting an even mode of the magnetic field across a symmetry plane, the direction of the magnetic field reverses because it is a pseudo vector and that is why it will make an even mode appear like as if it were odd and the odd modes would appear even. So, conversely an odd mode will appear to maintain symmetry under reflection similar to an even mode because the inversion inherent to the pseudo vector nature of magnetic field okay would counteract the expected inversion from the modes odd symmetry.

Linear defects in slabs

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- Symmetry and Mode Classification:**

The system exhibits invariance under reflections in the $y=0$ plane, allowing for classification of modes as either odd or even with respect to this reflection.

- Fundamental Mode Characteristics:**

The fundamental mode is classified as y -odd.

- Excitation Preference:** Y -odd modes are more readily excited by a planewave input beam, thus receiving more focus in analyses.

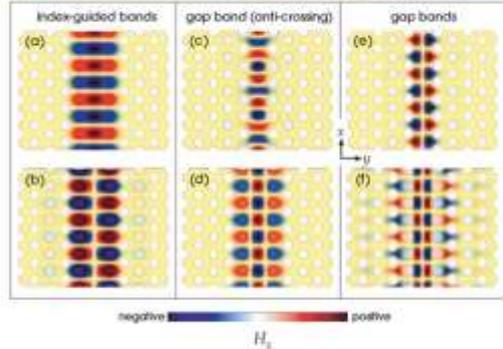


Figure 7: H_z field cross section for the missing-hole waveguide, with (a-f) corresponding to the labelled points of figure 5. Dielectric material is shown as translucent yellow.

So, with that you can further look into this and understand that the system exhibits invariance under reflections in the y equals 0 plane. So, it is a this kind of a plane across this So, you can now see that the modes can be classified with respect to odd or even based on the reflections. So, the fundamental mode that you see here A is basically Y odd mode though it looks appears even.

Linear defects in slabs

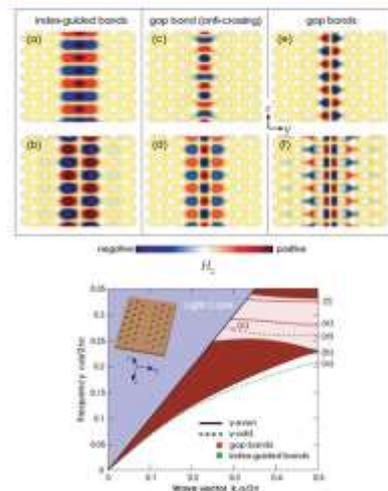
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- Field Profile Evolution of Y -odd Modes:**

Evolution with Wave Vector: The field profiles for the second y -odd band, located in the band gap, show notable changes as the wave vector varies from the light cone to the Brillouin zone edge.

Specific Patterns: Figure (b) displays the field pattern at $k_x=0.3\pi/a$

Figure (c) shows the field pattern at $k_x=0.5\pi/a$, noting an additional pair of nodes in the y -direction.



And why odd modes are more readily excited by plane wave input beam, thus they receive more focus in analysis. So, if you try to evaluate the field profile of y odd modes. So, the field profiles of the second wired band which is located in the band gap, it will show, so it is the B1, this. So, it will show notable changes as the wave vector varies from the light cone to the brilliant zone edge. So, as I mentioned the figure b, so this one displays the field pattern at k_x equals 0.3π by a . So, you can understand where it will stand. So, this axis is basically $k_x a$ by 2π . So, you can actually calculate

that. So, $k_x A$, so A will cancel it out by 2π .

So, you are basically at 1.15. So, figure C here, it shows the field pattern for k_x equals 0.5π by A , noting an additional pair of nodes in the y direction. So, what are the anticrossing effect? This significant change in the field pattern arises due to an anticrossing event where the two bands are expected to intersect instead couple and repel each other. So, that is the anticrossing effect. So, they would alter the trajectories unless a you know specific symmetry prevents this interaction.

Linear defects in slabs

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- **Anti-Crossing Effect:**

Description: This significant change in the field pattern is due to an anti-crossing event where two bands that are expected to intersect instead couple and repel each other.

This alters their trajectories unless a specific symmetry prevents this interaction.

Source: J. D. Joannopoulos et al., "Photonic Crystals: Molding the Flow of Light", Princeton Univ. Press, 2008.

Linear defects in slabs

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- **Initial Condition (Nonperiodic Waveguide):**

Visualization in Figure 8 (left side): Initially, consider the y -odd bands for a nonperiodic waveguide.

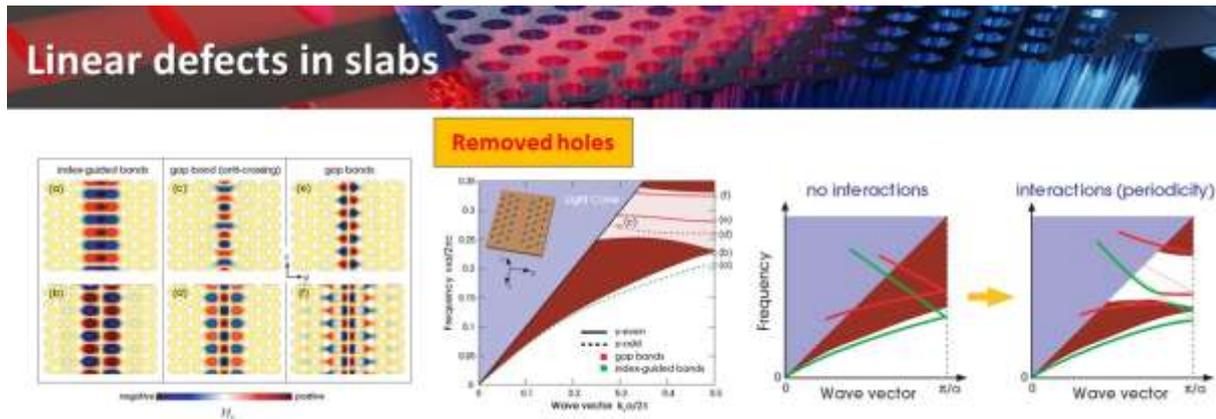
Mode Characteristics:
 The index-guided mode (depicted in green) is folded back at the "artificially" imposed edge of the Brillouin zone. A higher-order y -mode (depicted in red) is located at a higher frequency and is similarly folded back.

Source: J. D. Joannopoulos et al., "Photonic Crystals: Molding the Flow of Light", Princeton Univ. Press, 2008.

So, we will understand this better. So, this we have already seen. Now, here is a case where you consider no interaction. So, that is the initial condition. You consider non-periodic waveguide. So,

this is the schematic of anticrossing that occurs when periodicity is added.

So, here it is no interaction because it is not periodic. So, you just consider the y odd bands for a non-periodic waveguide and it looks like this. So, here the index guided mode is depicted in green is folded back at the artificially imposed edge of the Brillouin zone here. and the higher order Y mode which is depicted in red is located at a higher frequency and it is also similarly folded back. Now if you introduce this is a non periodic waveguide ok. Now if you introduce periodicity in that waveguide that is so this figure on the right this basically shows that case when the periodicity is introduced in the slab.



▪ **Introduction of Periodicity (Periodic Waveguide):**

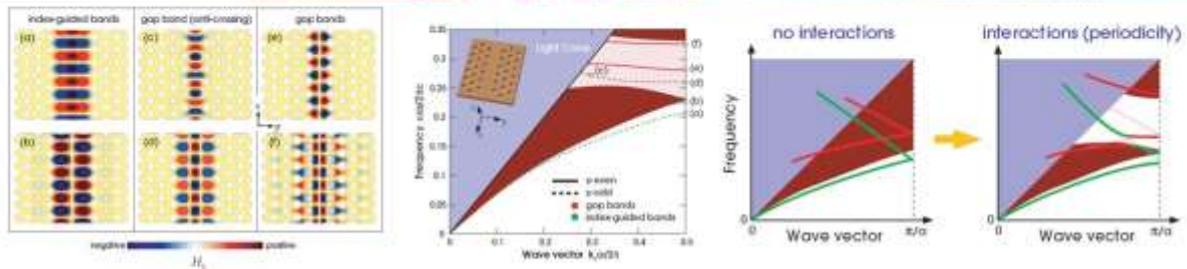
Visualization in Figure 8 (right side): When periodicity in the slab is introduced, band gaps emerge, and the bands begin to repel each other.

Interaction Points:

This repulsion occurs not only at the edge of the Brillouin zone but also at points where the red and green bands intersect.

So, as soon as periodicity is there band gap will emerge and the bands will start to repel each other and this is what is shown here. So, what are the interaction points? So, this repulsion occurs not only at the edge of the brilliant zone, but also at points where the bands basically intersect. So, where the red and the green bands are intersecting. So, in terms of you know the wave vector k_x you will see evolution of the field patterns with k_x you can see a continuous transformation.

Linear defects in slabs



Evolution of Field Patterns with k_x :

Continuous Transformation:

As k_x increases along the second band, the field pattern transitions continuously from the red mode [figure (b) of field plot] to the green mode [figure (c) of field plot].

Phenomenon Utilization:

This behavior can be leveraged to produce unusual dispersion effects, including **ultraflat quartic band edges** and **zero-dispersion inflection points**.

So, k_x increases along the second band. The field pattern basically transitions continuously from the red mode to the green mode. So, if you see the field pattern basically transitions from continuously from the red mode. So, you can also look here in the figure B to the green mode which is basically figure C of the field plot. So, that is where from the red mode to green mode it is transitioning. So, these are now this behavior can be leveraged to produce unusual dispersion effect including the ultra-flat quadratic sorry, quartic ultra-flat quartic band edges and zero dispersion inflection points.

what are these ultra-flat quartic band edges? So, ultra-flat would refer to band edges where the curvature of the band that means the second derivative with respect to the wave vector k would approach 0 over a relatively wide range of k and this would result in very low group velocities for photons or electrons. And then what is quartic band? So, the term quartic implies the band edge follows a fourth degree polynomial dependence near the edge. So, quartic band edges are basically categorized by dispersion relation that looks like this. So, $E(k)$ can be written as $E_0 + \beta(k - k_0)^4$. So, here E_0 is basically the energy at the band edge, k_0 is the wave vector at the band edge and β is a constant, ok.

Linear defects in slabs

Ultraflat Quartic Band Edges

1. Definition: "Ultraflat" refers to band edges where the curvature of the band (second derivative with respect to wavevector k) approaches zero over a relatively wide range of k . This results in very low group velocities for photons or electrons.
2. Quartic Band: The term "quartic" implies that the band edge follows a fourth-degree polynomial dependence near the edge.
3. Quartic band edges are characterized by a dispersion relation of the form $E(k) \approx E_0 + \beta(k - k_0)^4$, where E_0 is the energy at the band edge, k_0 is the wavevector at the band edge, and β is a constant.
4. Significance: Ultraflat bands are particularly interesting for enhancing light-matter interactions because photons in these bands travel slowly, increasing the interaction time with the medium.
5. This can enhance various nonlinear optical effects, increase the efficiency of light emission processes, or improve the sensitivity of sensors.

So, you can see the you know fourth order polynomial dependence. And what is the significance? This kind of ultra flat bands are particularly interesting for enhancing light matter interaction because photons in these bands would travel slowly. So, that would interact that would increase the interaction time with the medium. So, this can also enhance various types of non-linear effects increasing the efficiency of light emission processes or it could improve the sensitivity of different sensors.

Linear defects in slabs

Zero-Dispersion Inflection Points

1. Definition: A zero-dispersion inflection point occurs where the dispersion curve's second derivative with respect to k changes sign (from concave to convex or vice versa), and the first derivative (group velocity) is zero. This point marks a transition in the curvature of the band structure.
2. Mathematical Expression: At a zero-dispersion inflection point, $\frac{d^2 E}{dk^2} = 0$ and $\frac{dE}{dk} = 0$ at some value of k .
3. Implications: At such points, the group velocity dispersion (GVD) is zero, which means that the spread of wave packet velocities (group velocities) is minimal.
4. This property is crucial in applications like pulse propagation in optical fibers where minimal pulse broadening is desired.

The second one is zero dispersion inflection points. So, when you say zero dispersion inflection point that occurs where the dispersion curves second derivative with respect to k changes sign.

So, it changes sign means from concave it becomes convex or vice versa and the first derivative

which represents group velocity becomes 0. So, this point marks a transition in the curvature of the Venn diagram. So, mathematically you can represent this as you know at the 0 deflection 0 dispersion inflection point, $\frac{d^2\omega}{dk^2} = 0$ or you can say $\frac{d\omega}{dk} = 0$ and $\frac{d^2\omega}{dk^2} = 0$ as well at some point of inflection. or at some value of k . So, what is the significance or application? So, at such points the group velocity dispersion is 0.

So, GVD is 0 and that means the spread of wave packet velocity is minimal. And this property is very important for applications like pulse propagation within optical fibers where you know minimal pulse broadening is basically desired.



Linear defects in slabs

Substrates, dispersion, and loss

- **Index Contrast and Reflection Symmetry:**
 - Original Configuration:** Photonic-crystal slabs are typically suspended in air, which maximizes the index contrast between the slab and the surrounding air and preserves $z=0$ reflection symmetry.
- **Effects of Placing on a Substrate:**
 - Symmetry Break:** Introducing a substrate beneath the slab breaks the $z=0$ reflection symmetry.
 - Mode Coupling:** This asymmetry causes TE-like and TM-like modes to couple, disrupting any band gaps that are exclusive to either mode.
 - Leakage of Modes:** Waveguide modes, previously confined by the gap when the slab was floating in air, become leaky when the slab is placed on a substrate.

IIT Guwahati | NPTEL | swayam Source: J. D. Joannopoulos et al., "Photonic Crystals: Molding the Flow of Light", Princeton Univ. Press, 2008.

Now we will look into the effects of substrates and into dispersion and discuss about losses. So, index contrast and reflection symmetry. So, if you remember that the original configuration of the Photonic crystal slabs were assumed to be suspended in air because it maximizes the index contrast between the slab and the surrounding medium.

And it also preserves the $z = 0$ reflection symmetry. But that is not possible because you need to place a substrate below your thing. And as soon as you place the substrate, the symmetry breaks. So, you no longer have your $z = 0$ reflection symmetry. And this asymmetry also causes TE like and TM like modes to couple.

So, it disrupts any band gap which were actually exclusive to either of the modes. So, that way you know the waveguide modes which were previously confined by the gap when the slab was basically floating in air, those gaps becomes leaky when the slab will be placed on the substrate. So, that brings some application concerns something like the leakage rate, the issue of leakage is significant only if the rate at which the energy leaks from the modes is unacceptable for a particular application.

Linear defects in slabs

- **Application Concerns:**

Leakage Rate: The issue of leakage is significant only if the rate at which energy leaks from the modes is unacceptable for a specific application.

Empirical Evidence: Numerous experiments have demonstrated that effective waveguides can still be fabricated on oxide substrates, which have a relative permittivity (ϵ) of approximately 2.

- **Mitigation Strategies:**

Etching Patterns: Reducing polarization mixing by etching the periodic pattern into both the slab and the substrate can help maintain effective guidance of the modes.

Restoring Symmetry: Depositing a "superstrate" material on top of the slab that has similar properties to the substrate can help restore some degree of the lost z symmetry.

So, eventually every mode will disappear, but then the rate at which it disappears or leaks out the energy loses that is also important. There are numerous experiments which have demonstrated that effective waveguides can still be fabricated on oxide substrate.

So, that have relatively low permittivity which is close to 2. So, here we will have permittivity of 1 and you have this material like oxides, oxide based substrate where the permittivity can be kept as low as possible. So, what are the other mitigation strategies? So etching patterns, so reducing polarization mixing is possible by etching the periodic pattern into both the slab and the substrate and that can help, you know, maintain effective guidance of the modes. So do not take a solid substrate rather try to drill holes at the places where you already have holes on your slab. So that will try to, you know, help guidance of the modes and despite a super straight material, sorry and how do you restore symmetry? you can deposit a super straight material that means you put a material on top of your slab that has got similar properties as your substrate and that will help you restore some degree of the lost Z symmetry because now the top and the bottom of the slab will look identical okay.

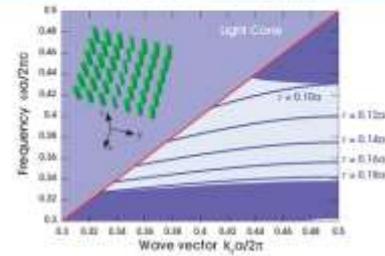
Linear defects in slabs

- **Location of Guided Modes:** In symmetric photonic structures, guided modes within the band gap typically lie near the edge of the Brillouin zone.

- **Behavior Near the Brillouin Zone Edge:**

Band Flattening: As the modes approach the edge of the Brillouin zone, the bands tend to flatten.

Group Velocity Reduction: The group velocity (v_g) of these modes approaches zero.



about the location of the guided modes. So in symmetric photonic structures, guided modes within the bandgap typically lie near the edge of the Brillouin zone. And what happens near the brilliant zone you can see band flattening that means as the modes approach the you know here you can see as the modes approach the edge of the brilliant zone the bands tend to flatten and that means the group velocity of this modes approaches 0 okay

Linear defects in slabs

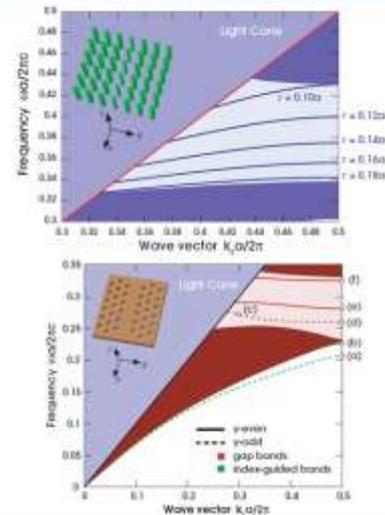
- **Group-Velocity Dispersion:**

Definition: Group-velocity dispersion ($\sim \frac{dv_g}{d\omega}$) indicates the rate at which pulse spreads temporally in the waveguide.

Behavior at Zone Edge: This dispersion parameter diverges at the edge of the Brillouin zone, leading to significant pulse spreading.

- **Observations from Band Diagrams:**

Figures Referenced: As depicted in figures, the guided waveguide modes exhibit both low group velocity and strong dispersion across most of their operational bandwidth.



what about group velocity dispersion that is basically calculated as dV_g over $d\omega$ that indicates the rate at which the pulse will spread temporarily in the waveguide and when you think of this region, okay. the behavior at zone edge. So, this dispersion parameter diverges at the edge of the brilliant zone and that would basically lead to significant pulse spreading.

you can observe from the band diagrams that you know as shown in the figures the guided waveguide modes exhibit both low group velocity and strong dispersion across most of the operational bandwidth. So, what are the utilization of this low group velocity? As we discussed that when you have low group velocity it is also known as slow light and they can be employed to enhance optical non-linearities which are very important for certain applications. So, you can also enhance the light-matter interaction in sensors or in other materials non-linear materials using this concept. So alternate design requirements would be you know you it is in some applications it is advantageous to have a broad bandwidth with low dispersion paired with more typical with a more typical group velocity so this can be achieved by doing some modification to the waveguide design. Some of these design strategies are you know surrounding a dielectric strip waveguide with a photonic crystal slab will allow you to adjust the dispersion characteristics by you know avoiding terminations with surface states.

Linear defects in slabs

- **Utilization of Low Group Velocity:**

Enhancement of Optical Effects: Low group velocity, also known as "slow light," can be employed to enhance optical nonlinearities, making it desirable for certain applications.

- **Alternative Design Requirements:**

Broad Bandwidth and Low Dispersion: In some applications, it is advantageous to have a broad bandwidth with low dispersion paired with a more typical group velocity.

Design Modifications: This can be achieved by modifying the waveguide design.

- **Specific Design Strategy:**

Integration with Photonic-Crystal Slab: Surrounding a dielectric strip waveguide with a photonic-crystal slab can adjust dispersion characteristics while avoiding terminations with surface states.

Waveguide Mode Characteristics: The resulting waveguide mode in this structure closely resembles that of an isolated strip, yet it effectively functions as a conduit for reaching point defects and other devices embedded within the photonic crystal.

So, integrating with photonic crystal slabs okay it helps. And what about the waveguide characteristics? So, the resulting waveguide modes in this particular structure will closely resemble that of an isolated strip, yet it will effectively function as a conduit for reaching the point defects and other devices embedded within the photonic crystal.

Linear defects in slabs

- **Inherent Losses in Real Systems:**

Imperfections: Even though waveguide modes in a perfect and symmetric slab are theoretically lossless, practical implementations invariably introduce some degree of loss.

- **Sources of Loss:**

Substrate Losses: As previously discussed, placing the slab on a substrate can break symmetry and introduce coupling losses between different modes.

Material Absorption: Inherent properties of the waveguide material can lead to absorption losses.

Radiative Scattering: Irregularities from the fabrication process can cause scattering losses. These irregularities disrupt the translational symmetry of the slab.

So, now let us focus on the losses. So, there are some inherent losses in real systems. And even though the waveguide modes in a perfect and symmetric slab are basically theoretically lossless, practical applications invariably will introduce some degree of loss in them.

And what are the sources of this losses? The first one will be substrate. So as discussed previously, when you place a slab on a substrate that breaks the symmetry and introduces coupling losses between different modes. And when there is coupling between different modes, the power or the energy gets shared. There would be material absorption, so inherent properties of the waveguide material will lead to absorption losses. There would be radiative scattering coming from the irregularities from the fabrication process. So, these irregularities will disrupt the translational symmetry of the slab.

Linear defects in slabs

- **Effects of Disorder:**

Coupling and Reflections: Disorder allows waveguide modes to couple to states with different wavevector values (k), and can cause reflections into the reverse-direction waveguide mode at $-k$.

- **Scaling Relation for Losses:**

Near $v_g = 0$ Band Edge: There is a specific scaling relation where the loss per unit distance due to disorder scattering into the reflected mode increases as $1/v_g^2$.

Other Loss Modes: Loss rates due to other mechanisms like radiation or absorption also increase, but they scale as $1/v_g$.

What would be the effect of disorder? coupling and reflections. So, disorder allows waveguide modes to couple to states with different wave vector values and can cause reflections in the reverse direction waveguide mode at minus k . And we can also find a scaling relation for losses. So, near v_g equals 0 band edge, there is a specific scaling relation where the loss per unit distance due to disorder scattering into the reflected mode increases as $1/v_g^2$.

And other loss modes would be something like they will basically scale as $1/v_g$. So, other when I say the other ones, the other ones are basically the radiation or the absorption. So, losses because of radiation and absorption will also increase, but they will scale as $1/v_g$. So, with that we conclude our discussion on the Leiden defects in slabs.



Point defects in slabs

Point defects in slabs

Localized Mode Trapping by Point Defects:

Resemblance to Infinite Crystals: A point defect in a photonic-crystal slab traps a localized mode similar to the corresponding mode in an infinite two-dimensional crystal.

Leaky Resonances: Due to the presence of the light cone in the slab, these localized modes are inherently leaky resonances with intrinsic vertical radiation losses.

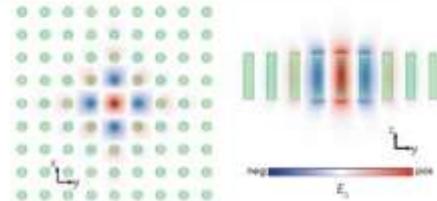


Figure 9: E_z cross sections for resonant "monopole" mode of a point defect in the rod slab (dielectric material shown as translucent green), formed by reducing the dielectric constant of the center rod and its four nearest neighbors from $\epsilon=12$ to $\epsilon=9$.

Now, we will briefly touch upon point defects in slabs. So, you can see here, these are localized mode trapping by point defects. And a point defect in a photonic crystal slab traps a localized mode similar to the corresponding mode in a infinite two-dimensional crystal. And due to the presence of the light cone in the slab, these localized modes are inherently leaky resonances with intrinsic vertical radiation loss. And you can think of creation of a monopole state in the rod slab. So, by simply removing a rod from this kind of a structure or in a 2-dimensional crystal or a 3-dimensional crystal is not very effective due to inadequate vertical confinement. Rather, what you have to do, you can either reduce the radius or the dielectric constant of the rod and its nearest neighbors and that could create a desirable monopole state. For instance, you can think of reducing the dielectric constant from epsilon equals 12 to epsilon equals 9 for a particular rod and its four neighbors.

Point defects in slabs

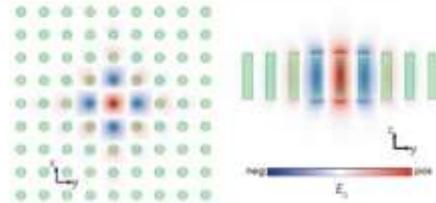
Creation of a Monopole State in the Rod Slab:

Ineffective Strategy: Simply removing a rod, as might be done in two- or three-dimensional crystals, is ineffective due to inadequate vertical confinement.

Effective Modification: Reducing either the radius or the dielectric constant of a rod and its nearest neighbors can create a desirable monopole state. For instance, reducing the dielectric constant from $\epsilon=12$ to $\epsilon=9$ for a rod and its four nearest neighbors.

Field Pattern: Figure illustrates the field pattern of this defect mode, characterized as a monopole-pattern TM-like mode.

Radiative Lifetime: The modified defect mode exhibits a radiative lifetime with a quality factor $Q_r=13000$



So, here you can actually see the field pattern for this kind of a defect mode. So, this is that rod and these are the four neighbors for whom the dielectric constant is changed. And this is the side view, this is the top view. So, they basically show you a TM like mode. So, this is Z and Y and what we are plotting is basically E_z and this modified defect mode would exhibit a radiative lifetime with a quality factor of QR that is around 13000.

Point defects in slabs

Influence of Substrate on Mode Lifetime:

Observations from "Quality Factors of Lossy Cavities" (Lecture 20): The presence of a substrate significantly impacts the lifetime of resonant modes in a slab.

Lifetime Comparison: Table 1 compares the radiative lifetime Q_r of the monopole state across various substrate choices.

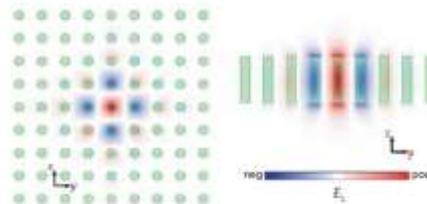
Substrate Configuration Impact:

Reduced substrate losses are noted when the substrate shares the same cross-section as the slab.

Placing a layer of the substrate material on top of the slab restores z symmetry, which helps prevent polarization mixing and reduces in-plane radiative losses.

	Symmetric	Asymmetric
Suspended membrane	13000	—
$\epsilon = 2.25$ pillars	7200	8100
Solid $\epsilon = 2.25$ substrate	380	370

Table: Intrinsic radiative lifetimes Q_r for the point-defect structure of figure 9 (below) resting on various substrates: air, $\epsilon = 2.25$ pillars with the same cross section as the rod slab, and solid $\epsilon = 2.25$.



In the next lecture, we will come into the discussion of how this large quality factors can be obtained. So, here we will just show you the influence of substrate on the mode lifetime. So, if you try to recall something from lecture 20, observations from the quality factors of lossy cavities, you would recall that the presence of substrate would significantly impact the lifetime of resonant modes in a particular slab. And the table 1 actually shows this kind of a comparison.

suspended membrane in air. So, they have like 13000, if you have you know epsilon equals 2.25 kind of pillars you have means you are still having those holes. So, they you slightly it reduces, but if you have a solid substrate of the same material that were used for making those you know pillars earlier. So, in that case it significantly drops. So, reduced substrate losses are noted when the substrate shares the same cross section as the slab. So, you also need to drill those periodic holes in the substrate or whatever is the you know if it is a rod structure your substrate should also have the same kind of cross section.

So, simply placing you know a solid substrate will not help. Placing a layer of substrate material on top of the slab will also help restore the Z symmetry and that would help prevent enough polarization mixing and would reduce in-plane radiative losses to some extent. So, there are some trade off here. So, restoring jet symmetry increases the mean dielectric constant. So, enhancing the local density of the radiative states which can counteract the benefits of you know reduced polarization mixing.



- **Trade-off:** Restoring z symmetry increases the mean dielectric constant, enhancing the local density of radiative states, which can counteract the benefits of reduced polarization mixing.

- **Strategic Considerations:**

No Clear Advantage to Symmetrization: The effects of increasing the mean dielectric constant and reducing polarization mixing nearly counteract each other, providing no compelling reason to symmetrize the system in this scenario.

So what is important? So no clear advantage to symmetrization. So when you put something like the super state, it is actually the benefit is getting counterbalanced. So the effect of increasing the mean dielectric constant and reducing polarization mixing nearly counteract each other. And that is why there is no compelling reason to put a super straight and try to make the system symmetric in this kind of scenario. So, you can actually avoid doing that because that does not bring anything good on the table.



Thank You

So, with that we will stop here. Thank you and we will be going for detailed discussion of engineering high Q resonant cavities in the next lecture. If you have any query regarding this lecture, you can drop an email to this email address mentioning MOOC, photonic crystal and the lecture number on the subject line. Thank you.

