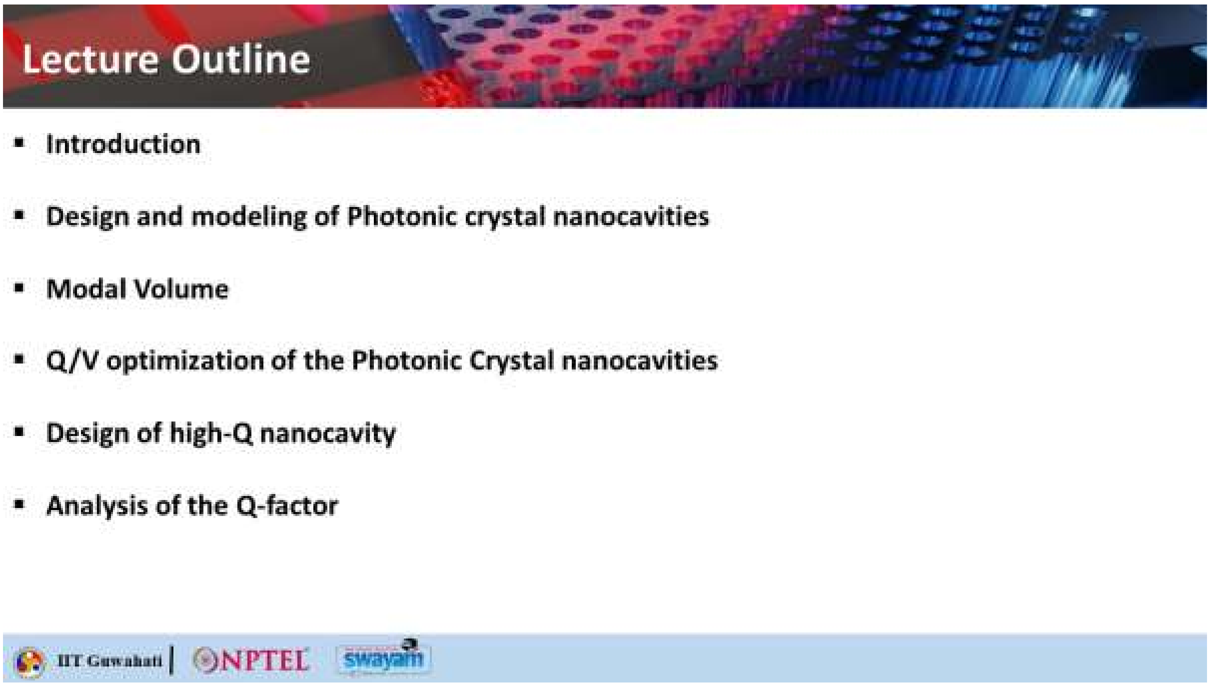
Lec 24: Engineering High-Q resonant Cavity

Hello students, welcome to lecture 24 of the online course on Photonic Crystals Fundamentals and Applications. Today's lecture will be on engineering high quality factor resonant cavities or you can say high Q resonant cavities.



. So, here is the lecture outline, we will have a brief introduction to the topic, we will discuss about designing and modeling of photonic crystal nano cavities. We will see how do we calculate the modal volume, then quality factor by volume ratio that is Q/V ratio optimization for the photonic crystal nano cavities. What is the method for designing high Q nano cavity and we will look into the methods for analyzing the quality factor.

Introduction

Introduction

- Photonic crystal-based resonant cavities with point defects are key components in manipulating and controlling light at the nanoscale.
- These defects, intentionally introduced variations in the periodic structure of a photonic crystal, localize light by disrupting the uniformity that usually allows photons to propagate freely.

Localized Modes: Point defects in photonic crystals create localized modes where light is trapped in the vicinity of the defect.

This confinement is due to the disruption of the photonic band structure, which otherwise would guide light through the crystal.

Resonant Frequencies: The defect acts as a resonant cavity, where only certain frequencies of light, determined by the size, shape, and refractive index of the defect, are allowed to resonate.

This selective frequency response is crucial for applications in filters, lasers, and sensors.

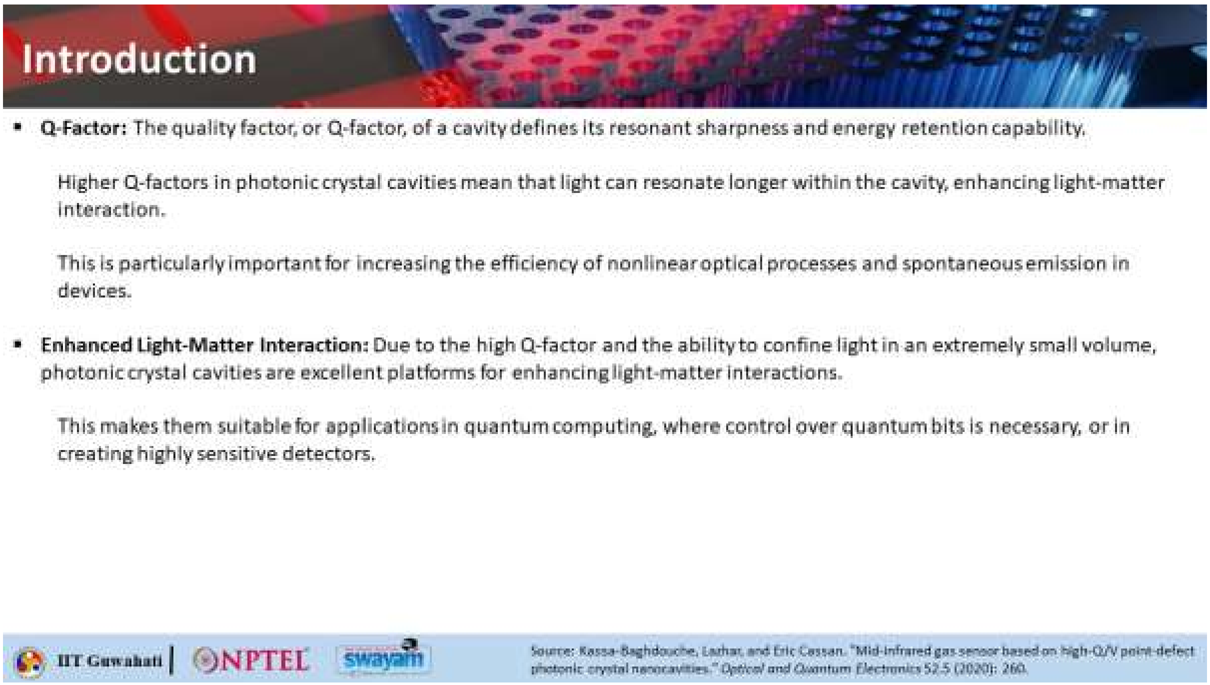
Source: Kassa-Baghdouche, Lazhar, and Eric Cassan. "Mid-infrared gas sensor based on high-Q/V point-defect photonic crystal nanocavities." *Optical and Quantum Electronics* 52.5 (2020): 260.

So, to begin with.

photonic crystal based resonant cavities are formed with point defects, right. So, they are the key components for manipulating and controlling light at the nanoscale. So, these defects which are basically intentionally introduced right and they are a variation to the otherwise normal periodic structure that is the photonic crystal. So, these defects allow you to localize light by disrupting the uniformity that usually allows photons to propagate freely.

So, point defects which are created in photonic crystals can create localized modes where light is trapped in the vicinity of the defect. And this confinement is due to the disruption of the photonic band structure. So, there will be allowed mode within the photonic band gap. that would which you know otherwise would guide light through the crystal. So, what are the resonant frequencies of this cavity? So, any cavity will exhibit resonance.

So, when you consider a defect as a resonant cavity there also you know for a certain frequency of light which is determined by the size, shape and the refractive index of the you know defect those frequencies are allowed to resonate. So, this selective frequency response becomes critical in applications such as filters, lasers and sensors.



So, when you analyze a cavity, the important parameter is quality factor or the Q factor. So it basically defines the sharpness of the cavity resonance and also the energy retention capability of that cavity or the resonator. So higher quality factor in photonic crystal cavities mean that light can resonate longer within the cavity and that enhances the light matter interaction.

So this is particularly important for increasing the efficiency of nonlinear optical processes because you will now have more light matter interaction and also it can enhance the spontaneous emission in the devices. So what we know that these cavities are excellent platform for enhanced light matter interactions, right? And this is mainly because this high quality factor cavities have the ability to confine light in a very small volume, okay? Thus this photonic crystal cavities becomes excellent platform for enhancing the light matter interaction. And this makes them suitable for applications such as quantum computing where control over quantum bits or qubits is necessary or in creating you know highly sensitive detectors. So, there these are the applications where you know the cavity which is basically a defect in a photonic crystal can come very handy.

Design and modeling of Photonic crystal nanocavities

Design and modeling of Photonic crystal nanocavities

- **Types of Planar Photonic Crystal Cavities:** The H1 cavity is created by removing one lattice point at the centre, enhancing optical confinement (Yoshie et al., 2004).

  The L3 cavity involves removing three adjacent lattice points.

  L3 cavities are extensively studied for its effective reduction in radiation losses and improved Q factor (Chalcraft et al., 2007)

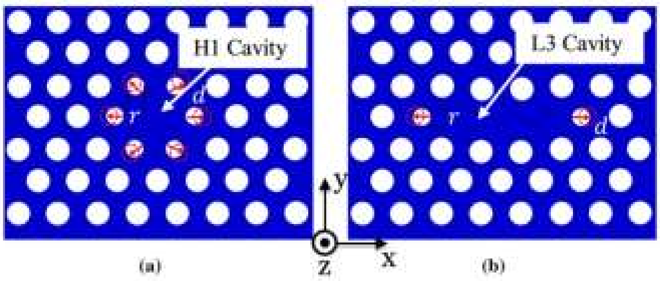


Figure 1: Schematic structures of two point-defect photonic crystal nanocavities. (a) H1 PhC cavity and (b) L3 PhC cavity

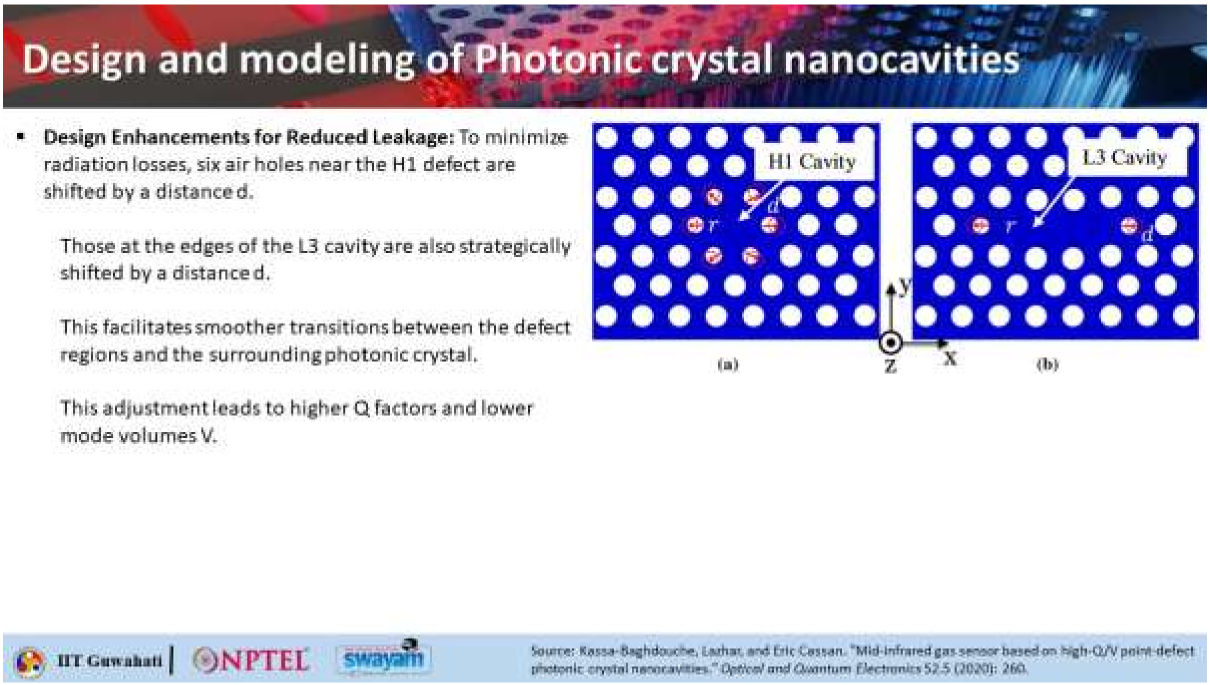
Source: Kassa-Baghdouche, Lazhar, and Eric Cassan. "Mid-infrared gas sensor based on high-Q/V point-defect photonic crystal nanocavities." *Optical and Quantum Electronics* 52.5 (2020): 260.

Now let us look into the design and modeling aspects of photonic crystal nano cavities.

So here you can see the two figures. So this first one is a cavity where one hole is missing at the center. You can see the hole is basically filled with the material of the background. So you can say that this is basically H1 cavity. One hole is missing and it is created by removing one lattice point at the center and it could enhance the optical confinement.

So it was reported by Yoshi et al in 2004. And if you see the next one, this is also another point defect cavity, but here instead of 1, we are basically removing 3 holes in a particular line. So, this L3 cavity, L you can say it is for line, 3 means there are 3 holes which are basically missing. So, L3 cavity involves removing of 3 adjacent lattice points here. And these cavities were studied extensively for its effective reduction in the radiation losses.

So, you can actually get very high Q factor from this kind of cavities and they were studied by Chalcraft et al in 2007. So, if you simply create a point defect, it is not going to work as a very high quality you know cavity, HQ cavity. You have to do some adjustment to minimize the radiation losses. So, what has been seen that in the case of H1 cavity, all the 6 neighbouring air holes need to be slightly shifted by a distance of d which you can see here. So, there is a you know dislocation of this hole, this hole, this hole and all these holes, okay, radially you can shift them outside, okay, by a distance of d, okay.



That actually allows you to minimize the radiation losses from this cavity. In the case of L3 cavity, the holes which are at the edges, they need to be strategically shifted by a distance of D. So, here you can see that you actually need to just shift two holes and that will actually help you minimize the radiation loss. So, why we are doing this? This facilitates smoother transitions between the defect region and the surrounding photonic crystal. So, there is a defect, this is a otherwise normal photonic crystal.

So, this shift helps you in getting a better matching impedance matching between these two areas or domains. So, this adjustment will lead you to higher quality factor and lower mode volume V.

Modal Volume

Modal Volume

- **Modal Volume (V)**

  Modal volume V is a fundamental concept in the field of photonics, particularly relevant in the study of optical cavities and waveguides.
  It quantifies the spatial extent over which an electromagnetic mode is confined.

  Definition
  Modal Volume V: This term describes the volume within which the electromagnetic field associated with a particular mode of an optical resonator or waveguide is significantly concentrated.

  It essentially measures how much space the mode occupies.

$$V \triangleq \int d^3\mathbf{r}\, \varepsilon|\mathbf{E}|^2/\max(\varepsilon|\mathbf{E}|^2)$$

  This integral gives a sense of how spread out the mode is; the smaller the volume, the more tightly the mode is confined.

Source: J. D. Joannopoulos et al., "Photonics Crystals: Molding the Flow of Light", Princeton Univ. Press, 2008.

Now this is some important factor modal volume V and we will see now what is that modal volume. So, modal volume you can represent it by V is basically a fundamental concept in the field of photonics which is relevant in the study of optical cavities and waveguides. So, it basically quantifies the spatial extent over which an electromagnetic mode is confined.

So, what is the formal definition? So, the modal volume V, this particular term defines the volume within which the electromagnetic field associated with a particular mode of an optical resonator or waveguide is significantly concentrated. So, it is essentially measuring how much space a particular mode is going to occupy. So, how do you calculate this? You can calculate this by as V, which is a volume integral, okay? And then you do $V = \int d^3\mathbf{r}\varepsilon|\mathbf{E}|^2 / \max\left(\varepsilon|\mathbf{E}|^2\right)$ okay? So, this basically gives you the modal volume. So, as you can see, this integral gives a sense of how spread out your mode is. So, smaller the volume, the tighter is the mode confinement.

Modal Volume

- The Q/V ratio, representing the quality factor (Q) divided by the modal volume (V), is a critical parameter in the design of resonant photonic crystal cavities using point defects. Here are some key points illustrating its importance:

1. Enhanced Light-Matter Interaction: A high Q/V ratio indicates strong confinement of light within a small volume, enhancing the interaction between light and matter.

This is crucial for applications that rely on efficient absorption, emission, or scattering of light, such as in lasers, sensors, and nonlinear optical devices.

2. Increased Purcell Factor: The Q/V ratio directly influences the Purcell factor, which quantifies the enhancement of spontaneous emission rates within a cavity.

A higher Q/V ratio leads to a greater Purcell effect, thereby increasing the emission efficiency of embedded quantum emitters like quantum dots or molecules.

IIT Guwahati | NPTEL swayam Source: J. D. Joannopoulos et al., "Photonics Crystals: Molding the Flow of Light", Princeton Univ. Press, 2008.

So, what becomes important in your case is this Q/V ratio that is the quality factor divided by the modal volume ratio. So, this becomes a critical parameter in the design of this resonant photonic crystal cavities based on point defects.

Modal Volume

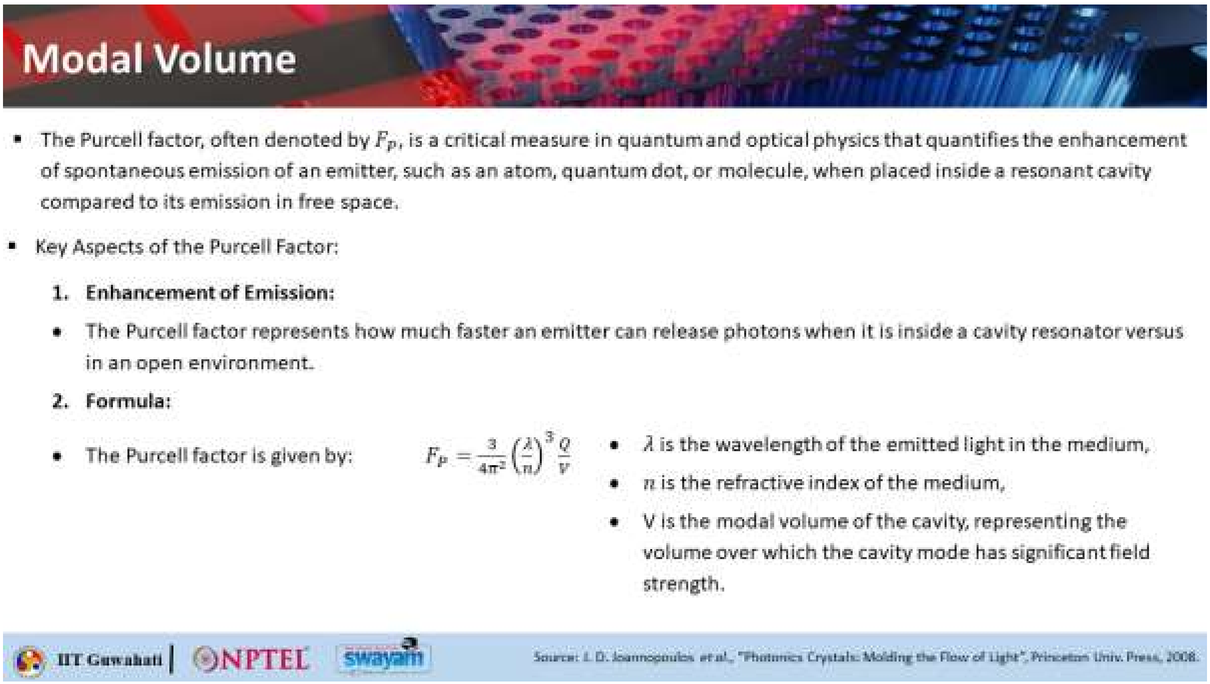
3. Improved Device Performance: In optical communication and computing, devices designed with a high Q/V ratio can achieve superior performance characteristics, including lower threshold lasers, more sensitive detectors, and more effective modulators.

This is due to the intensified field interactions within the minimized modal volume.

4. Energy Efficiency: Devices with a high Q/V ratio are generally more energy-efficient.

The strong confinement of the electromagnetic field reduces the power requirements for achieving significant optical effects, thus lowering operational costs and enhancing device longevity.

IIT Guwahati | NPTEL swayam Source: J. D. Joannopoulos et al., "Photonics Crystals: Molding the Flow of Light", Princeton Univ. Press, 2008.



Now we will discuss some of the important points that will illustrate the you know significance of this particular concept. The first thing is enhanced light matter interaction. So, a high Q/V ratio will indicate strong confinement of light within a small volume and that will definitely enhance the interaction between the light and matter.

And this is critical for applications that rely on efficient absorption, emission, or scattering of light, such as you will find in lasers, sensors, and different nonlinear optical devices. Secondly, it will help you increase the Purcell factor. So the Q/P ratio directly influences the Purcell factor, which quantifies the enhancement of spontaneous emission rate within a cavity.

So, the emission mentioned here in the context of Purcell factor refers to the spontaneous emission of photons by quantum emitters such as quantum dots or molecules when the transition from excited energy state to a lower energy state or their ground state. So, this process is a fundamental phenomena in quantum optics and photonic devices.

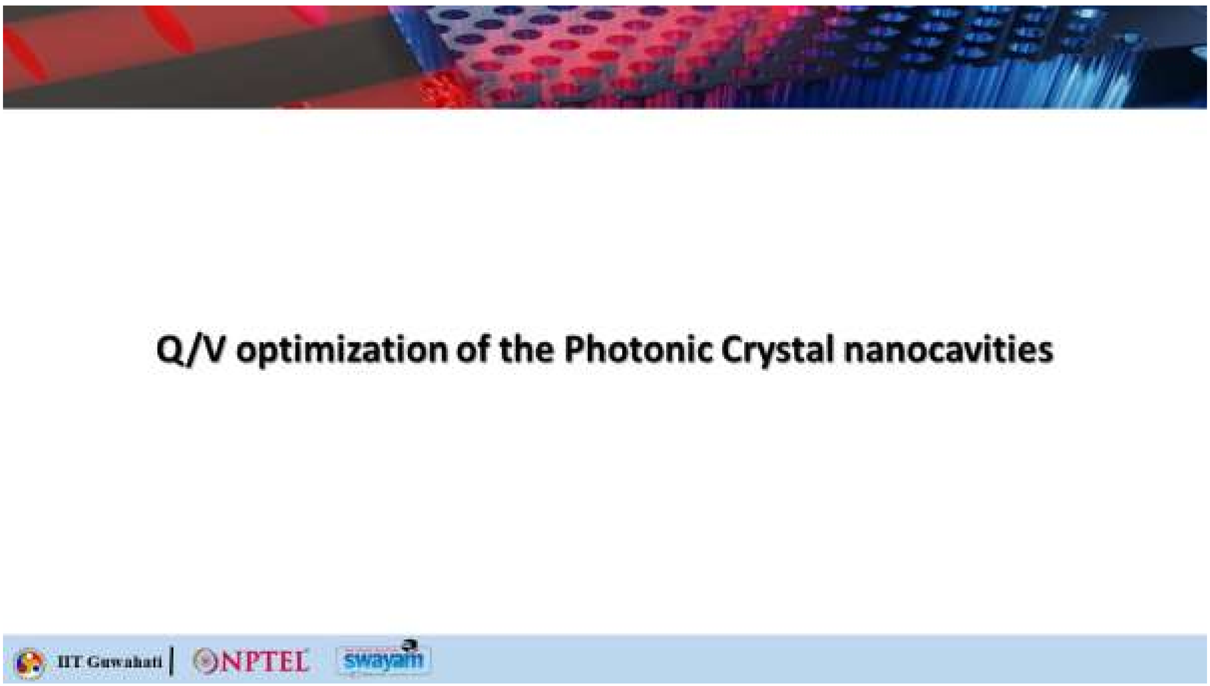
Okay you can also think about you know quantum emitters like what are the different quantum emitters? Quantum emitters like quantum dots, atoms or dye molecules they can absorb energy which would excite them to a higher energy state and after a certain time which is typically known as the excited state lifetime they will return to a lower energy state typically to the ground state. by doing spontaneous emission. So, they will release a photon. So, this process is known as spontaneous emission. And what will be the nature of the emitted photons? The energy of the emitted photon will correspond to the energy difference between the excited state and the lower state of the emitter.

And typically the direction, phase and the moment of emission are random due to the quantum nature of this process. So, what we can see that you know a point defect in photonic crystal cavity with higher Q/V ratio can lead to a greater Purcell effect thereby increasing the emission efficiency of embedded quantum emitters such as quantum dots or molecules. Third important point is that this will essentially improve the device performance. So, in optical communication and computing, if you consider devices designed with high Q/V ratio, they can achieve superior performance. Something like you know low threshold lasers, you can design more sensitive detectors and even more effective modulators.

this is due to the intensified field interactions within the minimized modal volume. So, in a very small volume you can have much more intense interaction of light and matter. The fourth factor is energy efficiency. So, if you have devices based on high Q/V ratio, they are generally more energy efficient. The strong confinement of the electromagnetic field would basically reduces the power requirements for achieving significant optical effects and that will lower the operational cost and enhances device longevity.

So, the Purcell factor that we have discussed can be denoted by Fp. It is basically a critical measure in quantum and optical physics that quantifies the enhancement of spontaneous emission of an emitter such as an atom, quantum dot or molecule when placed inside a resonant cavity compared to its emission in free space. So, that is how the emitters will also become more efficient when you put them inside a high Q/V ratio cavity. So, what are the key aspects of the Purcell factor as we discussed? The first one is enhancement of emission. The Purcell factor basically represents how much faster an emitter is able to release photons when it is inside a cavity resonator versus the same emitter in an open environment.

what is the formula that gives you the Purcell factor? It is fp given by $F_P = \frac{3}{4\pi^2}\left(\frac{\lambda}{n}\right)^3 \frac{Q}{V}$. So, n is basically the refractive index of the medium. What is lambda? That is the wavelength of the emitted light in the medium. and V is nothing but the modal volume of the cavity and it represents the volume over which the cavity mode has significant field strength.



Q/V optimization of the PhC nanocavities

- **Optimization of Point-Defect PhC Nanocavities:** Geometrical optimizations, including reducing the radius of air holes near the defect and introducing lateral displacement (d), enhance the Q/V ratio by minimizing reflection mismatch and propagation losses (as shown in figure).
- **Effect of Lateral Displacement on Q/V Ratio:** As shown in Fig. 2, the Q/V ratio initially increases with the lateral displacement (d), reaches a maximum at an optimal value, and then decreases if d exceeds this optimal value, indicating the importance of precise geometrical adjustments.

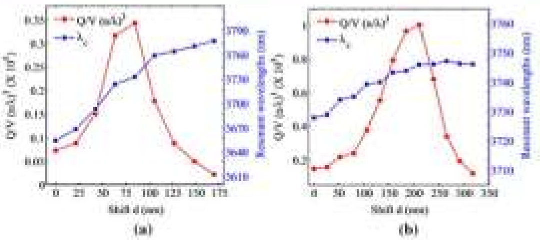


Figure: 2 The evolution of the Q/V ratio and resonant wavelength (λ_c) of the designed point-defect PhC nanocavities mode as a function of the displacement of single air holes d for reduced air-hole radii r' = 212 nm. (a) H1 PhC cavity and (b) L3 PhC cavity

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Source: Kassa-Baghdouche, Lazhar, and Eric Cassan. "Mid-infrared gas sensor based on high-Q/V point-defect photonic crystal nanocavities." *Optical and Quantum Electronics* 52.5 (2020): 260.

So, with that we move on to the methods of optimization of Q/V ratio in the case of photonic crystal nano cavities.

So, when you talk about optimization we have to play with the you know geometrical parameters something like the radius or the shift that can significantly change the q by v ratio so we have to do geometrical optimization including reducing the radius of the air holes near the defect and you can also introduce you know lateral displacement which is D as we discussed earlier. These two things can enhance the Q/V ratio by minimizing the reflection mismatch and the propagation losses okay. So, here in figure 2 you can see that you know Q/V the red one show the Q/V ratio and lambda c shows you the cavity resonance wavelength and it is basically function of d. So, the blue one is plotted here and the red curve corresponds to this axis. So, it is basically times 10 to the power 5.

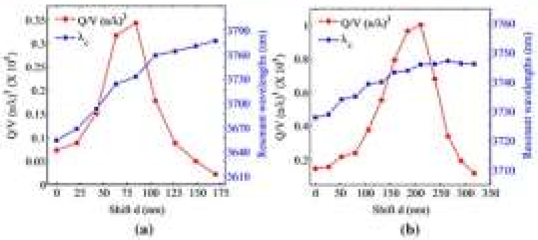
So, what you can see here that the Q by V ratio initially increases with the lateral displacement D, it will reach a maximum at some optimal value and then it will drastically decrease again if you go further. That means you need to do precise geometrical adjustment to find out this optimal shift that can give you very high Q/V ratio. Now, what are these two curves? The first one is for the H1 photonic crystal cavity and the second one is basically for L3 photonic crystal cavity. So, what is seen here is that even with small adjustment from here to here which is something like 10 nanometer shift, you can significantly enhance the Q/V ratio.

It means it is pretty sensitive. If you consider the performance of L3 photonic crystal cavity that is this particular figure B. Here also you can see that with about 10 to the power 5 order of Q/V ratio you can achieve. You can go here. that is much higher than this H1 cavity.

Q/V optimization of the PhC nanocavities

- **Optimal Displacement and Q/V Improvement:** Specific adjustments like a 10 nm increase in d significantly enhance the Q/V ratio.

- **Performance of L3 PhC Nanocavities:** L3 PhC nanocavities also demonstrate high Q/V values, with about 10^5 for a displacement of 212 nm at a resonant wavelength of 3740 nm , underscoring the effectiveness of this approach for different nanocavity designs.



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Source: Kassa-Baghdouche, Lazhar, and Eric Cassan. "Mid-infrared gas sensor based on high-Q/V point-defect photonic crystal nanocavities." *Optical and Quantum Electronics* 52.5 (2020): 260.

So, here the maximum value is 0.35 into 10 to the power 5, here the maximum value is 1 into 10 to the power 5. So, it is much stronger resonance and higher Q/V value and you can see that the shift here is also larger, it is showing a displacement of 212 nanometer and it corresponds to a resonant wavelength of 3740 nanometer. So, it actually highlights or underscores the effectiveness of this approach of you know optimization to achieve a very good resonant cavity which can give you extremely high Q/V ratio, okay. We understood that because of very high Q/V values, the L3 cavities leading to a potential design for various applications of resonators based on photonic crystals. So, L3 becomes the obvious choice, right.

Design of high-Q nanocavity

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Design of high-Q nanocavity

- **Design Rule for High-Q Nanocavities:**
  The envelope function of the electric field profile within the cavity plays a critical role.

  The in-plane mode profile's envelope should vary gently but remain spatially localized, resembling a Gaussian function.

  This helps achieve strong optical confinement in a small area, increasing the Q factor while maintaining a compact volume (V).

- **Mechanisms of Light Confinement:**
  In-plane Direction: Utilizes the photonic-bandgap effect to confine light, typical for a 2D photonic crystal (PC) slab.

  Vertical Direction: Employs total internal reflection (TIR) at the slab-air clad interface to confine light.
  This is essential for the functionality of high-Q nanocavities.

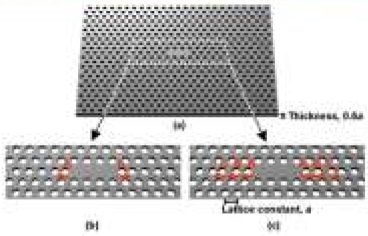

Figure 3: (a) Schematic of the point-defect nanocavity in a 2D photonic crystal (PC) slab. (b)The designed cavity structure created by displacing two air holes at both edges in order to obtain high-Q factor (c) The designed cavity structure created by fine-tuning the positions of six air holes near both edges to obtain an even higher Q factor.

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Source: Yoshihiro Akahane, Takashi Asano, Bong-Shik Song, and Susumu Noda, "Fine-tuned high-Q photonic-crystal nanocavity," Opt. Express **13**, 1202-1214 (2005)

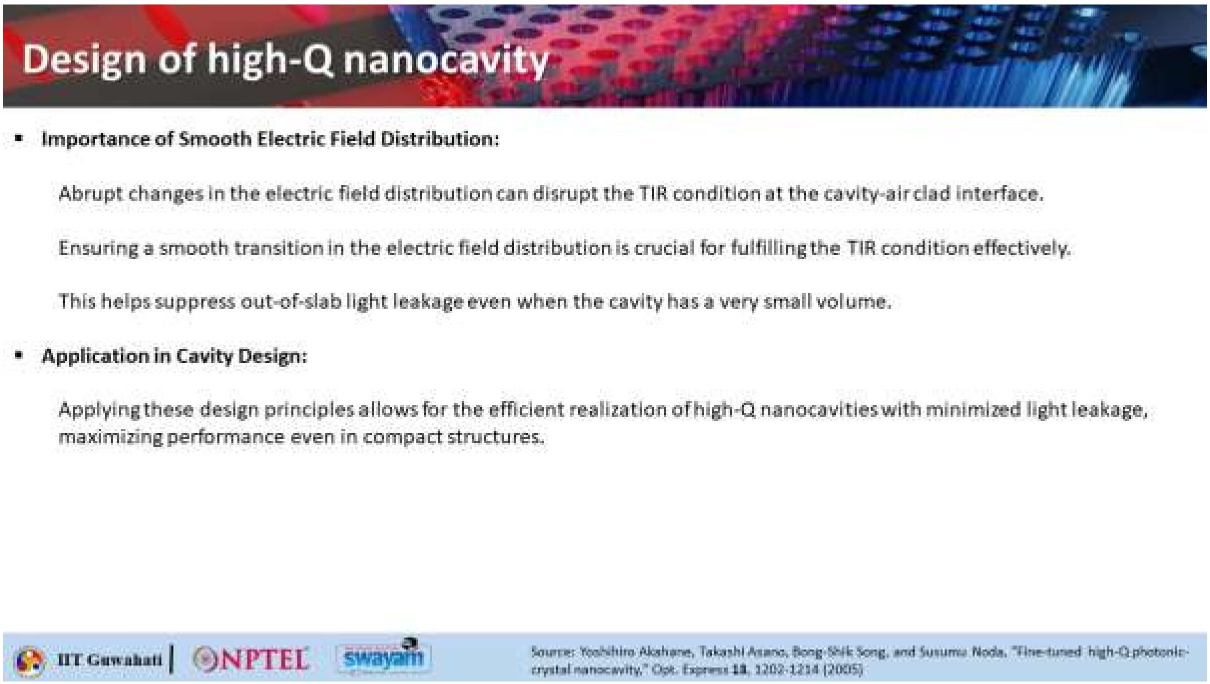
So, in the following discussion, we will take different strategies of designing this L3 photonic crystal cavities and we will also see how the quality factor is being calculated. So, now we go into the design aspect of this high Q nano cavity and as mentioned we will be focusing more on the L3 kind of cavity. So, the Q factor per modal volume that is the key parameter here that is the Q/V ratio. That determines the strength of various cavity interactions and an ultra-small cavity enables large scale integration along with single mode operation over a broad range of wavelength. However, the high Q cavities with dimensions of the order of optical wavelengths are difficult to realize since radiation losses are basically inversely proportional to the cavity size.

So if you see here, this one shows the schematic of a point defect nano cavity. So, this has got point defect in a 2D photonic crystal. And this one, the figure B shows the design cavity which is created by displacing two air holes, one this side, one this side at both edges. So, this is again a L3 cavity. And here you are basically displacing 1, 2, 3 on one side, these 3 holes are being displaced A, B, C.

Here on the other side also you are displacing 3 holes. So, 6 air holes are basically displaced here near the 2 edges. So, that could give you even higher quality factor. So, if you see the basic structure, it is basically composed of three missing holes in a particular line.

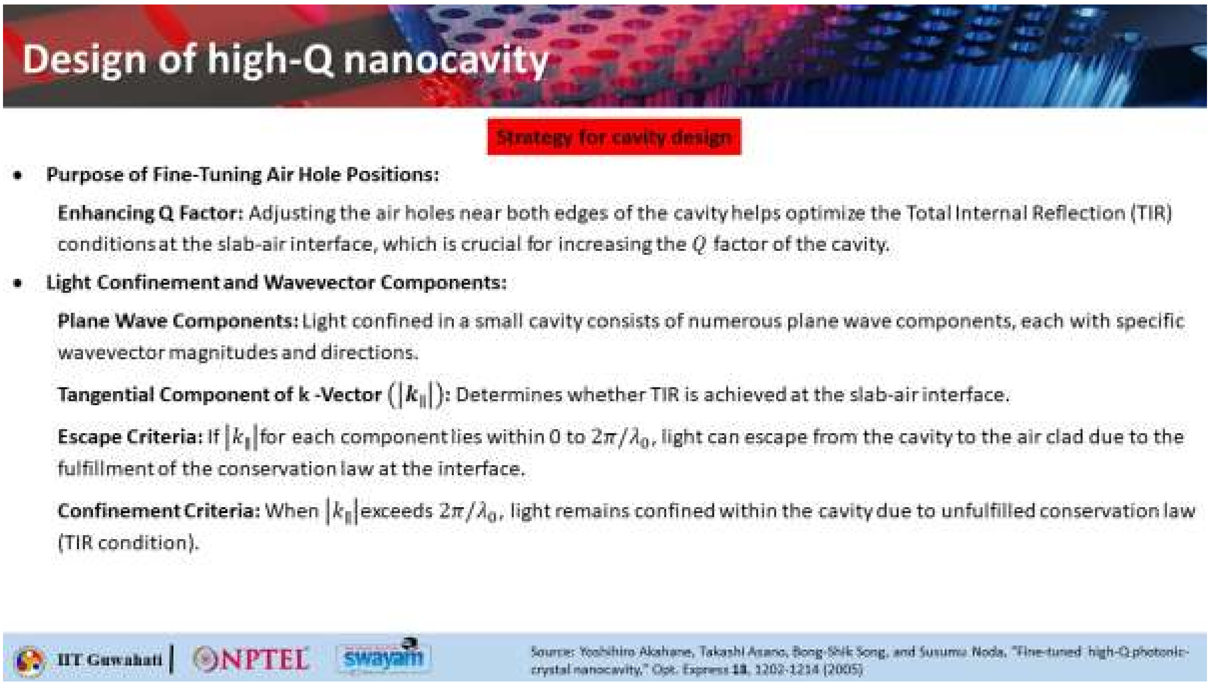
So, it is again a L3 cavity. The photonic crystal structure, you can see it has got a triangular lattice of air holes with a lattice constant of **a**. It is marked here and the thickness of this slab is considered to be 0.6a. and the radius of the air holes are considered to be 0.29A. These are the physical parameters. Now, what are the important design rules for achieving high Q nano cavities? The envelope function of the electric field profile within the cavity will play an important role. So, the in-plane mode profiles envelope should vary gently but remain spatially localized resembling a Gaussian function. So, this helps achieve strong optical confinement in a small area thus increasing the quality factor while maintaining a very compact volume. So, what are the mechanisms of light confinement? First, in the in-plane direction, it utilizes the photonic bandgap effect to confine light. So, you will actually place a light in the frequency of the bandgap of this photonic crystal so that light cannot escape out.

So, that is how you typically can get from this 2D photonic crystal slab. You know what is the bandgap and you can put a, you can choose the wavelength which is within the bandgap of this slab. dimension, it basically employs total internal reflection at the slab and air cladding interface and that will give you the confinement of light, okay. So, this confinement is essential for the functionality of the high Q nano cavities or else like will leak out and you will not get a very good quality factor of resonance.



Next, what is the importance of smooth electric field distribution? So, if you have abrupt changes in the electric field distribution that can disrupt the total internal reflection condition which occurs at the cavity and the air clad interface.

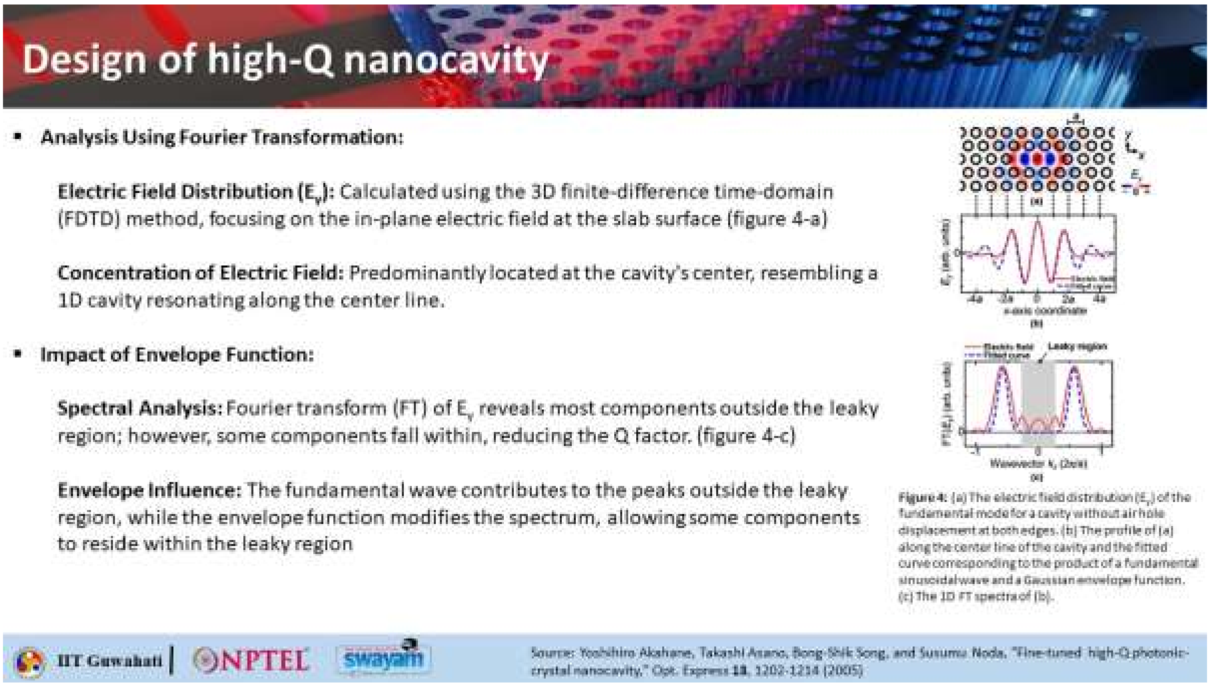
So, to ensure smooth transition in the electric field is crucial for fulfilling the total internal reflection conditions effectively. And this will also help suppressing out of slab light leakage even when the cavity is having a very tiny volume.



Now, let us apply these conditions and understanding in the cavity design. So, if you are applying these design principles in making a cavity that will allow you to realize a high Q nano cavity with minimized light leakage and maximizing the performance even within a compact structure. So, the important strategies for cavity designs are the fine tuning of air hole positions.

We have seen that D that small displacement plays a very important role. So, that allows you to enhance the quality factor. So, what you have to do there adjusting the air holes near both edges of the cavity. So, we are right now only focused about the L3 cavity because we have seen that L3 cavity is giving us much higher quality factor as compared to H1 cavity that is a single hole cavity. So, adjusting the air holes near both edges of the cavity helps to optimize the total internal inflection condition at the slab air interface and that is important for increasing the quality factor of the cavity.

Second aspect important aspects are light confinement and wave vector components. So, light confined in a small cavity will consist of numerous plane wave components each with specific wave vector magnitudes and directions. So, these are for the plane wave okay. However, the tangential component of the k vector that is $\boldsymbol{k}_{\parallel}$ this will determine whether the total internal reflection is achieved at the slab air interface. Now, what is the escape criteria? If $\boldsymbol{k}_{\parallel}$ or you can say mod k parallel for each component lies between 0 and $2\pi/\lambda_0$, light can escape from the cavity to the air cladding due to the fulfillment of conservation of law or you can say momentum conservation law at the interface.



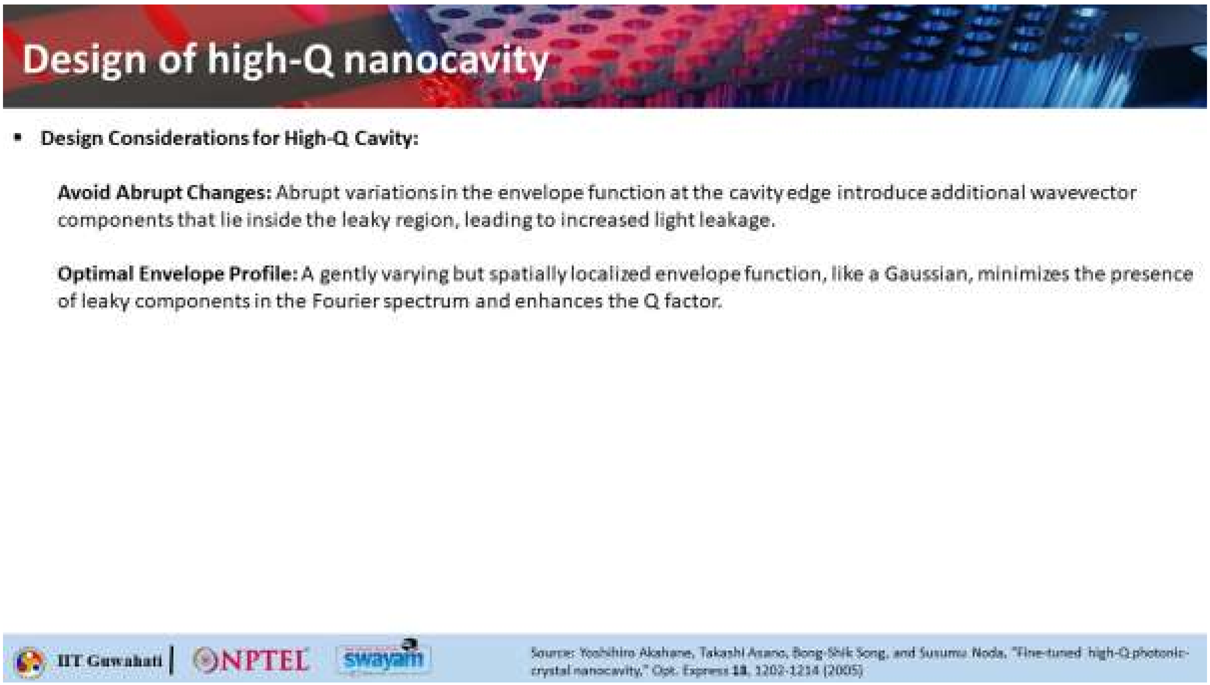
So, how you can make the light confined within the cavity? So, what is the confinement criteria? You can confine light when modulus k parallel okay that is the tangential wave vector okay needs to be larger than $2\pi / \lambda_0$. That means in that case light will remain confined within the cavity due to unfulfilled conservation law that is total internal refraction. Now, we can analyze this using Fourier transformation. So, here is a figure that shows first the electric field distribution that is Ey of the fundamental mode of the cavity without a air hole displaced at both edges. So, here you are not displacing the air holes at the both edges and this figure B is the profile of A along the center line.

So, you are just taking a line and this is how the variation will look like. So, blue means a dip, red means a peak because that is how the values are defined. So, here you get a peak, next you move you will get a dip and so on. So, you are basically going across the center. So, what are here basically you are having a electric field light and also the fitted curve which corresponds to the product of fundamental sinusoidal wave and a Gaussian envelope function.

And this figure C is basically a 1D Fourier transform spectra of this one. So, this is in case space, this is in physical space fine. So, here you can actually see that this region the gray region from here to here is being marked as the leaky region. So, what we have seen here is a electric field distribution plot in 3D FDTD. So, you have used this 3D FDTD method to obtain this in-plane electric field distribution at the slab surface.

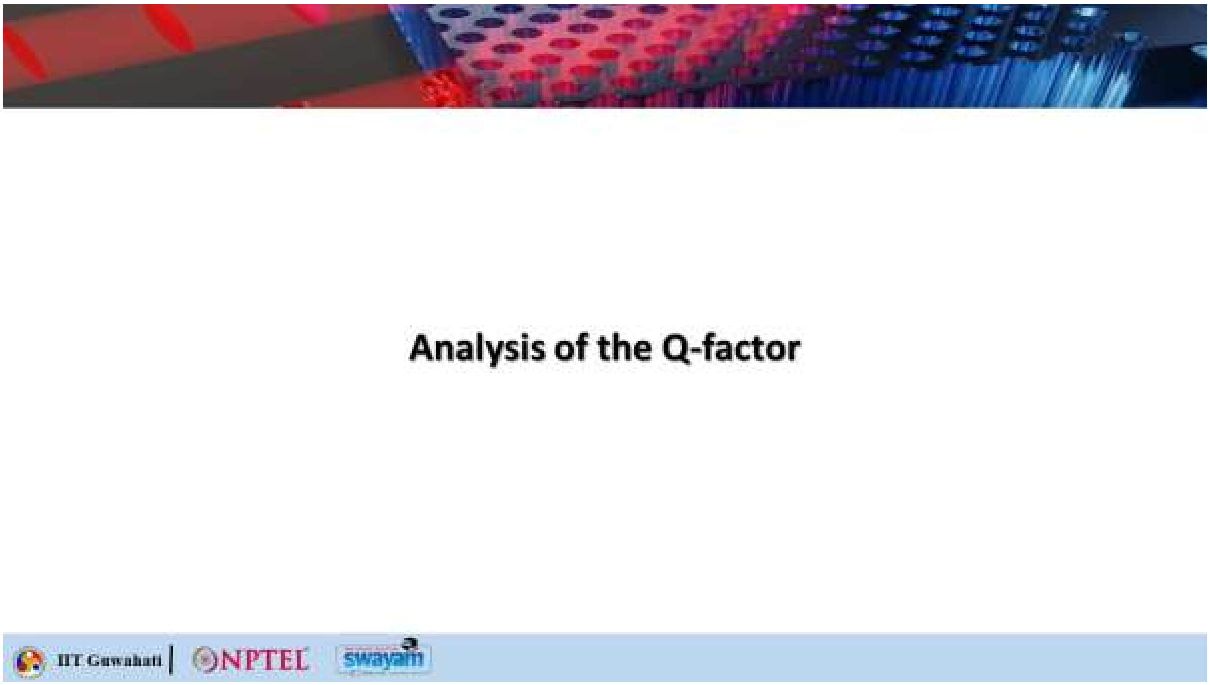
So, you can also see that the electric field is predominantly located at the cavity center. So, it resembles a 1D cavity resonating along the center line as you can see here. Now what is the impact of the envelope function? So the Fourier transform of this Ey has revealed that the most components outside the leaky region and this will actually help you to hold the resonance. But you can see that some components still fall within the leaky region which is basically the shaded region and that will reduce the quality factor. So, what is the envelope influence? The fundamental wave contributes to the peaks which are outside the leaky region.

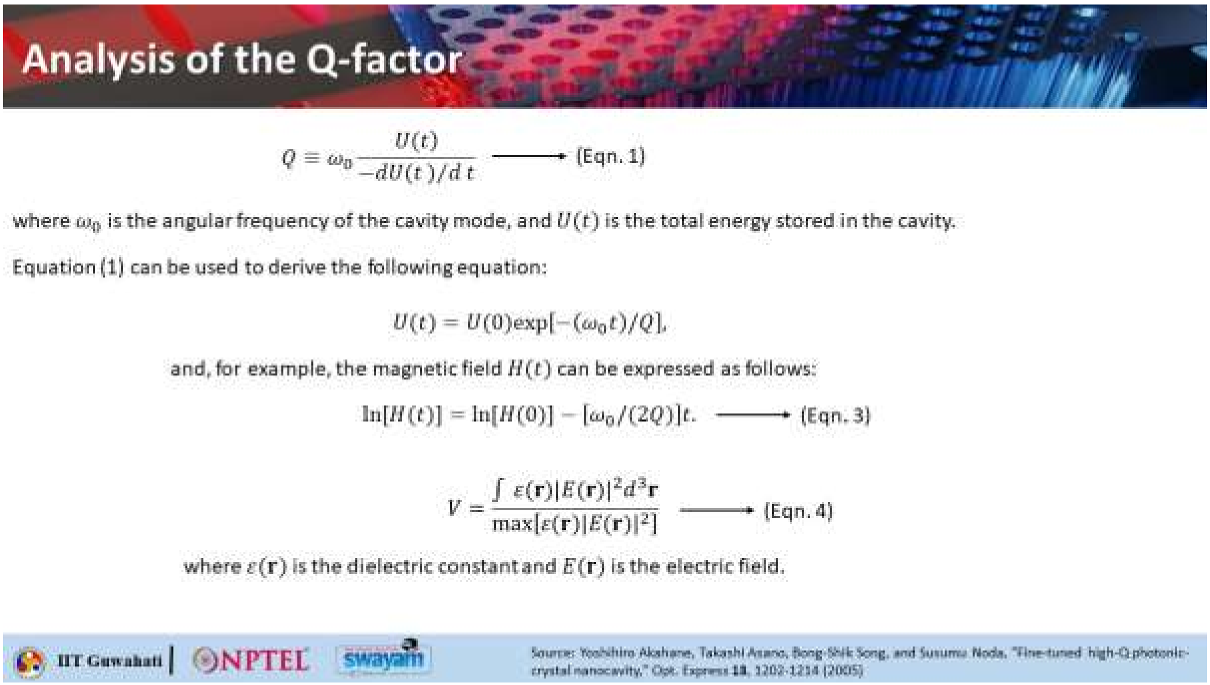
And while the envelope function basically modifies the spectrum allowing some component to be within the liquid region.



So, what are the design considerations for high Q cavity? First thing is you have to avoid any abrupt changes. So, any abrupt variation in the envelope function at the cavity edge will introduce additional wave vectors or wave vector components and that would lie typically inside the leaky region and anything inside the leaky region will basically increase the light leakage. So, what should be the optimal envelope profile? It should be a gently varying but specially localized envelope function something like Gaussian that could minimize the leaky component in the Fourier spectrum.

within that liquid region and then it can enhance the quality factor.



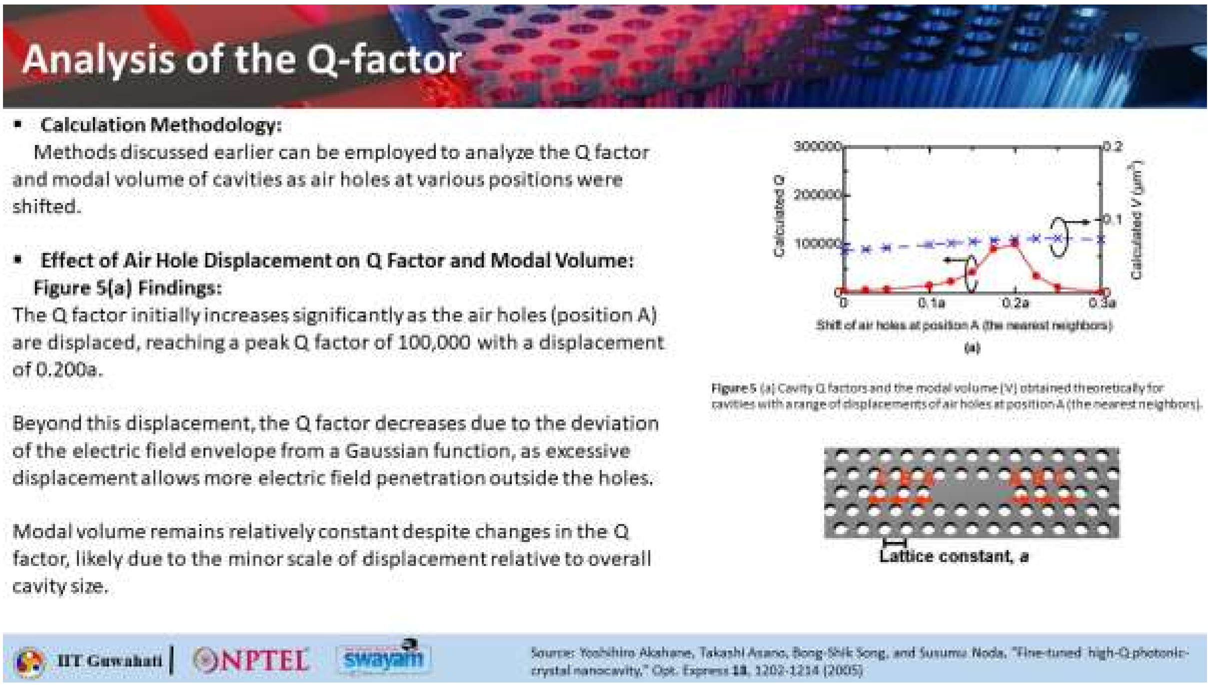


Now, we move on to the methods of analyzing the quality factor. So, here we will describe the methods that can be used to calculate the quality factors of the cavity modes. The Q factor can be expressed as this. $Q \equiv \omega_0 \frac{U(t)}{-dU(t)/dt}$. So, as you can see from this equation 1, the Q factor can be calculated by measuring the slope of the decay of any field. So, what is ω_0 here? It is the angular frequency of the cavity mode and $U(t)$ is basically the total energy stored in the cavity.

And the equation 1 can be derived, can be used to derive the following equation. So, you can write

$U(t) = U(0)\exp\left[-\left(\omega_0 t\right)/Q\right]$, . And for example, you know, the magnetic field can be expressed as, $\ln[H(t)] = \ln[H(0)] - \left[\omega_0/(2Q)\right]t$, okay. So, what we understand from here is that you can find out the modal volume of the cavity by inserting the calculated electric field distribution into this particular equation. So, epsilon r is basically the dielectric constant and E is basically the electric field.

right. So, this is how you can first obtain the magnetic field from that you can obtain the electric field and you can also put it in this equation and obtain what is V.



So, you can calculate Q and V and you can get the Q/V ratio. So, what are the calculation methodology? So, the methods discussed earlier can be employed to analyze the Q factor and the model volume of the cavities as air holes at various positions are shifted. So here you can see slightly enlarged figure.

So ABC are the three holes which move this way. Here also ABC are the three holes which are moving this side. So, this is the first case where the first case is basically where you know only the displacement is happening at the air holes which are at position A. So, only these two air holes are being shifted. So, shift of air holes at position A. So, this blue dot line tells you about the calculated modal volume which is in the unit of micrometer cube and the red lines tells you about the calculated cube.

So, what you can see that with shift in this only first hole A, your quality factor can reach 10 to the power 5. when the displacement is 0.2a. And beyond this displacement if you go further the quality factor will decrease and that is because of the deviation of the electric field envelope from the Gaussian function. So, if the envelope distorts from a Gaussian function, you will see that there will be more fields into the leaky region and the quality factor will basically get reduced, okay.

So, here the modal volume remains relatively constant despite the change of Q and this is likely due to the minor scale of displacement as compared to the overall cavity size.

Analysis of the Q-factor

- **Additional Experiments with Displacements:**

Position B (Second Nearest Neighbors) - Figure (b):

Air holes at position A fixed at optimum displacement of 0.200a.

Maximum Q factor achieved was 130,000 with a displacement of air holes B of 0.025a, suggesting that fine-tuning position B can further enhance the Q factor.

Position C (Third Nearest Neighbors) - Figure (c):

Air holes at positions A and B fixed at their optimum displacements.

The Q factor peaked at 260,000 for a displacement of air holes C of 0.200a, indicating significant potential for increasing Q factor through adjustments at more distant neighbor positions.

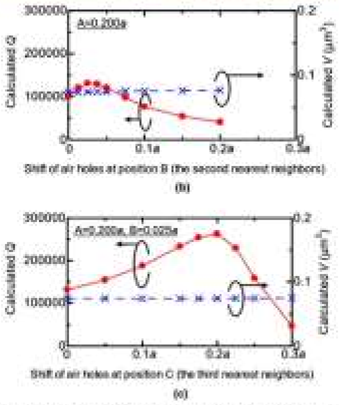


Figure 5 (b) at position B (the second nearest neighbors), whilst fixing the position of air holes A at their optimum value of 0.200a. (c) at position C (the third nearest neighbors), whilst fixing the positions of air holes A and B at their optimum values of 0.200a and 0.025a, respectively.

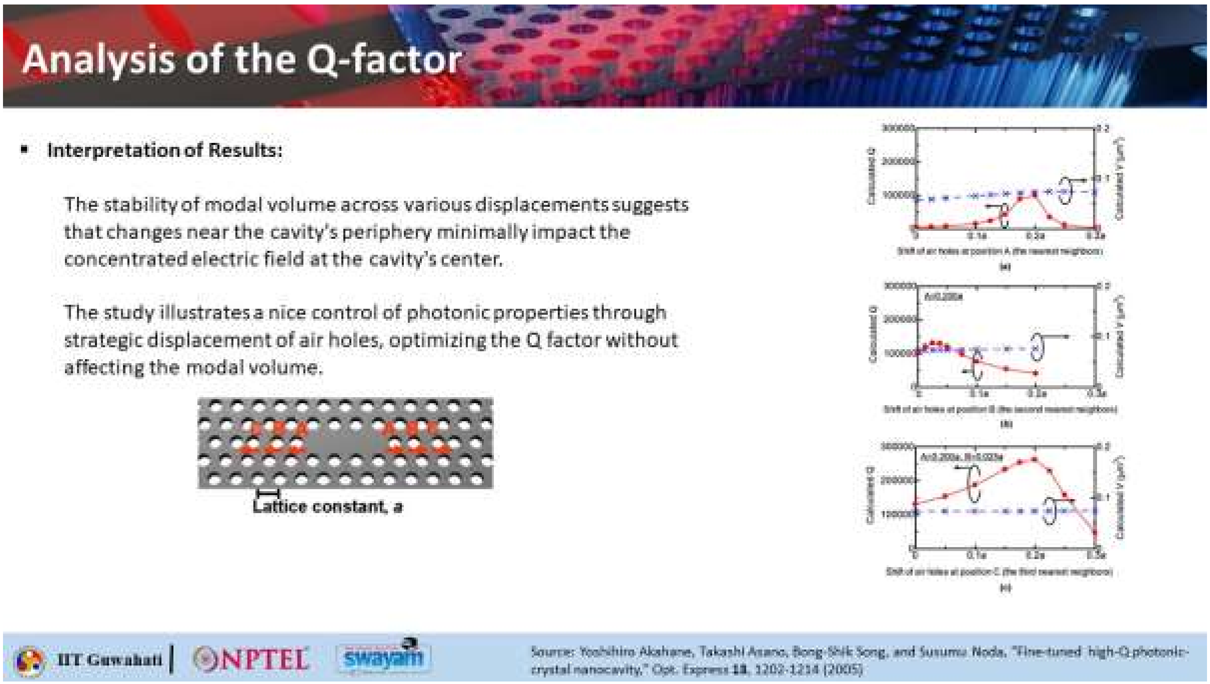
Source: Yoshihiro Akahane, Takashi Asano, Bong-Shik Song, and Susumu Noda, "Fine-tuned high-Q photonic-crystal nanocavity," Opt. Express **13**, 1202-1214 (2005)

Now you repeat the same with the second hole. The second hole is B on both side. So now you just move the air holes in B.

You see this is the method or this is how it changes. So here you can see. that you are keeping the air holes at position a to be fixed at the optimum place that is 0.2a and now you are shifting the air holes at position b and you can see that at 0.025a means here you are getting the best quality factor.

ok. So, this is telling you that you are actually going beyond 10 to the power 5. So, it means by tuning the second hole you are now being able to improve your quality factor. That is good. What happens to the modal volume? More or less same. Now you fixed A and B to their optimum position and only tune the third hole that is the C holes on both side and you can see that you can further push it away.

So this is the position C and if you can place it at 0.2a, you can go up to 260,000. will be your new quality factor and this is the fixed value of A and B in this case. So, this further helps you improve the quality factor right and this is the blue crosses tells you about the calculated modal volume.



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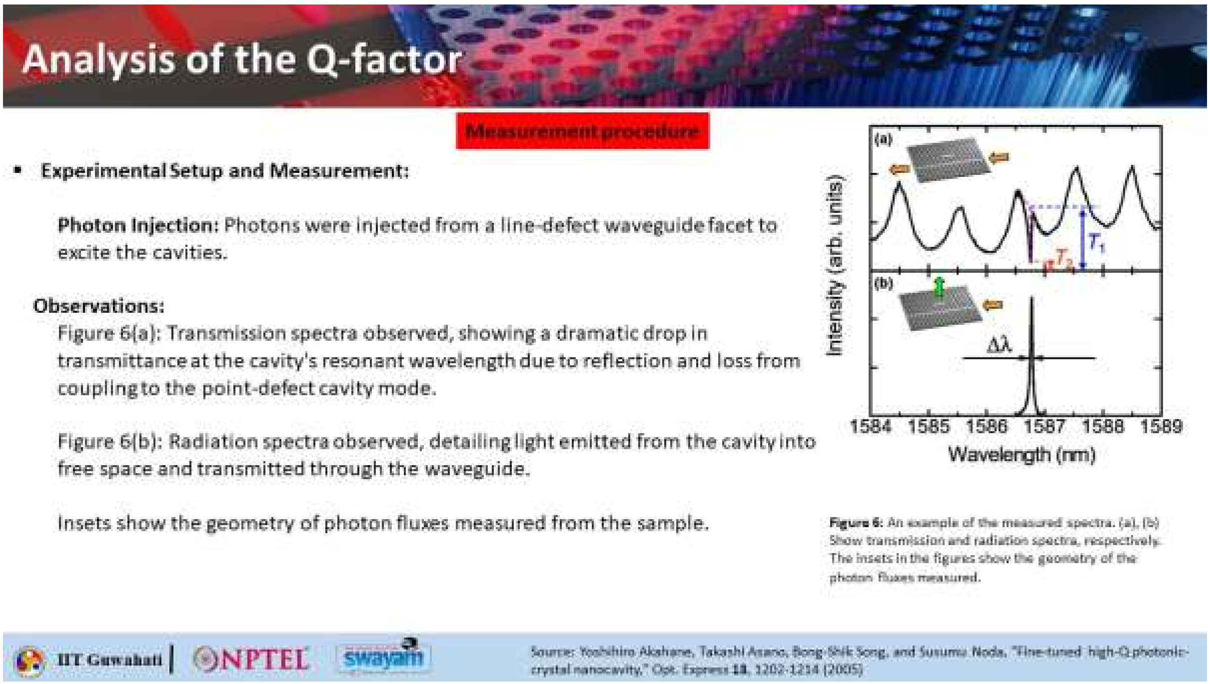


. So, now how do you experimentally measure heat? So to give you all a brief idea of how the quality factor is calculated experimentally, we shall now see a short discussion on the experiments of this L3 cavity.

So what you do, you basically take a line waveguide which is next to the cavity. So photons are basically injected from this line waveguide. facet to excite the cavity. So, you can see in this case, this is the transmission spectrum observed. So, there is a dramatic drop in the transmission of the cavity at the place of cavity resonance here.

So, this is the transmission spectrum. So, there is a drop means that light is getting basically coupled to the cavity. okay. And if you look into B, this is the radiation spectrum that is basically telling you the details of the light being emitted from the cavity into the free space and transmitted through the waveguide. So, it actually happens at the same position, right.

Analysis of the Q-factor

- **Q Factor Evaluation:**

Total Q Factor (Q_{total}):

Defined from the linewidth of the radiation spectrum in Figure (b).

Represents the Q factor influenced by both the intrinsic properties of the cavity and the coupling losses to the waveguide mode.

The Q-factor of a resonator is the ratio of the total energy stored in the cavity to the energy loss per cycle.

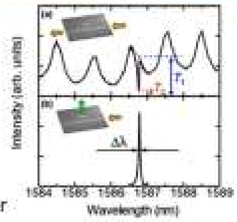
Formula: $Q = \frac{\omega_0}{\Delta\omega}$

Explanation: This formula represents the ratio of the resonant frequency of the stored energy (ω_0) to the spectral width (linewidth) of the energy loss per cycle ($\Delta\omega$).

Alternate Expression: $Q = \frac{\lambda_0}{\Delta\lambda}$

λ_0 : Central wavelength of the resonance.

$\Delta\lambda$: Full-width half-maximum (FWHM) value of the resonance, indicating the spectral range over which the energy of the mode is significantly above half its maximum value.



So, this wavelength is getting trapped in the cavity. So, here also it is shown schematically, fine. So how do you calculate the Q factor? So the Q factor can be calculated like this. You can first obtain the line width of the spectrum, radiation spectrum from this figure B that is delta lambda. So, this basically represents the quality factor getting influenced by both the intrinsic property of the cavity and the coupling losses to the waveguide mode.

So, that is why it is called Q_{total}. So Q factor of the cavity is basically the ratio of the total energy stored in the cavity to the energy loss per cycle. So you can actually write it in terms of Q is nothing but omega naught divided by delta omega. So this formula represents the ratio of the resonant frequency of the stored energy that is omega naught to the spectral width of the energy loss per cycle that is delta omega. clear. So, if you alternatively you can write this in terms of wavelength as Q equals lambda naught by delta lambda.

So, what is λ_0 ? That is the central wavelength of the resonance and delta lambda is basically the FWHM full width half maximum of the resonance that indicates the spectral range over which the energy of the mode is significantly above half of it maximum value.

Analysis of the Q-factor

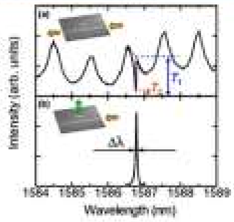
- **Intrinsic Q Factor (Q_v):**

  Isolated from total Q factor

  It reflects losses due to coupling to free space only.

  Calculated using the formula

  $Q_v = \frac{Q_{total}}{\sqrt{T}}$, where T is the transmittance at the resonant wavelength. ⟶ Eqn (5)



Source: Yoshihiro Akahane, Takashi Asano, Bong-Shik Song, and Susumu Noda, "Fine-tuned high-Q photonic-crystal nanocavity," Opt. Express 13, 1202-1214 (2005)

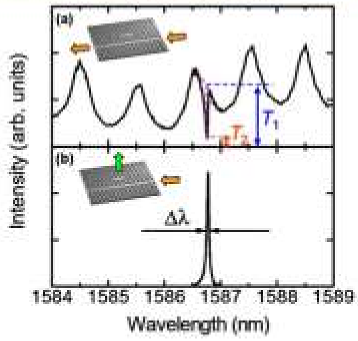
So with that, how do you obtain the intrinsic quality factor that is Qv? So, intrinsic quality factor is isolated from the total quality factor and it actually reflects the losses due to coupling to free space only. So, it is not associated with the loss towards the waveguide mode. So, you can actually calculate Qv using this formula. So, you know $Q_v = \frac{Q_{\text{total}}}{\sqrt{T}}$, T is basically the transmittance at the resonant wavelength.

Analysis of the Q-factor

- **Transmittance Measurement (T) :**

  Definition: Transmittance T at the resonant wavelength is the ratio of the transmitting energy in the presence of the cavity near the waveguide to the energy in the absence of the cavity.

  Calculation: Experimentally determined as T_2/T_1 using data from Figure .

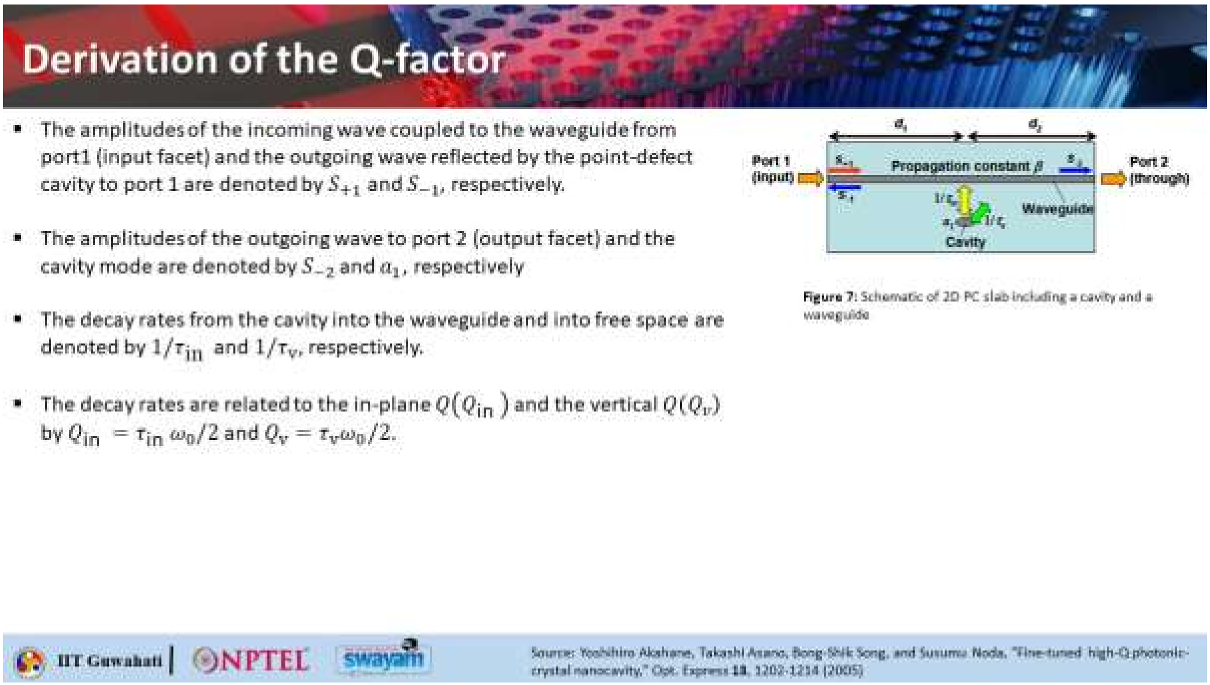


Source: Yoshihiro Akahane, Takashi Asano, Bong-Shik Song, and Susumu Noda, "Fine-tuned high-Q photonic-crystal nanocavity," Opt. Express 13, 1202-1214 (2005)

So, how do you measure this transmittance? The transmittance T at resonant wavelength is basically measured as a ratio of the transmitting energy in the presence of the cavity near the waveguide to the energy in the absence of the cavity. So, you can actually determine experimentally what is your

T1 and what is your T2, these two heights. So, it is like if the cavity is not there, it would have been here.

So, that is T1. And because of the cavity, it has now gone down to this level that is T2. So, these two transmission levels are known. So, you can actually obtain what is T2/T1. Now, with that we understand how we can calculate the q factor experimentally.



Now, before concluding this lecture, we shall briefly discuss about an important theory called couple mode theory that can be used to derive the formula that is given in equation 5.

So, let us have a quick look at that. I will not go into very much details, but briefly tell you how it is done. So, here is the method of deriving the quality factor. So, this is a schematic of that setup. So, it is a 2D photonic crystal slab that includes a cavity here and a waveguide.

So, this is the port 1 and this is the port 2, input port and the throughput. And we have now named the waves coming into and going out of the ports. So, the amplitude of the incoming wave coupled to the waveguide from this port 1 which is the input phase and the output wave reflected by the point defect, cavity to port 1. So, they can be named as S plus 1, S minus 1. So, this one is coming from the cavity, this one is going towards this.

from port 1 and this one is reflected from the cavity. Similarly, the amplitude of the outgoing wave to port 2 that is the output phase A can be num leveled as S minus 2 and the cavity mode will be A1. So, you can see A1 over here. So, the decay rates from the cavity into the waveguide and the free space are considered as 1 by tau in and into the free space it is considered as 1 over tau v, ok. So, with the two decay rates you can also find out the two quality factor that is q in which is tau in omega naught by 2 and then q v will be tau v omega naught by 2.

Analysis of the Q-factor

$$\frac{da_1}{dt} = \left(j\omega_0 - \frac{1}{\tau_v} - \frac{1}{\tau_{in}}\right)a_1 + \sqrt{\frac{1}{\tau_{in}}}e^{-i\beta d_1}S_{+1}$$

$$S_{-1} = -\sqrt{\frac{1}{\tau_{in}}}e^{-i\beta d_1}a_1$$

$$S_{-2} = e^{-i\beta(d_1+d_2)}\left(S_{+1} - \sqrt{\frac{1}{\tau_{in}}}e^{i\beta d_1}a_1\right)$$

$$T = \left|\frac{S_{-2}}{S_{+1}}\right|^2 = \frac{(\omega-\omega_0)^2 + \left(\frac{\omega_0}{2Q_v}\right)^2}{(\omega-\omega_0)^2 + \left(\frac{\omega_0}{2Q_v} + \frac{\omega_0}{2Q_{in}}\right)^2}$$

⟶ Eqn (6)

Under the condition of $\omega = \omega_0$, Eq. (6) becomes

$$T = \frac{\left(\frac{1}{Q_v}\right)^2}{\left(\frac{1}{Q_v} + \frac{1}{Q_{in}}\right)^2}$$

⟶ Eqn (7)

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Source: Yoshihiro Akahane, Takashi Asano, Bong-Shik Song, and Susumu Noda, "Fine-tuned high-Q photonic-crystal nanocavity," Opt. Express 13, 1202-1214 (2005)

Then the equations for the evaluation of cavity modes in time and the outgoing waves are given as follows. So, these are obtained using the coupled mode theory. We will not go into much detail, just write down the expressions here and tell you that once you obtain S minus 2 and S minus 1, you can calculate what is transmittance, ok. As S minus 2 over S plus 1 modulus square and this turns out to be this, ok. So, when you take the condition of resonance that is omega equals omega naught, these terms cancel out and you simply get this ratio and that becomes 1 over q v whole square divided by 1 over q v plus 1 over q in whole square, ok.

Analysis of the Q-factor

- Since the total loss from the cavity is equal to the sum of the radiation loss to free space and the coupling loss to the waveguide, total Q can be expressed as follows:

$$\frac{1}{Q} = \frac{1}{Q_v} + \frac{1}{Q_{in}}$$

⟶ Eqn(8)

- Substituting Q_{in} from Eq. (8) into Eq. (7), we get the following:

$$T = \left(\frac{Q}{Q_v}\right)^2$$

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Source: Yoshihiro Akahane, Takashi Asano, Bong-Shik Song, and Susumu Noda, "Fine-tuned high-Q photonic-crystal nanocavity," Opt. Express 13, 1202-1214 (2005)

So, since the total loss from the cavity is equal to as you can see here. Since the total loss from the cavity is equal to the sum of the radiation loss to the free space plus the coupling loss to that line waveguide, you can also write the total Q as 1 over Q that is equal to 1 over Q v plus 1 over Q in. So, once you substitute Q in from this equation into the previous equation, you can get T equals the transmission equals $\left(\frac{Q}{Q_v}\right)^2$ ok. So, this is the same equation that you have seen in equation 4 earlier that has been used for the measurement procedure section. So, this is how you can obtain the quality factor and that can and V we have already shown how the V is calculated and that is how you are able to make high Q resonant filters.

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So, this is all for this lecture. We shall start the discussion of overview of photon crystal fibers in the next lecture. If you have any queries or doubt regarding this lecture, you can drop an email to this email address deb.sikdar at iitg.ac.in mentioning MOOC photon crystals and this lecture number on the subject line.