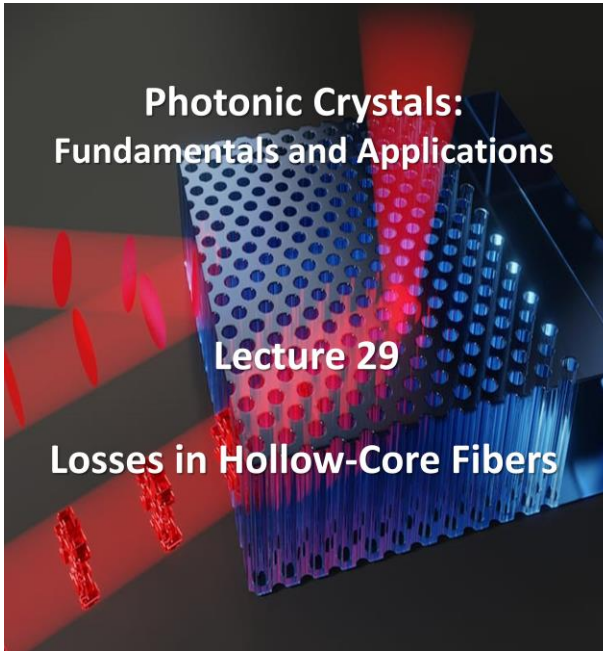


Lec 29: Losses in Hollow-core Fibers



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Hello students, welcome to lecture 29 of the online course on Photonic Crystals Fundamentals and Applications.

## Lecture Outline

- **Losses in Hollow-Core Fibers**

- Cladding losses:**

- **$1/R^3$  power law**
- **Radiative Leakage**
- **Rayleigh Scattering**

- Inter-modal coupling**



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Source: J. D. Joannopoulos, S. G. Johnson, J. N. Winn & R. D. Meade, "Photonic Crystals: Molding the Flow of Light", Princeton Univ. Press, 2008.

Today's lecture we will be discussing about the losses in hollow core fibers. So, here is the lecture outline we will be discussing mainly about the cladding losses, the  $1/R^3$  power law. Radiative leakage, Rayleigh scattering and we will also discuss intermodal coupling in this context.



## Losses in Hollow-Core Fibers



### Losses in Hollow-Core Fibers

- Optical fibers are used to transport light over distances ranging from meters to thousands of kilometers.
- Over such distances, even small imperfections can lead to substantial effects.
- Conventional silica fibers have attained such an amazing degree of perfection that their losses (only about 0.2 dB/Km at 1.55  $\mu\text{m}$ ) are limited by a combination of intrinsic material absorption and scattering from microscopic density fluctuations.
- At longer wavelengths, on the other hand, such as the 10.6  $\mu\text{m}$  high-power lasers used for many industrial and medical applications, silica and other common fiber materials are not transparent at all.



Source: J. D. Joannopoulos, S. G. Johnson, J. N. Winn & R. D. Meade, "Photonic Crystals: Molding the Flow of Light", Princeton Univ. Press, 2008.

So, let us first have an overview of the losses in hollow core fibers. So optical fibers are used to transport light over distances that typically range from several meters to thousands of kilometers.

So over such distances, small perturbations can also lead to substantial effects. Conventional silica fibers have attained such an amazing degree of perfection that their losses are very minimal. So, it is only about 0.2 dB per kilometer at the telecom wavelength of 1550 nanometer or 1.55  $\mu\text{m}$ .

55 micron and are limited by a combination of intrinsic material absorption and scattering from microscopic density fluctuations. So, these are the two important factors that contribute to losses that you can understand here which is intrinsic material absorption that is the inherent property of the material to absorb some portion of light. And the other contribution would come from scattering that arises from microscopic fluctuation in the density. So at long wavelengths, on the other hand, you will see that for 10.6 micron high power lasers used in many industrial and medical applications, silica and other common fiber materials are not transparent at all.

## Losses in Hollow-Core Fibers

- One of the promises of hollow-core photonic-bandgap fibers is that they may allow for lower losses than are possible with solid-core fibers, by relaxing the fundamental limitations imposed by solid material properties.
- In order to design fibers for such a goal, however, one must comprehend the different loss mechanisms in realistic fibers.
- Although a detailed study of these losses is outside the scope of this course, a broad understanding may be gained by dividing fiber losses into two categories depending on how they scale with the radius  $R$  of the air core.
- There is a tradeoff between losses that *decrease* with  $R$  (losses associated with field penetration into the cladding), and losses that *increase* with  $R$  (losses associated with coupling between modes).
- This makes the choice of  $R$  a delicate balancing act that is crucial for fiber performance.

They're not transparent means they're basically absorbing, right? So, one of the promises of the hollow core photonic band gap fiber is that they may allow for lower losses than are possible with the solid core fibers by relaxing the fundamental limitations which are basically imposed by the solid material properties. So, in order to design fibers for such a goal, one must understand that you know different loss mechanisms which are existing in the realistic fibers. So, although you know the detailed study of each of these losses is not included in the scope of this course, but a broad overview or understanding may be you know gained by dividing the fiber losses in two categories depending on how they basically scale with radius  $R$  of the air core. So you will see that there is a trade-off between losses that decrease with  $R$ .

Those are basically losses associated with the field penetration into the cladding and the other losses that basically increase with  $R$ . So these are the losses associated with the coupling between different modes. So, this makes the choice of  $R$  a very delicate balancing act that is crucial for the fiber performance. So, the radius of the air core plays a very

significant role in striking that delicate balance where you will minimize both the losses. Interestingly, not all losses are bad.

## Losses in Hollow-Core Fibers

- Interestingly, not all losses are bad.
  
- As we have seen, most of the proposed hollow-fiber designs have been multi-mode.
  
- They support multiple guided modes that propagate at different speeds.
  
- If unchecked, this would result in **modal dispersion**: since it is impossible to avoid exciting multiple modes, the differing velocities cause pulses to spread and information transmission to be scrambled.
  
- However, this problem is reduced in a hollow-core fiber by *differential attenuation*:
  - Some modes (typically the lower-order modes) have much lower losses than others, and thus transmission in everything but the lowest-loss mode will be filtered out after propagation over a long distance.



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Source: J. D. Joannopoulos, S. G. Johnson, J. N. Winn & R. D. Meade, "Photonics Crystals: Molding the Flow of Light", Princeton Univ. Press, 2008.

So you can actually use some losses for your benefit as well. We'll see how that can be done. As we have seen, most of the proposed hollow fiber designs have been multi-mode, right? So they basically support multiple guided modes which propagate at different speeds inside the fiber. So if they are not controlled or if they are not checked properly this would result in modal dispersion ok. Since it is impossible to avoid exciting multiple modes and the differing velocities cause pulses to spread and information transmission to be scrambled.

However, this problem is reduced in a hollow core fiber. by differential attenuation. Now what is that some modes typically the lower order modes have much lower loss than others and thus transmission in everything but the lower lowest loss mode will basically get filtered out after propagation over a long distance. So, only the modes which have you know the lowest loss that mode will be able to you know sustain. So, that is something why we are saying that you know not all losses are bad.



## Cladding losses



### Losses in Hollow-Core Fibers: Cladding losses

- Three important loss mechanisms are associated with the amount of field penetration into the cladding:
  - ❑ Material absorption
  - ❑ Radiative leakage due to the finite crystal size
  - ❑ Scattering from disorder
- All of these will tend to decrease as the core radius  $R$  increases.
- We will show that they typically decrease asymptotically as  $1/R^3$ .
- Of these three loss mechanisms, the simplest one to analyze is material absorption.
- This can be described by a small imaginary part  $i\kappa$  that is added to the real refractive index  $n$  ( $\kappa$  is called the **extinction coefficient**).



Source: J. D. Joannopoulos, S. G. Johnson, J. N. Winn & R. D. Meade, "Photonic Crystals: Molding the Flow of Light", Princeton Univ. Press, 2008.

So, although there are possibility of multiple modes getting propagated into the fiber, but while propagation you know the lowest loss mode only survives and the other modes basically you know get cancelled out ok or they just attenuate or die down ok. So, that is how you know the lowest loss mode will be filtered out after propagating a long distance. Now let us look into different cladding losses. So three important loss mechanisms can be associated with the amount of field that penetrates into the fiber cladding. So the first mechanism is material absorption.

Second is radiative leakage due to finite crystal size. And the third one is basically

scattering from disorder. So all of these losses basically tend to decrease as the core radius  $R$  increases. And we will show that they typically decrease asymptotically as  $1/R^3$ .

## Losses in Hollow-Core Fibers: Cladding losses

- Because  $\kappa \ll n$  for transparent materials, one can obtain essentially exact results for the loss by starting with the eigenmode of the lossless structure and employing perturbation theory.
- The imaginary change  $\Delta\omega$  in the frequency due to  $\kappa$ , and to obtain the loss rate per unit distance we compute  $\Delta k_z = -\Delta\omega/v_g$  via the group velocity  $v_g = d\omega/dk_z$ .
- Letting decay rate  $\alpha \triangleq 2 \operatorname{Im} \Delta k_z$ , this describes a decay of  $e^{-\alpha z/2}$  in fields and  $e^{-\alpha z}$  in intensity.
- Combining these equations, the decay rate  $\alpha$  due to a single absorbing material with a complex refractive index  $n + i\kappa$  is:

$$\alpha = \frac{2\omega\kappa}{v_g n} \cdot (\text{fraction of } \int \epsilon |\mathbf{E}|^2 \text{ in absorbing material})$$

So of these three loss mechanisms, the simplest one to analyze is the material absorption.

And this can be described by a small imaginary part  $i\kappa$  that gets added to the real refractive index  $n$ . So the kappa is known as the extinction coefficient. Now because  $\kappa$  in most transparent materials is much, much smaller than  $n$ , one can obtain essentially exact results for the loss by starting with the eigenmodes of the lossless structure and employ a perturbation theory. So the imaginary change in frequency can be written as  $\Delta\omega$  and this is coming because of this  $\kappa$ , okay. And to obtain the loss rate per unit distance, we could compute  $\Delta k_z$  that comes out to be  $-\Delta\omega/v_g$  where you know the  $v_g$  is the group velocity that is given by  $d\omega/dk_z$ .

So, if you assume the decay rate  $\alpha$  which can be written as  $2 \operatorname{Im} \Delta k_z$ , this basically describes a decay of  $e$  to the power you know minus  $-\alpha z/2$  in the case of fields and if you take the square of it that will give you the decay. in intensity. So,  $e^{-\alpha z/2}$  if you take whole square you will get  $e$  to the power  $-\alpha z$ . So, combining these equations the decay rate  $\alpha$  due to a single absorbing material which has got a complex refractive index that can be represented as  $n + i\kappa$  you can write alpha equals basically  $2\omega\kappa/(v_g n)$  and then you can have fraction of you know this integration of  $\epsilon |\mathbf{E}|^2$  in the absorbing material. So, this is basically you know for the case of multiple materials one can simply add up the decay rate that is coming from the different material depending on the fraction of that material in that composite structure.

## Losses in Hollow-Core Fibers: Cladding losses

- As a special case, if the field energy propagates entirely within the material with a group velocity  $v_g = c/n$  as for a plane wave (neglecting material dispersion), then **bulk absorption** loss:

$$\alpha_0 \triangleq \frac{2\omega\kappa}{c} = \frac{4\pi\kappa}{\lambda} \quad \alpha = \frac{2\omega\kappa}{v_g n} \cdot (\text{fraction of } \int \epsilon |\mathbf{E}|^2 \text{ in absorbing material})$$

- Therefore, a useful dimensionless figure of merit for a hollow-core fiber mode is the ratio  $\frac{\alpha}{\alpha_0}$
- This is called the absorption suppression factor, the factor by which loss is decreased due to the portion of light in air.



Source: J. D. Joannopoulos, S. G. Johnson, J. N. Winn & R. D. Meade, "Photonic Crystals: Molding the Flow of Light", Princeton Univ. Press, 2008.

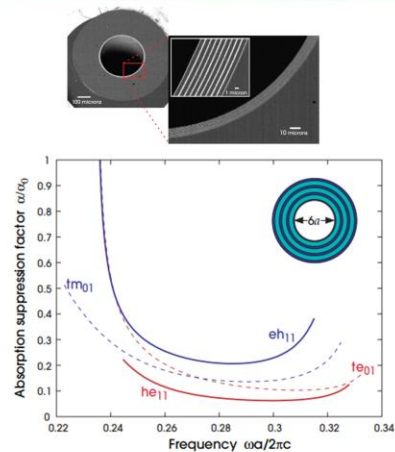
So, as a special case if the field energy propagates entirely within the material with a group velocity  $v_g$  is given by  $c/n$  as for a plane wave that neglects you know material dispersion then the bulk absorption loss can be expressed as  $\alpha_0$  to be  $2\omega\kappa/c=4\pi\kappa/\lambda$  So, once again if you remember that  $\kappa$  is. So, this is the bulk absorption loss and you already know what is your decay rate alpha ok. So, in that case you can define a useful dimensionless figure of merit that can be used for hollow core fiber mode ok. And that is basically the ratio of this  $\alpha/\alpha_0$ . So,  $\alpha/\alpha_0$  this particular ratio can be used as a dimensionless figure of merit.

So, this is also called the absorption suppression factor. So, the factor by which loss is decreased due to the portion of light in air.



## Losses in Hollow-Core Fibers: Cladding losses

- For example, let us consider the hollow-core Bragg fiber, and suppose that the *low*-index material ( $n_1 = 1.6$ ) has some absorption.
- This is motivated by the experimental fiber, which is designed to operate at  $10.6 \mu\text{m}$ .
- Its absorption loss is dominated by the effect of the low index polymer, which has a bulk absorption of around  $50,000 \text{ dB/m}$  ( $\kappa = 0.01$ ).
- Plot of the absorption suppression factor  $\frac{\alpha}{\alpha_0}$  of four modes for the core radius  $R = 3a$ .



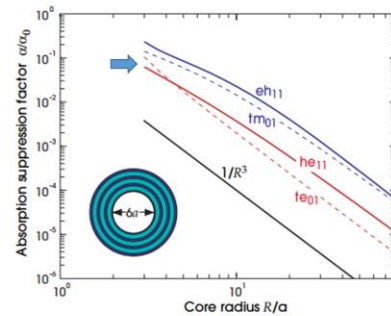
For example, let us consider the hollow core Bragg fibre and suppose that the low index material has got a refractive index of  $n_1$  that is 1.6 and that has got some absorption associated right. So, this is typically that polymer refractive index we discussed in the last lecture ok.

And this this is motivated by the experimental fibre which was designed to operate at 10.6 micron and its absorption loss is dominated by the effect of the low index polymer which has a bulk absorption of around 50,000 dB per meter. So, corresponding to a  $\kappa$  value of 0.01. So, if you plot the absorption suppression factor that is  $\alpha/\alpha_0$  for the 4 modes of this hollow core fiber where the core radius is taken as  $R$  equals  $3a$ , you will be able to see this different modes for different frequencies appearing.

So, the absorption loss as you can see is basically dominated by those of the low index material. So, here  $n$  equals 1.6 which are basically the shaded green parts in the inset and if you see here.  $he_{11}$  so at you know very starting here ok  $he_{11}$  of the 4 modes the solid red line shows you  $he_{11}$  ok. So, this is basically showing you the lowest loss ok.

## Losses in Hollow-Core Fibers: Cladding losses

- The absorption losses are dominated by those of the *low*-index material ( $n = 1.6$ , shaded green in inset).
- $he_{11}$  (fundamental) mode is the lowest loss.
- For larger  $R$ , the  $te_{01}$  mode becomes the lowest.
- Even for this small radius, absorption losses can be suppressed by more than a factor of 10.
- Notice also that the absorption losses *diverge* as the zero-group-velocity band-edge of the  $te_{01}$  or  $eh_{11}$  mode is approached.
- This radius, however, is much smaller than that of the experimental structure, which has a dramatic effect on the loss.



**Figure:** Scaling of the absorption suppression factor  $\frac{\alpha}{\alpha_0}$  versus core radius  $R$ , at quarter-wave frequency  $\omega a/2\pi c \approx 0.30$ , for several modes of the hollow-core Bragg fiber with indices 2.7/1.6 (blue/green in inset).

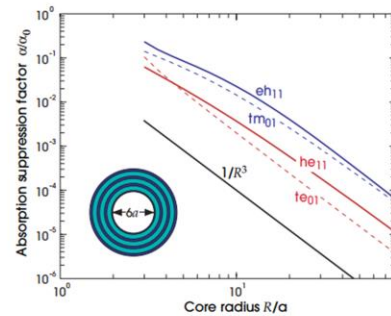
what is this figure just to tell you once again. So, this is basically the scaling of absorption suppression factor that is  $\alpha/\alpha_0$  versus the core radius again it is normalized as you can see it is  $R$  by  $a$ . And this graph is shown at quarter wave frequency which is  $\omega a/2\pi c$  equals 0.30, okay. And you can correlate that from here, okay.

So, you are actually plotting it here, okay, 0.3, okay, only for that part. fine and what you could see we could see four fibers and and this all this calculation are for these two different material which is 2.7 and 1.6 chalcogenide glass and the polymer right.

So now when you start analyzing you will see that you know  $he_{11}$  is the fundamental mode that is called the lowest loss And as we keep on increasing the radius,  $te_{01}$  mode becomes the lowest one which has got the lowest loss. even for this kind of small radius you can see that the absorption losses can be suppressed by 10 to the power minus 1 that is a factor of 10 ok. You can suppress the losses by 10. So, notice that the absorption losses basically diverge as the 0 group velocity band edge of the  $te_{01}$  and or  $eh_{11}$  ok, they they basically approaches ok. And this this radius however is much smaller than the experimental structure which is no dramatic effect on the loss.

## Losses in Hollow-Core Fibers: Cladding losses

- Regardless of polarization, the mode absorptions all approach a  $1/R^3$  dependence, with the  $te_{01}$  mode having the lowest asymptotic loss.
- At the experimental radius  $R \approx 80a$ , the  $he_{11}$  suppression factor is almost  $10^{-5}$
- Indeed, losses less than 1 dB/m were observed experimentally, representing a suppression of the polymer absorption by over four orders of magnitude.



**Figure:** Scaling of the absorption suppression factor  $\frac{\alpha}{\alpha_0}$  versus core radius  $R$ , at quarter-wave frequency, for several modes of the hollow-core Bragg fiber with indices 2.7/1.6 (blue/green in inset).

So, regardless of the polarization, so here you can see that the modes all this modes basically approach a  $1/R^3$  dependence which is also shown here okay and you can overall see that  $te_{01}$  mode has basically the lowest asymptotic loss and at the experimental radius of  $R$  equals  $80a$ , you can see that  $he_{11}$  mode is giving you a separation of almost  $n$  to the power  $-5$ , that is the separation factor. So, indeed losses less than 1 dB per meter were observed experimentally representing a separation of the polymer absorption by over .4 orders of magnitude right. So, as I mentioned already that this is basically a you know reference line that shows you know the asymptote for scaling as  $1/R^3$  and all of this modes basically follow that. So, that is what we will be discussing that  $1/R^3$  power law.



## Cladding losses: $1/R^3$ power law

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### Losses in Hollow-Core Fibers: $1/R^3$ power law

- What is the source of this  $1/R^3$  power law?

$$\alpha = \frac{2\omega\kappa}{v_g n} \cdot (\text{fraction of } \int \epsilon |E|^2 \text{ in absorbing material})$$

- The key fact from the *equation* is that the contribution to the loss from a particular absorbing material is proportional to the fraction of the electric-field energy in the material.
- For a core-guided mode (not a surface state), the losses will scale as  $1/R$ .
- If the field penetrates a certain distance  $d_p$  into the cladding, then the fraction of field in the cladding goes as the penetration area  $2\pi R d_p$  divided by the core area  $\pi R^2$ , yielding  $\sim 1/R$ .

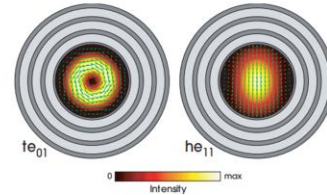
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Source: J. D. Joannopoulos, S. G. Johnson, J. N. Winn & R. D. Meade, "Photonic Crystals: Molding the Flow of Light", Princeton Univ. Press, 2008.

So what is the source of this  $1/R^3$  power law? If you revisit this equation, that gives you the decay rate. You have seen that you know the key fact from this equation is the contribution of the loss from a particular absorbing material and it is basically proportional to the fraction of the electric field energy in the material. So, this is the electric field energy and it is proportional to the fraction of that material in the whole structure. So for a core guided mode not a surface state the losses will basically scale as  $1/R$  and if the field penetrates a certain distance  $d_p$  inside the cladding then the fraction of the field in the cladding goes as the penetration area. okay that is  $2\pi R d_p$  divided by the core area which is  $\pi R^2$  and that yields  $1/R$  okay.

## Losses in Hollow-Core Fibers: $1/R^3$ power law

- Such an argument, however, assumes that the field *amplitude*  $|\mathbf{E}|$  in the cladding compared to the core is independent of  $R$ , and in fact this is not the case.
- This is easiest to see for the  $te_{01}$  mode, which by analogy with the metal waveguide has a node in  $\mathbf{E}$  near  $r = R$  (see **figure**).
- As a consequence, the  $te_{01}$ 's cladding  $|\mathbf{E}|$  is proportional not to the field at  $r = R$  (which is  $\approx 0$ ) but rather to the *slope*  $d|\mathbf{E}|/dr$  at  $r = R$ , which scales as  $1/R$  for a fixed  $\max |\mathbf{E}|$  in the core.
- As a result, the  $te_{01}$ 's  $|\mathbf{E}|^2$  in the cladding picks up an additional  $1/R^2$  factor, and the net absorption losses scale as  $1/R^3$ .



So, for such an argument however, we can assume that the field amplitude  $E$  in the cladding compared to the core is basically independent of  $R$ . And in fact, this is typically not the case. So, you can actually see for the field intensity distribution of  $te_{01}$  mode, okay. which by analogy with the metal waveguide has a mode of you know. So, we discussed that it has got a the metal waveguide has got a node at the border or the boundary.

So, here also you can see that you know the field is almost 0 when it is this particular boundary that is small  $r$  equals capital  $R$ . So, as a consequence you can think of  $te_{01}$ 's cladding electric field is basically proportional to the field not to the field at small  $r$  equals  $R$  because the field here is basically 0 rather it is proportional to the slope of this one that is the  $dE$  over  $dr$  at  $r$  equals  $R$  and which basically scales as  $1/R$  over a fixed maximum electric field in the core. So, that way you know you can see that  $te_{01}$ 's electric field intensity in the cladding will be able to pick up another  $1/R^2$  factor. So, that will actually make the net absorption loss to scale as  $1/R^3$  okay. So, you actually get the  $1/R$  factor from here and you pick up this additional  $1/R^2$  factor and finally the absorption loss scales as  $1/R^3$ .

## Losses in Hollow-Core Fibers: $1/R^3$ power law

- In fact, a similar argument holds for all core modes, because of the scalar limit.
- For any given mode, in the limit of large  $R$  the mode becomes more and more similar to a plane wave propagating along the  $z$  axis.
- Its dispersion relation approaches the air light line, and its penetration depth into the cladding becomes negligible compared to the scale of the transverse oscillations.
- These were precisely the conditions in which the scalar limit applies.
- In this limit, we can describe the mode as a linear polarization multiplied by a scalar amplitude  $\psi(x, y)$  that is zero in the cladding.
- In reality, there is some small nonzero amplitude in the cladding, but because of the approximate zero boundary condition at  $r = R$ , the amplitude of the cladding field goes as  $1/R$  just as we explained for  $te_{01}$ .

*Thus, all modes approach a  $1/R^3$  scaling*



Source: J. D. Joannopoulos, S. G. Johnson, J. N. Winn & R. D. Meade, "Photonic Crystals: Molding the Flow of Light", Princeton Univ. Press, 2008.

So, in fact a similar argument holds for all core modes because of the scalar limit. For any given mode in the limit of large  $R$ , the mode will become more and more similar to a plane wave propagating along the  $z$ -axis. So, its dispersion relation would approach that of the light line in air and its penetration depth into the cladding will become negligible compared to the state of the transverse oscillations. These were precisely the conditions in which the scalar limit applies.

And in this limit we can describe the mode as a linear polarization multiplied by a scalar amplitude  $\psi(x, y)$ .  $x, y$  that is 0 in the cladding. So, in reality there is some non-zero amplitude in the cladding, but because of the approximate zero boundary condition at you know  $r$  equals  $R$ , okay that is exactly on the boundary okay of the cladding and core. So, you can see that the amplitude of the cladding field goes as  $1/R$  just as we explained for  $te_{01}$  and that is how you know all the modes basically approach a  $1/R^3$  scaling.



## Cladding losses: Radiative Leakage



### Losses in Hollow-Core Fibers: Radiative Leakage

- Another cladding-related loss is radiative leakage.
- Because a real photonic crystal fiber cannot have an infinite number of crystal periods, the fields will have a small exponential tail beyond the edge of the crystal, which will couple to radiating modes.
- Again, this loss scales as  $1/R^3$ :
  - $1/R$  factor from the surface area/volume ratio
  - $1/R^2$  factor from the field amplitude scaling in the scalar limit.
- As a practical matter, however, such radiation can be more easily reduced by simply increasing the number of periods.
- For a high-contrast band-gap fiber, the radiative leakage typically decreases by a factor of ten for every period (or two) that is added to the cladding.
- As a result, even the most stringent loss requirements can be met by including a few dozen periods at most.



Source: J. D. Joannopoulos, S. G. Johnson, J. N. Winn & R. D. Meade, "Photonic Crystals: Molding the Flow of Light", Princeton Univ. Press, 2008.

So, next important cladding loss is the radiative leakage.

So, because a real photonic crystal fiber cannot have an infinite number of periodic crystal or crystal periods. So, it has to be terminated somewhere. So, the fields will have a small exponential tail beyond the edge of the crystal and that will couple to the radiating modes. Again this loss also scales as  $1/R^3$ . So,  $1/R$  factor is coming from the surface area by volume ratio if you take that ratio that is  $1/R$  and then you have  $1/R^2$  factor coming from field amplitude scaling in the scalar limit.

So, as a practical manner, however, such radiation can be more easily reduced by simply increasing the number of periods, okay. So, for the case of high contrast bandgap fiber, you will see that the radiative leakage typically would decrease by a factor of 10 for every period or 2 that is added to the cladding. So, as a result even the most stringent loss requirements can be met by including just a few dozen periods at max. So the absorption loss, yeah, at most.



## Cladding losses: Rayleigh scattering



### Losses in Hollow-Core Fibers: Rayleigh scattering

- Finally, one can also have losses from disorder, which causes light to scatter and radiate by breaking translational symmetry.
- Because photonic-crystal fibers typically have many high-contrast interfaces, the most serious problem seems to be due to surface roughness, especially in silica-based fibers at wavelengths where the absorption is small.
- A detailed analysis of disorder is complicated but a few general statements can be made for the usual case in which the length scale of the roughness is much smaller than the wavelength.
- In this case, known as **Rayleigh scattering**, the scattered (lost) power is roughly proportional to the  $|\mathbf{E}|^2$  at the scattering location and to the square of the scatterer's volume.
- The  $|\mathbf{E}|^2$  dependence produces the same  $1/R^3$  scaling for disorder-induced loss as for absorption loss, since disorder again affects only the small fraction of the field inside the cladding where the interfaces or materials are.





So the third one that decides the third mechanism for the cladding losses is the Rayleigh scattering.

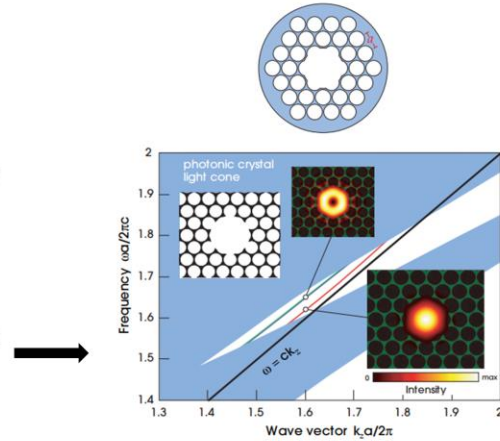
So finally, one can also have losses that originates from disorders, which causes light to scatter and radiate by breaking the translational symmetry. Because photonic crystal fibers typically have many high contrast interfaces. Those are like you know every boundary between the high and the low dielectric medium. The most serious problem seems to be due to you know the surface roughness. Especially in silica based fibers at wavelengths where the absorption is small.

So, a detailed analysis of this kind of disorder is indeed very complicated, but you know a few general statements can be made for the usual case in which the length scale of the roughness is much smaller than the wavelength ok. So, this kind of scattering from roughness which are much smaller than the wavelength can be covered within the scope of Rayleigh scattering. The scattered power is basically the lost power is roughly proportional to the intensity of the electric field okay. So, that is  $|\mathbf{E}|^2$  at the scattering location.

and to the square of the scatterers volume ok. So, you can see that this  $|\mathbf{E}|^2$  dependence produces the same  $1/R^3$  kind of scaling for disorder induced loss as for the absorption loss. disorder again affects only a portion of the field that is inside the cladding where the interfaces or materials are present.

## Losses in Hollow-Core Fibers

- What about two-dimensionally periodic photonic-crystal fibers, such as the hollow-core holey structure?
- Overall, the same asymptotic  $1/R^3$  scaling should apply.
  - The core interface/area ratio goes as  $1/R$
  - Additional  $1/R^2$  factor from the cladding field amplitude in the scalar limit
- However, an additional wrinkle is provided by the proliferation of surface states.
- Unless a crystal termination is chosen that eliminates surface states as in the figure, as the core size is increased, we will get more and more surface states.



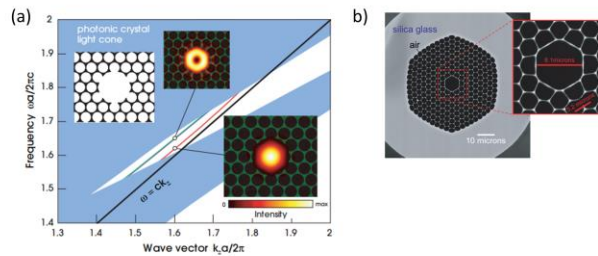
So, what about you know a two-dimensionally periodic photonic crystal fiber such as you know the hollow core holey structure. So, overall the same asymptotic  $1/R^3$  scaling will apply here. the core area by the core interface by area ratio again goes as  $1/R$  and you will get a additional  $1/R^2$  factor coming from the cladding fill amplitude in the scalar limit.

So, now there may be an additional wrinkle coming from the proliferation of the surface states right. So, in this case you will get some surface states if you remember. So, unless you know the crystal termination is chosen in this kind of a way which we have already discussed. This this basically make sure the air gaps are also you know cut in halves ok. And this would help us to eliminate the surface state as you can see in the figure as the core size basically increases here and you know you will have very nice air core modes, right.

## Losses in Hollow-Core Fibers

- These surface states cross the guided band and chop up its usable bandwidth.
- Precisely such a phenomenon was observed experimentally when the air core of fiber (figure b) is replaced with one of about 2.2 times the diameter:
  - The losses were reduced by a factor of eight (from 13 dB/km to 1.6 dB/km), but the bandwidth was reduced by a factor of five because the surface states were not eliminated.

- The surface states below the light line do *not* have absorption/leakage/scattering losses that decrease with  $R$ .
- They remain localized at the cladding surface regardless of  $R$ .



Now, this surface states cross the guided band and chop up its usual bandwidth. So, precisely such a phenomena can be observed experimentally when an air core of the fiber that you can see here is replaced by a size of you know air core which is 2.2 times the diameter okay. So, what happens in that case that is like this kind of a case. So, from here you are actually trying to make a larger air core that cuts through this air holes periodic air holes.

So, because of that you are eliminating the surface states and the losses will also reduce by a factor of 8. So, you can straight away come down from 13 dB per kilometer to 1.6 dB per kilometer. But you know the bandwidth was also reduced by a factor of 5 because you know some surface states were not eliminated.

ok. So, the surface states below the light line do not have absorption, leakage scattering that would decrease with  $R$  ok. So, in the previous case you have seen that kind of a not this one I think the figure is not shown here for this one you could if you remember there is a surface state over here ok. So, that was the surface state below the light line that do not have this absorption leakage scattering losses that would decrease with  $R$  ok. So, they basically remain localized at the cladding surface regardless of the value of the radius  $R$ .



## Inter-modal coupling

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### Inter-modal coupling

- Given the  $1/R^3$  dependencies of the loss mechanisms described before, it may seem that increasing the core radius  $R$  is always a winning strategy.
- This is not the case. As  $R$  grows, we worsen losses and other problems due to inter-modal coupling: the transfer of energy from one mode to another at the same  $\omega$  but different  $k_z$ .
- This is caused by fiber non-uniformities that break the translational symmetry in  $z$ .
- This is a problem because higher-order modes will typically have higher losses (e.g., stronger penetration into the cladding), and will also produce modal dispersion.
- As we mentioned above, the differential losses of higher-order modes will suppress modal dispersion, but not if we couple into them faster than they are filtered out.

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Source: J. D. Joannopoulos, S. G. Johnson, J. N. Winn & R. D. Meade, "Photonic Crystals: Molding the Flow of Light", Princeton Univ. Press, 2008.

So the last topic in this lecture will be intermodal coupling. So given the  $1/R^3$ , given the  $1/R^3$  dependencies of the loss mechanisms that we have described before, it may seem that increasing the core radius  $R$  is always a winning strategy because the loss will decay as  $1/R^3$ . But this is not always the case. As radius  $R$  will grow, we worsen the losses and other problems due to the intermodal coupling. Now, when you increase the radius of the fiber core, you are actually allowing more and more modes to exist. And that will allow different modes to couple with other and that would you know, make the transfer of energy possible from one mode to another at the same, you know, frequency but at different  $k_z$ .

So, this is typically caused by the fiber non-uniformities that break the translational symmetry in  $z$ . And this is a problem because higher order modes will have higher losses because of stronger penetration into cladding, and will also produce modal dispersion. And as we mentioned above the differential losses of the higher order modes will suppress the modal dispersion but not if we couple them you know couple into them faster then they are basically filtered out. So, before they are filtered out if you couple with them it is like your the energy will get transferred to those modes and then finally it will escape.



- Inter-modal coupling tends to worsen with increasing core radius  $R$ , for two reasons:

**First:**

- The number of core-guided modes increases.
- The number of modes scales with the area  $\sim R^2$ .
- Correspondingly, the mode spacing  $\Delta k_z$  decreases.
- This makes it easier for a non-uniformity to couple different modes.
- Roughly speaking,  $\pi/\Delta k_z$  is a minimum length scale for non-uniformities.
- If the fiber changes shape (e.g. due to ellipticity or stresses) over distances of this length scale or shorter, then coupling between modes may be substantial.

So, that is not going to help us that will be counted as a loss. So, intermodal coupling tends to worsen with increasing core radius  $R$  and there are two main reasons. The first one is that The number of core guided modes will increase. The number of modes basically scales with area. So, it is proportional to  $R$  square. Correspondingly, the mode spacing that is  $\Delta k_z$  would decrease and this makes it easier for a non-uniformity to couple between different modes.

So, roughly speaking you know  $\pi/\Delta k_z$  is a minimum length scale for non-uniformities. Now, if the fiber changes shape okay that could be due to you know ellipticity or say stresses that has slightly bend the curvature. So, over distance of the length scale or shorter the coupling between the modes would become substantial okay.

## Inter-modal coupling

### Second:

- Even for a fixed  $\Delta k_z$  between modes, the inter-modal coupling due to fiber *bends* will worsen as  $R$  increases.
- Intuitively, as  $R$  grows, the difference in path length between the part of the core on the inside of the bend and the part on the outside will also grow.
- The result is a bigger “centrifugal force” that distorts the modes.
- The mathematical treatment of bending is rather complicated, and involves a coordinate transformation of the bent waveguide to a straight waveguide.

## Inter-modal coupling

### Second:

- However, the final result is elegant, and we summarize it here.
- If we take  $x$  to be the direction away from the center of the bend and  $x = 0$  to be the center of the fiber at the bend radius  $R_b$ , the bend effectively adds a perturbation  $\Delta\epsilon$  and  $\Delta\mu$  proportional to  $x/R_b$ .
- This acts like a potential ramp that pushes the fields towards the outside of the bend.

and the second reason would be you know that even if you maintain a fixed  $\Delta k_z$  between the modes you will see intermodal coupling due to fiber bending okay so if the fiber bands will worsen as radius  $R$  will increase So intuitively you can say that as radius  $R$  grows, the difference in path length between the different parts of the core on the inside of the band and the part on the outside, that difference will also grow because your radius is large. So the result is a bigger centrifugal force that basically distorts the mode.

the two modes right or multiple modes what because of large radius you will have actually multiple modes. So, the mathematical treatment of bending is a complicated one and it involves a coordinate transformation of bent waveguide into a straight waveguide.

## Inter-modal coupling

### Second:

- Fiber performance then deteriorates for two reasons:
  - 1) far enough away from the core, the exponential tails of the guided modes will “see” a perturbation so large that the band gap is shifted to a different frequency.
    - ✓ This results in radiation losses that increase exponentially with  $1/R_b$ .
  - 2) for sufficiently large  $R_b$ , the bend radiation is negligible, and instead the losses in high-contrast photonic-crystal fibers are dominated by coupling between guided modes.
    - ✓ This coupling varies as  $(R/R_b)^2$  for large  $R_b$  (since  $R$  is the maximum value of  $x$  in the core), not including changes in  $\Delta k_z$ .



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Source: J. D. Joannopoulos, S. G. Johnson, J. N. Winn & R. D. Meade, "Photonic Crystals: Molding the Flow of Light", Princeton Univ. Press, 2008.

## Polarization Mode Dispersion :

- Finally, another important form of inter-modal coupling is known as **polarization-mode dispersion (PMD)**.
- PMD arises in an ordinary fiber because the operating mode is doubly degenerate, with two orthogonal polarizations.
- Any imperfection or stress in the fiber, however, can break the symmetry and split these two polarizations into modes that travel at different speeds.
- This produces a form of modal dispersion, where pulses spread due to random imperfections in the fiber.
- The same thing can happen in a photonic-crystal fiber *if* we operate in a doubly degenerate mode.
- The differential losses of a hollow-core fiber, however, allow one to operate in a low-loss higher-order mode like  $te_{01}$  that is *nondegenerate* and hence immune to PMD (no perturbation can split the mode into two).
- Alternatively, one can design a core that is so asymmetric that it supports only a single nondegenerate mode.



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Source: J. D. Joannopoulos, S. G. Johnson, J. N. Winn & R. D. Meade, "Photonic Crystals: Molding the Flow of Light", Princeton Univ. Press, 2008.

So, we will not go into that details. So, let us look into the final result which is pretty elegant and we can summarize it here. So, if we take  $x$  to be the direction away from the centre of the band and if you consider  $x$  equals 0 as the centre of the fibre at the band radius  $R_b$ , the band will effectively adds you know a perturbation  $\Delta\epsilon$  and  $\Delta\mu$  that will be proportional to  $x/R_b$ .

Now this basically acts as a potential ramp that pushes the field towards the outside of the band. So the fiber performance will then deteriorate because of two reasons. The first one is that far away from the core, the exponential tails of the guided modes will see a perturbation so large that the band gap is shifted to different frequency. So, this will result in a radiation loss and that increases exponentially with  $1/R_b$ .

So,  $r_b$  is the band radius and secondly for sufficiently large  $R_b$ , the band radiation is negligible and instead the losses in high contrast photonic crystal fibers would be dominated by the coupling between the guided modes. And this coupling would vary as  $(R/R_b)^2$  for the case of large  $R_b$  since you know  $R$  basically the maximum value of  $x$  in the core. It does not include you know the changes in  $\Delta k_z$ . So finally another important form of intermodal coupling we will also discuss that is the polarization mode dispersion. So as the name suggests you know it has to do something with the polarization of the light that is traveling.

So PMD arises in an ordinary fiber because the operating mode is doubly degenerate with two orthogonal polarizations and if there is any imperfection or stress in the fiber that will break the symmetry and split these two polarizations into two modes that travel at different speeds. So, this will also form you know this will produce a form of modal dispersion where the pulses spread due to random imperfection of the fiber. The same thing can happen in photonic crystal fiber if we can operate in a doubly degenerate mode. So, the differential losses of a hollow core fiber however allow one to operate in a low loss higher mode like you know like  $te_{01}$  and that is non-degenerate and in that way it will be immune to polarization mode dispersion. That means no perturbation or imperfection in the fiber will be able to split that mode into two.

Alternatively, one can also design a core that is so asymmetric that it only supports one non-degenerate mode. So, that is also a way to you know mitigate this kind of challenges. So that is all for this lecture.





Thank You

We will start discussing about the applications of photonic crystals fibers in the next lecture. If you have got any query regarding this lecture, you can drop an email to this particular email address [deb.sikdar@iitg.ac.in](mailto:deb.sikdar@iitg.ac.in) plus please mention MOOC Photonic Crystal and the lecture number on the subject line for a quick response. Thank you. Thank you.