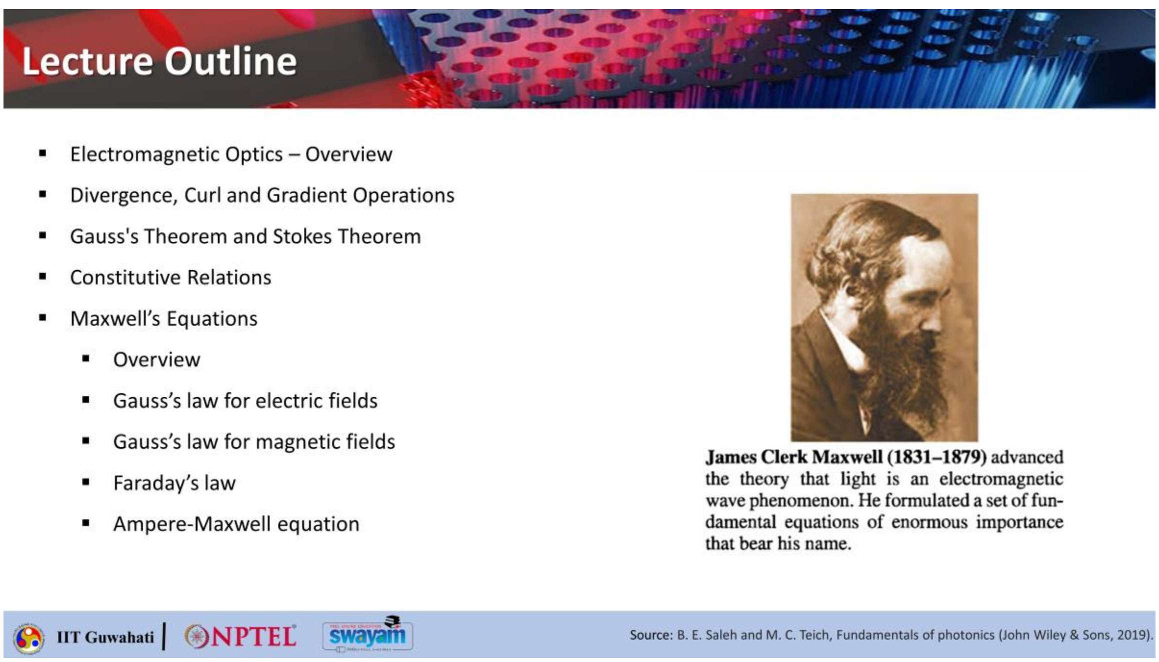
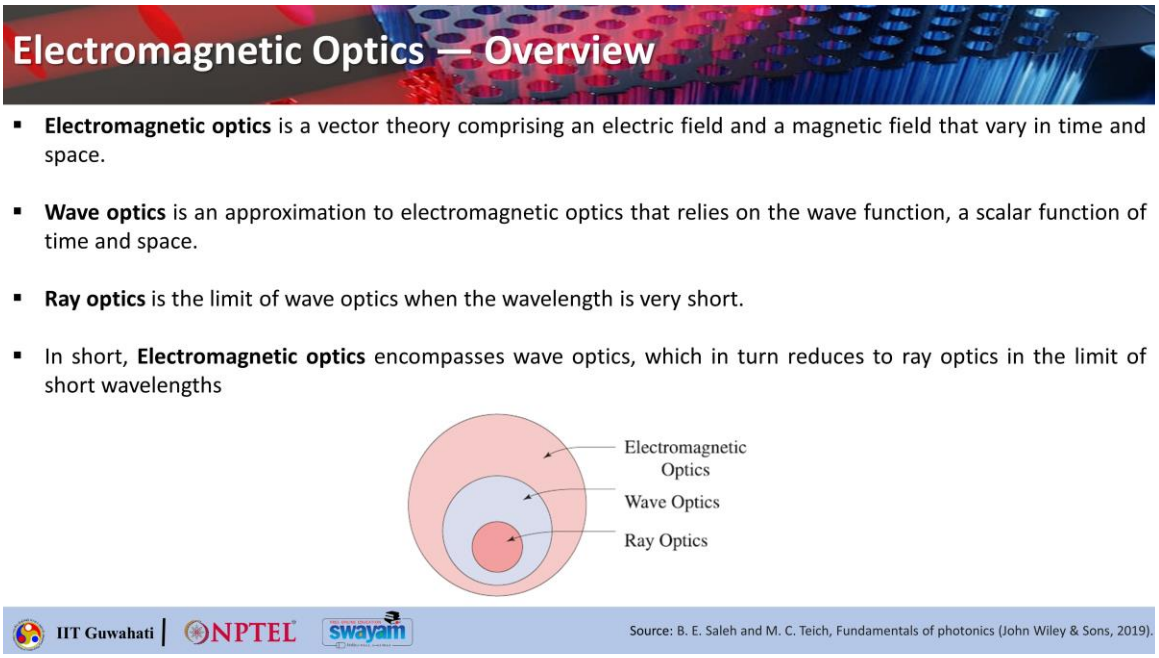
Lec 3: Fundamentals of EM theory of Light



Hello students, welcome to lecture 3 of the online course on Photonic Crystals, Fundamentals and Applications. Today's lecture will be covering the fundamentals of electromagnetic theory of light. So, this lecture provides a detailed interaction of the electromagnetic theory of light and we will see how the classical Maxwell's equations are valid for wave optics. So, here is the lecture outline, a brief overview of electromagnetic optics. We will discuss about divergence, curl and gradient operations. We will also discuss about Gauss theorem and Stokes theorems, then the constitutive relations which we will help us describe light matter interaction.

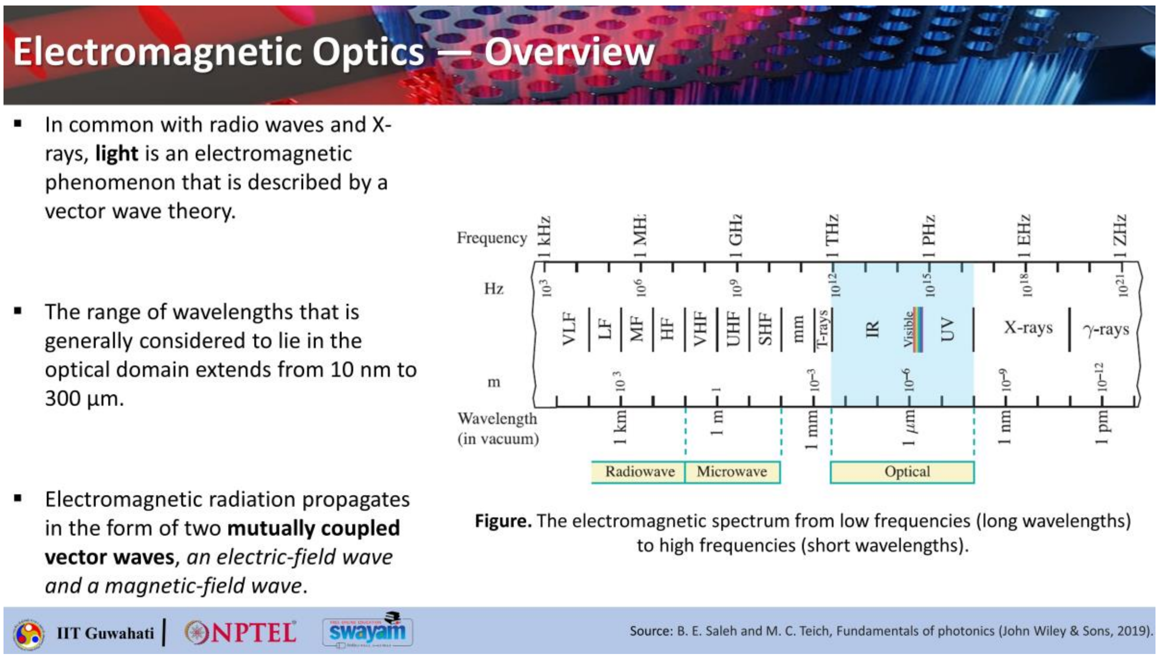
And then we'll go into Maxwell's equations and discuss all the four Maxwell's equations. So here is a photograph of James Clerk Maxwell. So he formulated a set of fundamental equations of enormous importance that bear his name. So with the use of Maxwell's equation, you can actually describe light as electromagnetic wave and we can actually see later on that this electromagnetic optics actually describes lot more physical phenomena than wave optics and ray optics.



So, electromagnetic optics so it is basically a vector theory comprising an electric field and a magnetic field that vary in time and space. So, in electromagnetic optics light is basically a mix of electric field and magnetic field. So, wave optics is basically an approximation to this electromagnetic optics that relies on the wave function. So, it is basically a scalar function of time and space ok. And when we talk about ray optics, it is basically the limit of wave optics when the wavelength is very short.

So, in short you can say that electromagnetic optics encompasses wave optics which in turn reduces to ray optics in the limit of short wavelengths. So, wave optics has a far greater reach than ray optics. Remarkably, both approaches provide similar results for many simple optical phenomena involving paraxial waves, such as focusing of light by lens and a behavior of light in gradient index media and periodic systems. but wave optics offers something that ray optics cannot. It is like the ability to explain phenomena such as interference and diffraction.

So, for explaining interference and diffraction you have to use wave optics, ray optics will not be sufficient there. However, wave optics is also unable to quantitatively account for some simple observations in optics experiment, such as division of light at beam splitter. Now the refraction of light that is reflected and transmitted turns out to be dependent on the polarization of the incident light which means the light must be treated as a you know vector rather than in the context of scalar theory and that is where electromagnetic optics enters the picture okay. So that is how you can see that electromagnetic optics can explain much more things than wave optics and ray optics alone. Now, in common with radio waves and X-rays light is also an electromagnetic phenomenon that is described by vector wave theory.

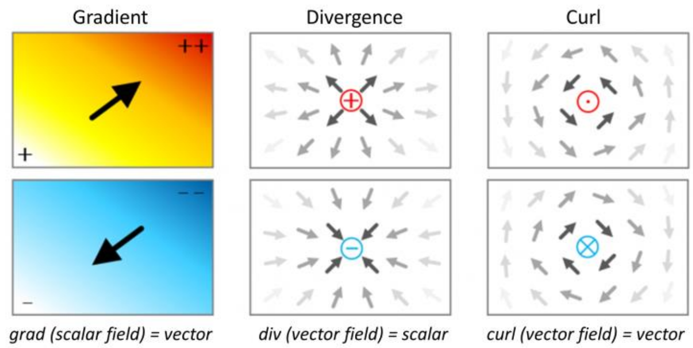


So, optical frequencies they basically occupy a band in this electromagnetic spectrum that extends from far infrared to visible to ultraviolet. So, this is basically the range of wavelengths. So, if you talk in terms of the range of optical wavelengths you can talk in terms of 10 nanometer to 300 micrometer. And in that vast range, you can see this is a very small region which is marked by 300 nanometer to 700 nanometer. That basically tells us about the visible range because we humans can see that only particular those frequency range or the wavelength range.

okay. So, electromagnetic radiation propagates in the form of two mutually coupled vector waves which are electric field wave and magnetic field wave. So, they are basically coupled to each other and that is how electromagnetic radiation propagates.

Divergence, Curl and Gradient Operations

- The three main operators in **vector calculus** quantify changes in fields:
  - **Gradient** - change in magnitude of scalar field
  - **Divergence** - source of vector field
  - **Curl** - rotation of vector field.
- The basic operations allow extracting information about the distribution of electromagnetic fields, energy associated with the field, electromagnetic radiation, and so on.
- The four Maxwell's equations are typically written in the vector calculus notation.



Now, before proceeding to Maxwell's equation for electromagnetics, we need to go through some important operators used in vector calculus. And these operators used widely in the calculation of electromagnetic phenomena. And these are nothing but gradient, divergence and curl. So, gradient basically tells you about change in magnitude of a scalar field, divergence tells you about the source of vector field and curl tells you about the rotation of a vector field. So here you can actually see that these basic operations allow extracting information about the distribution of electromagnetic field, energy associated with the field and electromagnetic radiation and so on. So here you can see that there is some positive charge here and there are more positive charges here. So, the gradient is in this particular direction. Similarly, it is more negative here and then it is less negative here.

So, the gradient is in this direction. So, if you take gradient of a scalar field, you actually see a vector. Now, this is divergence. So, if you take divergence of a vector field, you basically get a scalar. And if you take the curl of a vector field, you again get a vector.

So, we will get into this in more details in the next slides. So, the four Maxwell's equation that are useful for describing an electromagnetic wave are basically written in the vector calculus notation. That is why it is very important to understand these three basic operators of vector calculus that is gradient, curl and divergence.

Divergence, Curl and Gradient Operations

Nabla ∇ operator

- In a 3D space, vectors can be split into orthogonal components, and partial derivatives can be calculated accordingly for each directional component.
- The del operator ∇ is a vector differential operator written as:

$$\vec{\nabla} \equiv \boldsymbol{\nabla} \equiv \nabla \equiv \hat{\imath}\frac{\partial}{\partial x} + \hat{\jmath}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$$

Laplacian operator

- Nabla can be used in the "Laplacian" operator, referred to sometimes as "nabla squared" or "del squared", denoting effectively double differentiation:

$$\vec{\nabla}.\vec{\nabla} \equiv \boldsymbol{\nabla}.\boldsymbol{\nabla} \equiv \nabla.\nabla \equiv \vec{\nabla}^2 \equiv \boldsymbol{\nabla}^2 \equiv \nabla^2 \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

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Source: https://e-magnetica.pl/vector calculus

So, in the differential form of Maxwell's equation you will see that the equations are written in the form of vector differential operator nabla okay or it is also called as del okay. So, in 3D space the vectors can be split into orthogonal components and the partial derivatives can be calculated accordingly for each directional component.

And this nabla or del operator is basically a vector differential operator which can be written as this. So, this is ∇ it is basically $\partial/\partial x\ \hat{\imath} + \partial/\partial y\ \hat{\jmath} + \partial/\partial z\ \hat{k}$. So, then the next operator useful operator is Laplacian operator which is nothing but you know nabla squared ok. So, nabla that is del operator is also used in Laplacian operator and it is sometimes called nabla squared or del squared. So, this is basically showing double differentiation right.

So, if you can also write it as $\nabla.\nabla$ ok you can write them in the vector form boldface ok it turns out to be ∇^2. which is basically multiplying this dot product with the same thing. So, you will get $\partial^2/\partial x^2 + \partial^2/\partial y^2 + \partial^2/\partial z^2$. So, this is basically double differentiation okay. Now, let us look into the gradient in more details.

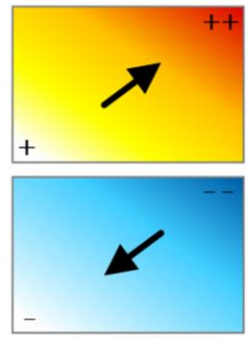
Divergence, Curl and Gradient Operations

Gradient of a Scalar

- A scalar field's gradient is a **vector field** whose magnitude represents the rate of change and which points in the general direction of the scalar field's greatest rate of increase.
- If ∇ is made to operate on a scalar function *F* (such as scalar field), then the following notation for the **gradient** is used, with the **result being a vector**:

In a Cartesian system of coordinates	
(simplified notation)	$\text{gradient (F)} \equiv \text{grad (F)} \equiv \vec{\nabla}\text{F} \equiv \boldsymbol{\nabla}\text{F} \equiv \nabla\text{F}$
(full notation)	$\nabla \text{F} \equiv \left(\hat{\imath}\frac{\partial}{\partial x} + \hat{\jmath}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)\text{F} \equiv \hat{\imath}\frac{\partial F}{\partial x} + \hat{\jmath}\frac{\partial F}{\partial y} + \hat{k}\frac{\partial F}{\partial z}$

Gradient



grad (scalar field) = vector

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Source: https://e-magnetica.pl/vector calculus

So, a scalar field a scalar fields gradient is basically a vector field. Whose magnitude represents the rate of change? And which points in the general direction of the scalar fields greatest rate of increase? So here you can see as discussed that there is some positive charge here say and then there are more charges over here. So this is the way the gradient will be showing. Similarly, there is negative charge here and then more negative charges here.

So this is how it is. the gradient in this particular direction. So, now if ∇ is made to operate on scalar field say F is a scalar field here ok. So, when you want to take the gradient of that scalar field what you will get is a vector. So, it is represented like this in Cartesian coordinate system you can write gradient of field F ok.

So F is a scalar field. So you can write grad F or like this ∇F directly. So ∇F will be nothing but this that is the del and then you have F. So you can actually write $\partial F/\partial x\,\hat{\imath} + \partial F/\partial y\,\hat{\jmath} + \partial F/\partial z\,\hat{k}$. So F is nothing but a scalar field, but then when you take the gradient this becomes a vector. So, that is why it is written here you see gradient of scalar field gives you vector.

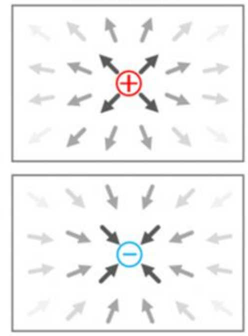
Divergence, Curl and Gradient Operations

Divergence of a Vector

- Divergence quantifies the magnitude only (no direction) of the amount of a vector field which "flows" out or into a specific region. In other words - the divergence calculates the amount of source (or sink) for a given field.
- If ∇ is made to operate on a vector function ***F*** (such as vector field), then the following notation for the **divergence** is used, with the **result being a scalar** (even though the input is a vector field).

In a Cartesian system of coordinates	
(simplified notation)	$\text{divergence}\,(\vec{F}) \equiv \text{div}\,(\vec{F}) \equiv \vec{\nabla}.\vec{F} \equiv \mathbf{\nabla}.\mathbf{F} \equiv \nabla.\mathbf{F}$
(full notation)	$\mathbf{\nabla}.\mathbf{F} \equiv \left(\frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}\right)$

Divergence



div (vector field) = scalar

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Source: https://e-magnetica.pl/vector calculus

Next important one is divergence of a vector. So divergence quantifies the magnitude only, not the direction of the amount of vector field that flows out or into a specific region. In other words, the divergence calculates the amount of source or sink. for a given field. So if this is like electric field lines, electric field lines originate from a positive charge.

So if you see that the fields are coming out, it means there is a positive charge over there. And if you see that the fields are going in, it means there is a sink over there. That means a negative charge is over there. ok. So, how do you calculate this? So, if del is made to operate on a vector function F.

So, here electric field or any other vector field ok. So, you can actually see this is a bold phase **F** ok. So, you can calculate this as a dot product with your del operator. So, divergence of F is nothing but you know ∇. the vector field F, F also now will have three components F_x, F_y and F_z. So, you can write ∇. F equals $\partial F_x/\partial x + \partial F_y/\partial y + \partial F_z/\partial z$.

So, it tells you that what you get is a scalar ok. So, divergence of a vector field gives you a scalar.

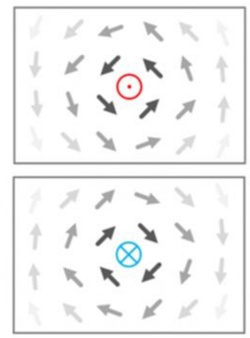
Divergence, Curl and Gradient Operations

Curl of a Vector

- The calculation of curl quantifies the amount and direction of rotation of a vector field, with the result being a vector perpendicular to the plane of rotation (in a similar sense as when a pseudo-vector is used to represent rotation in physics).
- In a 3D Cartesian system, the curl of a vector field can be calculated from its orthogonal components, as follows:

In a Cartesian system of coordinates	
(simplified notation)	$\text{curl}\,(\vec{F}) \equiv \vec{\nabla} \times \vec{F} \equiv \mathbf{\nabla} \times \mathbf{F} \equiv \nabla \times \mathbf{F}$
(full notation)	$\mathbf{\nabla} \times \mathbf{F} \equiv \left(\hat{\imath}\left(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z}\right) + \hat{\jmath}\left(\frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x}\right) + \hat{k}\left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right)\right)$

Curl



curl (vector field) = vector

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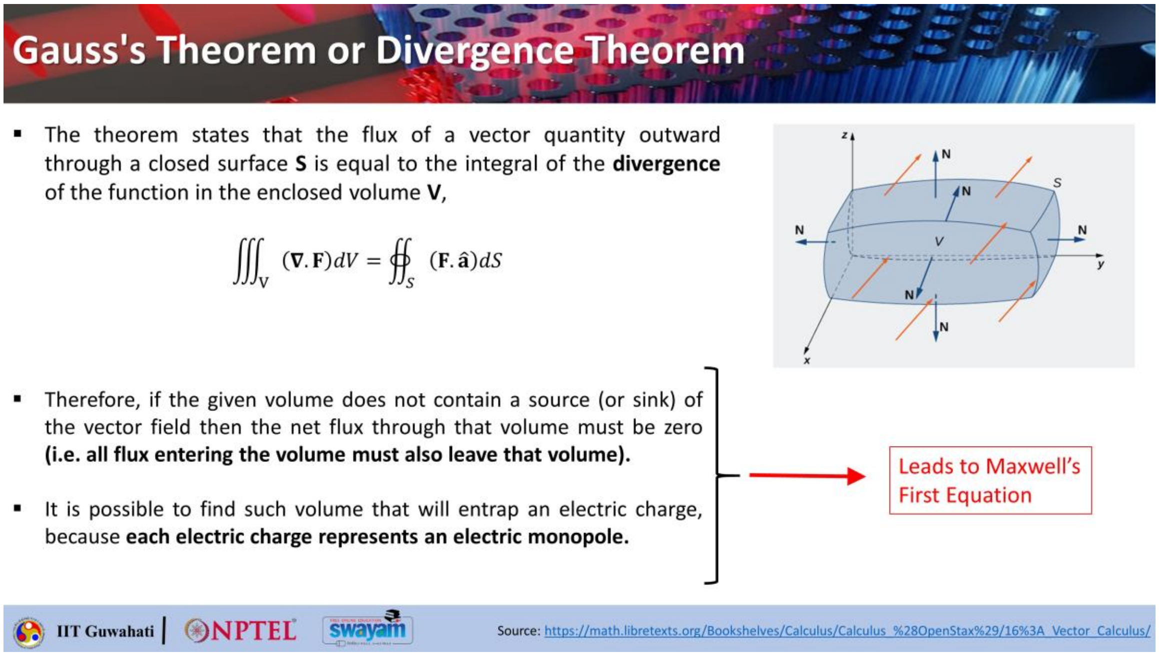
Source: https://e-magnetica.pl/vector calculus

So, the calculation of curl quantifies the amount and direction of rotation of a vector field okay. So, the curl is always associated with the amount and direction of rotation okay. So, with the result being a vector perpendicular to the plane of the rotation.

So, This is similar to a sense when a pseudo vector is used to represent rotation in physics okay. So, if there is a charge coming out okay. So, you can see that if there is a field that is vector field that is coming out okay. If you take the curl it will be like this in this direction okay and if that the vector field is going in the curl will be in the clockwise direction okay.

So, you can calculate curl. So, curl of a vector field F will be nothing but the cross product ok. So, when you take this is a vector this is a vector you take the cross product and this is what you get you get a vector. So, curl of a vector field also gives you a vector ok. So, curl of F ($\nabla \times$ F) will be nothing but $\hat{\imath}((\partial Fz)/\partial y-(\partial Fy)/\partial z)+ \hat{\jmath}\ ((\partial Fx)/\partial z-(\partial Fz)/\partial x)+ \hat{k}\ ((\partial Fy)/\partial x-(\partial Fx)/\partial y)$. So, this is simply a cross product of you know ∇ and F these two vector fields.



So, with that we can now move on to discuss the Gauss theorem or the divergence theorem which will be also useful in understanding you know Maxwell's equation. So, this particular theorem states that the flux of a vector quantity outward through a closed surface S. So, this is a closed surface S ok. So, this is the flux that is coming outward is basically equal to the integral of the divergence of that function in the enclosed volume V.

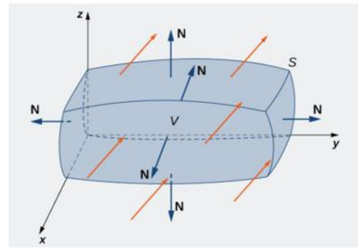
So, graphically you can see here. So, if you think of calculating the flux that comes out of the surface of this particular enclosed volume ok. So, you can actually take F and then a cap is basically the normal of the surface. you integrate it over all the closed surfaces that will be same as taking the divergence of this field F and then integrate it over the volume. So, how do you interpret this result? So, if the given volume does not contain a source or a sink. So, what do you expect then the net flux through that volume must be 0 it means whatever will enter must also exist ok.

So, the net flux will be 0 and it is possible to find such volume that will enter up an electric charge. because an electric charge represents an electric monopole. So, it is also possible to have you know only lines coming out of this okay if there is a positive charge over here or you can also have only a negative charge. So, in that case electric field lines will only enter this particular surface. So, you can actually have monopoles electric monopoles and this observations will lead to the first equation of Maxwell okay.

Gauss's Theorem or Divergence Theorem

- The theorem states that the flux of a vector quantity outward through a closed surface **S** is equal to the integral of the **divergence** of the function in the enclosed volume **V**,

$$\iiint_V (\nabla.\mathbf{F})dV = \oiint_S (\mathbf{F}.\hat{\mathbf{a}})dS$$



- But it is not possible to find a volume which entraps a magnetic charge, so the magnetic field is "divergenceless" and thus there are **no magnetic monopoles**.

→ Leads to Maxwell's Second Equation

Source: https://math.libretexts.org/Bookshelves/Calculus/Calculus_%28OpenStax%29/16%3A_Vector_Calculus/

Now, if you consider the same thing for magnetic monopoles that is not the case ok. So, for magnets magnetic field you will see that it is not possible to find a volume which entraps a magnetic charge that means you know You have to always have, you know, north and south pole together. It means there are no magnetic monopoles and that is why you can say that the magnetic field is basically divergence less. Okay. So whatever enters this volume will be exiting the volume.

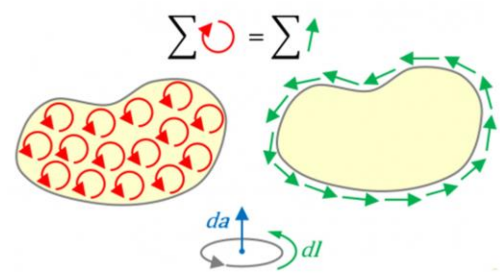
So there will be no divergence. So this leads to Maxwell's second equation.

Stokes Theorem

- Stokes' theorem states that the surface integral of the curl of the vector field **F** over an open surface **S** is equal to the closed line integral of the vector along the contour enclosing the open surface.

$$\iint_S (\nabla \times \mathbf{F}) \cdot d\mathbf{a} = \oint_C \boldsymbol{F} \cdot \mathrm{d}\boldsymbol{l}$$

- In other words, the circulation of a vector around a given boundary is equal to net curl over the whole surface of the patch limited by that boundary.



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Source: https://e-magnetica.pl/vector_calculus

We then look into Stokes theorem. So Stokes theorem, it basically relates the surface integral of the curl of a vector to the line integral of the vector itself. So, if the Stokes theorem basically states that the surface integral of the curl of the vector field over an open surface S will be equal to the closed line integral of the vector along the contour enclosing that open surface.

So you can see this one. So it is like summation of all these curls will be equal to the summation of this particular line integral, closed line integral. So you can actually write it like this. So say you have curl of a vector F, and that is there over this entire surface. So and A is basically a vector which is normal to the surface. So this will be equal to a closed line integral of the vector along the surface.

So, in other words the circulation of a vector around a given boundary is equal to the net curl over the whole surface of the patch limited by that boundary.

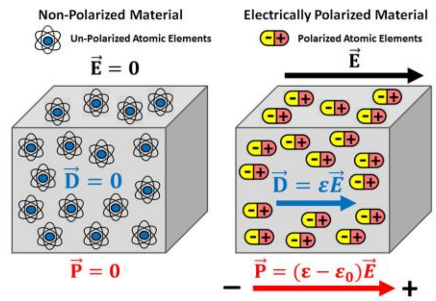
Constitutive Relations

- **Dielectric permittivity** (ε) is defined as the ratio between the electric field ($\boldsymbol{E}$) within a material and the corresponding electric displacement ($\boldsymbol{D}$):

$$\boldsymbol{D} = \varepsilon_0 \boldsymbol{E}$$

ε_0 = Dielectric constant of vacuum = 8.85×10^{-12} [F/m]

- When exposed to an electric field, bounded electrical charges of opposing sign will try to separate from one another.



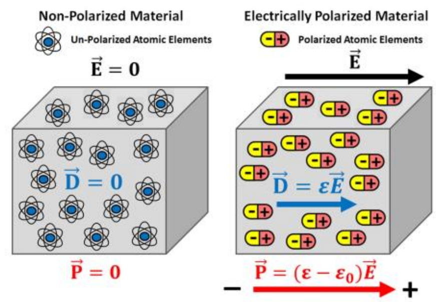
Next, we come to constitutive relations. So, in order to apply Maxwell's macroscopic equation, it is necessary to specify the relationship between the displacement field D and electric field E as well as the magnetic field H. and the magnetic flux density that is B. So, equivalently we have to specify the dependence of polarization ok.

So, that is P and there we will have this bound current on the applied electric and magnetic field. So, this equation specifying this range response are called the constitutive relations. So, now first we will talk about the dielectric permittivity which is basically defined as a ratio of the electric field within a material and the corresponding electric displacement. So, D is basically written as epsilon naught E right. what happens when now so epsilon naught is basically the vacuum permittivity.

Constitutive Relations

- For example, the electron clouds of atoms will shift in position relative to their nuclei.
- The extent of the separation of the electrical charges within a material is represented by the **electric polarization** (***P***).
- The electric field, electric displacement and electric polarization are related by the following expression:

$$\boldsymbol{D} = \varepsilon_0 \boldsymbol{E} + \boldsymbol{P} = \boldsymbol{\varepsilon E}$$



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So, if you have a non polarized material without any electric field. So, here any there is no electric field. So, all these atoms are basically unpolarized and you can see the electron cloud dancing around the nucleus ok. So, there is no displacement as well, but as long as you as soon as you put this material under the influence of electric field vector like this. What happens is atomic elements get polarized because the electron cloud moves away from the nucleus ok in the direction opposite of the electric field and that will create a kind of charge separation of this bound charges and they basically work like dipoles and this is the polarization that is happening in this particular material.

So, the material becomes electrically polarized okay. So, in that case you can basically explain the amount of polarization okay which is proportional to the electric field. So, more the amount of electric field you apply the charge separation will be larger. So, the electric field and the electric displacement and electric polarization all of this can be related by this particular equation where D can be written as epsilon naught E plus P ok. So, this P is nothing but epsilon minus epsilon naught E.

So, when you put that here you can actually get that. D equals epsilon E. So, this epsilon is basically the permittivity of this material.

Constitutive Relations

- When exposed to an applied magnetic field, the collection of individual magnetic dipole moments within most materials will attempt to reorient themselves along the direction of the field.

- This generates an induced magnetization, which contributes towards the net magnetic flux density inside the material.

$$\boldsymbol{B} = \mu_0 \boldsymbol{H} + \mu_0 \boldsymbol{M} = \boldsymbol{\mu} \boldsymbol{H}$$

$\mu_0 = permeability\ of\ free\ space = 4\pi \times 10^{-7}$ H/m

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Now, when exposed to an external magnetic field, the collection of individual magnetic dipole moments within most materials will also attempt to reorient themselves in the direction of the field. So, this will generate an induced magnetism or you can say magnetization, okay. So, this will contribute to the net magnetic flux density inside the material.

So, in that case you can write B which is the magnetic flux density as nothing but mu naught (μ_0) H. μ_0 is a vacuum permeability and H is the magnetic field strength plus μ_0M. So, this is the magnetization ok. So, when you do that together you write μ_0 H.

Constitutive Relations

- The degree in which the induced magnetization impacts the magnetic flux density depends on the material's magnetic permeability.

- **Magnetic permeability** (μ) defines the ratio between the magnetic flux density ***B*** within a material, and the intensity of an applied magnetic field ***H***; provided the fields are sufficiently weak.

- **Note:** For now, we will focus on material for which

$$\boldsymbol{M} = 0 \longrightarrow \boldsymbol{B} = \mu_0 \boldsymbol{H}$$

Source: https://em.geosci.xyz/content/physical_properties/dielectric_permittivity/index.html

Now, the degree in which the induced magnetization impacts the magnetic flux density depends on the materials magnetic permeability that is mu (μ).

So, what is μ? μ magnetic permeability is basically the ratio of the magnetic flux density B within a material and the intensity of the applied magnetic field H okay provided the fields are sufficiently weak and that is typically the case. Now, if you consider a material which is non magnetic, so their magnetization cannot take place. So, you can write M equals 0 in that case your B will be simply mu naught H. So, here in this particular course we will be mostly dealing with that kind of material which falls in this particular category of M equals 0 ok.

Maxwell's Equations — Overview

- An **electromagnetic field** is described by two related vector fields that are functions of position and time:

  Electric field *E*(*r, t*) and Magnetic field *H*(*r, t*)

- After the myriad of researches carried out for fundamental reasons behind the source of electromagnetic field and relation between electric and magnetic fields by pioneer scientists-

  Ampere, Coulomb, Faraday and Gauss

  the revolution in the Electromagnetic Fields happened when **James Clerk Maxwell** proposed a set of fundamental equations in 1865.

James Clerk Maxwell (1831–1879) advanced the theory that light is an electromagnetic wave phenomenon. He formulated a set of fundamental equations of enormous importance that bear his name.

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Source: https://em.geosci.xyz/content/physical_properties/dielectric_permittivity/index.html

Now with that understanding of constitutive relations now let us go and look into the Maxwell's equation.

Now Maxwell's equation, so there an electromagnetic field is basically described by two related vector fields which are functions of position and time. So, you have electric field E as function of R and T, you also have a magnetic field H which is function of R and T. Now, after the myriad of researches carried out for fundamental reasons behind the origin or source of electromagnetic field and the relationship between electric and magnetic fields by pioneer scientists like Ampere, Coulomb, Faraday and Gauss. Finally, the revolution happened when James Clerk Maxwell proposed a set of these fundamental equations in 1865 which are used to describe the electromagnetic properties of light.

Maxwell's Equations — Overview

Maxwell's Equations

$$\nabla.\mathbf{E} = \frac{\rho_v}{\varepsilon} \quad \text{(Gauss's Law)}$$

$$\nabla.\mathbf{H} = 0 \quad \text{(Gauss's Law for Magnetism)}$$

$$\nabla\times\mathbf{E} = -\mu\frac{\partial\mathbf{H}}{\partial t} \quad \text{(Faraday's Law)}$$

$$\nabla\times\mathbf{H} = \mathbf{J} + \varepsilon\frac{\partial\mathbf{E}}{\partial t} \quad \text{(Ampere's Law)}$$

$\boldsymbol{E}$ = Electric field vector
$\boldsymbol{H}$ = Magnetic field vector
$\boldsymbol{D}$ = Electric flux density
$\boldsymbol{B}$ = Magnetic flux density
ρ = charge density
J = current density

The Maxwell's equations are valid for both static and dynamic electromagnetic fields in a media.

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So Maxwell's equation these are basically collection of Gauss law, Faraday's law and Ampere's law but then put them together they are Maxwell's equation okay.

So In general, there are six scalar functions of position and time required to describe an electromagnetic field in a medium. And fortunately, these six functions are interrelated such that they satisfy the celebrated set of coupled partial differential equations which are known as Maxwell's equations. So, this is the first equation which says $\nabla.\mathbf{E} = \frac{\rho_v}{\varepsilon}$. So, that is the Gauss law and $\nabla.\mathbf{H} = 0$ is basically the Gauss law for magnetism okay $\nabla\times\mathbf{E} = -\mu\frac{\partial\mathbf{H}}{\partial t}$ or you can simply write $-\frac{\partial\mathbf{B}}{\partial t}$ that is Faraday's law and $\nabla\times\mathbf{H} = \mathbf{J} + \varepsilon\frac{\partial\mathbf{E}}{\partial t}$ that is also can be written as $\frac{\partial\mathbf{D}}{\partial t}$, so that is Maxwell Ampere's law okay. So here you can see that ε and E can be combined together in the form of D that is the electric flux density.

So there are 6 important parameters here. So E is the electric field vector, H is the magnetic field vector, D is the electric flux density, B is magnetic flux density, ρ is charge density and J is current density. So these are the important parameters okay. And as discussed that these are basically four laws derived by Gauss, Faraday and Ampere. But when you put them together to describe electromagnetic field that is the Maxwell's equation.

Maxwell's Equations — Overview

- Gauss's law for electric fields: While the *area integral of the electric field* gives a measure of the net charge enclosed, the *divergence of the electric field* gives a measure of the density of sources.

Maxwell's Equations	
Integral Form	**Differential Form**
$Q_e(t) = \oiint_S \vec{D}(t).d\vec{s} = \iiint_V \rho_V(t)dv$	$\nabla.\vec{D}(t) = \rho_V(t)$

So, let us look into these equations one by one. The first one is Gauss law for electric field. So, here you can say that while the area integral of the electric field okay gives a measure of the net charge and closed so this is the area integral of the electric field okay is giving you about the net charge and closed so this is the Maxwell's equation in So, you can integral form. So, you can write the net charge is Q, okay. That is nothing but the charge density integrated over the volume, okay.

In differential form, it looks pretty neat. So, you can simply write $\nabla.\vec{D}(t) = \rho_V(t)$, okay. That is the charge density, okay.

Maxwell's Equations — Overview

- Gauss's law for Magnetism: The *net flux* will always be zero for dipole sources.

Maxwell's Equations	
Integral Form	**Differential Form**
$\oiint_S \vec{B}(t).d\vec{s} = 0$	$\nabla.\vec{B}(t) = 0$

Similarly, Gauss law for magnetism. So, there the net flux is always 0 for dipole sources.

So, you can see that $\vec{B}(t).d\vec{s}$ over a closed surface is 0. That means the flux that is entering is also exiting and this is possible because there is no monopole in magnetism right. So, there is no magnetic monopole and that is why the flux that will be entering will also leave that particular closed surface. In differential form, it looks like this. So, it is $\nabla.\vec{B}(t) = 0$.

Maxwell's Equations — Overview

- Faraday's law: The line integral of the electric field around a closed loop is equal to the negative of the *rate of change of the magnetic flux* through the area enclosed by the loop.

Maxwell's Equations	
Integral Form	**Differential Form**
$V_{\text{emf}}(t) = \oint_L \vec{E}(t).d\vec{l} = -\iint_S \left[\frac{\partial \vec{B}(t)}{\partial t}\right].d\vec{s}$	$\nabla \times \vec{E}(t) = -\frac{\partial \vec{B}(t)}{dt}$

Now, Faraday's law tells us that the line integral of the electric field around a closed loop is equal to the negative rate of change of the magnetic flux through the area enclosed by that loop.

So, in differential form this can be written as $\nabla \times \vec{E}(t) = -\frac{\partial \vec{B}(t)}{dt}$ ok. So, this is also the electromagnetic force that is described in this particular form, but this differential form tells us that whenever you have a time varying you know magnetic field okay, you will have a rotation in the electric field vector.

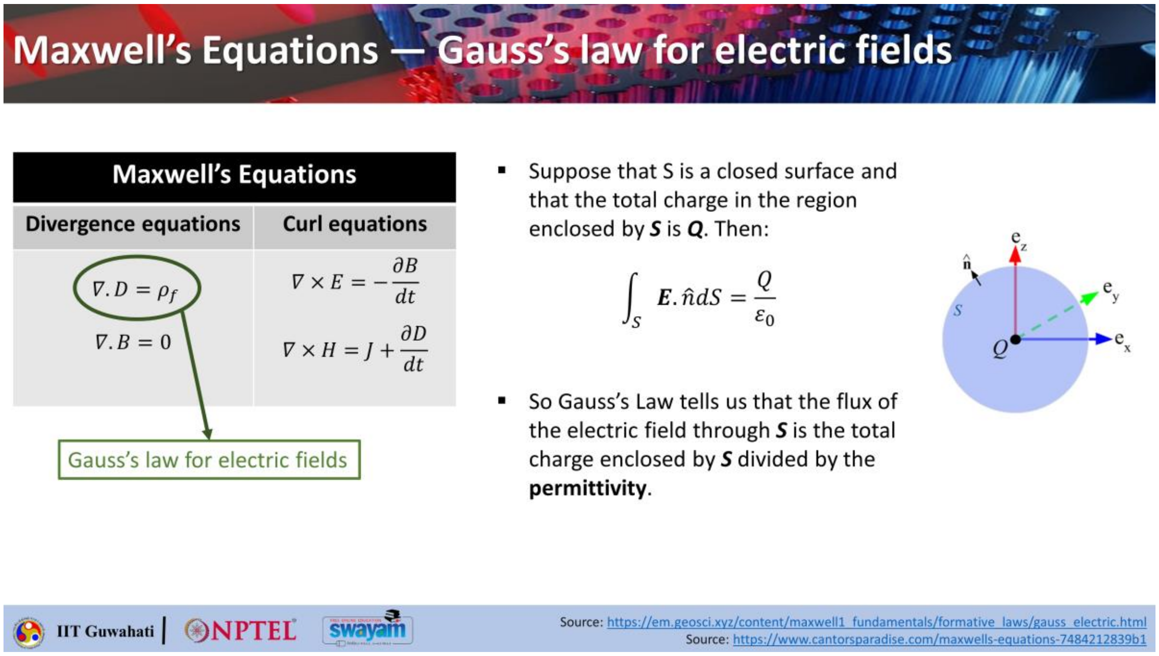
Maxwell's Equations — Overview

- Ampere-Maxwell equation: This gives the total magnetic force around a circuit in terms of the *current through the circuit*, plus any *varying electric field* through the circuit (that's the "displacement current").

Maxwell's Equations	
Integral Form	**Differential Form**
$I\ (t) = \oint_L \vec{H}(t).d\vec{l} = -\iint_S \left[\vec{J}(t) + \frac{\partial \vec{D}(t)}{\partial t}\right].d\vec{s}$	$\nabla \times \vec{H}(t) = \vec{J}(t) + \frac{\partial \vec{D}(t)}{dt}$

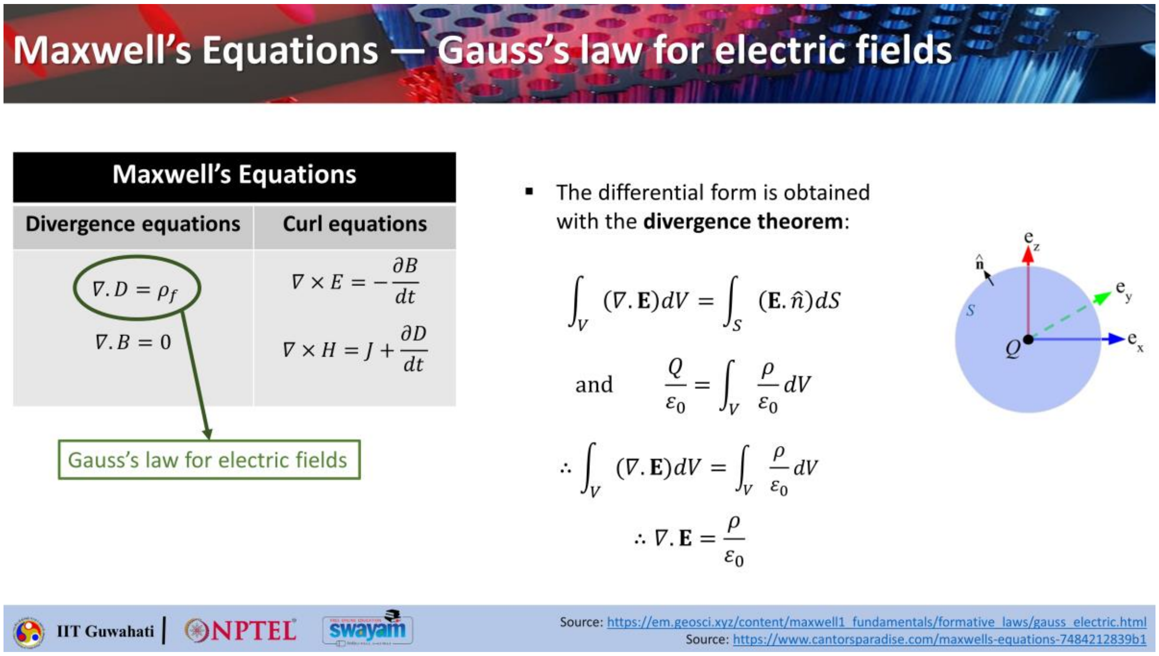
Similarly, when you look into Ampere Maxwell's equation, so this gives that the total magnetic force around the circuit in terms of the current through the circuit plus varying electric field through the circuit that is basically the displacement current. So, I (t) in integral form can be written as closed line integral of H(t). dl ok and that can be written as ok surface integral of the displacement current plus the rate of change of the displacement electric field or the varying electric field ok.

and then you take the surface integral of it. So, differential equation or differential form is much more simpler. So, you can write as $\nabla \times \vec{H}(t) = \vec{J}(t) + \frac{\partial \vec{D}(t)}{dt}$ ok. It means if you have a current flowing. So, you will have a magnetic field ok which is rotating ok and then that also equates to the time varying electric field. So, these two together will give you the rotation of the magnetic field.



So, a brief description of the Gauss law the first one ok. The law was published posthumously in 1867 as part of collection of work by the famous German mathematician Gauss ok. this particular $\nabla . D = \rho_f$ ok. These are different notations, but it stands for the free charge density ok.

So, this is the Gauss law for electromagnetism. Now, we can suppose that you know S is a closed area which has got a charge q at the center ok. So, you can actually write that the electric field that is coming out okay and $\hat{n}$ is basically the normal vector to this closed surface okay. When you do this integration you will get Q over ε_0. So, Guass law tells us that the flux of the electric field through S is basically total charge enclosed divided by the permittivity.



So, this is the total you know flux that you are getting. in the differential form you can obtain by using the divergence theorem. So, instead of this particular surface integral you can convert this into a divergence and then volume integral right and $\frac{Q}{\varepsilon_0}$ can be written as volume integral of $\frac{\rho}{\varepsilon_0}$. So, when you take instead of this left hand side if you take this as the left hand side and instead of this right hand side in the previous equation if you take this as the right hand side. So, this is how you can put them together ok. So, it is like the volume integral of the divergence of E will be equal to the volume integral of rho by epsilon naught.

So, you can actually take this quantities and equate them together. So, you get divergence of E equals $\frac{\rho}{\varepsilon_0}$. So, this is how you can write this particular equation. You can take ε_0 and multiply to E and you can write that also as D fine.

Maxwell's Equations — Gauss's law for magnetic fields

Maxwell's Equations	
Divergence equations	**Curl equations**
$\nabla . D = \rho_f$	$\nabla \times E = -\frac{\partial B}{dt}$
$\nabla . B = 0$	$\nabla \times H = J + \frac{\partial D}{dt}$

Gauss's law for magnetic fields

- Gauss's law for magnetism states that **no magnetic monopoles exists** and that the total flux through a closed surface must be zero.
- ∇ . **B** = 0 is derived from:

$$\int B . dS = 0$$

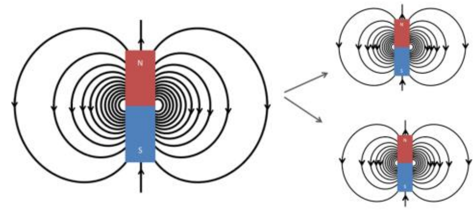


Figure. When a bar magnet is cut in two, you get two bar magnets.

Source: https://em.geosci.xyz/content/maxwell1_fundamentals/formative_laws/gauss_magnetic.html

So, let us look into the Gauss law of magnetic field. So, Gauss law for magnetism states that no magnetic monopole exist and therefore, the total flux through a closed surface must be 0.

ok. So, you can actually see that any surface the amount of you know flux that is entering and exiting will be equal because if you start cutting a magnet into half each of those half magnets will also have the 2 poles ok. So, divergence of B equals 0 can be actually derived from surface integral of B equals

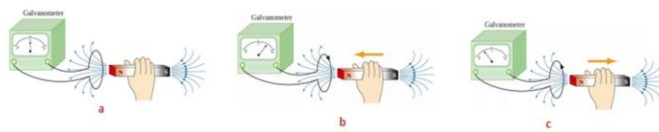
Maxwell's Equations — Faraday's law

Maxwell's Equations	
Divergence equations	Curl equations
$\nabla . D = \rho_f$	$\nabla \times E = -\frac{\partial B}{dt}$
$\nabla . B = 0$	$\nabla \times H = J + \frac{\partial D}{dt}$

Faraday's law

Faraday's Law of Induction: Integral Form

- The line integral of the electric field around a closed loop is equal to the negative of the rate of change of the magnetic flux through the area enclosed by the loop.



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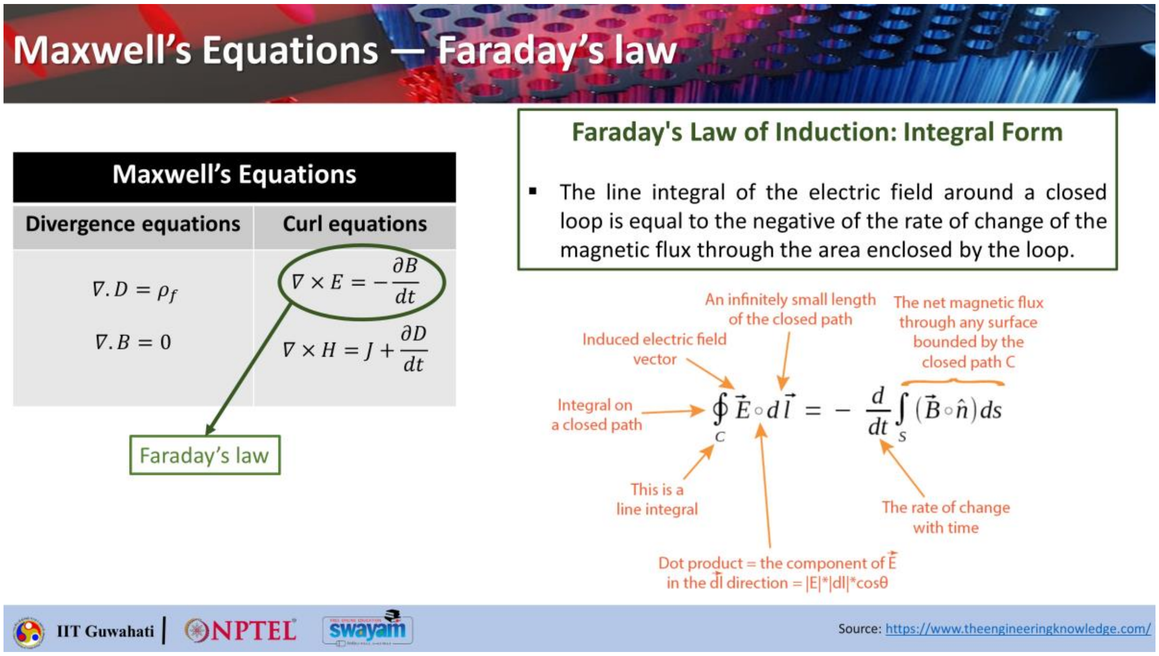
Source: https://www.theengineeringknowledge.com/

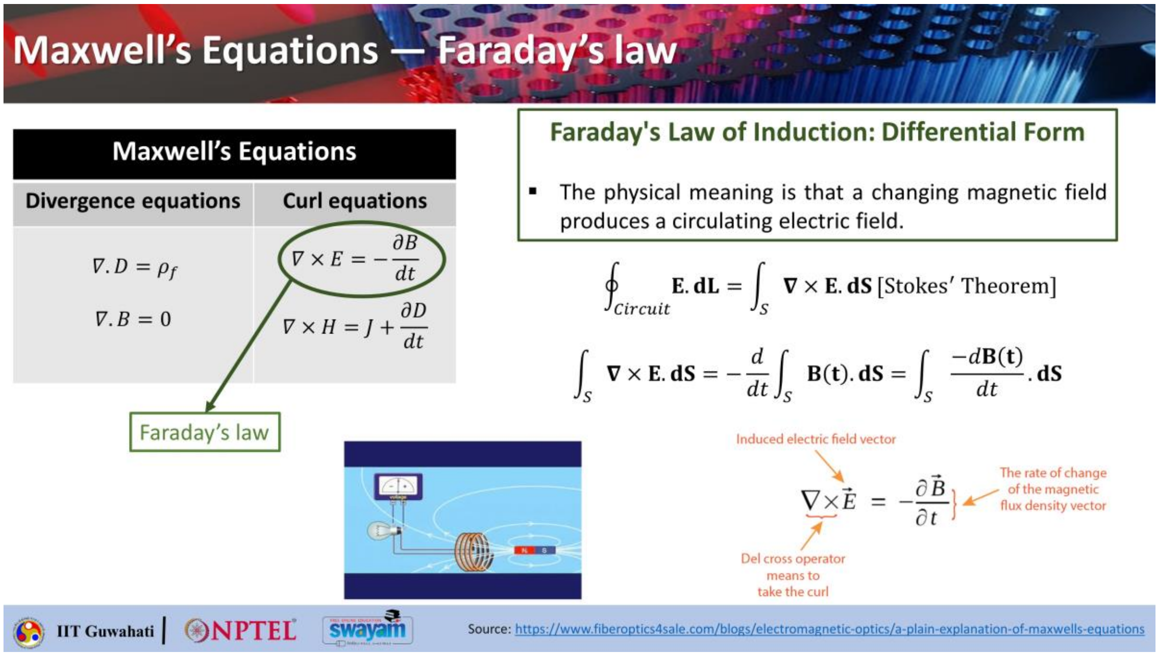
Now, the third equation is coming from Faraday's law of induction, right. So, this is one of the first two equations that connect E and B. okay. So, that is very interesting because here you are actually getting a relationship between magnetic flux density and electric field.

And in the last equation, the fourth equation you will get the relationship between magnetic field and electric flux density. So, let us look into the first one here. So, the electromagnetic induction was discovered independently by Michael Faraday in 1931 and Joseph Henry in 1932. But Faraday published his results first and so the law is known as Faraday's law of induction right. So, here it says that line integral of an electric field around a closed loop is equal to the negative rate of change of the magnetic flux through that area and closed by the loop.

So, you can actually take a make a loop like this and take a magnet through this one ok and you will can see that with the direction when you go this way the galvanometer shows positive current if you take it away it shows the negative current and so on. So, there is this negative sign coming to picture.



So, you can also see that you know this integral form tells us about couple of interesting things that you can take induced electric vector field and then take a closed loop line integral ok, where ∂l stands for a very small length of that closed path. So, what do you get is nothing but $-\partial/\partial$t ok, that is the rate of change of the magnetic flux density through that closed surface. So, you do $\vec{B}.\hat{n}.\hat{n}$ is the unit vector which is normal to the surface and this is how you obtain this ok.

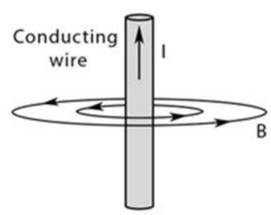


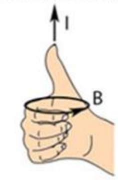
So, you can use you can try to see the magnet Faraday's law of induction in differential form. So, here the physical meaning remains same it says that you know the changing magnetic field produces a circulating electric field. So, this curl of E means it is a circulating electric field ok. $\partial B/\partial t$ means changing magnetic field with time when I say changing magnetic field is basically changing with time. So, you can start with this equation that over a closed circuit $E.dl$ ok and then you can use the Stokes theorem and write that you know this is equal to $\nabla \times E.dS$ ok. So, you can write $\nabla \times E.dS$ over surface integral is same as $-\frac{\partial B}{dt}$ integral over that closed surface ok.

So, from this you can equate these two and say this quantity $\nabla \times E$ is basically equal to $-\frac{\partial B}{dt}$. So, this is how you can obtain the differential form.

Ampere's Law - no time dependence (Incomplete)

- Suppose you have a conductor (wire) carrying a current, **I**. Then this current produces a Magnetic Field which circles the wire.
- Ampère had shown how to make magnetism from electricity.
- **Right Hand Thumb Rule:**
  - Thumb points in the direction of the electric current and fingers curl around the current indicating the direction of the magnetic field.





Source: https://www.sciencefacts.net/amperes-law.html

Now, coming to the fourth equation where actually Maxwell made a very important contribution. So, before Maxwell the world only knew that half of this equation and this half of the equation was known as Ampere's law. Okay and what is that law it says that you know a electric current going through the wire turns this current turns this wire into a magnet because when the current flows the magnetic field will be generated right.

And the direction of the magnetic field is obtained by right hand rule so where the thumb go in the direction of the current flow and the fingers basically tell you about the direction of the rotating magnetic field. So, Ampere had shown how to make magnetism from electricity right. So, right hand thumb rule also works well here.

Ampere's Law - no time dependence (Incomplete)

Ampere's Law – Integral Form

- From **Biot-Savart's Law**, the magnetic field due to a long straight wire is:

Since, $\vec{B}$ and $d\vec{l}$ are in the same direction,

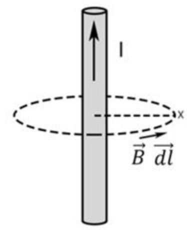
$$\vec{B}.d\vec{l} = B\,dl\cos 0 = B\,dl$$

Therefore,

$$\oint \vec{B}.d\vec{l} = \frac{\mu_0 I}{2\pi r}\oint dl$$

$$\Rightarrow \oint \vec{B}.d\vec{l} = \frac{\mu_0 I}{2\pi r}\oint (2\pi r)$$

$$\boxed{\Rightarrow \oint \vec{B}.d\vec{l} = \mu_0 I}$$



Source: https://www.sciencefacts.net/amperes-law.html

So, Ampere's law when there is no time dependence ok. it basically becomes Biot-Savart law you can directly write it from this particular Biot-Savart law that $B.dl$ ok.

So, this is B ok. and dl is a very small path in this closed loop. So, when you do the integration you get $\frac{\mu_0 I}{2\pi r}$ ok. So, finally you can write that $B.dl$ closed loop integral is nothing but the kind and closed $\mu_0 I$ ok.

Ampere's Law - no time dependence (Incomplete)

Ampere's Law – Differential Form

- We can put $\boldsymbol{B} = \mu_0 \boldsymbol{H}$ in the integral form of Ampere's law $\oint \vec{B}.\,d\vec{l} = \mu_0 I$.
- Then, the differential form of Ampere's law can be determined by applying **Stokes Theorem**:

$$\oint \mathrm{H}.\,\mathbf{dL} = \boldsymbol{I}_{enc} \quad \Rightarrow \quad \oint \mathbf{H}.\,\mathbf{dL} = \int_S (\nabla \times \mathbf{H}).\,d\mathbf{S}$$

$$\boldsymbol{I}_{enc} = \int_S \mathbf{J}.\,\mathbf{dS} \quad \Rightarrow \quad \int_S (\nabla \times \mathbf{H}).\,\mathrm{d}\mathbf{S} = \int_S \mathbf{J}.\,\mathrm{d}\mathbf{S}$$

$$\boxed{\Rightarrow \nabla \times \mathbf{H} = \mathbf{J}} \longrightarrow \text{Incomplete and not valid for electrodynamics}$$

We can put B equals $\mu_0 H$ in this integral form of Ampere's law. That is cyclic integral or you can say closed loop integral $\oint \vec{B}.\,d\vec{l} = \mu_0 I$.

And then the differential form of Ampere's law can be determined by using the Stokes theorem. So you can say that μ_0 can be adjusted here and you can write H. dL is nothing, but I enclosed and closed loop integral of H. dL can be written as curl of H into a surface integral right. And current enclosed can be written as surface integral of the current surface current density right. So, these two can be equated and you can simply write curl of H is nothing, but current density ok surface current density.

Ampere's Law - no time dependence (Incomplete)

Maxwell's contribution to Ampere's law – time-dependence

- When Maxwell wrote down Ampere's law, he found out **that it is incomplete**. So let's take the divergence of Ampere's Law.

$$\nabla \times \mathbf{H} = \mathbf{J}$$

$$\nabla.(\nabla \times \mathbf{H}) = \nabla.\mathbf{J} \quad \text{"Divergence of the Curl is Zero"}$$

$$0 = \nabla.\mathrm{J} \quad \text{[the divergence of J is always Zero?]}$$

- But this not the case. Electric currents obey the continuity equation:

$$\frac{\partial \rho}{\partial t} = -\nabla.\mathbf{J}$$

- In other words, mathematically the curl of $\boldsymbol{H}$ must equal something more than just $\boldsymbol{J}$. Maxwell knew that a time-varying magnetic field gave rise to a solenoidal Electric Field (i.e. Faraday's Law). **So, why is not that a time varying D field would give rise to a solenoidal $\boldsymbol{H}$ field.**

So, curl of H becomes J. So, this is the Ampere's law without any time dependence. So, this is incomplete and it is not valid for electrodynamics. This is good for electrostatics, but not electrodynamics. So, now this is where you know Maxwell's law goes sorry Ampere's law goes wrong and incomplete and where Maxwell actually made his contribution to this Ampere's law by bringing into time dependence. So when Maxwell wrote down Ampere's law, he found that something is incomplete.

So how do you see that? You can take divergence of the Ampere's law. So Ampere's law is simply curl of H equals J. So you take divergence of it. So you get divergence of J. Now divergence of a curl is always 0. So, does it mean that divergence of J is also always 0, but that is not the case always right, electric currents obey the continuity equation.

And we have seen that you know, $\frac{\partial \rho}{\partial t} = -\nabla.\mathbf{J}$. It means Mathematically, the curl of H should have something more than just this J, okay. So, Maxwell knew that a time varying magnetic field gives rise to a solenoidal electric field which is the Faraday's law. So, why not a time varying D field, okay, give rise to a solenoidal H field. So, that is possible.

Ampere's Law - no time dependence (Incomplete)

Maxwell's contribution to Ampere's law – time-dependence

- When Maxwell wrote down Ampere's law, he found out **that it is incomplete**. So let's take the divergence of Ampere's Law.

$$\nabla \times \mathbf{H} = \mathbf{J}$$

$$\nabla . (\nabla \times \mathbf{H}) = \nabla . \mathbf{J}$$ "Divergence of the Curl is Zero"

$$0 = \nabla . \mathrm{J}$$ [the divergence of J is always Zero?]

- The universe loves symmetry, so Maxwell introduced the term named as the *displacement current density*:

$$\frac{\partial \mathbf{D}}{\partial t} = \boldsymbol{J}_{\mathrm{d}}$$ [Displacement Current Density]

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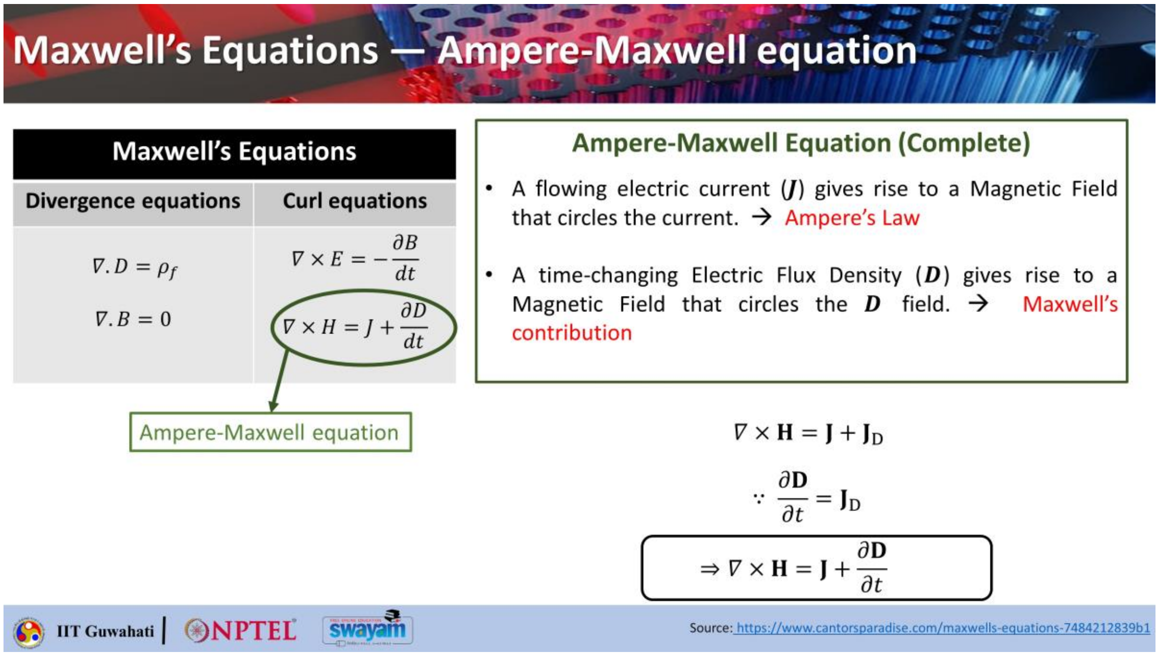
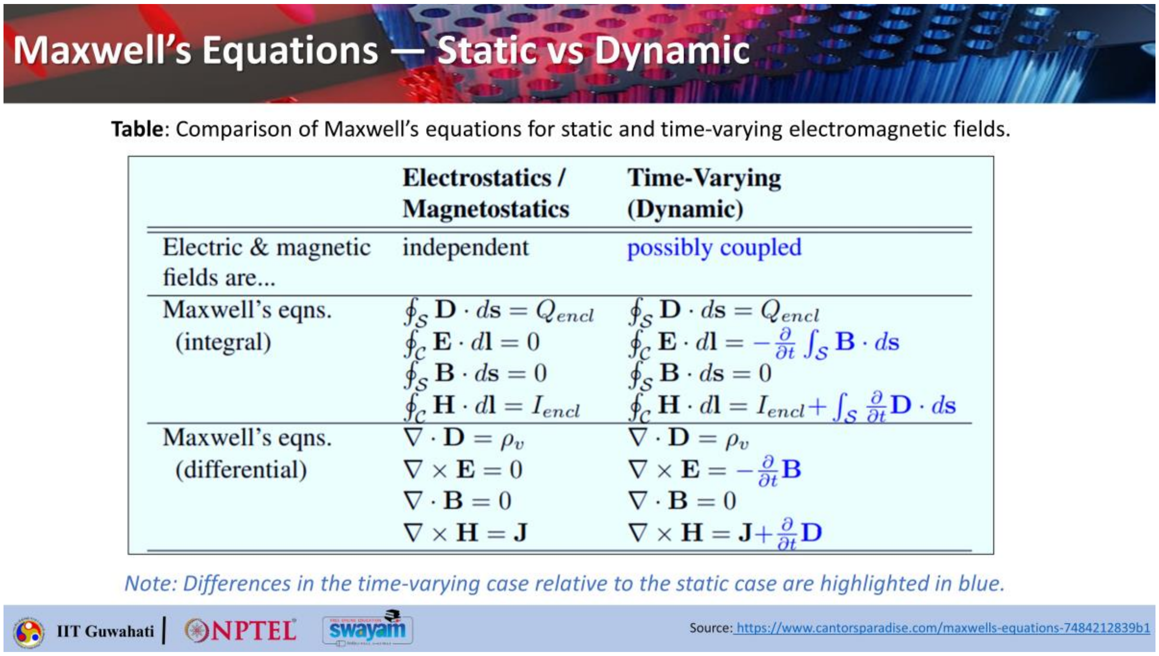
The universe loves symmetry so Maxwell introduced this term as displacement current density which is J_d that is nothing but $\partial D/\partial t$.



So, the fourth Maxwell's equation now states that the generation of magnetic field can be done in two methods. So, you can apply a surface current or electric current and you can also have changing electric field. So, that makes you know the ampere Maxwell equation complete. So, a flowing current J gives rise to a magnetic field that circles the current. So, that is pure Ampere's law and a time varying electric flux density D gives rise to a magnetic field that circles the D field ok.

So, that is the Maxwell's contribution. So, you can now write this as curl of H equals J plus J_D where J_D is nothing but $\partial D/\partial t$. So, you can write it in terms of you know this is the relationship between a electric flux density and magnetic field ok.



Maxwell's Equations — Static vs Dynamic

Table: Comparison of Maxwell's equations for static and time-varying electromagnetic fields.

	Electrostatics / Magnetostatics	Time-Varying (Dynamic)
Electric & magnetic fields are...	independent	possibly coupled
Maxwell's eqns. (integral)	$\oint_S \mathbf{D} \cdot d\mathbf{s} = Q_{encl}$ $\oint_C \mathbf{E} \cdot d\mathbf{l} = 0$ $\oint_S \mathbf{B} \cdot d\mathbf{s} = 0$ $\oint_C \mathbf{H} \cdot d\mathbf{l} = I_{encl}$	$\oint_S \mathbf{D} \cdot d\mathbf{s} = Q_{encl}$ $\oint_C \mathbf{E} \cdot d\mathbf{l} = -\frac{\partial}{\partial t} \int_S \mathbf{B} \cdot d\mathbf{s}$ $\oint_S \mathbf{B} \cdot d\mathbf{s} = 0$ $\oint_C \mathbf{H} \cdot d\mathbf{l} = I_{encl} + \int_S \frac{\partial}{\partial t} \mathbf{D} \cdot d\mathbf{s}$
Maxwell's eqns. (differential)	$\nabla \cdot \mathbf{D} = \rho_v$ $\nabla \times \mathbf{E} = 0$ $\nabla \cdot \mathbf{B} = 0$ $\nabla \times \mathbf{H} = \mathbf{J}$	$\nabla \cdot \mathbf{D} = \rho_v$ $\nabla \times \mathbf{E} = -\frac{\partial}{\partial t}\mathbf{B}$ $\nabla \cdot \mathbf{B} = 0$ $\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial}{\partial t}\mathbf{D}$

Note: Differences in the time-varying case relative to the static case are highlighted in blue.

So, we can now summarize the Maxwell's equation in terms of static and dynamic fields where electric and magnetic fields are independent of each other in the case of electrostatics whereas they are coupled to each other in the case of So, you can see the equations in integral form over here.

So, the first equation remains same. Second equation is where you bring in the time varying magnetic flux density. Similarly, in the fourth equation, you also incorporate this new term. So, the new terms are shown in blue. If you want to remember only the differential form, you can say that ∇.D equals ρ_v that remains same for electrostatic and electrodynamics. But curl of E equals 0, but curl of E is $-\partial B/\partial t$ in case of electrodynamics.

So, a time varying magnetic flux density gives rise to a rotating electric field. right. Similarly, the Gauss law for magnetism that remains same, but then the Ampere's law is corrected as Ampere's Maxwell's equation okay, where this new term in blue is introduced. right. So, you can actually see the differences in the time varying case as compared to the static case which are basically highlighted in this blue colour.

So, with that we will come to an end to this basic discussion. So, thank you everyone. In case you have got any doubt regarding this lecture you can always drop an email to me at this email address mentioning MOOC and photon crystal on the subject line. Thank you. Thank you.