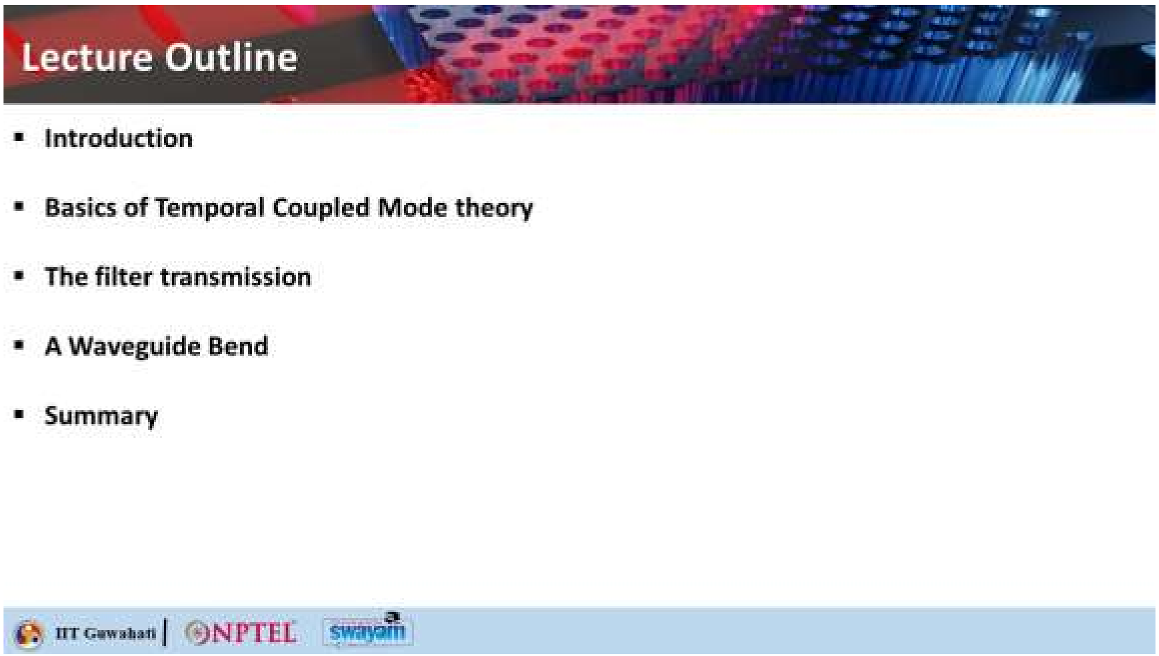
Lec 32: Temporal Coupled Mode Theory: Fundamentals and Applications

Hello students, welcome to lecture 32 of the online course on Photonic Crystals Fundamentals and Applications. Today's lecture will be on temporal couple mode theory. We will discuss the fundamentals and some analysis. So here is the lecture outline. We will have a brief introduction to the topic. We will discuss about the basics of temporal couple mode theory.



Then we'll take the example of a filter transmission, a waveguide band, and then summarize our findings. So first, an overview of this topic. So why this coupled mode theory is interesting, and then how this theory works.

Introduction

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So here we'll be discussing the concept of coupled mode theories, which are basically essential for analyzing complex photonic systems including devices such as narrowband filters, web guides. So as you can see, these are basically the passive devices which are very important in any photonic integrated circuits. We will also draw parallels between the couple mode theories in photonics and also the time dependent perturbation theory in quantum mechanics. So, when you discuss the fundamentals of temporal couple mode theory, in short we will be calling it as TCMT. There we will first make an abstract formulation.

Overview

- **Introduction to Coupled-Mode Theories**

  Overview: Discuss the concept of coupled-mode theories, which are essential for analyzing complex photonic systems, including devices like narrowband filters, waveguides etc.

  Analogous Concepts: Draw parallels between coupled-mode theories in photonics and time-dependent perturbation theory in quantum mechanics.

- **Fundamentals of Temporal Coupled-Mode Theory (TCMT)**

  Abstract Formulation: Unlike other methods, temporal coupled-mode theory uses an abstract formulation rather than a concrete physical model.

  Expansion in Eigenmodes: TCMT often involves an expansion in the exactly computed eigenmodes of idealized systems (such as isolated waveguides and cavities).

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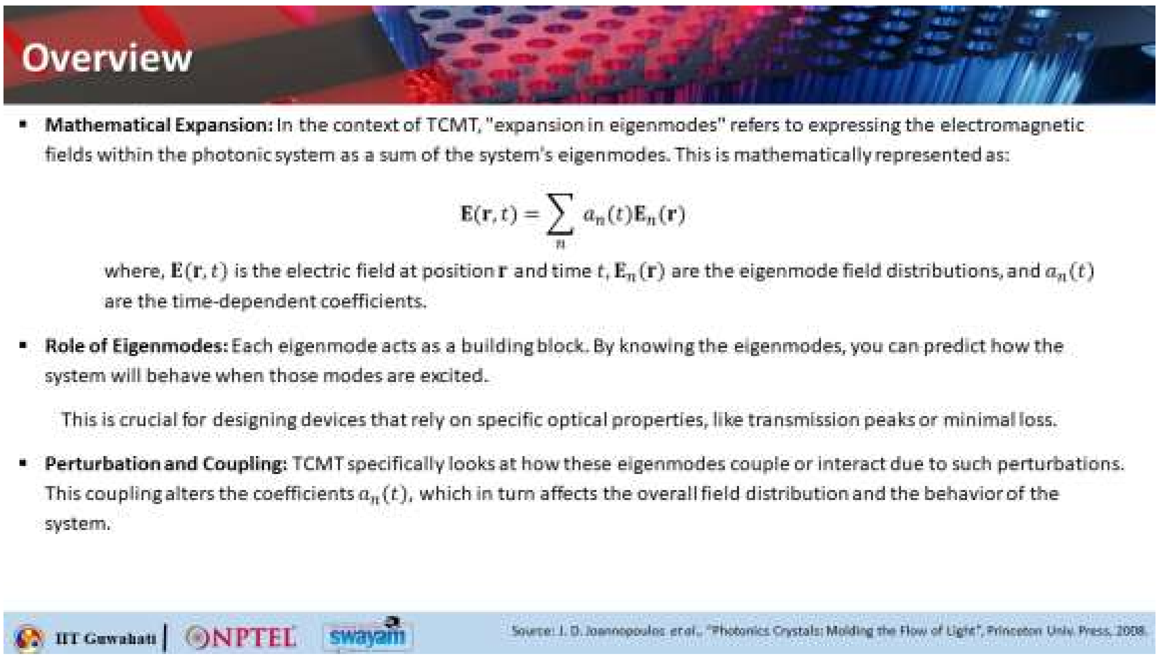
Source: J. D. Joannopoulos *et al.*, "Photonics Crystals: Molding the Flow of Light", Princeton Univ. Press, 2008.

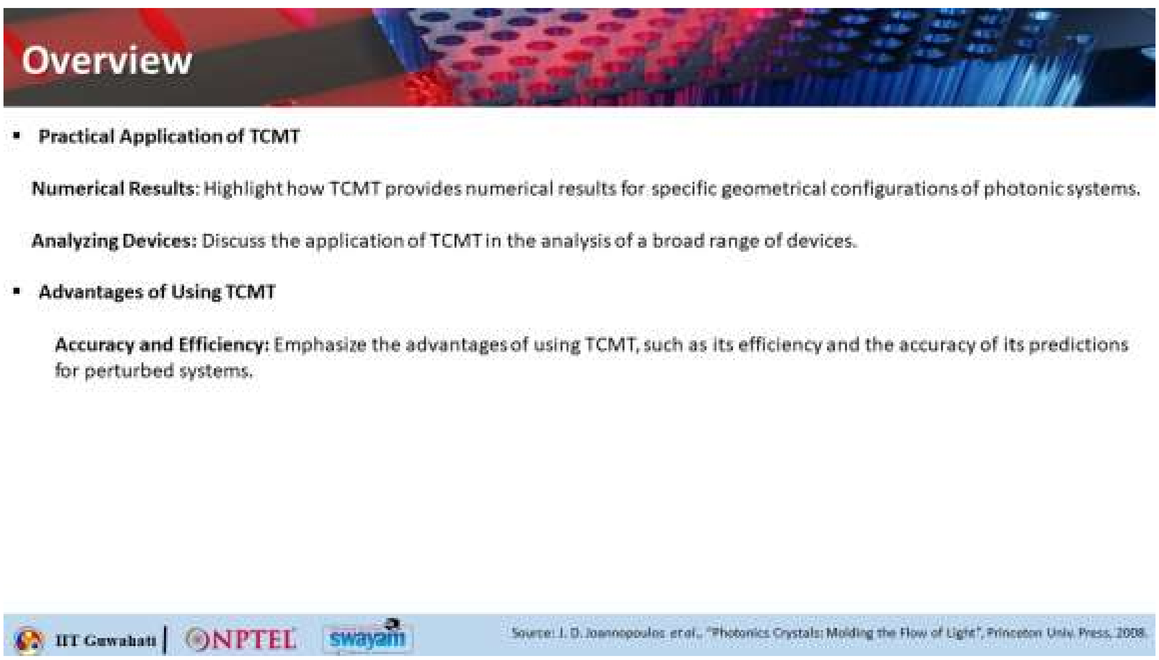
So here, unlike other methods, temporal couple mode theory will be using an abstract formulation rather than a concrete physical model. So it will involve expansion in exactly computed eigenmodes of idealized systems such as isolated waveguides and cavities. So basically it will work on abstract models rather than the actual physical system. So the concept of expansion of eigenmodes as used in temporal couple mode theory is a fundamental approach that helps in understanding and predicting the behavior of complex photonic systems. We'll also look into the mathematical expansion.



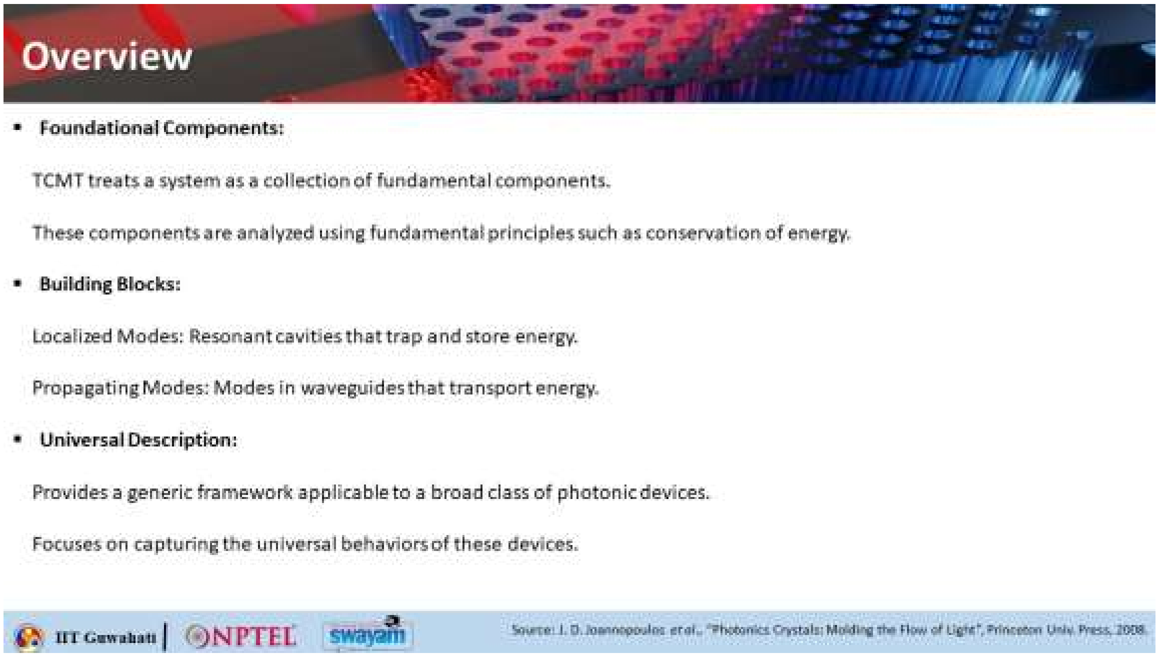
So here in the context of temporal couple mode theory, the expansion in eigenmodes will refer to expressing the eigenmodes that is like the electromagnetic fields within a particular photonic system as a sum of the system's eigenmodes. So if you want to present it mathematically, this is how it will look like. you can write E (r, t) okay so electric field is basically function of both position and time and you can write it as $\sum_n a_n(t)\mathbf{E}_n(\mathbf{r})$. So here okay as you can see this is the electric field e and t are basically the eigen mode field distribution and a and t are basically the time dependent coefficients and here you can see it is basically sum of different eigenmodes. So this is how basically you are expressing the electric field at any position r and t as an expansion of the system eigenmodes. So what is the role of the eigenmodes here? Each eigenmode basically acts as a building block.

So by knowing the eigenmodes, you can predict how the system will behave when those modes will get excited. So this is also crucial for designing devices that rely on specific optical properties, something like transmission peaks or minimal losses. The third important point would be like perturbation and coupling. So, in real world applications, the idealized eigenmodes are perturbed due to their interaction with other components or due to features like bands, splits or some material inconsistencies that you will see in your design. So temporal couple mode theory specifically looks at how these eigenmodes would couple or interact in the presence of this kind of perturbations and the coupling will alter this coefficients a and t which in turn will basically affect the overall field distribution and hence the behavior of the system will also change.

So all these things you can analyze using the temporal couple mode theory. Now the question is what are the practical applications as you have seen that this is basically an abstract model other than actual direct modeling. So first thing is that you know we have to understand how TCMT provides numerical results for specific geometrical configurations ok. How it can take care of those geometry specific information and give you some useful result and how it can help you in analyzing devices ok. That is where the application of TCMT will come into picture while you are discussing broad range of devices.

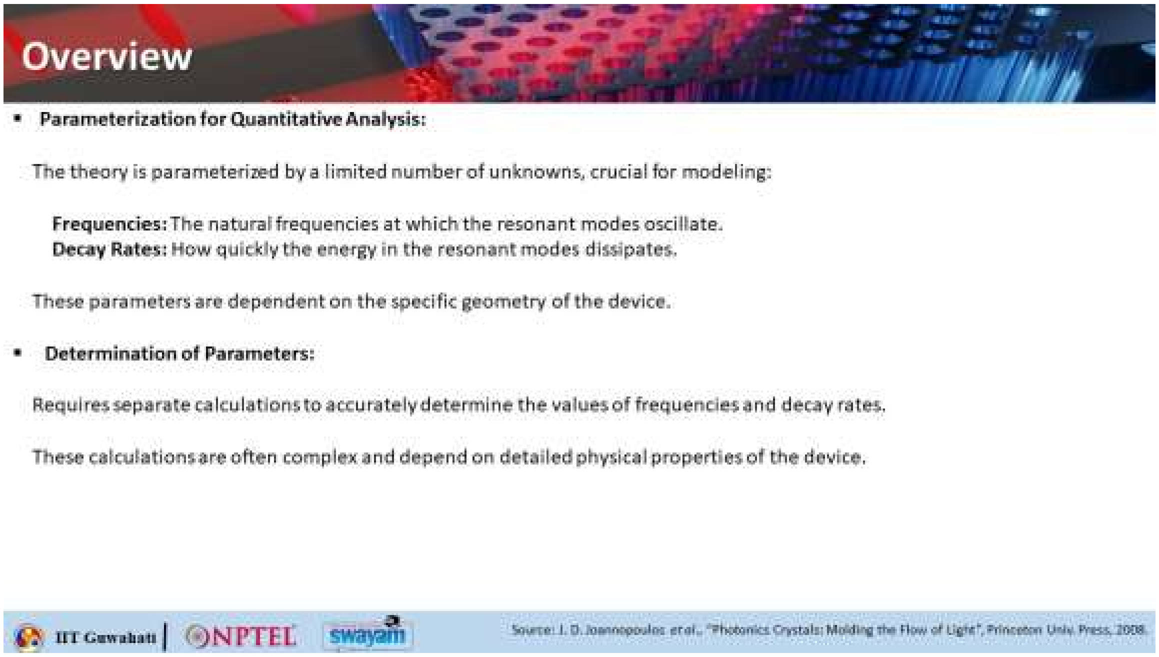


Next what is the advantage of using this theory? First thing is the accuracy and the efficiency as you understand you know we can emphasize on the advantages of using TCMT is its efficiency and accuracy of its prediction of the part of systems. So, it can do it very accurately and efficiently. So, what are the foundational components? So TCMT basically treats the system as a collection of fundamental components. And these components are analyzed by fundamental principles such as conservation of energy, And then we will consider the building blocks. So there can be two types of modes that we all know.

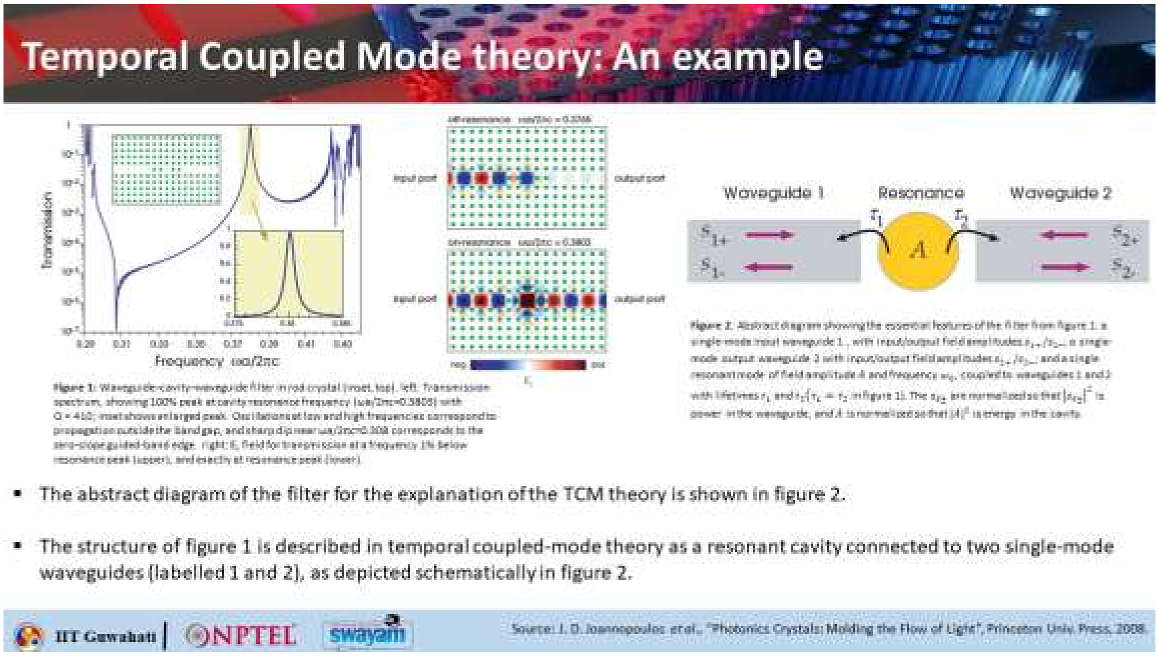


One is the localized modes. These are basically formed inside resonant cavities that trap and store energy. The other type of mode is propagating mode. These are the modes in waveguides that can transport energy. So these two are the building blocks.

And now this theory can also provide a generic framework which is applicable to a broad class of photonic devices and that is why this kind of system is also giving us a universal description. And it focuses on capturing the universal behavior of these devices. Next, how do you do parameterization of the quantitative analysis? The theory is basically parameterized by a limited number of unknowns which are crucial for modeling. First thing is frequencies. So, we will be talking about the natural frequencies at which the resonant modes oscillate.



And then we will also concern about the decay rates that tells us how quickly the energy in the resonant modes could dissipate. So these parameters are dependent on the specific geometry of the device and this is where the device specific information gets into the theoretical framework. How do you determine the parameters? This requires separate calculation to accurately determine the values of frequencies and the decay rates. And these calculations are usually complex and they depend on the detailed physical properties of the system. So let us start the example of applying temporal coupled mode theory to a practical example of narrowband filter.



So here you can see the structure of figure 1. is described in temporal couple mode theory as a resonant cavity connected to two single mode waveguides which are leveled as waveguide 1 and waveguide 2 okay. So, this is basically a filter okay. So, what is this filter? This is a waveguide cavity waveguide filter in a rod crystal. So, as you can see here in the inset ok this this dark spots are all rods ok.

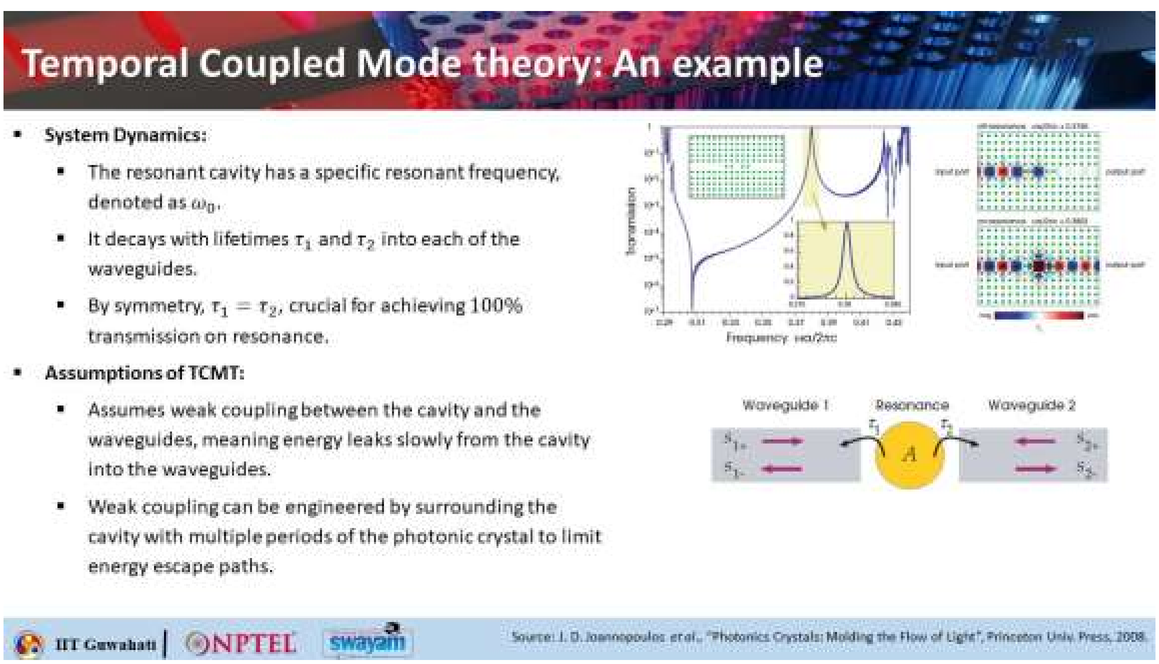
So, this is a waveguide then you have a cavity then you have waveguide ok. So, this is a waveguide cavity waveguide kind of filter and this is based on a rod crystal ok. And what you see here, this is the transmission spectrum that shows around 100% transmission peak at the cavity resonance frequency which comes out to be $\frac{\omega a}{2\pi c}$ equals 0.3803 that is exactly this point. So, they have highlighted this part and actually drawn over here.

So if you only focus on this small section, the highlighted part, this is how the transmission peak looks like. So you get a 100% peak over here, okay? And it has got a quality factor of 410, okay? So this is the inset that shows the enlarged peak. And the oscillations that you see here at the low and the high frequencies, they correspond to the propagation which are outside the band gap. And the sharp dip at $\frac{\omega a}{2\pi c}$ equals 0.308, it corresponds to the zero slope guided band edge.

And that is where there is no transmission at all at this particular point. So, on the right here you can see these are the electric field plots of the device. So, this is for the off resonance case. So, here you are considering $\frac{\omega a}{2\pi c}$ equals 0.3765.

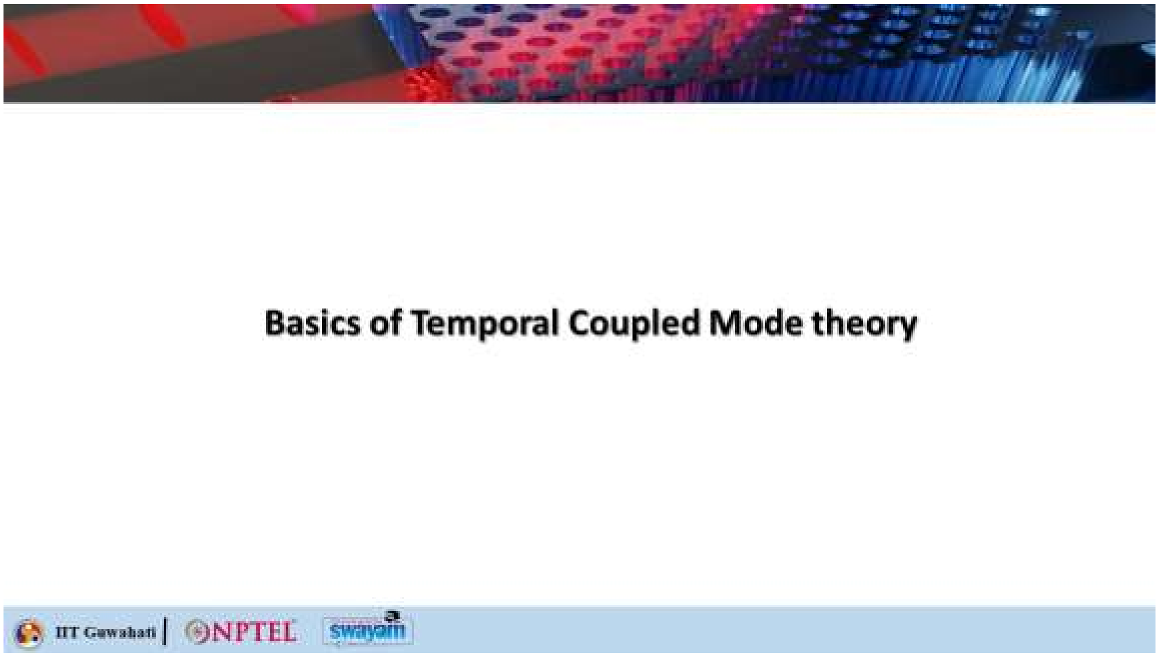
So, this is typically here somewhere. So, here you can see the transmission is very low. So, this is the input port, this is the output port. practically nothing comes out at the output port. But if you see at the resonance that is the on resonance okay, so you look at the parameter here $\frac{\omega a}{2\pi c}$ equals 0.3803 that is exactly where the peak is and you can see beautifully everything getting transferred to the output port. So, this is the electric field distribution telling you what is happening at off resonance and on resonance case. Now, if this system, this is an exact system, this has to be modeled using temporal coupled mode theory and this is the abstract diagram of the filter that actually helps us establishing this temporal coupled mode theory.

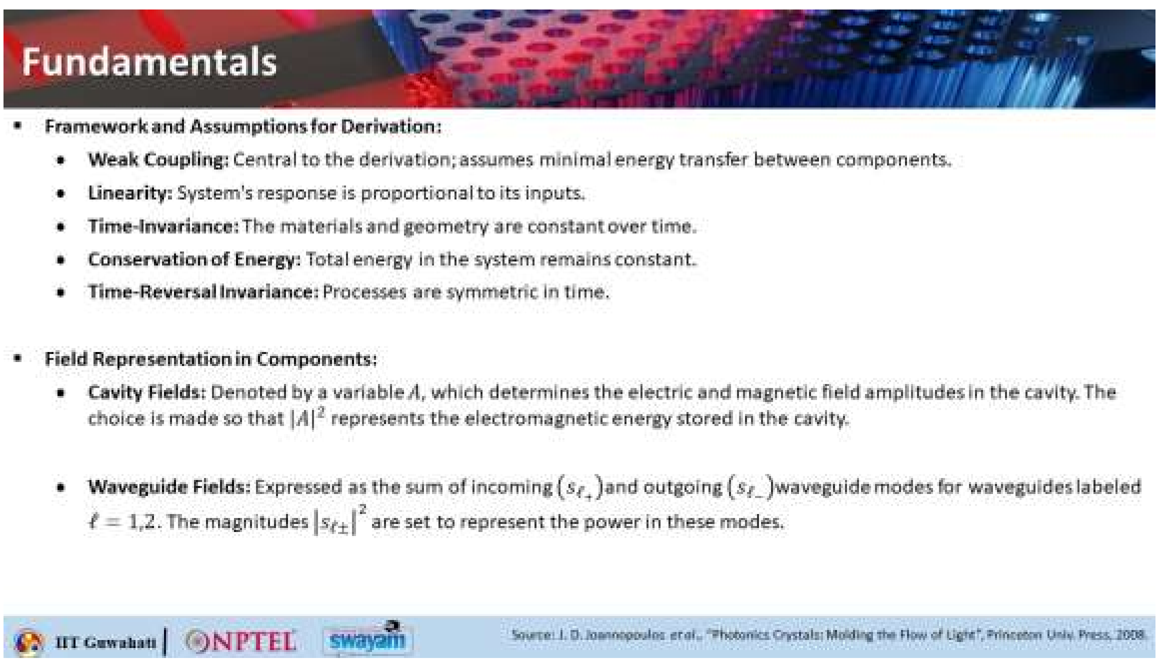
So this abstract diagram is showing the essential features of the filters from this figure 1. The first thing here is a single mode input waveguide that is waveguide 1 okay and it has got the input and output field amplitude which is S_{1+} and S_{1-} those this is input this is output okay. Similarly you have the output waveguide that is waveguide 2 it has got input and output field amplitudes given as S_{2+} and S_{2-}. So, the plus ones are the inputs and the minus ones are the outputs okay. And then you have a single resonant mode of field amplitude A and frequency omega naught coupled to waveguides 1 and 2 and they have lifetimes of τ_1 and τ_2. So, in this particular case okay τ_1 and τ_2 are equal okay and the $s_{\ell\pm}$ are normalized so that $|s_{\ell\pm}|^2$ is basically representing the power in the waveguide ok. And A is basically normalized so that modulus A square is basically giving you the energy in the cavity. What is ℓ ? ℓ is basically 1 or 2 ok. So, with that we can start analyzing the system slowly. Let's first focus on the system dynamics.



So the resonant cavity has a specific resonant frequency, which is denoted by omega naught. And what is understood that this cavity decays with lifetime τ_1 and τ_2 into each of the waveguides and because the waveguides are symmetrical, you can also take τ_1 and τ_2 to be symmetrical and that is crucial for achieving 100 percent transmission on resonance. Now, there are certain assumptions you need to make before applying this temporal couple mode theory.

So you have to assume weak coupling between the cavity and the waveguides, which means that the energy leaks slowly from the cavity into the waveguides. And the weak coupling can be engineered by surrounding the cavity with multiple periods of the photon crystal to limit the energy escape path. So that briefly you have seen. So now we will go into the basics of temporal couple mode theory. So we will now derive a set of equations describing the coupling of the cavity to the waveguides in terms of the field amplitude in those components.

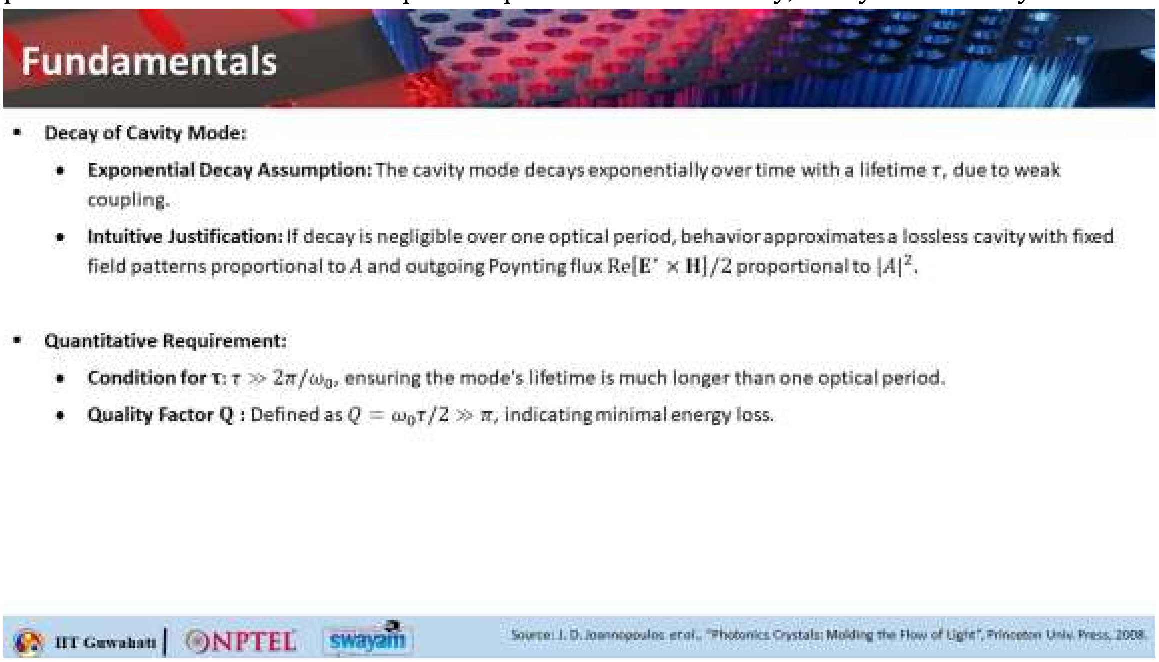




So here is the framework and the assumptions required for the derivation. First is weak coupling. So this is central to the derivation because it assumes minimal energy transfer between the components. We also consider linearity that means the system's response is proportional to the inputs. Time invariance, the materials and the geometry are constant over time.

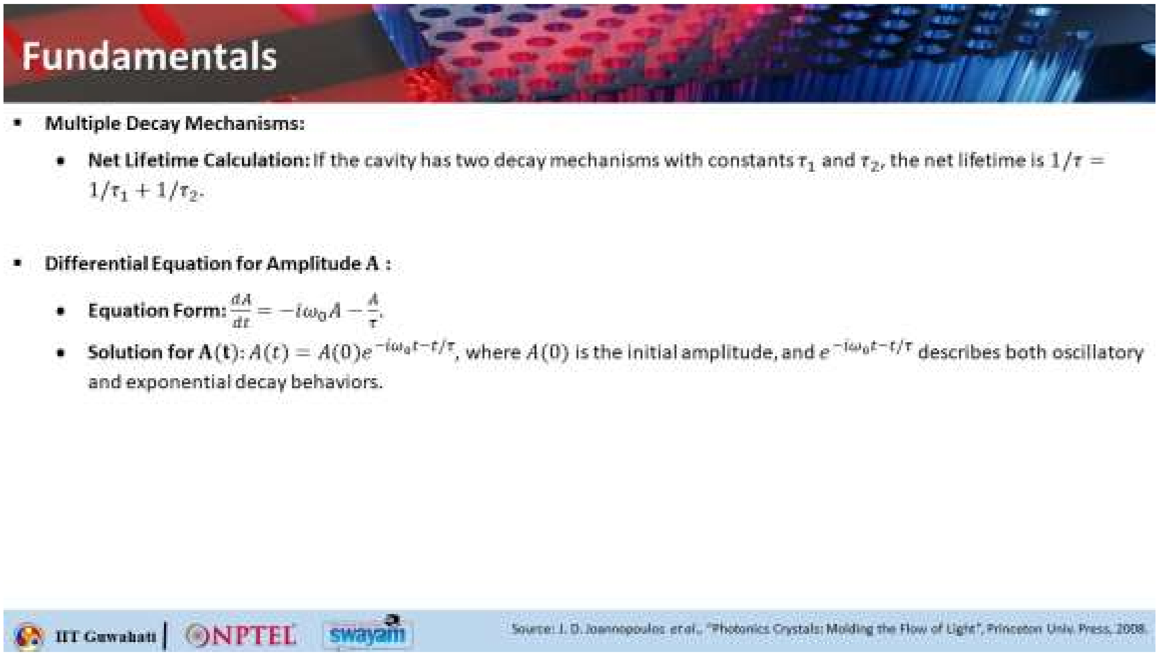
Conservation of energy that means the total energy in the system remains constant. and time reversal invariance that means the processes are symmetric in time. So, with that we can make the field representation in components. The first field will be the cavity field which can be denoted by a variable A that determines the electric and magnetic field amplitudes in the cavity. So, you can The choice is made in such a way that modulus of a square basically gives you the electromagnetic energy stored in the cavity.

Next important field is the waveguide field. These are basically expressed as the sum of the incoming that is s_{ℓ_+} and the outgoing that is $S_{\ell\_}$ waveguide modes for the waveguides which are levelled as ℓ equals 1 and 2. We have seen that previously. So the magnitudes which are calculated as modulus SL plus minus whole square, they basically represent the power in those modes. Third important parameter is the decay, decay of the cavity modes.



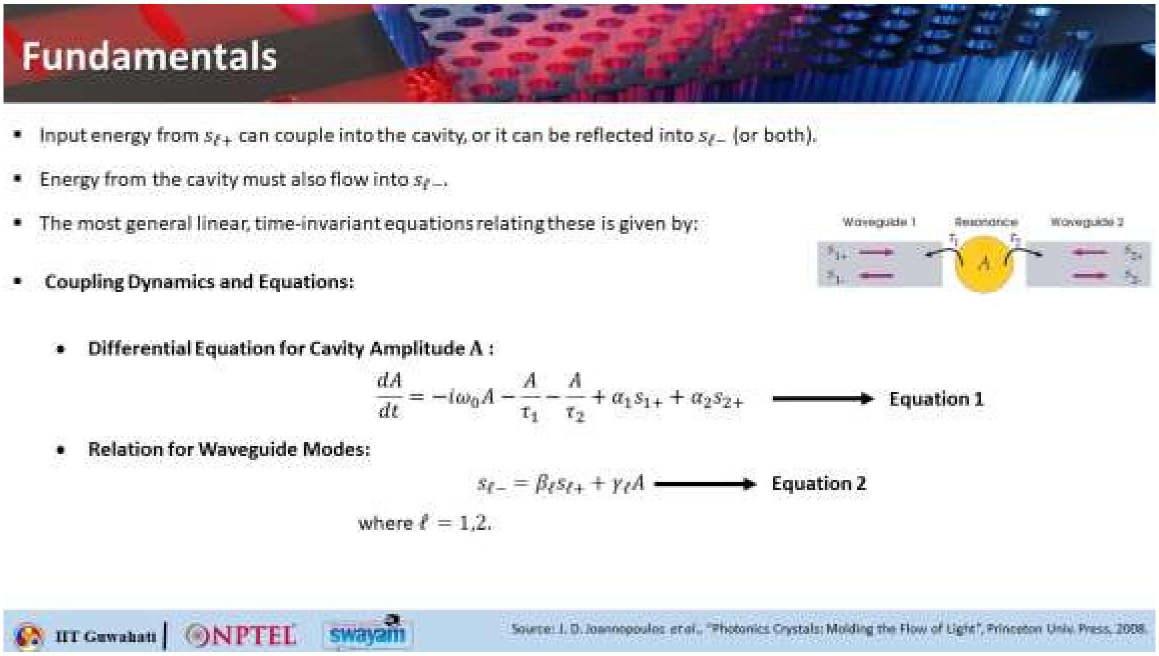
So we assume exponential decay. So the cavity modes basically decays exponentially over time with a lifetime of tau due to weak coupling. And if the decay is negligible over one optical period, the behavior approximates a lossless cavity with fixed field patterns proportional to A and the outgoing pointing flux that is real of E conjugate times H, this is the cross product of the two vectors by 2 will be proportional to modulus A square. So, to remember that to begin with we are considering the cavity mode by itself with no incident power coming from the waveguides. Then comes some important quantitative requirement.

First one is the condition for tau. Your tau should be $\tau \gg \frac{2\pi}{\omega_0}$, that ensures the modes lifetime is much longer than one optical period. okay and then the quality factor Q. So, here you can define the quality factor $Q = \omega_0\tau/2$ that is much greater than π and that indicates minimal energy loss. There can be multiple decay mechanism. So, you have to estimate the net lifetime. So if the cavity has got two decay mechanism with constants τ_1 and τ_2, the net lifetime can be given as $\frac{1}{\tau} = \frac{1}{\tau_1} + \frac{1}{\tau_2}$.



From that, you can obtain the differential equation for amplitude A. So in the equation form, you can write it as $\frac{dA}{dt}$ will be $-\ i\omega_0 A - \frac{A}{\tau}$. So if you want to solve this, the solution for $A(t)$ will have this particular form. So you can write $A(t)\ = A(0)e^{-i\omega_0 t - t/\tau}$, where $A(0)$ is basically the initial amplitude and $e^{-i\omega_0 t - t/\tau}$ basically describes the oscillatory behavior So, this is the oscillator behavior and this part actually tells you about the exponential decay behavior.

So, now we will include the waveguides. So, when you have the waveguides, so there is an input energy of SL plus that can couple into the cavity. or it can be reflected into SL minus or both can happen, okay. So, the energy from the cavity must flow into SL minus. So, this is the direction of the energy flow from waveguide, from cavity to waveguide, right. So, the most general linear time invariant equations relating all these phenomena is given by this coupling dynamics equation.



So, you can write this differential equation for cavity amplitude A as $\frac{dA}{dt} = -i\omega_0 A - \frac{A}{\tau_1} - \frac{A}{\tau_2} + \alpha_1 s_{1+} + \alpha_2 s_{2+}$. So, this is equation 1 and if you try to establish the relation for waveguide modes, so you can write $s_{\ell-}$ is basically $\beta_\ell s_{\ell+} + \gamma_\ell A$.

A is the amplitude of the cavity mode, okay. So, here ℓ is again it can be 1 or 2. Now, we have seen the two new parameters α_ℓ and γ_ℓ. So, these are basically the coupling strengths. So α_ℓ and γ_ℓ represents the strength of cavity to waveguide coupling okay and β_ℓ is basically the reflection coefficient at each waveguide okay. So now let us look into the conservation of energy analysis.

Fundamentals

- **Parameter Interpretations:**
  - **Coupling Strengths:** α_ℓ and γ_ℓ represent the strength of cavity to waveguide coupling.
  - **Reflection Coefficients:** β_ℓ represents the reflection coefficient at each waveguide.
- **Conservation of Energy Analysis:**
  - **Determining γ_1 and γ_2 :**

    When $\tau_2 \to \infty$ and $s_{1+} = s_{2+} = 0$, the cavity decays solely through τ_1, giving:

$$-\frac{d|A|^2}{dt} = \frac{2}{\tau_1}|A|^2 = |\gamma_1|^2|A|^2 \Rightarrow |\gamma_1|^2 = \frac{2}{\tau_1} \longrightarrow \textbf{Equation 3}$$

$$\gamma_1 = \sqrt{\frac{2}{\tau_1}} \quad \text{and} \quad \gamma_2 = \sqrt{\frac{2}{\tau_2}}$$

assuming weak coupling, changes in γ_1 due to τ_2 and vice versa are considered as second-order effects and are neglected.

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Source: J. D. Joannopoulos et al., "Photonics Crystals: Molding the Flow of Light", Princeton Univ. Press, 2008.

So first determining γ_1 and γ_2 okay. So when τ_2 tends to infinity that is the end you have s_{1+} and s_{2+} both 0, the cavity decays solely through τ_1 So, you can understand that there is nothing input. So, these are the inputs from the waveguide to the cavity, they are 0, τ_2 is infinity, that means that cavity is solely decaying through this τ_1. So, in that case, you can write that equation $-\frac{d|A|^2}{dt} = \frac{2}{\tau_1}|A|^2$ and this can be represented as 1 over tau gamma 1 square modulus okay and then you have this modulus this a square. So, from this what you can understand if you equate these two you can find out the relationship between the γ_1 and τ_1 .

So, you can see that $|\gamma_1|^2 = \frac{2}{\tau_1}$. So, that way you can obtain $\gamma_1 = \sqrt{\frac{2}{\tau_1}}$ and similarly you can write $\gamma_2 = \sqrt{\frac{2}{\tau_2}}$. So that is important equation number 3. And here we are assuming weak coupling. that means changes in gamma 1 due to gamma 2 and vice versa are all neglected.

So, gamma 1 is only changing because of τ_1 and there is change in γ_2 only because of τ_2, ok. Now, we have to determine the constants α_ℓ and β_ℓ that can be determined by the time reversal symmetry. So, if you use the time reversal symmetry, that can be done by running the original solution backward in time and conjugating. So, we can derive another valid solution that looks like this, okay. So, here $A(t) = A(0)e^{-i\omega_0 t + t/\tau}$.

Fundamentals

- The constants α_ℓ and β_ℓ can be determined by time-reversal symmetry.

- **Use of Time-Reversal Symmetry:**
  - **Creating a Time-Reversed Solution:** By running the original solution backward in time and conjugating, we can derive another valid solution:
  - **Form of the solution:** $A(t) = A(0)e^{-i\omega_0 t + t/\tau}$, representing exponentially growing amplitude.
  - **Input fields:** $s_{\ell+} = \sqrt{2/\tau_\ell}A$, assuming zero output fields ($s_{\ell-} = 0$).

- **Derivation of Reflection Coefficients (β_ℓ) :**
  - **Calculation of β_ℓ :** From the above time-reversed conditions, it leads to $\beta_\ell = -1$ (from equation 2).
  - **Interpretation of 100% Reflection:** As $\tau_\ell \to \infty$, the relation $s_{\ell-} = -s_{\ell+}$ implies total reflection.

 Source: J. D. Joannopoulos *et al.*, "Photonics Crystals: Molding the Flow of Light", Princeton Univ. Press, 2008.

So, that is basically representing an exponentially growing amplitude. So, here the input fields are $s_{\ell+}$ can be written as $\sqrt{2/\tau_\ell}A$ that assumes 0 output fields, that means $s_{\ell-}$ equals 0. So, when I am using L, I am talking about both 1 and 2, both the waveguides. So, from that you can derive the reflection coefficient β_ℓ. So, the calculation of β_ℓ can be done using this time reversed condition and it will give you from equation 2, if you go back and see equation 2, yeah, this one, okay, it gives you that β_ℓ equals minus 1.

And how do you interpret 100 percent reflection? So, when you have τ_ℓ going to infinity that means you know the relationship will be something like this between the two input and the output becomes like this. So, $s_{\ell-} = -s_{\ell+}$ that means whatever is getting incident is basically getting reflected that means total input. total reflection is taking place. Next, we determine the coupling coefficient α_ℓ . So first, we consider the special case where you are taking τ_2 to be infinity.

Fundamentals

- **Determination of Coupling Coefficients (α_ℓ) :**
  - **Special Case Analysis:** Consider when $\tau_2 \to \infty$ (disconnecting waveguide 2):
  - In this setup, with no input energy ($s_{1+} = s_{2+} = 0$), the decay of the cavity mode provides a clear solution.
  - **Expression for α_1 :** Plugging $A(t)$ into the **equation 1** leads to $\alpha_1\sqrt{2/\tau_1}A = 2A/\tau_1$, simplifying to $\alpha_1 = \sqrt{2/\tau_1}$.
  - **Generalization to All α_ℓ :** Extending this, $\alpha_\ell = \sqrt{2/\tau_\ell} = \gamma_\ell$ for both waveguides.

That means you just move it away or disconnect your waveguide 2. So in this case with no input energy, so you are considering that this input and this input both are 0. So the decay of the cavity mode provides a clear solution. So, how do you find out the expression for α_1 ? So, whenever I am say α_ℓ, so there will be α_1 and α_2. So, when you plug this A(t) into equation 1, so you have seen equation before, you can see that it comes out to be $\alpha_1\sqrt{2/\tau_1}A = \frac{2A}{\tau_1}$. So, if you simplify this expression, you will get $\alpha_1 = \sqrt{2/\tau_1}$.

So if you generalize this expression for alpha L, it will be $\alpha_\ell = \sqrt{2/\tau_\ell}$. Or you can say this is same as γ_ℓ . So this is the generic expression for α_ℓ and gamma L for both the coefficients. So finally, we have obtained the temporal coupled mode equations for the system that is shown in the figure. So next we shall consider the two cases where this theory is applied to explain the behavior of some waveguides.

Fundamentals

$$\frac{dA}{dt} = -i\omega_0 A - \sum_{\ell=1}^{2} A \big/ \tau_\ell + \sum_{\ell=1}^{2} \sqrt{\frac{2}{\tau_\ell}}\, s_{\ell +} \longrightarrow \textbf{Equation 4}$$

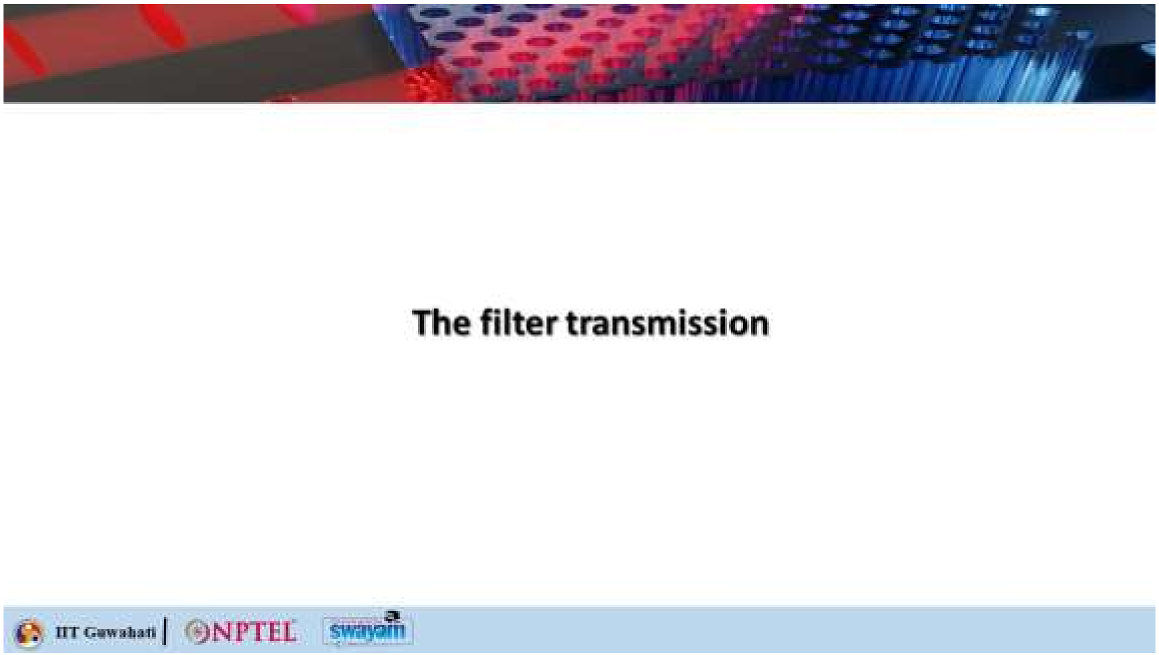
$$s_{\ell -} = -s_{\ell +} + \sqrt{\frac{2}{\tau_\ell}}\, A \longrightarrow \textbf{Equation 5}$$

- These equations are valid for any filter satisfying the assumptions; the details matter only in determining the values of ω_0 and τ_ℓ.
- This approach is easily generalized to include more than two waveguides, radiative losses, and so on.

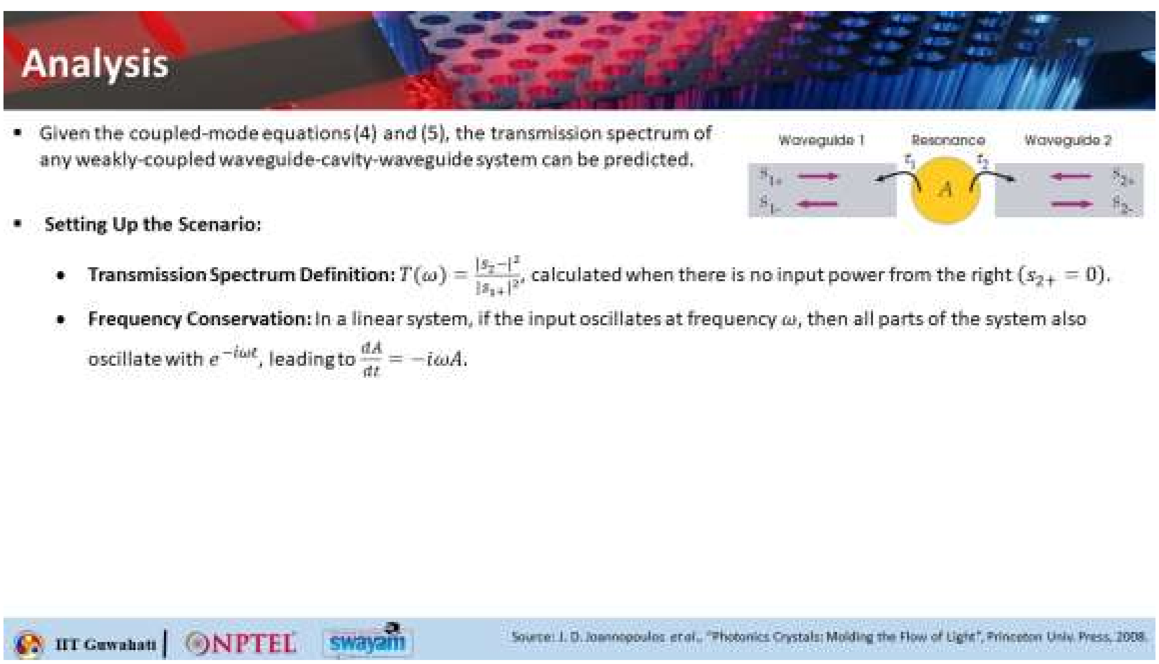
Source: J. D. Joannopoulos et al., "Photonics Crystals: Molding the Flow of Light", Princeton Univ. Press, 2008.

So these are the two important equations. So, we have seen $\frac{dA}{dt} = -i\omega_0 A - \frac{\sum_{\ell=1}^{2} A}{\tau_\ell} + \sum_{\ell=1}^{2} \sqrt{\frac{2}{\tau_\ell}}\, s_{\ell+}$. So, this is how you write the expression for $\frac{dA}{dt}$ that tells you about the time varying amplitude ok. And the relationship between the input and outputs of the waveguides are given as this. So, $s_{\ell-} = -s_{\ell+} + \sqrt{\frac{2}{\tau_\ell}} A$. So, these expressions are valid for any filter satisfying the the assumptions that we have made okay and the details will matter only in determining the values of omega naught and tau L.

So this approach is generalized to include more than two waveguides, radiative losses and so on.

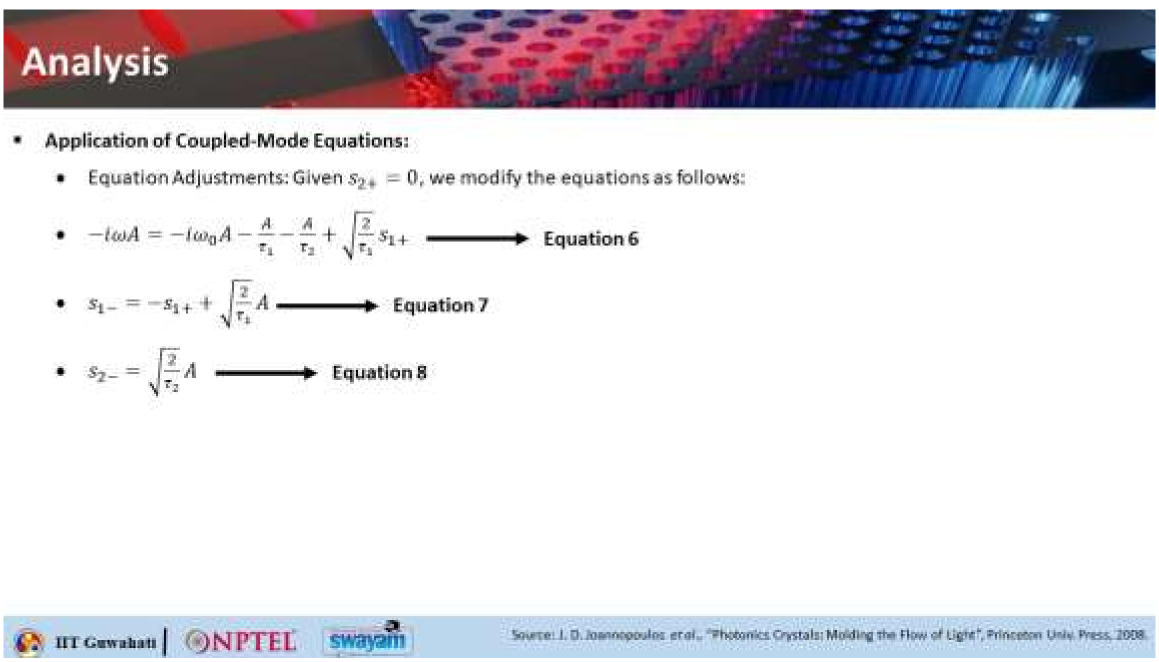


Now we will see how can we use this theory to find out the filter transmission. So the previous two equations if you remember the coupled mode equation number 4 and 5, from those you can obtain the transmission spectrum of any weakly coupled waveguide cavity, waveguide kind of system. okay and if you set up the scenario the transmission spectrum can be defined like this $T(\omega)$ will be $\frac{|s_{2-}|^2}{|s_{1+}|^2}$.



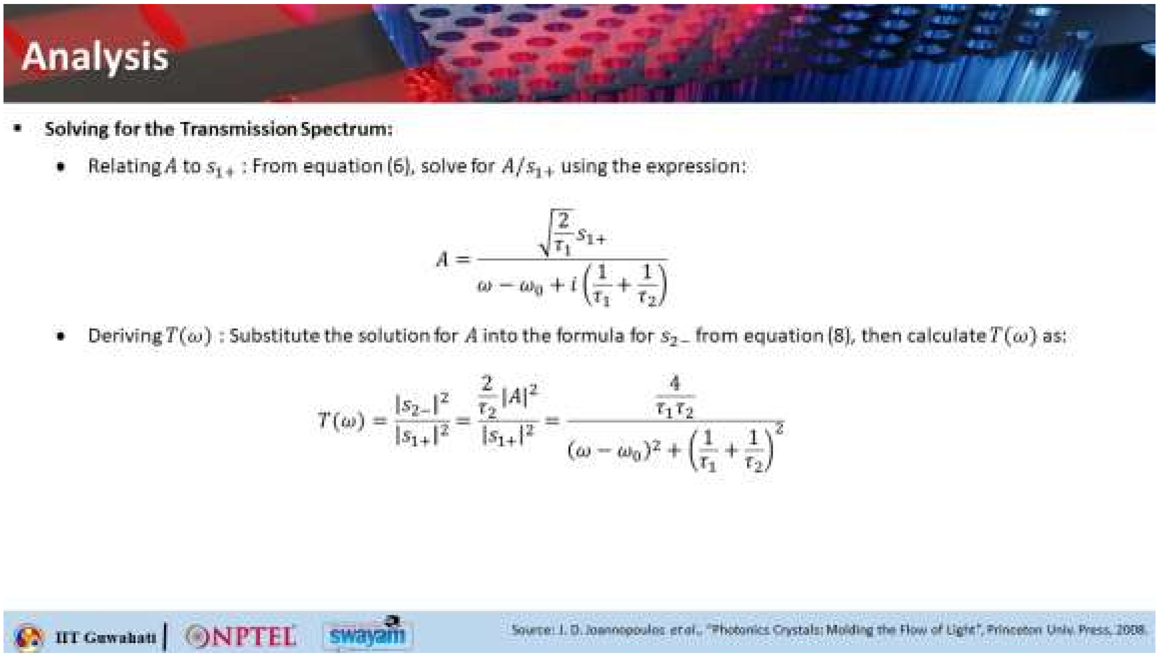
So, this is the input and this is the transmitted. So, this is the filtered output. So, this is what you are bothered about. So, this you have to calculate when there is no input from the right that means S2 plus will be 0. Regarding frequency conservation in a linear system, if the input oscillates at frequency omega, then all parts of the system will also oscillate with e to the power minus i omega that means it will lead to $\frac{dA}{dt} = -i\omega A$ okay.

So, this is fine. Now, let us apply coupled mode equations. So, first adjustment that you make in the expression is that you put s_{2+} equals 0. that will help us modify the equations. Now, the equations will look like this $-i\omega A = -i\omega_0 A - \frac{A}{\tau_1} - \frac{A}{\tau_2} + \sqrt{\frac{2}{\tau_1}} S_{1+}$ ok. So, this becomes your equation 6 from that you can find out the relationship between the reflected beam in the first waveguide that is $s_{1-} = -s_{1+} + \sqrt{\frac{2}{\tau_1}} A$ and s_{2-} that is the final output will be simply $\sqrt{\frac{2}{\tau_2}} A$.

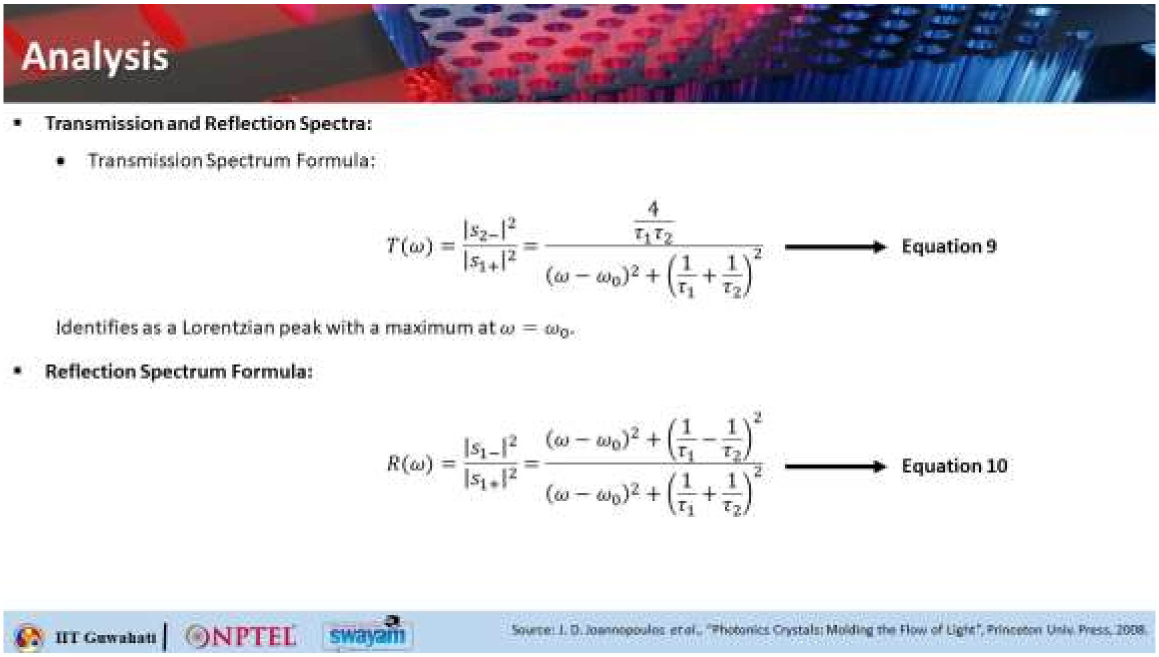


So, these are your equation 6, 7, 8 and that tells you the transmission characteristics. Now, if you solve for the transmission spectrum you relate $\frac{A}{s_{1+}}$. So, from equation 6, so from this equation you solve for $\frac{A}{s_{1+}}$ using the expression okay and you get $A = \frac{\sqrt{\frac{2}{\tau_1}} s_{1+}}{\omega - \omega_0 + i\left(\frac{1}{\tau_1} + \frac{1}{\tau_2}\right)}$. This is pretty simple maths, it is not a complicated one, you just try it on paper and from that you can derive what is T that is the transmittance, okay. So, you substitute the solution for A that you have found here into the formula for S2 minus of equation 8. So, you put it here, okay.

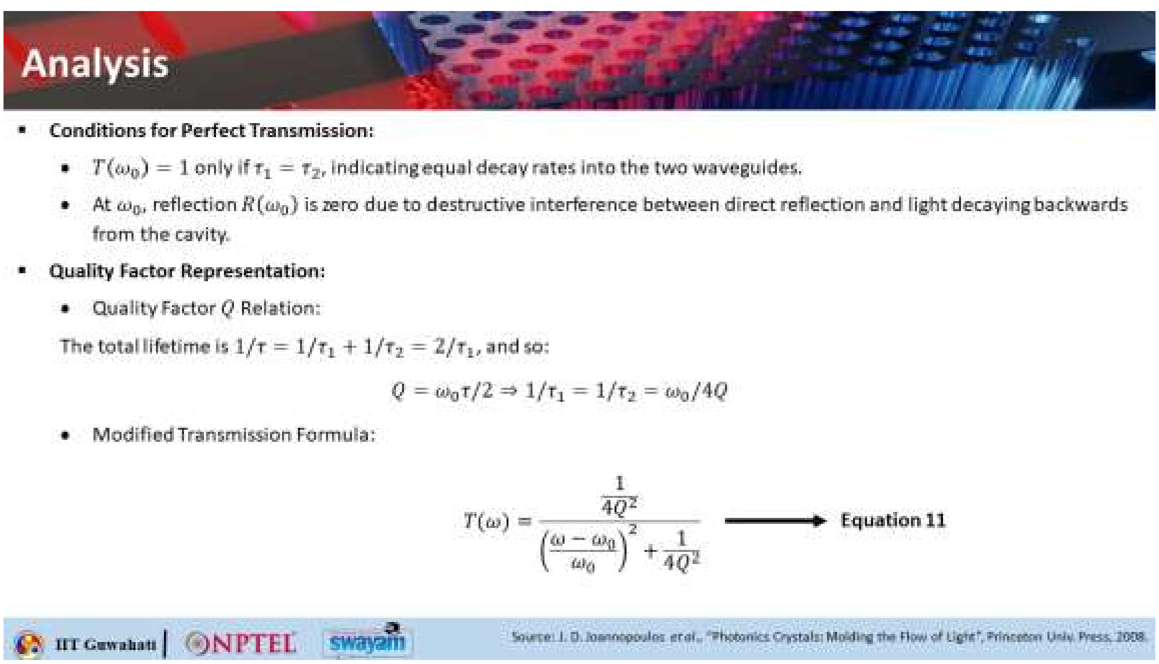
So, that will help you to get a relation expression of S2 minus and S1 plus and from that you can have this one that is your transmittance you can simplify and you can get this particular expression, okay. So, here also you can see that when you know omega equals omega naught this guy blows up and it becomes very very large transmission, right.



This becomes 0, okay and then you can do the maths and find out how it works. So, the transmission formula is basically this one. So, you can put omega which is a variable and when omega equals omega naught you will have the peak transmission. So, this is the expression for transmission spectrum that is your equation 9. So, it actually gives you a Lorentzian peak with a maximum at omega equals omega naught. So, however the reflection formula from this kind of theory you can also find out what is the reflection or reflectance.



So, $R(\omega)$ will be simply $\frac{|s_{1-}|^2}{|s_{1+}|^2}$. So, only difference is that here you are interested in the power that is coming out of the waveguide 2, here you are basically interested in the power in the case of reflection. you are interested in the power that is coming back to waveguide 1. So, this is how the expressions are related. So, what are the conditions for perfect transmission? As you can see, if I want that T at omega naught to be equals 1 means you want 100 percent transmittance.



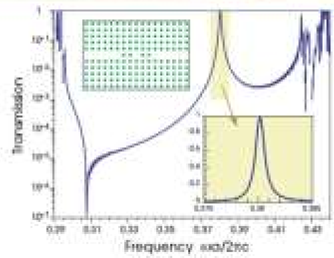
That means that can occur only when you will have $\tau_1 = \tau_2$ $t$hat means you should have equal decay rate into the two waveguides. And at ω_0, the $R(\omega_0)$ should also be 0 and that should come from the destructive interference between the direct reflection and light that is decaying backwards from the cavity. Those two things should cancel out the reflection so that you get 100 percent transmission. Now, how do you represent quality factor? Quality factor The total quality factor is Q and the total lifetime here is represented as $\frac{1}{\tau}$ which is nothing but $1/\tau_1 + 1/\tau_2$ and we have seen that for perfect transmission you want τ_1 and τ_2 to be equal.

So, you can write $\frac{2}{\tau_1}$ and hence, you can represent $Q = \frac{\omega_0 \tau}{2}$. So, that actually allows you to write $\frac{1}{\tau_1}$, which is also equal to $\frac{1}{\tau_2}$ to be $\frac{\omega_0}{4Q}$. So, why you are looking here? Because it is sometimes useful to write the transmission spectrum in terms of the quality factor instead of using tau. So if you do that in that case your equation 9 the transmission formula will change in terms of quality factor and it will look like this $T(\omega) = \frac{\frac{1}{4Q^2}}{\left(\frac{\omega-\omega_0}{\omega_0}\right)^2 + \frac{1}{4Q^2}}$.

So, that way also you can. So, these are same things just that you know tau and Q are related and that way the formula also now appears in the in terms of Q quality factor. Now if we were to plot that equation 11 that we have seen in the figure like this that is possible by plug-in in the omega naught and the Q as determined. So, it would nearly give you a very indistinguishable from what has been computed here, okay. So that is how you will see that the theory pretty much works well for temporal couple mode theory predicting the waveguide, cavity, waveguide kind of filters. So what is the main design criteria for narrow end filter? The first thing is the system should be symmetric, waveguide, cavity, waveguide configuration.

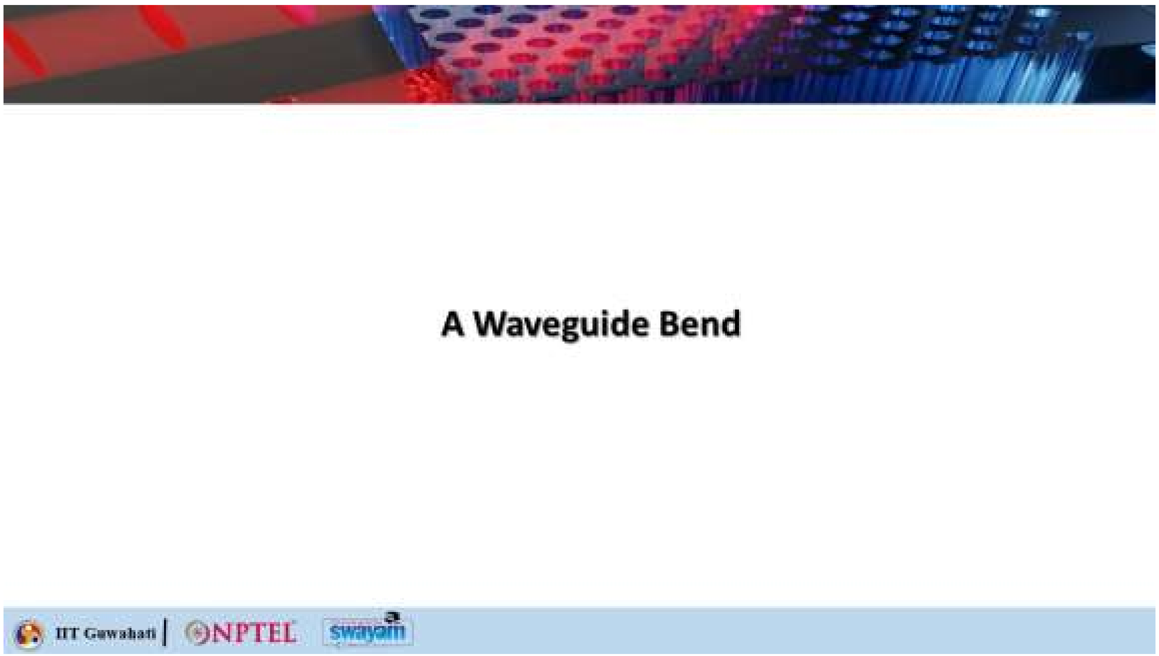
Analysis

- **Design Criteria for Narrow-Band Filter:**
  - The system should be:
  1. Symmetric waveguide-cavity-waveguide configuration.
  2. Single-mode.
  3. Free from other loss mechanisms like radiation or absorption.
- **Role of Photonic Crystal:**
  - Ideal for minimizing losses as it prohibits radiative modes beyond its bandgap, thus enhancing the performance of the system as a narrow-band filter.



Source: J. D. Joannopoulos *et al.*, "Photonics Crystals: Molding the Flow of Light", Princeton Univ. Press, 2008.

The waveguides must be single mode, so is the cavity, and it should be free from other loss mechanisms such as radiation or absorption. Now, what is the role of the photonic crystal here? Ideal for minimizing losses as it will prohibit the radiative modes beyond its band gap. Thus, it will enhance the performance of the system as a narrowband filter. So to summarize, we have basically derived the sufficient conditions for us to achieve a narrowband filter with 100% transmission. So that is the goal of using temporal couple mode theory to develop a narrowband 100% transmission filter based on photonic crystal.



Next, we'll move on to designing a waveguide bend. The applicability of temporal couple mode theory to the photonic crystal filter has been clear now. So, similar ideas can help us to understand the situations that seems very different at first. So, another such example would be you know how do you depict a sharp 90 degree bend okay in our missing rod waveguide. So, here you can see that figure 3 here shows a sharp 90 degree band made of missing rows of rods.

So, that is a right angle band. for a waveguide, okay. And here you can see this has been modeled using that temporal couple mode theory where you have waveguide 1, you have a resonator cavity and then you have waveguide 2 giving you that 90 degree band. Okay, so what are the assumptions here you have considered in like T1 tau 1 equals tau 2 by symmetry and that's 100% transmission, which is although a low Q. Okay, because you want it to be broad.

Okay, and you are actually able to achieve it. So this is how. the theory and the experimental ones overlap and gives you pretty good match right so the dots here the red dots here basically tells you the experimental transmission for a 90 degree bend okay and it is basically made in a square lattice where You have used alumina rods which have permittivity of 8.9 and the latest period is basically a 1.27 millimeter. So, what are the effects of bending in a ordinary dielectric waveguides? We have discussed this earlier as well.

Analysis

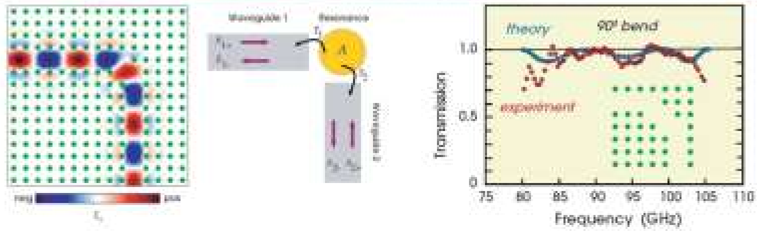


Figure 3: left: E_z field in right-angle bend, for a waveguide. Middle: Roughly, the bend can be thought of as weakly resonant filter, with $\tau_1 = \tau_2$ by symmetry and thus 100% transmission with a broad (low-Q) resonance. Right: Experimental transmission spectra for 90° bend, along with theoretical prediction from simple 1D model, in a square lattice of $\varepsilon = 8.9$ alumina rods with a=1.27mm

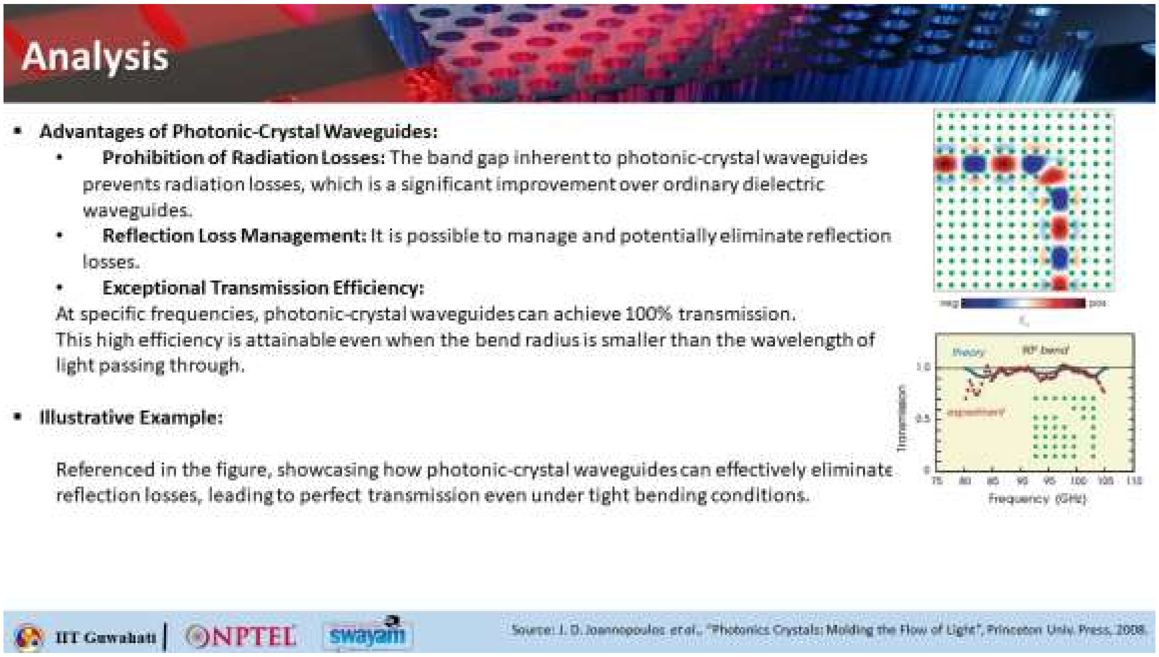
- **Effects of Bending in Ordinary Dielectric Waveguides:**
  - **Reflection and Radiation Loss:** Bending a dielectric waveguide results in both reflection of some light and radiation loss.
  - **Influence of Bend Sharpness:** The sharper the bend, the greater the radiation loss.
  - **Contrast Dependence:** Low-contrast optical fibers experience significant radiation losses with bend radii less than a few centimeters, potentially resulting in nearly complete radiation loss.

  High-contrast waveguides on chips show minimal radiation losses even for bends close to the wavelength scale.

Source: J. D. Joannopoulos et al., "Photonics Crystals: Molding the Flow of Light", Princeton Univ. Press, 2008.

First thing is the reflection and the radiation loss. So, bending a dielectric waveguide would result into both reflection of some light and then radiation loss. The influence of the band sharpness comes from the fact that sharper the band the greater will be the radiation loss. And there is a contrast dependence as well. So low contrast optical fibers experience significant radiation losses with band radii less than a few centimeters, potentially resulting in a nearly complete radiation loss. And this is the reason why when you have low contrast, your mode the majority of the mode energy is not confined only in the core.

If you have a high contrast means the refractive index difference between the core and the cladding is very high. In that case the mode is mainly concentrated within the core. So, only the tails of the mode goes to the So, there is they may leak out when you bend they may not satisfy the condition for total refraction. But if you are using low contrast optical fibers then the modes are significantly going out towards your cladding and in that case when there is a sharp bend modes will simply leak out. So high contrast waveguides on chips, they have shown minimal radiation loss even for bands close to the wavelength scale.



So what are the advantages of photon crystal waveguides? So these are photon crystal waveguides. We have discussed before that they prohibits radiation loss. So, the band gap that is inherent to the photon crystal waveguides will help you prevent the radiation loss which is a significant improvement over the ordinary dielectric waveguides. And it is also possible to manage and potentially eliminate reflection losses. So, that gives exceptional transmission efficiency. So, at specific frequencies, photon crystal waveguides can achieve 100 percent transmission and this high efficiency is attainable even when the band radius is smaller than the wavelength of the light passing through.

So, here you can actually see that how photon crystal waveguide can eliminate the reflection losses and you can see nothing basically reflects and nothing leaks out. So, you almost have 100% transmission even under this tight bending condition. Now, you can conceptualize the band as a resonant cavity. So, the band's corner, this part is analogous to a weak that is a low Q resonant cavity within the waveguide spectrum. So, you can write this as waveguide 1, waveguide 2 similar kind of input and output powers okay S1 plus, S1 minus, S2 plus, S2 minus okay.

Analysis

- **Conceptualizing the Bend as a Resonant Cavity:**

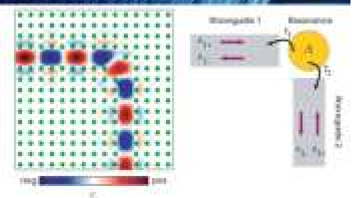
  The bend's corner is analogous to a weak (low-Q) resonant 'cavity' within the waveguide system.

- **Coupling Dynamics:**

  This "cavity" is connected to two waveguides, depicted schematically in the upper-right diagram of figure.
  The geometry of the bend, being bent, does not impact the core analysis derived from coupled-mode theory.

- **Symmetry and Decay Rates:**

  By symmetry, the corner-resonator must decay at equal rates into both the horizontal and vertical waveguides.
  There are no additional radiation channels available, focusing energy solely into the connected waveguides.



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Source: J. D. Joannopoulos et al., "Photonics Crystals: Molding the Flow of Light", Princeton Univ. Press, 2008.

So, this cavity is also connected to the two waveguides Okay, the geometry of the band, big band does not impact the core analysis derived from the couple mode theory. So, you could have had it in different angles as well. But right now, we just put it in this way to match it with the kind of system you are designing. So, by symmetry the corner resonator must decay at equal rates into both horizontal and the vertical waveguides and there are no additional radiation channel available that means the energy solely goes into the connected waveguides. So, what are the resonance and coupling impacts? So, first of all remember we will be assuming with coupling that means the system is predicted to exhibit transmission peaks at 100 percent on resonance and the resonance will likely to be broad that is attributed to the low quality factor of this resonant cavity.

Analysis

- **Resonance and Coupling Impact:**

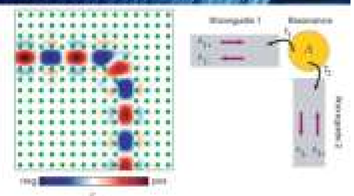
  Assuming weak coupling, the system is predicted to exhibit transmission peaks at 100% on resonance.
  The resonance will likely be broad, attributed to the low quality factor (Q) of the resonant "cavity".

- **Limitations of Coupled-Mode Theory for Bend Analysis:**

  The bend in the waveguide does not constitute a weak coupling to the waveguides, contradicting some assumptions of coupled-mode theory.
  The "cavity" (bend) does not trap light for extended periods, exhibiting a low quality factor ($Q < 10$).

- **Qualitative vs. Quantitative Accuracy:**

  While coupled-mode theory may not provide quantitatively accurate results due to these deviations, its qualitative predictions remain valid.



Source: J. D. Joannopoulos *et al.*, "Photonics Crystals: Molding the Flow of Light", Princeton Univ. Press, 2008.

So, what are the limitations there in coupled mode theory for band analysis? First thing is the band in the waveguide does not constitute a weak coupling to the waveguides. that sometimes contradicts the kind of basic assumptions you made into the coupled mode theory because here the coupling has to be strong to continue the light propagation. Second thing is the cavity that is the band does not trap light for extended periods because the light propagation should continue like this. So, it should ideally exhibit a very low quality factor, the quality factor should be less than 10.

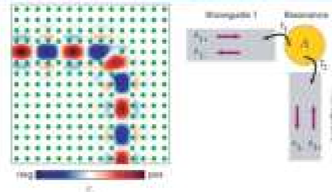
So, in that case you can actually think about the quantitative and qualitative versus quantitative accuracy. So, we can understand that while couple mode theory may not provide quantitatively accurate results due to these deviations, but its qualitative predictions remain valid. So, we can also think of advanced theoretical modeling.

Analysis

- **Advanced Theoretical Modeling:**

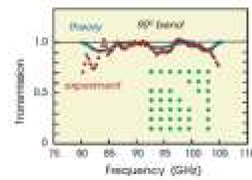
  A more precise model is suggested by treating the problem as essentially one-dimensional, where light can only travel forward or backward.

  This scenario is akin to a quantum-mechanical model of scattering from a symmetric one-dimensional potential well, known for exhibiting 100% transmission resonances.



- **Empirical Validation:**

  The bottom panel of figure illustrates the predicted transmission spectrum using this more accurate one-dimensional model, supporting the theoretical expectations.



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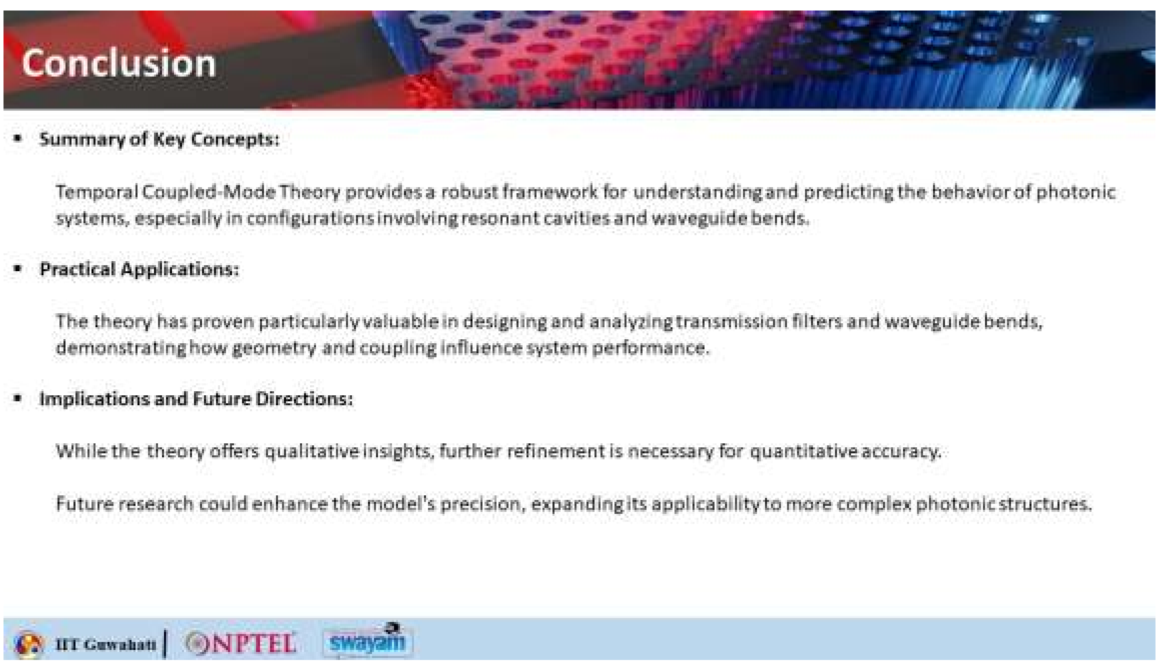
Source: J. D. Joannopoulos et al., "Photonics Crystals: Molding the Flow of Light", Princeton Univ. Press, 2008.

So, a more precise model is suggested by treating the problem as essentially one dimensional where the light can only move forward or backward. So, this kind of scenario is basically similar to the quantum mechanical model of scattering from a symmetric one dimensional potential well, which are known for exhibiting 100 percent transmission characteristics. So based on that, you can also do some empirical validation, which is shown here. So this illustrates the predicted transmission spectrum theory one in the blue. We have also already discussed this using this more accurate one dimensional model. So that supports the theoretical calculations. So what do you understand from here that similar to the filter, the high transmission is primarily facilitated by the waveguides single mode nature and the symmetry of the band.

However, unlike the filter the low quality factor in this case is essentially becoming a good thing because it means a high transmission can be achieved over a large bandwidth.

So, let us now summarize the things that we understood. So, here is the summary of the key concepts. So, first thing Temporal couple mode theory provides a robust framework for understanding and predicting the behavior of photonic systems, especially in configurations involving resonant cavities and waveguide bands. There are some practical applications that we have seen that the theory has proven particularly valuable in designing and analyzing transmission filters and waveguide bands and demonstrating how geometry and coupling could influence the system performance.



So, what are the implications and future direction? So, you need to keep this in mind that the theory basically provides qualitative insights So, further refinement is necessary for quantitative accuracy and future research could enhance the models precision expanding its applicability into more complex photonic systems. So, to conclude this lecture we summarize the entire analysis into all these important key points. So thank you that is all for this lecture if you have any queries or doubt regarding this lecture you can always drop an email to me at this particular email address mentioning MOOC and the lecture number on the subject line. Thank you.