Lec 33: Waveguide Splitters, Non-linear Filters, and Bistability

Hello, students. Welcome to lecture 33 of the online course on photonic crystals, fundamentals, and applications. Today's lecture will be on waveguide splitters, nonlinear filters, and biostability. So here's the lecture outline. We'll have a brief introduction to the topic. We'll analyze a waveguide splitter using the temporal couple mode theory.

Lecture Outline

- **Introduction**
- **Analysis of waveguide splitter**
- **Nonlinear Filters**
- **Optical Bistability**
- **Summary**

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We'll also discuss about nonlinear filters, optical biostability, and then finally conclude this lecture. So, we have already seen the analysis of two devices using temporal coupled mode theory. In the previous lecture one was a filter, another was one was a sharp waveguide band. So, here in this particular lecture we will look into another important waveguide device which is a splitter.

Introduction

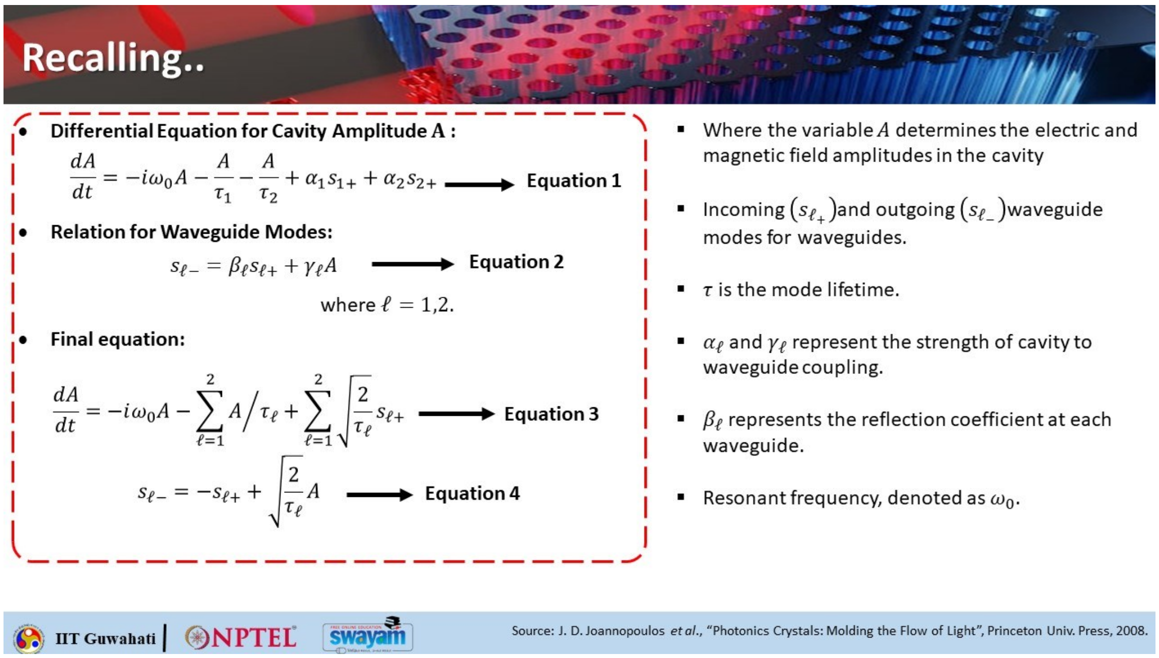
Now what does the splitter do? It divides the power in an input waveguide equally between the two output waveguides. So like the band, the photonic band gap here also eliminates the radiation loss and we need to only deal with the possibility of reflection. So unlike the band, it turns out that we cannot eliminate reflection by a symmetry argument and we must do something counterintuitive. So here we need to abstract the output waveguides in order to increase transmission. So before we go into the details, let us recall the basic formulae that have been used in the temporal couple mode analysis in the previous lecture.

Overview

- An example of a useful waveguide device is a splitter, which divides the power in an input waveguide equally between two output waveguides.
- Like the bend, the photonic band gap eliminates radiation loss and we need to only deal with the possibility of reflection.
- Unlike the bend, it turns out that we cannot eliminate reflections by a symmetry argument, and must do something counterintuitive.
- We need to obstruct the output waveguides in order to increase transmission

Source: J. D. Joannopoulos *et al.*, "Photonics Crystals: Molding the Flow of Light", Princeton Univ. Press, 2008.

So, the first one was the differential equation for cavity amplitude A. So, $\frac{dA}{dt}$ was given as $-i\omega_0 A - \frac{A}{\tau_1} - \frac{A}{\tau_2} + \alpha_1 s_{1+} + \alpha_2 s_{2+}$ and that was equation 1. We have already seen the relation for waveguide modes. So, how the outgoing and the incoming waves in the waveguide modes they are related ok. So, $s_{\ell-}$ was given as $\beta_\ell s_{\ell+} + \gamma_\ell A$, ℓ represents 1 or 2 that tells you whether it is waveguide 1 or waveguide 2.



And the final equation was this if you remember that was our equation 3. So, $\frac{dA}{dt}$ was basically $-i\omega_0 A - \sum_{\ell=1}^{2} A/\tau_\ell + \sum_{\ell=1}^{2} \sqrt{\frac{2}{\tau_\ell}} s_{\ell+}$ ok and you can write this in terms of the parameters here. So, $s_{\ell-}$ will be equal to $-s_{\ell+} + \sqrt{\frac{2}{\tau_\ell}} A$. So, these are the four important equations.

So, just to give you a brief what is A? It was basically the variable that determines the electric and magnetic field amplitudes in the cavity. $s_{\ell+}$ are the incoming waves or incoming modes. $s_{\ell-}$ represent the outgoing waveguide modes for the waveguides. Tau is basically the mode lifetime. $\alpha\ell$ and γ_ℓ they basically represent the strength of the cavity to waveguide coupling and β_ℓ is basically the reflection coefficient at each waveguide and another term is also there that is omega naught that is basically the resonant frequency with all these basics if you have any doubt on this particular things you can go back and revisit lecture 32 that will give you a better understanding of all these equations how they are obtained and what are they doing now with that understanding let us analyze web guide splitter so an example of a t-shaped uh splitter structure which is basically formed by missing rods in a photonic crystal waveguide that is shown in this particular figure okay. So, here you see the Ez field distribution is shown for this T splitter. So, this is input, this is output port 3 and this is output port 2. So, this is port 1, port 2 and port 3 that is how they numbered it ok.

Analysis of waveguide splitter

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So, what is expected here that you should get 100 percent transmission that is 50-50 from each of your output modes and that happens when $\frac{\omega a}{2\pi c}$ equals 0.4. So, this is the exact structure and this is the abstract model using the temporal couple mode theory. So here you have one incoming waveguide that is waveguide one and two output ports Okay. But then the incoming and output outgoing waveguide modes are identified in the same fashion. S1 plus, S1 minus. Then you have S2 plus. Okay, S2 minus. Okay. This is the cavity. These are the decay constant tau 1, tau 2, tau 3 Okay. And so on. So, this is a generic formula.

So, here to get 100 percent transmission we cannot use a symmetric junction and we must insert rods something like white rods that you see over here. These are basically white rods of permittivity epsilon equals 3.5. okay and that could obstruct the output waveguides okay and that basically improves the transmission.

Waveguide splitter: Analysis

- To analyze it qualitatively, as for the bend, we treat the junction point as a low- Q resonant cavity and apply temporal coupled-mode theory, as depicted schematically in figure.
- Since this is now a three-port system, the coupled-mode equation (3) is modified to become

$$\frac{dA}{dt} = -i\omega_0 A - \sum_{\ell=1}^{3} A/\tau_\ell + \sum_{\ell=1}^{3} \sqrt{\frac{2}{\tau_\ell}} s_{\ell+} \longrightarrow \textbf{Equation 5}$$

- Using equations (5) and (4) with $s_{2+} = s_{3+} = 0$ and solving for the reflection and transmission spectra as before, gives

$$R(\omega) = \frac{|s_{1-}|^2}{|s_{1+}|^2} = \frac{(\omega-\omega_0)^2 + \left(\frac{1}{\tau_1} - \frac{1}{\tau_2} - \frac{1}{\tau_3}\right)^2}{(\omega-\omega_0)^2 + \left(\frac{1}{\tau_1} + \frac{1}{\tau_2} + \frac{1}{\tau_3}\right)^2} \longrightarrow \textbf{Equation 6}$$

For reflection back into waveguide 1,

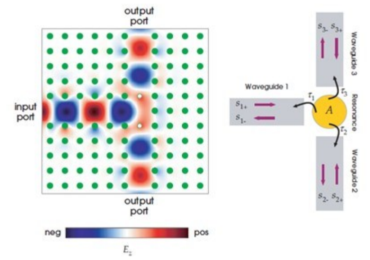


Figure 1: Left: E_z field in "T" splitter, showing essentially 100% transmission (50% in each branch) at $\omega a/2\pi c \approx 0.4$. Right: Abstract model, treating junction as weak resonance, predicting 100% transmission when $1/\tau_1 = 1/\tau_2 + 1/\tau_3$.

Source: J. D. Joannopoulos *et al.*, "Photonics Crystals: Molding the Flow of Light", Princeton Univ. Press, 2008.

So, some extra thing that I have to do to basically get that 100 percent transmission and that was the one I mentioned about the counterintuitive thing okay. Now, what you see here that this is basically becoming a 3 port system. So, the coupled mode equation that you have seen that equation number 3 needs to be modified and then that typically becomes like this. So, now, the equation will look like $\frac{dA}{dt} = -i\omega_0 A - \sum_{\ell=1}^{3} A/\tau_\ell + \sum_{\ell=1}^{3} \sqrt{\frac{2}{\tau_\ell}} s_{\ell+}$. So, only thing that has changed is ℓ equals 1 to 3 because it is now a 3 port system.

Now, in this abstract model another important parameter is that you will be getting 100 percent transmission when you have 1 over tau 1 should be becoming equal to 1 over tau 2 plus 1 over tau 3. So, this is the incoming input port, you can say these are the two output ports. Now, using the equations 5 and 4, okay where you can put some because there are no waves which are basically incident from waveguide 2 and waveguide 3. So, you can write S_{2+} plus and S_{3+} both can be 0. So, you can put that into this equation and then you can solve for the reflection and transmission spectra like before and that will give you $R(\omega)$ that is the reflection reflectance as a function of frequency.

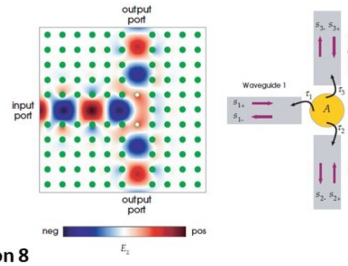
You can represent it as $\frac{|s_{1-}|^2}{|s_{1+}|^2}$ and this is how the equation looks like. So, this is giving you the estimate of the power getting reflected back to waveguide 1. So, this is our equation 6. Now how much is the transmission happening to from 1 to 2 ok. So, that you can represent as T 1 to 2 and that is also function of frequency.

Waveguide splitter: Analysis

- For transmission into waveguide 2,

$$T_{1\to 2}(\omega) = \frac{|s_{2-}|^2}{|s_{1+}|^2} = \frac{\frac{4}{\tau_1\tau_2}}{(\omega-\omega_0)^2 + \left(\frac{1}{\tau_1}+\frac{1}{\tau_2}+\frac{1}{\tau_3}\right)^2}$$

⟶ **Equation 7**



- For transmission into waveguide 3 .

$$T_{1\to 3}(\omega) = \frac{|s_{3-}|^2}{|s_{1+}|^2} = \frac{\frac{4}{\tau_1\tau_3}}{(\omega-\omega_0)^2 + \left(\frac{1}{\tau_1}+\frac{1}{\tau_2}+\frac{1}{\tau_3}\right)^2}$$

⟶ **Equation 8**

- From equation (6) it is clear that zero reflection can be achieved at $\omega = \omega_0$, and thus 100% transmission from waveguide 1 to waveguides 2 and 3, if

$$\frac{1}{\tau_1} = \frac{1}{\tau_2} + \frac{1}{\tau_3}$$

⟶ **Equation 9**

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Source: J. D. Joannopoulos *et al.*, "Photonics Crystals: Molding the Flow of Light", Princeton Univ. Press, 2008.

So, the formula will be $\frac{|s_{2-}|^2}{|s_{1+}|^2}$ and this is how the expression looks like. It is $\frac{\frac{4}{\tau_1\tau_2}}{(\omega-\omega_0)^2+\left(\frac{1}{\tau_1}+\frac{1}{\tau_2}+\frac{1}{\tau_3}\right)^2}$. So, the transmission will have a peak when omega equals omega naught. For transmission into waveguide 3 the same equation can be written just you replace 2 by 3 ok.

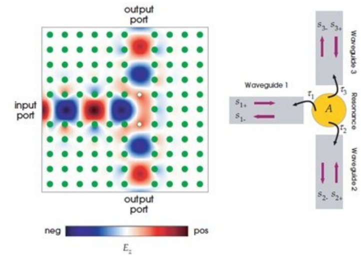
So, this denominator remains same just that in the numerator τ_2 will be now replaced by τ_3 and this is how the equation looks like. So, with this you can estimate the transmission happening to waveguide port 2 and port 3. So, from this equation 6 ok, it is clear that zero reflection can happen when you have this to be matched. So, omega equals omega naught and also there is another term that is when $\frac{1}{\tau_1} = \frac{1}{\tau_2} + \frac{1}{\tau_3}$ ok. So, from equation 6 you can also see that it is possible to get the zero reflection when omega equals omega naught and to ensure 100 percent transmission from waveguide 1 to the waveguides 2 and 3 ok, you have to also meet this particular condition that $\frac{1}{\tau_1} = \frac{1}{\tau_2} + \frac{1}{\tau_3}$.

So, if you meet this condition you could go back. here you will see you will basically get complete 0 reflection ok. And 100 percent power will be transmitted from waveguide 1 to waveguide 2 and 3 and they will get equally split ok. So, this particular relation $\frac{1}{\tau_1} = \frac{1}{\tau_2} + \frac{1}{\tau_3}$ is a very interesting relation for two reasons. First it can never be satisfied at a 120 degree rationally rotationally symmetric junction where you can that if you make it one 20 degree rotationally symmetric junction, means this one goes here and this one looks like this.

Waveguide splitter: Analysis

$\frac{1}{\tau_1} = \frac{1}{\tau_2} + \frac{1}{\tau_3}$ This relation is very interesting for two reasons.

- First, it can never be satisfied in a 120° rotationally symmetric junction with $1/\tau_1 = 1/\tau_2 = 1/\tau_3$
- Second, if we cannot satisfy equation (9) purely by symmetry, then we must force it "manually."
- In the case of figure 1(left), we accomplish this obstruction by adding a single rod (white) in before each of the output waveguides.
- Since we cannot satisfy equation (9) a priori, we must adjust the strength of this obstruction (varying the radius or ε of the rod) until numerical simulations yield maximum transmission at a desired frequency.



Source: J. D. Joannopoulos *et al.*, "Photonics Crystals: Molding the Flow of Light", Princeton Univ. Press, 2008.

In that case, $\frac{1}{\tau_1}$ will be equal to $\frac{1}{\tau_2}$ equal to $\frac{1}{\tau_3}$. So, you cannot can never make this one happen. And secondly, we cannot satisfy equation 9 purely by symmetry, then you must force it manually. So, you have to do some extraordinary things to get that. So, here what has been done in the case of figure 1, you can actually accomplish this obstruction by adding a single rod ok that you can see in white ok. each of the output waveguide.

So, just in the beginning of each of this output waveguide you add the single rod and that helps you to achieve that extraordinary fit. Since, we cannot satisfy that equation 9 a priori, we must adjust the length of this obstruction or by varying the radius or the permittivity of this rod until through numerical simulation you can see that you are actually achieving the highest transmission or maximum transmission at a desired frequency for the structure. So, this is some kind of tuning you have to do on top of the regular thing to get that splitter works. Now, we move on to the next topic which is non-linear filters. So, first let us look into some basics of non-linear optics.

So, to understand what is non-linear first let us look into what is linear ok. So, a linear system basically exhibits a response that is directly proportional to the external influences. So, if there are multiple influences such as f 1, f 2 and up to f j they are applied simultaneously The overall responses will be the sum of responses as if each are applied independently. So, that is basically the superposition principle. Now, what will be then the definition and characteristics of nonlinear system? So, if you think in contrast to the linear system, a non-linear systems response is not strictly proportional to the applied influences.

Nonlinear Filters

And the interactions between the influences can lead to energy transfer among them. And this is evident in the behaviour of certain materials under electromagnetic fields. So, in this context non-linear optics is basically those phenomena where the response of a material to applied electric field becomes non-linear. That means the material basically alters the field through mechanisms such as polarization and this can include a variety of behaviors depending on the intensity and the nature of the electric field. So, that can give rise to non-linear photonic crystals as well.

Basics on Nonlinear Optics

- **Definition of Linear Systems:**

  A linear system exhibits a response that is directly proportional to the external influences.

  If multiple influences $F_1, F_2, \ldots, F_j$ are applied simultaneously, the overall response is the sum of the responses as if each were applied independently.

- **Definition and Characteristics of Nonlinear Systems:**

  In contrast, a nonlinear system's response is not strictly proportional to the applied influence, and interactions between influences can lead to energy transfers among them.

  This is evident in the behavior of certain materials under electromagnetic fields.

- **Nonlinear Optics in Context:**

  Nonlinear optics involves phenomena where the response of a material to an electric field $\mathbf{E}$ is nonlinear, resulting in the material altering the field through mechanisms such as polarization $\mathbf{P}$.

  This can include a variety of behaviors depending on the intensity and nature of $\mathbf{E}$.

Source: Y. Benachour, "Nonlinear Optics of Photonic Crystals," *2020 Advances in Science and Engineering Technology International Conferences (ASET)*, Dubai, United Arab Emirates, 2020, pp. 1-8, doi: 10.1109/ASET48392.2020.9118251.

So, the periodic structure where the optical response varies with the intensity of the optical field okay. So, here the intensity that is mean that means the square of the electric field right. that offers enhanced functionalities which are typically not possible by the linear photonic crystal. So, linear photonic crystal is where the property varies with the amplitude of the electric field, but here in non-linear photonic crystal the optical responses varies with the amplitude square or the intensity of the electric of the optical field. So, some examples of tunability will include optical control through external fields, temperature induced refractive index changes, intrinsic material nonlinearities for fast response in advanced communication systems.

Nonlinear Photonic Crystals

- **Nonlinear Photonic Crystals (PCs):**

  Periodic structures whose optical response varies with the intensity of the optical field, offering enhanced functionalities not possible with linear PCs.

  Examples of tunability include optical control through external electric fields, temperature-induced refractive index changes, and intrinsic material nonlinearities for fast response in advanced communication systems.

- **Characteristics of Nonlinear Crystals:**

  Nonlinearity: Refractive index changes with light intensity, critical for nonlinear optical responses.

  Periodicity: Regular, repeating lattice structure crucial for the structured arrangement of atoms.

  Optical Anisotropy: Varying refractive indices along different axes of the crystal.

Source: Y. Benachour, "Nonlinear Optics of Photonic Crystals," *2020 Advances in Science and Engineering Technology International Conferences (ASET)*, Dubai, United Arab Emirates, 2020, pp. 1-8, doi: 10.1109/ASET48392.2020.9118251.

What are the characteristics of this nonlinear crystals? First is the nonlinearity that means their refractive index can change with light intensity. which is very crucial and critical for nonlinear optical responses and the periodicity that means the regular repeating lattice structure which are crucial for structure element of the atoms. Lastly, optical anisotropy that means you can have varying refractive index or refractive indices along different axis of the crystal. So, the propagation of electromagnetic wave in a medium is typically described by the Maxwell's equation. the electric displacement that is capital D and the electric field E are connected by the constitutive relation of the material that is where the epsilon r which is the relative permittivity that comes into the picture.

Nonlinear Photonic Crystals

- The propagation of an electromagnetic wave in a medium is described by the Maxwell equations.
- The electrical displacement **D** and the electric field **E** are connected by a constitutive relation of the material where ϵ_r is the relative dielectric permittivity which in a homogeneous anisotropic medium is a tensor quantity.
- This relationship can be written as:

$$\mathbf{D}_i = \epsilon_0 \epsilon_{r_{ij}} \mathbf{E}_j$$

ϵ_0 is a vacuum dielectric permittivity.

- The dielectric constant of the medium is $1 + \chi_{ij}^{(1)}$ and the refraction index n_{ij} :

$$n_{ij} = \mathrm{Re}\ \left(1 + \chi_{ij}^{(1)}\right)$$

Here $\chi_{ij}^{(1)}$ is the first-order nonlinear susceptibility, its imaginary part describes the losses in the medium.

$$\epsilon_{r_{ij}} = 1 + \chi_i^{(1)}$$

Source: Y. Benachour, "Nonlinear Optics of Photonic Crystals," *2020 Advances in Science and Engineering Technology International Conferences (ASET)*, Dubai, United Arab Emirates, 2020, pp. 1-8, doi: 10.1109/ASET48392.2020.9118251.

So, if you consider a homogeneous anisotropic medium this permittivity is basically a tensor and the relationship of the displacement field and the electric field can be written like this. So, $\mathbf{D}_i$ will be $\epsilon_0 \epsilon_{r_{ij}} \mathbf{E}_j$, ϵ_0 which is the vacuum permittivity. So, $\epsilon_{r_{ij}}$ is the relative permittivity which is the tensor and then it depends on Ej, ok. So, that way you can actually calculate what is the electric displacement in a particular direction. So, the dielectric constant of the medium can be written as $1 + \chi_{ij}^{(1)}$ So, χ is basically the electric susceptibility and refractive index $n_{ij} = \mathrm{Re}\left(1 + \chi_{ij}^{(1)}\right)$, which is the permittivity. So real part of the complex permittivity okay can help you this is the first order. So that can help you find the refractive index tensor as well. So as I mentioned so this $\chi_{ij}^{(1)}$ represents the first order non-linear susceptibility. So, the real part of it gives you the refractive index and the imaginary part will describe the loss in the medium. So, $\epsilon_{r_{ij}} = 1 + \chi_i^{(1)}$.

Nonlinear Photonic Crystals

- When the material is strongly disturbed, the linear approximation which considers independent of $\mathbf{E}$ is no longer valid, the relation between the volume polarization $\mathbf{P}$ or the displacement $\mathbf{D}$ and the electric field $\mathbf{E}$ is no longer linear.
- Then $\mathbf{P}$ can be written locally as:

$$\mathbf{P}_i = \epsilon_0 \left(\chi_{ij}^{(1)} \mathbf{E}_j + \chi_{ijk}^{(2)} \mathbf{E}_j \mathbf{E}_k + \chi_{ijkl}^{(3)} \mathbf{E}_j \mathbf{E}_k \mathbf{E}_l + \cdots \right)$$

- Nonlinear effects in photonic crystals have been studied to understand various phenomena like:
  - Kerr-like third order susceptibility
  - Two-photon absorption
  - Negative refraction
  - Optical memory and light storing

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Source: Y. Benachour, "Nonlinear Optics of Photonic Crystals," *2020 Advances in Science and Engineering Technology International Conferences (ASET)*, Dubai, United Arab Emirates, 2020, pp. 1-8, doi: 10.1109/ASET48392.2020.9118251.

So, when the material is strongly disturbed the linear approximation which considers independent of electric field is no longer valid. So, in that case the relation between the volume polarization P or the displacement D and the electric field E also stays no longer So, you can write the polarization P in terms of you know. the different components electric field components and the expression typically look like this. $\mathbf{P}_i = \epsilon_0 \left(\chi_{ij}^{(1)} \mathbf{E}_j + \chi_{ijk}^{(2)} \mathbf{E}_j \mathbf{E}_k + \chi_{ijkl}^{(3)} \mathbf{E}_j \mathbf{E}_k \mathbf{E}_l + \cdots \right)$.

So, when you actually see non-linear effect in photonic crystal you have to understand this different nonlinear phenomena something like the Kerr like third order susceptibility okay. So, this is the first order second order this is the third order susceptibility okay. Then other phenomena something like two photon absorption negative refraction optical memory and light storage all these things will come because of the nonlinear effects in photonic crystal. Now, in optical realm and specifically for 2D photonic crystals, various methods have been proposed for applying tuning abilities and controlling the properties of the desired devices. However, the approaches used so far basically suffer from a lot of disadvantages.

Tuning Methods in 2D Photonic Crystals

- **Thermo-Optic Methods:**
  Application: Used in 2DPhC switches.
  Disadvantages: Exhibits switching speeds on the microsecond order, which may not meet the requirements for ultrafast applications.

- **Electro-Optic Techniques:**
  Application: Reported use in various studies for fast switching.
  Limitations: The required voltage for operation can limit the overall operating speed of the device.

- **Directional Coupler Structures:**
  Issues: Noted for their large coupling lengths, which lead to higher consumption of optical power and may be impractical for compact device designs.

- **Advantages of Nonlinear Optical Processes:**
  Ultrafast Response: Demonstrates response times on the order of 10 picoseconds, suitable for ultrafast optical applications.
  Mechanisms:
  Strong nonradiative recombination of photo-carriers at etched holes contributes to fast responses.
  Carrier-induced nonlinear index changes can rapidly shift the wavelength of a PhC resonance.

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Source: Mansouri-Birjandi, Mohammad Ali, Alireza Tavousi, and Majid Ghadrdan. "Full-optical tunable add/drop filter based on nonlinear photonic crystal ring resonators." *Photonics and Nanostructures-Fundamentals and Applications* 21 (2016): 44-51.

We can see some of the tuning methods. One is thermo optic method. So, they are basically used in 2D photon crystal switches. Their disadvantage is that they exhibit switching speeds in the order of microsecond, which may not meet the requirement for ultrafast applications. The second one is electro optic techniques, here the application is are the reported cases in various studies for first switching okay. Here also the limitation is that the required voltage for operation can limit the overall operating speed of the device, you can also have directional coupler based structures So, what are the issues here? They have large coupling length which lead to higher consumption of optical power and may become impractical for compact device design.

Thus, there are some benefits coming from non-linear optical processes. First thing would be the ultrafast response So nonlinear effects could demonstrate response times of the order of 10 picoseconds, which are typically suitable for ultrafast optical applications. So strong non-radiative recombination of photocarriers at etched holes can contribute to fast responses and that becomes the mechanism for this kind of ultra-fast response and the carrier induced non-linear index changes can rapidly shift the wavelength of a photonic crystal resonance and that is how you can get the tuning. Now, let us discuss about an add-drop filter. So, in a add-drop filter there will be a ring And this ring and the cavity resonator, what is the function of that? I believe all of you have seen this before, the ring resonator or the head drop filter in this particular course as well.

An Add/Drop filter

- **Function of Ring and Cavity Resonators:**

  Designed to trap light, which propagates in a circular closed path, enhancing nonlinear optical effects.

- **Constructive Interference and Standing-Wave Formation:**

  Continuous circulation of light allows it to constructively interfere with itself in each round trip of the resonator.

  This process results in the formation of a standing-wave pattern, intensifying the optical field within the resonator.

- **Intensity Build-Up:**

  The repeated circulation of light coherently builds up the light's intensity inside the resonator to levels exceeding the incident optical power.

- **Enhancement of Nonlinear Effects:**

  The significant increase in intensity facilitates a strong AC Kerr effect, enhancing the nonlinear response of the resonator to incoming light.

Source: Mansouri-Birjandi, Mohammad Ali, Alireza Tavousi, and Majid Ghadrdan. "Full-optical tunable add/drop filter based on nonlinear photonic crystal ring resonators." *Photonics and Nanostructures-Fundamentals and Applications* 21 (2016): 44-51.

So they are basically designed to trap light which propagates in a circular closed path, enhancing the nonlinear optical effects. Then you can think of constructive interference of light allows it to, so constructive interference with itself after every round trip and that is how you can actually get a you know standing wave formation right. So, this process creates a standing wave pattern and that intensifies the optical field within the resonator. And this will lead up to intensity buildup as well. The repeated circulation of the light coherently builds up the light's intensity inside the resonator to levels exceeding the incident optical power, ok.

So, that is how we will be able to see the CAD-like non-linear full optical add-drop filter. So, this will lead to enhancement in the non-linear effects. As I mentioned, this significantly increase in intensity will facilitate a strong AC care effect enhancing the non-linear response of the resonator to the incoming light. The total polarization due to the second and the third order non-linearities can be given as $\vec{P}_x = \varepsilon_0 \left[1 + \chi^{(1)} \cdot \vec{E}_x + \chi^{(2)} \vec{E}_x \cdot \vec{E}_x + \chi^{(3)} \left|\vec{E}_x\right|^2 \cdot \vec{E}_x\right]$ ok.

An Add/Drop filter

- The total polarization due to second and third order nonlinearities are given by

$$\vec{P}_x = \varepsilon_0 \left[1 + \chi^{(1)} \cdot \vec{E}_x + \chi^{(2)}\vec{E}_x \cdot \vec{E}_x + \chi^{(3)}\left|\vec{E}_x\right|^2 \cdot \vec{E}_x\right]$$

where $n_L^2 = \varepsilon_L = 1 + \chi^{(1)}$ is the dielectric constant.

Here $\chi^{(2)}$ and $\chi^{(3)}$ are second order and third order nonlinear susceptibilities.

- At a single frequency of ω, only the cubic nonlinearity; i.e. Kerr effect, is associated with oscillating part of the polarization that is found to be:

$$P_{NL} = \frac{3}{8}\varepsilon_0\chi^{(3)}|E_0|^2 E_0 e^{-i\omega t}\hat{x}$$

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Source: Mansouri-Birjandi, Mohammad Ali, Alireza Tavousi, and Majid Ghadrdan. "Full-optical tunable add/drop filter based on nonlinear photonic crystal ring resonators." *Photonics and Nanostructures-Fundamentals and Applications* 21 (2016): 44-51.

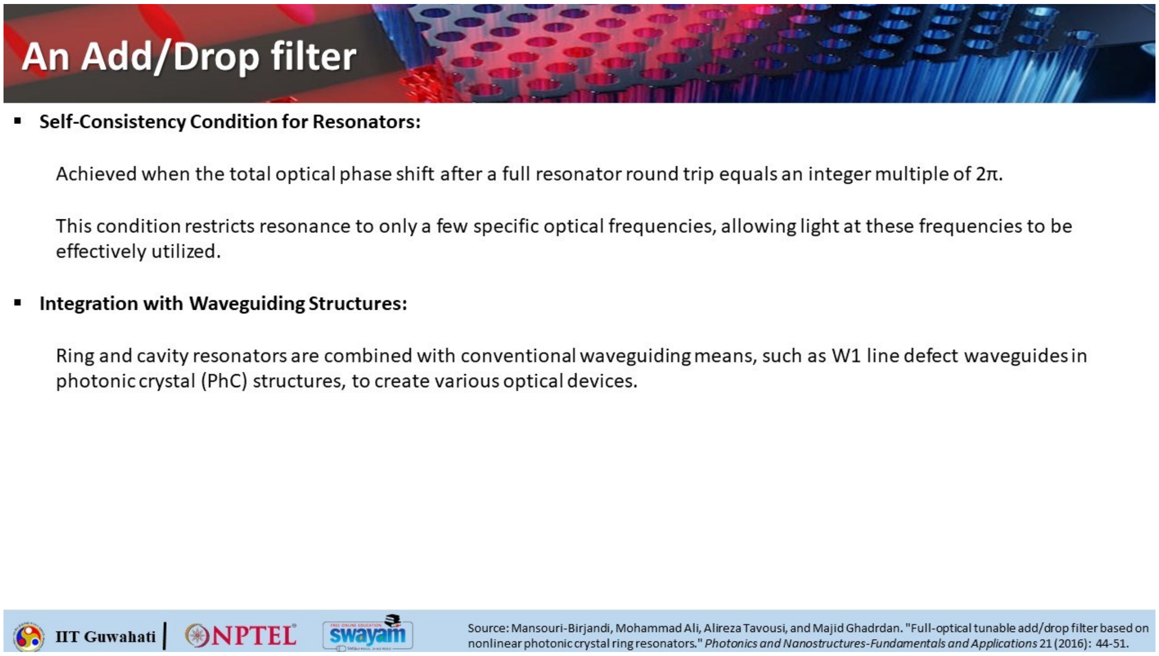
So, that way you are able to see the non-linear effect coming here and n_L^2 yeah that is the correct formula n_L^2 will be equal to $\varepsilon_L = 1 + \chi^{(1)}$ that is the dielectric constant. So, this is where I was also noticing the typo can just go back here yeah. So, this one is correct, but not this equation ok. So, I'll upload it later when I upload the slides. Because ideally, 1 plus susceptibility gives you the permittivity.

And if the permittivity is complex, it will have, say, epsilon prime. So it will have epsilon prime and epsilon double prime. That is the real and the imaginary part. And those two can be related to the real and the imaginary part of the refractive indices as well.

So, this equation does not look correct to me. I will just cross check. It has been taken from this particular paper. So, we will just look into this. There may be some typo by the authors over there.

So, disregard this particular equation. You go with this one. This is the correct equation. So, ideally it should be like this. that n_L^2 that is the refractive index square will be equal to the permittivity. So, this is typically when your permittivity is real. So, if it is complex as I mentioned the equations will slightly vary.

So, here $\chi^{(2)}$ and $\chi^{(3)}$are basically the second and third order nonlinear susceptibilities. And at a single frequency of omega only the cubic nonlinearity that is the Kerr effect this one is associated with the oscillating part of the polarization. So, that is to be found as this. So, you can write P_{NL} the nonlinear will be equal to $\frac{3}{8}\varepsilon_0\chi^{(3)}|E_0|^2 E_0 e^{-i\omega t}\hat{x}$ okay. So, what are the self-consistency condition for the resonators? This can be achieved when the total optical phase shift after a full resonator round trip will be equal to the integral multiple of 2 pi.



This condition restricts resonance to only those specific optical frequencies which meet this condition and that will allow the light at those frequencies to be effectively utilized. So, how to integrate this with the waveguiding structures? So, the ring and the cavity resonator are combined with the conventional waveguiding such as W1 line defect waveguides in photonic crystal and they can be used to create various optical devices. So, this is a typical optical ring resonator based device. OK, so this is a single ring.

So there are devices with the double rings also over here. OK, so this single ring is placed in between two parallel web guides. And as you can see, so this is how the mode is traveling. The ring can couple something over here and then it can couple back to this particular web guide and some will be so dropped from this particular waveguide. So what happens here from this you can actually make it as a add drop filter right. So this basically works as a building block for many optical devices something like switches, logic gates, optical limiters, analog to digital converters and so on.

An Add/Drop filter

- **Nonlinear Regime Applications:**
  Both configurations from Fig. 2a and 2b serve as building blocks for optical switches, logic gates, optical limiters, and analog-to-digital converters.
  The functionality in the nonlinear regime is influenced by the field intensity within the resonating mode, affecting the round-trip phase shift.

- The amount of nonlinear dependent refractive index that change due to an applied electric field is

$$\Delta n_{NL} = \frac{3\chi^{(3)}}{4Z_0 n_L}|E_0|^2 = n_2 I$$

where Z_0 is the free space impedance and n_2 is nonlinear refractive index expressed in term of intensity I

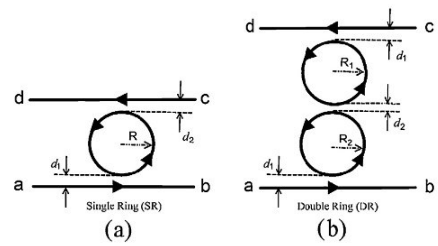


Figure 2: Common designs for an optical ring resonator: (a) single ring: coupling a single ring to two parallel waveguides, and (b) double rings: coupling two vertically-aligned rings to two parallel bus waveguides. The parameters R_1, R_2, d_1, and d_2 controls the device properties.

Source: Mansouri-Birjandi, Mohammad Ali, Alireza Tavousi, and Majid Ghadrdan. "Full-optical tunable add/drop filter based on nonlinear photonic crystal ring resonators." *Photonics and Nanostructures-Fundamentals and Applications* 21 (2016): 44-51.

And the functionality in the nonlinear regime is basically influenced by the field intensity within the resonating modes that affects the round trip phase shift okay. Because the round trip phase shift plays a very important role which frequency will be contained in this resonating cavity and then only it will get coupled to this particular waveguide okay. So, the amount of non-linear dependent refractive index that change due to the applied electric field can be given as this. So, Δn_{NL} can be written as $\frac{3\chi^{(3)}}{4Z_0 n_L}|E_0|^2$ and that can be written as $n_2 I$. you can go and look into this particular reference paper for more details on this structure ok.

So, here Z_0 is basically the free space impedance and n_2 is the non-linear refractive index which is expressed in terms of the intensity ok. So, here is a filter design. So, in this filter it combines a single ring resonator with two parallel line defect waveguides that creates a backward add drop filter. So, here you can see this is the input port, this is the throughput and this is the drop port and this this is showing the ring. So, these are basically all defect ok introduced in a otherwise uniform unique photonic crystal.

An Add/Drop filter

- **Filter Design:**

  Combines a single ring (SR) resonator with two parallel line defect waveguides to create a backward Add/Drop filter.
- **Purpose:**

  Used to tune the resonant wavelength of the ring resonator to a specific target.
- **Operational Wavelength:**

  Configured to target a resonance center wavelength of $\lambda_0 = 1550$ nm in free space.
- **Adjustment of Optical Properties:**

  Modifies the size of the silicon nanocrystal (Si-nc) rods in the ring to change its linear refractive index to $n_0 = 1.2$.

  Ensures that the transmission and drop peaks are precisely located at 1550 nm .

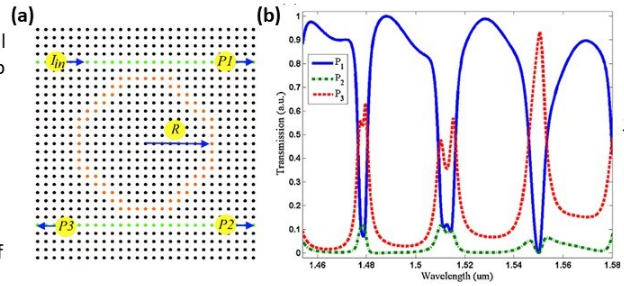


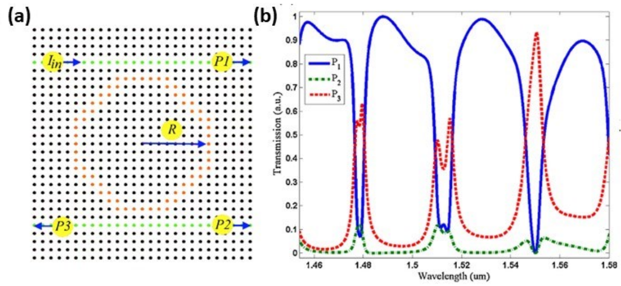
Figure: 3 (a) Single ring (SR) based PhC Add/Drop filter structure used for tuning the ring resonator resonant wavelength. (b) Transmission and drop efficiency of the structure tuned to work exactly at λ_0 = 1550 nm

Source: Mansouri-Birjandi, Mohammad Ali, Alireza Tavousi, and Majid Ghadrdan. "Full-optical tunable add/drop filter based on nonlinear photonic crystal ring resonators." *Photonics and Nanostructures-Fundamentals and Applications* 21 (2016): 44-51.

So this can be used to tune the resonant wavelength of the ring resonator to a specific target. So here you can consider operational wavelength to be 1550 in free space. So, what you can do you can modify the size of the photonic nanocrystal rods in the ring to change its linear refractive index to n equals 1.2 and that will basically ensure that the transmission and the drop peaks at precisely that location of 1550. okay so here you can see you are getting the so this is P3 okay you are getting very sharp transmission at P3 okay and this is the input line so P1 you see pure is a short drop over here at 1550 so all these things can be tuned by changing the diameter of the rods and the way you are introducing the defect in the crystal so this is basically it at the third telecom window that is in C band and this silicon nanocrystals they basically demonstrate very strong nonlinear properties and that is good because the silicon nanocrystals characteristic then strongly depend on the applied field intensity okay.

An Add/Drop filter

- At the third telecommunication window (C band), Si-ncs demonstrates a very strong nonlinear properties.
- Si-nc's characteristics strongly depends on applied field intensity.
- By applying a proper intensity, the real and imaginary parts of Si-nc's refractive index varies nonlinearly.
- The change in the real part is described by AC Kerr effect; i.e. n_2, and the imaginary part change is described by β.
- The real part of third order nonlinear susceptibility $\chi^{(3)}$; i.e. n_2 is higher than its imaginary part; i.e. β by 2 orders of magnitude, thus the most refractive index change is due to real part; i.e. n_2.



Source: Mansouri-Birjandi, Mohammad Ali, Alireza Tavousi, and Majid Ghadrdan. "Full-optical tunable add/drop filter based on nonlinear photonic crystal ring resonators." *Photonics and Nanostructures-Fundamentals and Applications* 21 (2016): 44-51.

And by properly applying the required intensity the real and imaginary part of the silicon crystals refractive index can be changed non-linearly. And the change in the real part is described by the AC Kerr effect and if you look for n_2 that is basically the imaginary part is described by beta, okay. And the real part of the third order non-linear susceptibility chi 3, okay there n_2 is higher than its imaginary part that is you know beta ok. So, n_2 is basically higher than beta by 2 orders of magnitude, thus the most refractive index change is happening mainly due to the real part. And if you want to enhance the quality factor ok, what you can do? You can actually align another ring over here vertically So, if you have more rings vertically aligned and then you bring your final waveguide.

So, in this case basically you have this is your input waveguide this is the first ring then you have to put another ring below and then you can actually put your final output waveguide and that will also give you a higher quality factor resonance. Now, we move on to the next topic which is optical bistability. So, as the name says optical bistability there are two stable states right. So, bi means two stability ok. So, it refers to the non-linear relation between the output and the input power in a filter with a non-linear cavity.

Optical bistability

Optical Bistability

- Optical bi-stability refers to the nonlinear relation between the output and input power in a filter with a nonlinear cavity.
- Example of this is shown in figure 5.
- Contrast this with a linear device, in which the output power is always a linear function of the input power.
- **Instability of the Middle Branch:**

  The middle (dashed) branch of the S-curve is unstable, causing the power to follow either the upper or lower stable branch.

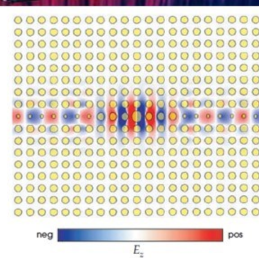


Figure 4: Waveguide-cavity-waveguide filter in square lattice of rods. Crystal: $\varepsilon = 5.4$ rods with radius $r = 0.2a$. Waveguide: row of reduced-radius ($r = 0.08a$) rods. Cavity: increased-radius ($r = 0.42a$) rod with a dipole-like state at $\omega_0 a/2\pi c = 0.25814$.

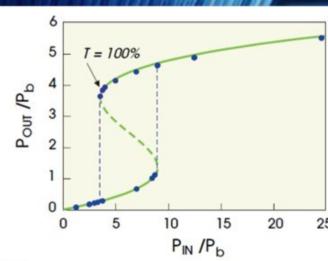


Figure 5: Output versus input power, in units of a characteristic power P_b, for the filter of figure 4 when a Kerr nonlinearity is included in the cavity

Source: J. D. Joannopoulos *et al.*, "Photonics Crystals: Molding the Flow of Light", Princeton Univ. Press, 2008.

You can see the example here in figure 5. So, in linear device where the output is always a linear function of the input power, here you can see that there is some instability in the middle branch. So, this is basically shown as the dashed curve of this S or you can say dashed branch of this S curve okay. So that causes the power to follow either upper or lower stable branch. So from here if you further increase the input power it will go here. and then if you are already here and then you have you are reducing the power after you have come here it will not come this way okay it will jump from here to here and then it will follow like this okay so you are actually getting discontinuous jump so power jumps discontinuously between branches when reaching the end of a stable branch so this branch This is the end of the stable branch.

Optical Bistability

- **Discontinuous Jump:**

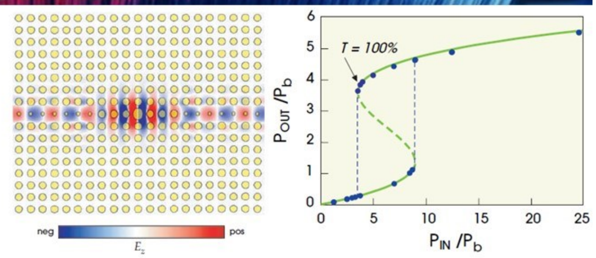
  Power jumps discontinuously between branches when reaching the end of a stable branch.

- **Dependence on Historical Power Values:**

  Output power depends on its historical values; it follows the lower branch when starting from low power and the upper branch when starting from high power.

- **Hysteresis Effect:**

  This behavior exemplifies a type of hysteresis effect in the power output response.



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Source: J. D. Joannopoulos *et al.*, "Photonics Crystals: Molding the Flow of Light", Princeton Univ. Press, 2008.

This is also the end of the stable branch. And this is where it will jump. So there is a dependence on the historical power values, right? It depends which way, you know, earlier power value was, okay? So if you are here, you will be coming here. But if you are here and you further reduce it, you come down, okay, your output power drastically drops. So, the output power basically depends on its historical values, okay. So, it follows the lower branch when starting from low power and it will follow the upper branch when you are starting from the high power right. So, you are also able to see examples of hysteresis effect in the power output response. So, in this case, what are the initial conditions? So, first you need a cavity that has a resonance at frequency omega naught and then you have the input power which is introduced at a frequency omega which is slightly below omega naught, okay. Now, you increase the power. So, in the linear regime the output power will be directionally directly proportional to the input power. And as the input power increases the nonlinearity in the cavity will cause the permittivity epsilon to increase and that will shift your omega naught to lower frequencies.

Optical Bistability

- **Initial Conditions:**
  - A cavity has a resonance at frequency ω_0.
  - Input power is introduced at a frequency ω slightly below ω_0.
- **Response to Increasing Power:**
  - In the linear regime, output power is directly proportional to input power.
  - As input power increases, nonlinearity in the cavity causes ϵ (permittivity) to increase, shifting ω_0 to lower frequencies.
  - This shift moves the resonant peak down through the input frequency ω, affecting transmission.
- **Feedback Effects:**
  - Positive Feedback: Enhanced coupling to the cavity as the system approaches resonance, leading to a sharper transition to high transmission ("on" transition).
  - Negative Feedback: Reduced coupling as the system moves away from resonance, resulting in a delayed transition to low transmission ("off" transition).

Source: J. D. Joannopoulos *et al.*, "Photonics Crystals: Molding the Flow of Light", Princeton Univ. Press, 2008.

And this shift moves the resonance peak down through the input frequency omega which affects the transmission. So, there are some feedback effects here. So, positive feedback something like you know there are enhanced coupling to the cavity as the system approaches resonance that leads to a sharper transition to higher transmission then that can be called as on transition. There are also negative feedback that means reduced coupling as the system moves away from the resonance.

This results in a delayed transition to low transmission. So, that can be denoted as off transition. So, if you think of a filter configuration, we can again think of this particular filter setup or model of the temporal couple mode theory that you have seen in the previous lecture right. So, this is basically the setup for a waveguide cavity waveguide kind of structure ok. So, here if you include the cairn nonlinearity ok. So, you can introduce the cairn nonlinearity where the dielectric constant epsilon is basically proportional to the square of the electric field's magnitude that is modulus of e square.

Optical Bistability

- **Filter Configuration:**
  - Utilizes the waveguide-cavity-waveguide setup from figure 6 .
- **Inclusion of Kerr Nonlinearity:**
  - Introduces a Kerr nonlinearity where the dielectric constant ϵ is proportional to the square of the electric field magnitude $|E|^2$.
- **Localization of Nonlinear Effects:**
  - Nonlinear effects are most significant in the cavity, where field intensity is highest.
- **Frequency Shift Due to Nonlinearity:**
  - The increase in ϵ within the cavity leads to a frequency shift $\Delta\omega_0$ in the cavity mode, proportional to $|A|^2$ (where A is the field amplitude).

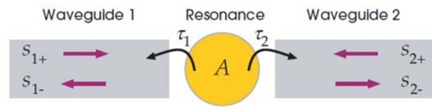


Figure 6: Abstract diagram showing the essential features: a single-mode input waveguide 1 , with input/output field amplitudes s_{1+}/s_{1-}; a single-mode output waveguide 2 with input/output field amplitudes s_{2+}/s_{2-}; and a single resonant mode of field amplitude A and frequency ω_0, coupled to waveguides 1 and 2 with lifetimes τ_1 and τ_2 ($\tau_1 = \tau_2$ in figure 1). The $s_{\ell\pm}$ are normalized so that $|s_{\ell\pm}|^2$ is power in the waveguide, and A is normalized so that $|A|^2$ is energy in the cavity.

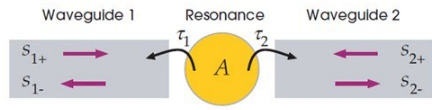
Source: J. D. Joannopoulos *et al.*, "Photonics Crystals: Molding the Flow of Light", Princeton Univ. Press, 2008.

If you I hope you remember this is the abstract diagram that shows the essential features of this model. That means you have a single mode. input waveguide 1 ok, where the input and the output field amplitudes are marked as S 1 plus and S 1 minus. Then you have this single mode output waveguide 2, where the input and the output are marked as S 2 plus and S 2 minus. Then you have a single resonant mode of field amplitude A and frequency omega naught and that is basically coupled to waveguides 1 and 2 with lifetimes of τ_1 and τ_2 . And remember in the first case we have considered τ_1 and τ_2 to be similar because they are symmetrical waveguides ok. Now, SL plus L is like 1 or 2. So, SL plus are normalized. So, that modulus SL plus minus whole square is the power in the waveguide. And the amplitude A of the resonant cavity is normalized so that modulus of A square is basically the energy of the cavity.

So, this was the setup abstract diagram that we used for describing the filters. Now, here how do you work on the localization of the non-linear effects? So, non-linear effects are most significant in the cavity because in the cavity the field is highest. So, as we understand that because of the nonlinearity there will be shift in resonance. So, the increase in permittivity within the cavity leads to a frequency shift that is delta omega naught in the cavity mode and that is proportional to modulus a square where a is basically the amplitude. So, when you take modulus square that is the intensity.

Optical Bistability

- **Coupling and Frequency Conversion:**
  - Kerr nonlinearity enables coupling between different frequencies, potentially causing third harmonic generation (converting frequency ω to 3ω).
- **Dominant Nonlinear Effect:**
  - The primary nonlinear effect is the shift in the resonance frequency ω_0; third-harmonic generation only becomes relevant if there is a resonant mode at $3\omega_0$.



IIT Guwahati | NPTEL swayam Source: J. D. Joannopoulos *et al*., "Photonics Crystals: Molding the Flow of Light", Princeton Univ. Press, 2008.

So, proportional to that there is a shift in the resonance that is delta omega naught. The K-nanolinearity enables coupling between different frequencies potentially causing the third harmonic generation that means it converts from frequency omega to 3 omega. And the primary nonlinear effect in the system will be the shift in the resonance frequency omega naught and the third harmonic generation that is generation of frequency component 3 omega also becomes relevant if there is any resonant mode present at 3 omega naught.

Optical Bistability

- This type of nonlinear transmission allows us to create an all-optical transistor: by adding or removing power from the input waveguide, we can switch from a low-transmission to a high-transmission state, and vice versa.
- Everything that can be done with an electronic transistor can also be accomplished with our optical device: switching, logic gates, signal rectification, amplification, and many other functions.

IIT Guwahati | NPTEL swayam Source: J. D. Joannopoulos *et al*., "Photonics Crystals: Molding the Flow of Light", Princeton Univ. Press, 2008.

So this type of nonlinear transmission allows us to create an all optical transistor. By adding or removing power from the input waveguide, you can switch from a low transmission to a high transmission state and vice versa. So everything that can be done with electronic transistor can be thus accomplished with our optical device as well.
So we can have switching, we can have logic gates, signal rectification, amplification and many other functions.

Summary

So with that we will try to conclude our findings. So first we have seen the waveguide splitter using temporal couple mode theory. We have understood that this utilizes the principle of temporal couple mode theory to effectively manage light propagation and distribution within the waveguide system. It employs constructive interference and resonance conditions to maximize transmission efficiency and minimize loss and it can dynamically adjust to the changes in the light intensity that enables flexible optical routing and switching. Next, we understood a drop filter using non-linear photonic crystals. So, here it combines the non-linear photonic crystals with conventional waveguide structures for enhanced filtering capabilities. This kind of devices exploits the Kerr effect, the Kerr nonlinear effects and other nonlinear properties to adjust the filter responses based on light intensity, allowing for tunable frequency selectivity.

Conclusion

- **Waveguide Splitter Using Temporal Coupled-Mode Theory:**

  Utilizes the principles of temporal coupled-mode theory to effectively manage light propagation and distribution within waveguide systems.

  Employs constructive interference and resonance conditions to maximize transmission efficiency and minimize loss.

  Can dynamically adjust to changes in light intensity, enabling flexible optical routing and switching.

- **Add/Drop Filter Using Nonlinear Photonic Crystals:**

  Combines nonlinear photonic crystals with conventional waveguide structures to enhance filtering capabilities.

  Exploits the Kerr effect and other nonlinear properties to adjust filter responses based on light intensity, allowing for tunable wavelength selectivity.

  Enhances device functionality with minimal physical adjustments, offering applications in dense wavelength-division multiplexing (DWDM) systems.

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So, that is very important, tunability in any kind of devices. So here the device materials remain same because the material is changing with intensity of the incoming light. It behaves differently. The same device can give you a different output wavelength. This enhances the device functionality with minimal physical adjustment. Thus, offering applications in dense WDM network or systems like wavelength division multiplexing systems.

Conclusion

- **Optical Bistability:**

  Achieved through setups like the waveguide-cavity-waveguide filter, incorporating Kerr nonlinearity.

  Bistability arises from nonlinear changes in the dielectric constant, leading to significant shifts in resonance frequencies within the cavity.

  Enables the system to exhibit two stable states (on/off) depending on the input power, useful for optical switches and memory devices.

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Then we have seen optical biostability. This can be achieved through setups something like webguide cavity, webguide kind of filter, but you have to incorporate CAN nonlinearity here in the cavity. So, biostability comes from another nonlinear changes in the optical or you can say dielectric constant. This leads to significant shift in the resonance frequency within the cavity and this will enable the system to exhibit two stable states on state and off state depending on the input power and this is very useful for optical switches and different memory devices.

So, with that we will conclude here. If you have got any doubt or query regarding this lecture, you can always send your queries to this email address mentioning MOOC and the lecture number on the subject line.