Lec 4: Electromagnetic Properties of Materials

Hello students, welcome to lecture 4 of this online course on photonic crystals fundamentals and applications. Today's lecture will be on electromagnetic properties of materials. So, in this week we will continue the electromagnetic theory of light starting from the derivation of wave equation from the Maxwell's equations. Then we will discuss some important electromagnetic properties of different materials.

Lecture Outline

- Maxwell's Equations — Recap
- The Wave Equation
- Boundary Conditions
- Electromagnetic properties of materials — Introduction
  - Dielectric permittivity (ε)
  - Magnetic permeability (μ)
  - Conductivity (σ)
- Classification of Materials — by Anisotropy
- Classification of Materials — by Linearity
- Classification of Materials — by magnetization
- Classification of Materials — by Conductivity

Source: B. E. Saleh and M. C. Teich, Fundamentals of photonics (John Wiley & Sons, 2019).

So here is the lecture outline, a quick recap of Maxwell's equation that you have seen in the last lecture. We will see how we can derive wave equations from that.

We will discuss about the boundary conditions and then we will go into the electromagnetic properties of material. We will discuss about dielectric permittivity ε, magnetic permeability μ conductivity σ and then we will look into the classification of materials by anisotropy, linearity, magnetization and conductivity. So, that will be our lecture outline today.

Maxwell's Equations — Recap

Table: Comparison of Maxwell's equations for static and time-varying electromagnetic fields.

	Electrostatics / Magnetostatics	**Time-Varying (Dynamic)**
Electric & magnetic fields are...	independent	possibly coupled
Maxwell's eqns. (integral)	$\oint_S \mathbf{D} \cdot d\mathbf{s} = Q_{encl}$	$\oint_S \mathbf{D} \cdot d\mathbf{s} = Q_{encl}$
	$\oint_C \mathbf{E} \cdot d\mathbf{l} = 0$	$\oint_C \mathbf{E} \cdot d\mathbf{l} = -\frac{\partial}{\partial t} \int_S \mathbf{B} \cdot d\mathbf{s}$
	$\oint_S \mathbf{B} \cdot d\mathbf{s} = 0$	$\oint_S \mathbf{B} \cdot d\mathbf{s} = 0$
	$\oint_C \mathbf{H} \cdot d\mathbf{l} = I_{encl}$	$\oint_C \mathbf{H} \cdot d\mathbf{l} = I_{encl} + \int_S \frac{\partial}{\partial t} \mathbf{D} \cdot d\mathbf{s}$
Maxwell's eqns. (differential)	$\nabla \cdot \mathbf{D} = \rho_v$	$\nabla \cdot \mathbf{D} = \rho_v$
	$\nabla \times \mathbf{E} = 0$	$\nabla \times \mathbf{E} = -\frac{\partial}{\partial t}\mathbf{B}$
	$\nabla \cdot \mathbf{B} = 0$	$\nabla \cdot \mathbf{B} = 0$
	$\nabla \times \mathbf{H} = \mathbf{J}$	$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial}{\partial t}\mathbf{D}$

Note: Differences in the time-varying case relative to the static case are highlighted in blue.

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Source: B. E. Saleh and M. C. Teich, Fundamentals of photonics (John Wiley & Sons, 2019).

So, in the last lecture we have seen that Maxwell's equations can be written in terms of static and dynamic fields where E and H fields are independent if we are talking about static conditions like electrostatics or magnetostatics.

Whereas the fields the electric and magnetic fields are coupled when we talk about dynamics, electrodynamics which actually tells about time varying fields. So, here you can see that these are the Maxwell's equation in integral form for electrostatics or magnetostatics and this is for electrodynamics. So, here what you can see that this blue terms are the addition when you are talking about time varying or dynamic case. These are the same equations but in differential form. So, I will not go and repeat these equations we have seen that in the last lecture.

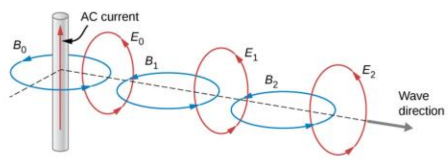
But just to remind you that there are a couple of changes when you go from static to dynamic cases ok. Something like curl of E is 0 when you are talking about electrostatics or magnetostatics ok. But in the dynamic case you can actually see that curl of E is nothing but $-\partial B/\partial t$. Similarly Ampere's law which gets modified to Ampere's Maxwell's equations okay. So it modifies in this particular form.

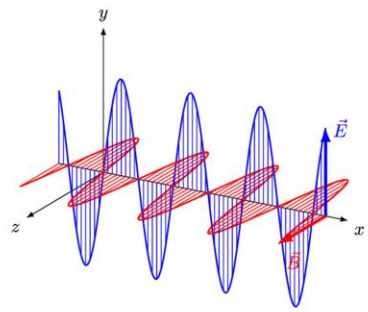
So curl of H is now given as J plus J_D the displacement current density or you can write the term as $\partial D/\partial t$ that is the time varying electric flux density. So, here in the note it is mentioned that this time varying cases in the time varying cases which are you know the changes that are basically highlighted in blue color for quick visualization.

Wave Equation

- The **electromagnetic wave equation** is a second-order partial differential equation that describes the propagation of electromagnetic waves through a medium or in a vacuum.
- The homogeneous form of the equation, written in terms of either the electric field $\boldsymbol{E}$ or the magnetic field $\boldsymbol{B}$.

$$\nabla^2 \boldsymbol{E}(r,t) = \frac{1}{v^2}\frac{\partial^2 \boldsymbol{E}(r,t)}{\partial \boldsymbol{t}^2}$$





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Source: https://phys.libretexts.org/Bookshelves/University_Physics
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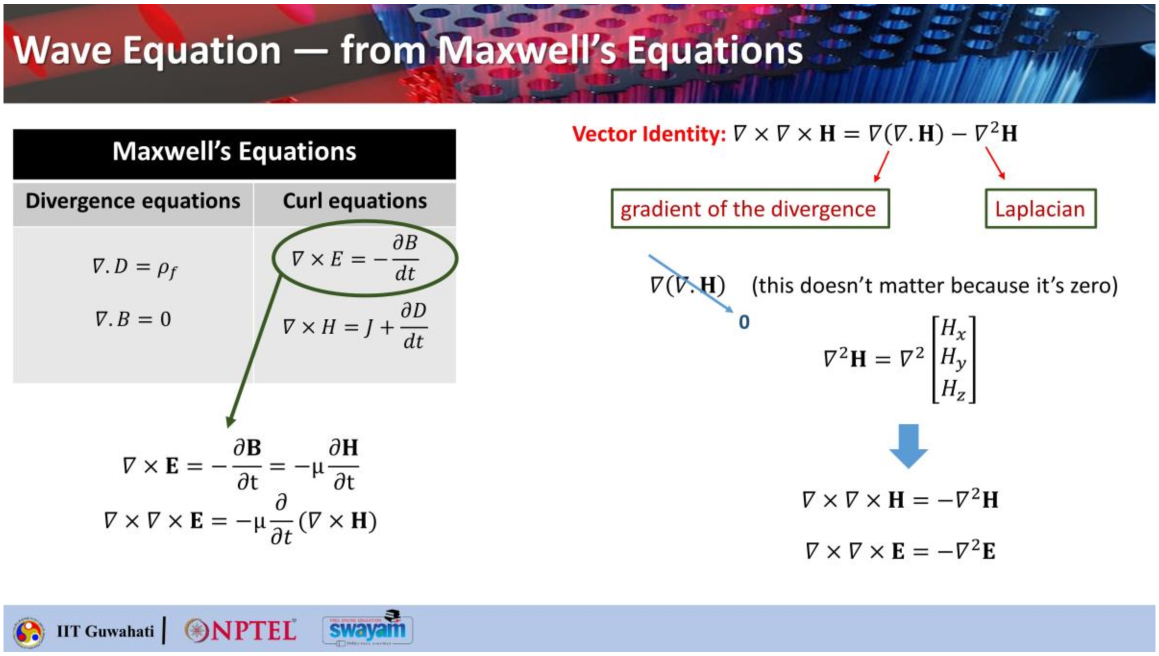
Now, let us start with how to determine. the electric and magnetic field propagation through a particular region right. So, this can be described in the form of wave equation.

So, when we talk about wave equation electromagnetic wave equation is nothing but a second order partial differential equation that describes the propagation of electromagnetic wave through a particular medium or in vacuum. So, here you can see that the electric and magnetic fields are coupled to each other and they are oscillating in their own plane and then this is the wave propagation direction. homogeneous form of the equation written in terms of either electric field or magnetic field is called the wave equation. So, typically it looks like this. So, you have Laplacian of electric field vector which is a function of position and time that is equal to $1/v^2$, v is basically the phase velocity of the wave propagating in a particular medium.

And then you have the second order partial derivative with respect to time of that particular vector field. So, the same equation could have been written in terms of magnetic field as well. So, instead of E you could have placed B and that will give you again the wave equation.

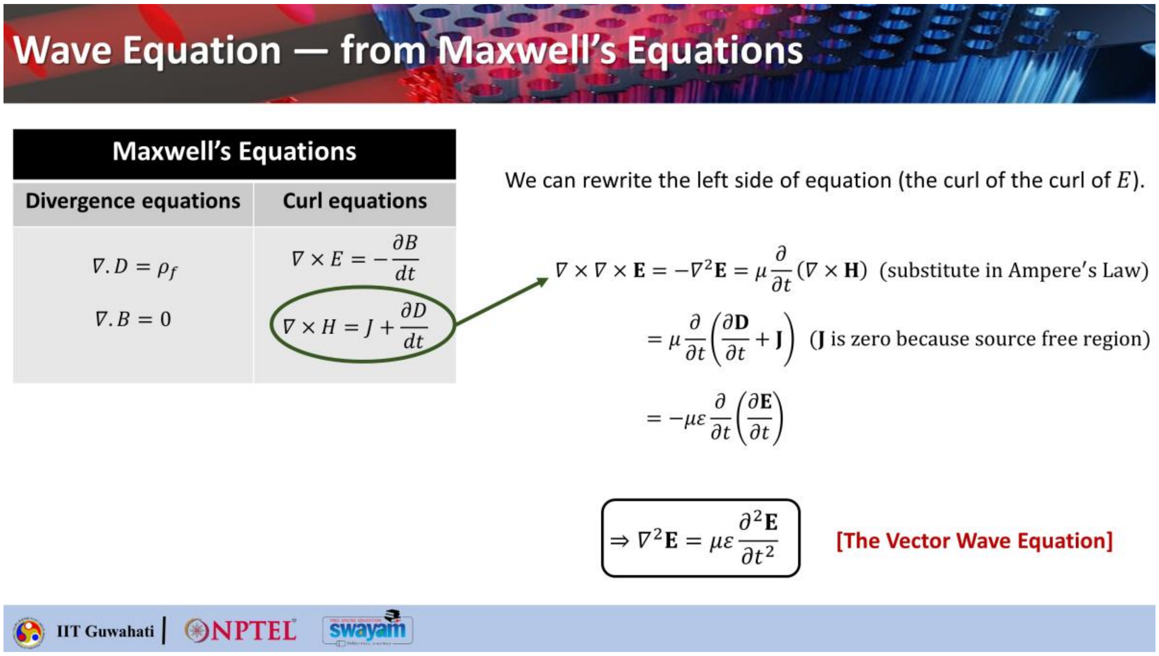


So, our goal here is to determine how the wave equation can be derived from Maxwell's equation. So, to start with let me throw a vector identity okay and this is basically a mathematical manipulation that is true for all magnetic fields.

So, curl of curl of H can be written as gradient of the divergence of the particular field minus the Laplacian of the field. So, once we know that vector identity we can actually use the curl equation. okay to obtain the wave equation. So, you start with curl of E which is -∂B/dt, B can be written as μH. So, you can also write it at -μ∂H/dt.

Now, in this equation if you take curl on both the sides you get $\nabla \times \nabla \times \mathbf{E}$ which is nothing but -μ∂/dt. H will now be replaced with curl of H. Now if you consider that we are in a source free region that means there is no charge or current flowing anywhere we can actually take this gradient term as 0 okay. So what happens this $\nabla \times \nabla \times \mathbf{H}$ can be simply written as $-\nabla^2 \mathbf{H}$. So this is how you can actually compute this H can be written in terms of the three components H_x, H_y and H_z.

So, you can finally write this equation in this particular form that $\nabla \times \nabla \times \mathbf{H}$ equals ∇^2H. Remember this is for the case where we are assuming that we are in a source free region that is there is no charge or no current is flowing. So, if you replace this by electric field you get $\nabla \times \nabla \times \mathbf{E}$ is nothing but $-\nabla^2$E.



So, how do you get that? So, what what we have till now? We we know that $\nabla \times \nabla \times \mathbf{E}$ is nothing but $-\nabla^2$E ok and that can be written from the previous one. So, $\nabla \times \nabla \times \mathbf{E}$ we have obtained that is basically this term the right hand side.

You can equate those two here. So, you equate those two here, okay? So, this is what we have. The right $\nabla \times \nabla \times \mathbf{E}$ is $-\nabla^2$E, which can be written as $\mu\partial/\partial t$ $(\nabla \times \mathbf{H})$. Now, the $\nabla \times \mathbf{H}$ can be replaced by $\partial D/\partial t + J$. Now remember, we are talking about a source-free region, so we can take **J** to be 0, okay? So, what we are left with is only this term, and D can be written as εE.

So, you can take out ε, which is a constant. You can take it out of this derivation. So, you get $-\mu\varepsilon \frac{\partial}{\partial t}\left(\frac{\partial \mathbf{E}}{\partial t}\right)$. Therefore, you can write $\nabla^2 \mathbf{E} = \mu\varepsilon \frac{\partial^2 \mathbf{E}}{\partial t^2}$.

Wave Equation — speed of EM waves

- The connection between electromagnetic optics and wave optics is now evident.
- The **wave equation**, which is the basis of wave optics, is embedded in the structure of electromagnetic theory.
- The speed of electromagnetic wave is related to the electromagnetic constants ε and μ.

Speed of the EM wave:

Compare:

$$\nabla^2 \boldsymbol{E} = \mu_0 \varepsilon_0 \varepsilon_\mathrm{r} \frac{\partial^2 \boldsymbol{E}}{\partial t^2} \text{ and } \nabla^2 \boldsymbol{E} = \frac{1}{v^2}\frac{\partial^2 \boldsymbol{E}}{\partial t^2}$$

$$\Rightarrow v^2 = \frac{1}{\mu_0 \varepsilon_0}\frac{1}{\varepsilon_\mathrm{r}} = \frac{c_0^2}{\varepsilon_\mathrm{r}}$$

In Free Space (Vacuum):

$$\mu_0 = 4\pi \times 10^{-7}\ \mathrm{H/m}$$
$$\varepsilon_0 = 8.854 \times 10^{-12}\ \mathrm{F/m}$$
$$c_0 = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 299{,}795{,}638\ \mathrm{m/s}$$

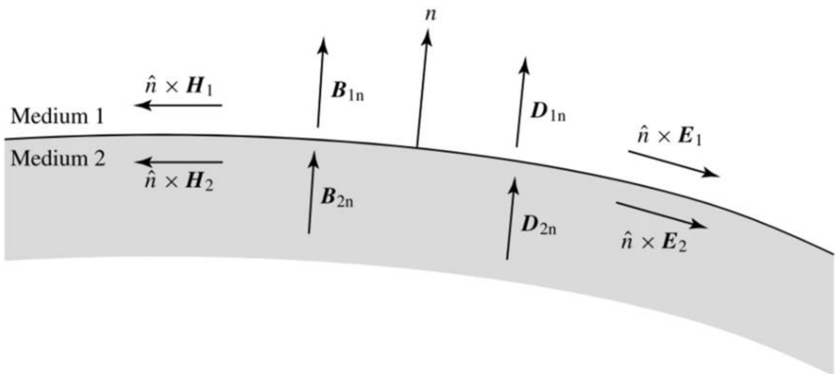
So, this is your vector wave equation. The connection between electromagnetic optics and wave optics is now more evident, as you can see that the wave equation, which is the basis of wave optics, is essentially embedded in the structure of electromagnetic theory. The speed of the electromagnetic wave is related to two important electromagnetic constants, ε and μ. We can actually compute the speed by comparing these two forms of the wave equation, where you can write $\nabla^2 \boldsymbol{E} = \mu_0 \varepsilon_0 \varepsilon_\mathrm{r} \frac{\partial^2 \boldsymbol{E}}{\partial t^2}$.

So, μ_0 is the vacuum permeability, ε_0 is the vacuum permittivity, and ε_r is the relative permittivity. So when you compare this wave equation with this particular form, you can see that v^2 is nothing but this term.

You can write $v^2 = \frac{1}{\mu_0 \varepsilon_0}\frac{1}{\varepsilon_\mathrm{r}}$. Thus, the term $\frac{1}{\mu_0 \varepsilon_0}$ can be expressed as c_0^2,where c_0 is the speed of light in a vacuum. So, that is how you can actually correlate the phase velocity with the velocity in that particular medium in vacuum divided by the permittivity of the medium in which the wave is propagating. So, it is basically v^2. So, from this, you can also see that if you take the square root of it, you can simply get $v = \frac{c_0}{\sqrt{\varepsilon_\mathrm{r}}}$.
The square root of ε_r is essentially the refractive index of the medium, which is the fundamental definition of refractive index. So, the refractive index tells you the ratio of the speed of light in vacuum to the speed of light in that particular medium. So, that is how you can actually get n. The relationship between n and ε_r in a non-absorbing medium is basically very simple: n equals the square root of $\sqrt{\varepsilon_\mathrm{r}}$. So, all these parameters you should remember; they are very basic things: μ_0 is $4\pi\times10^{-7}$ H/m, ε_0 is 8.854×10^{-12} F/m, and this is the speed of light. So, we typically take this as 3×10^8 meters per second. To keep the calculations easy, we should.
So, c_0 is basically computed as 1 over the square root of $\mu_0 \varepsilon_0$.

Boundary Conditions

- At the interface of two media of different optical properties, the optical field components must satisfy certain boundary conditions.



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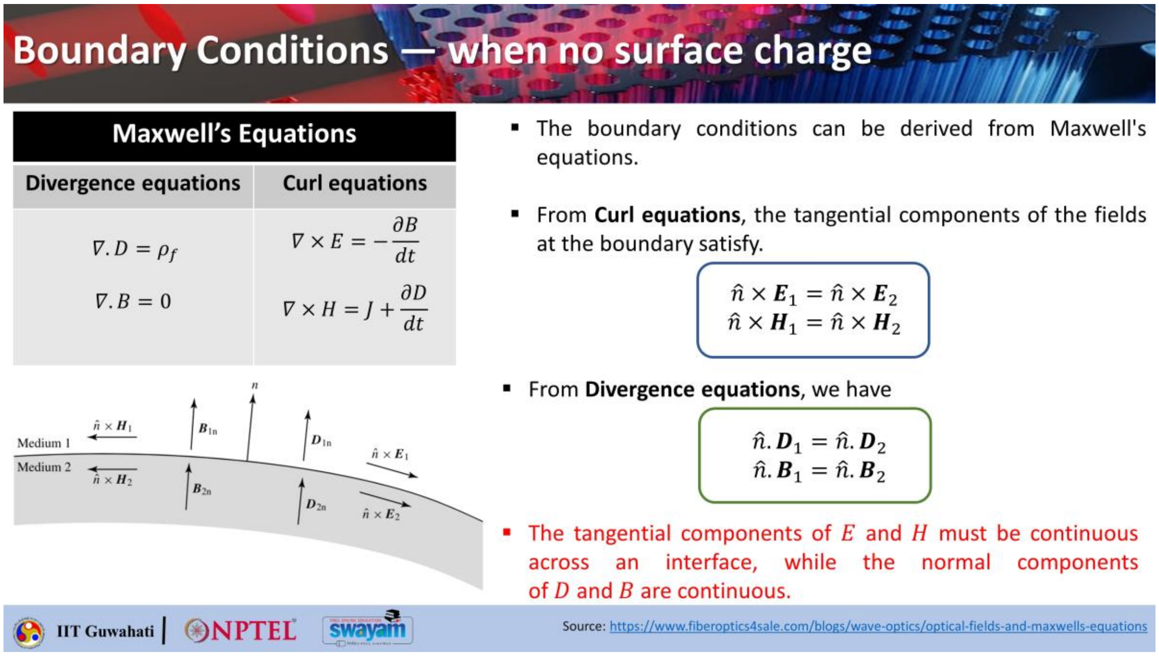
Source: https://www.fiberoptics4sale.com/blogs/wave-optics/optical-fields-and-maxwells-equations

Now, coming to the boundary conditions, this is very important when you want to solve the field equations at different boundaries of the medium. Whenever there are two mediums coming next to each other, there will be a boundary, and you have to determine the conditions that allow your fields to cross the boundary. At the interface of two media with different optical properties, the optical field component must satisfy certain boundary conditions. Now, what are those? The boundary conditions basically describe electromagnetic fields, such as the electric field.

The electric displacement field D, the magnetic field H, and the magnetic flux density B, also known as the magnetic displacement field, are all important concepts. Now, if you consider a source-free region, we can say that the tangential components of E and H must be continuous across the interface, while the normal components of The flux density or the displacement fields D and B must also be continuous. So, here you can see that this is medium 1, this is medium 2, and this is the boundary. So, what we are basically saying is that the tangential component of H can be calculated as the unit vector n crossed with H_1. Similarly, this is the tangential component of the magnetic field in the second medium; these two should be continuous.

Similarly, the electric field should also be continuous, and this is essentially the normal to the interface. So, the normal component of the electric flux density or electric displacement field. So, D_{1n} should be equal to D_{2n}; similarly, the magnetic flux density or magnetic displacement field B_{1n} should be equal to B_{2n}. So, these boundary conditions can also be derived from Maxwell's equations. So, always remember that in this particular case, we are considering that there is no surface charge.

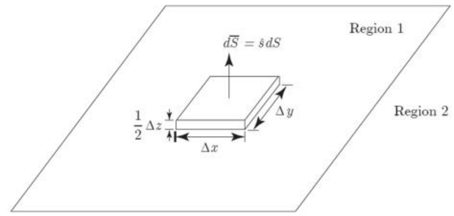


So, what we have seen is that the curl equations basically give us the tangential components of the field at the boundary. So, we are basically looking at $\hat{n} \times \boldsymbol{E}_1 = \hat{n} \times \boldsymbol{E}_2$. Similarly, $\hat{n} \times \boldsymbol{H}_1 = \hat{n} \times \boldsymbol{H}_2$. What is $\hat{n}$? $\hat{n}$ is basically this vector, okay. So, when you take the curl, you essentially obtain the tangential components. Similarly, if you look into the divergence equations, we are basically calculating the dot products with $\hat{n}$.

We are essentially examining the normal components of D and B. You can say that $\hat{n}$ D_1 should be equal to $\hat{n}$ D_2, and $\hat{n}$ B_1 should be equal to $\hat{n}$ B_2. When there is no surface charge, we understand that the tangential components of E and H must be continuous across the interface, while the normal components of D and B are continuous.

Boundary Conditions — presence of surface charge and current density

- It is often convenient, in particular mathematically, to define regions where the electric and magnetic fields are zero.
- Assume that there is a plane boundary surface at $z = 0$ separating Regions 1 and 2, we can derive the boundary condition for H by using a small pill-box [as shown in Fig.] and letting Δz go to zero.
- **The media occupying such regions are called perfect conductors**, which are idealizations of media where the fields inside are vanishingly small.
- We assume that all fields in Region 2 are zero, $\boldsymbol{E_2} = \boldsymbol{H_2} = \boldsymbol{B_2} = \boldsymbol{D_2} = \boldsymbol{0}$.
- **Electric charges and currents** are located primarily in a very thin layer on the **surface of perfect conductors**. Thus on the surface of perfect conductors, we assume $\boldsymbol{\rho}$ is infinite contained in a zero thickness.

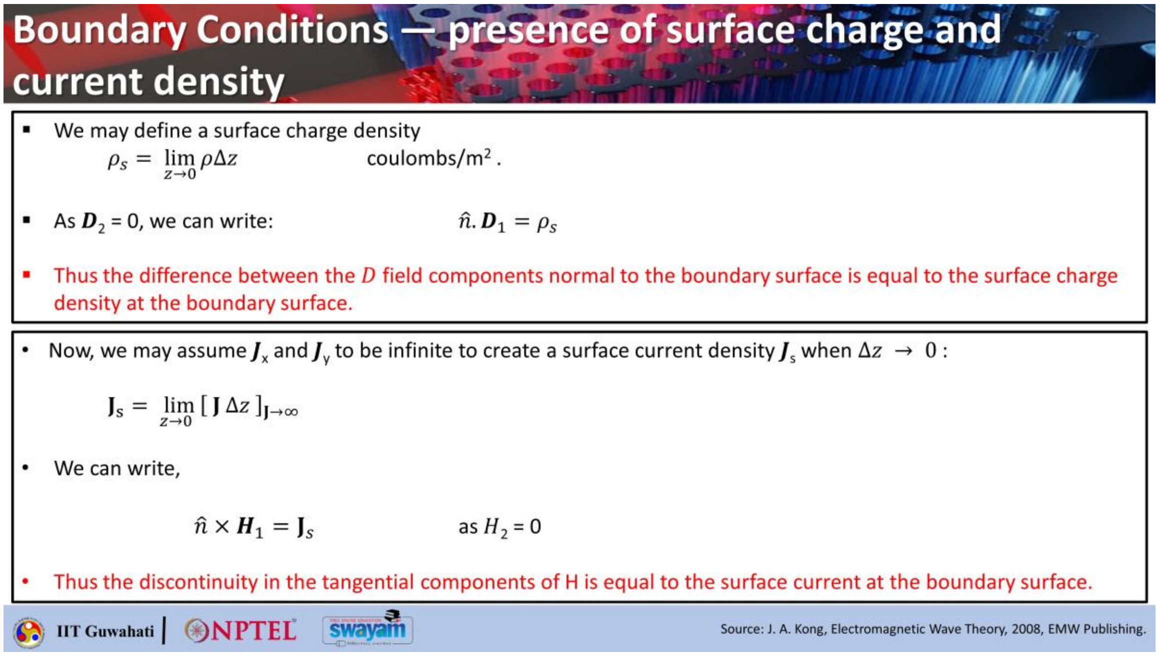


Source: J. A. Kong, Electromagnetic Wave Theory, 2008, EMW Publishing.

Now, what happens when there is a surface charge present or when there is some current density? So, now let us look into the boundary conditions and how they will change if there is some surface charge or charge density present. The concept of surface charge density will have practical usefulness in this case.

So, we have seen that it is often very convenient, mathematically, to define regions where you know the electric and magnetic fields are zero. Now, let us assume that there is a plane boundary condition or plane boundary surface at z equals 0, separating two regions: region 1 and region 2. So, region 2 is at the bottom, region 1 is at the top, and this surface is basically at z equals 0, okay? We can derive the boundary conditions for H by using a small pillbox, as shown in this figure, by letting Δz approach 0. Now, the media occupying such regions are called perfect conductors, which are basically idealizations of media where the fields inside are considered to be vanishingly small. So, in that case, we can assume that all fields in region 2 are basically 0.

You can say that E_2, H_2, B_2, and D_2 are all equal to 0. Now we assume that electric charges and currents are located primarily on a very thin layer on the surface of the perfect conductor. Thus, on the surface of the perfect conductors, we assume that ρ is infinite and is contained in a medium of zero thickness.



So, this is what the surface charge density in that case, ρ_s, will be: you know, $\rho\Delta z$, where z tends to 0. So, if we can, we know that D_2 is already 0 because there is nothing below the surface, whereas $\hat{n}$ D_1 is not 0 because there is surface charge density.

So, $\hat{n}$ D_1 is basically ρ_s. You can see that there is a difference between the D field components that are normal to the boundary surface, and this difference is equal to the surface charge density that lies at the boundary surface. Similarly, we can now assume that J_x and J_y are infinite to create a surface charge density J_s when Δz tends to zero. So, you can write J_s equal to $J\Delta z$, okay? So, the value of J tends to infinity; however, we are operating in a limit where the pillbox is infinitely thin. So, it is as z tends to 0. So, that is how you actually get the surface current density correct.

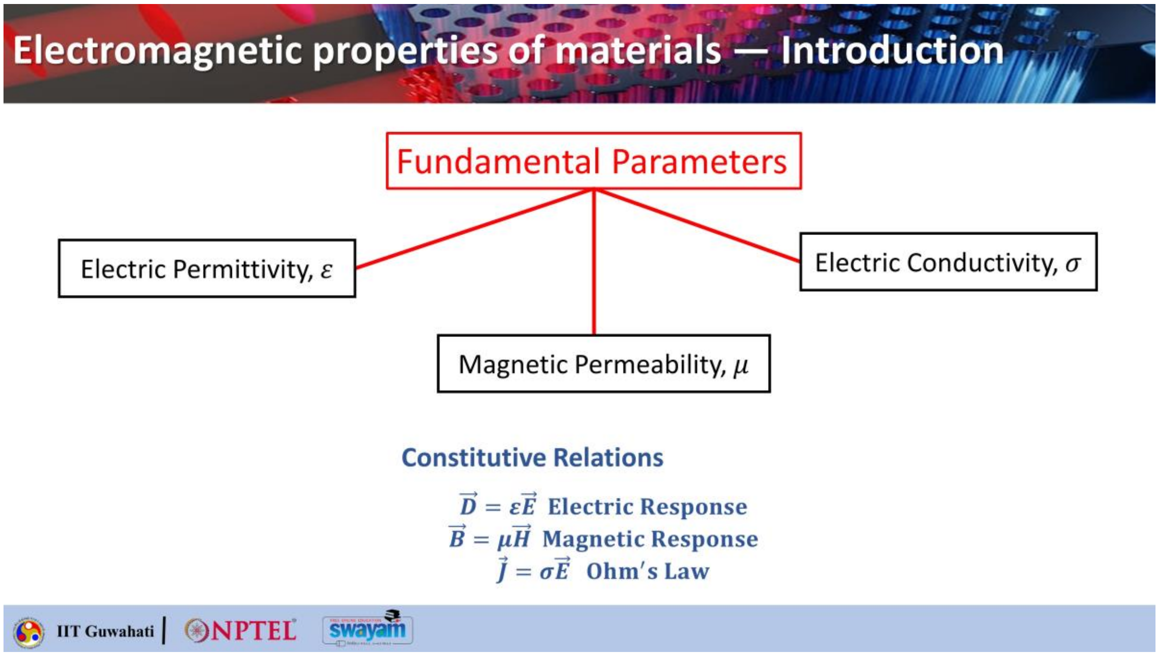
Boundary Conditions — presence of surface charge and current density

S.No.	Vector Form	Scalar Form	Description
1.	$\hat{e}_n \times (\vec{E}_1 - \vec{E}_2) = 0$	$E_{t1} - E_{t2} = 0$	Tangential electric field ($\vec{E}$) is continuous.
2.	$\hat{e}_n \times (\vec{H}_1 - \vec{H}_2) = \vec{J}_s$	$H_{t1} - H_{t2} = J_s$	The discontinuity of the tangential H field equals the surface current.
3.	$\hat{e}_n \cdot (\vec{D}_1 - \vec{D}_2) = \rho_s$	$D_{n1} - D_{n2} = \rho_s$	The discontinuity of the normal $\vec{D}$ equals the surface charge density.
4.	$\hat{e}_n \cdot (\vec{B}_1 - \vec{B}_2) = 0$	$B_{n1} - B_{n2} = 0$	The normal component of $\vec{B}$ is continuous.

Source: J. A. Kong, Electromagnetic Wave Theory, 2008, EMW Publishing.

So, in that case, you can write that while your H_2 shows the magnetic field in region 2 is 0, the tangential component of the magnetic field is not. In region 1, the intersection of n and H_1 is not 0; rather, it is the surface current density J_s. So, here we can see that there is a discontinuity in the tangential component of the magnetic field, which is equal to the surface current at the boundary surface. So, finally, we can rewrite the boundary conditions in the presence of surface charge and current density, as we can see in this particular table. So, if you write it in terms of a vector field. So, here you know these are continuous. You can write it as $\hat{n}$, or you can write $\hat{e}_n$. The cross product with $E_1 - E_2$ is 0. So, that is where the tangential component of the electric field is continuous, but it is disrupted whenever there is a surface current. The tangential component of the magnetic field shows a discontinuity, and the difference between the two is given by this current density.
So, you can simply write $H_{t1} - H_{t2}$ equals J_s; this is the scalar form. In the vector form, the tangential component is calculated using the cross product. And if you talk about the discontinuity of the normal electric flux density. So, you can simply write D_{n1} minus D_{n2} equals ρ_s; that is the scalar form, and in vector form, this is calculated as a dot product with the $\hat{n}$ vector. You could have simply written "$\hat{n}$," or you can write $\hat{e}_n$.

Both refer to the same thing: the unit vector along the surface normal. The difference between D_1 and D_2 gives you this. ρ_s is okay, and this is how your magnetic field is continuous across the surface and the boundary, and this is how you can write it. So, you can actually see that the presence of the surface charge density and surface current basically affects these two equations. The tangential component of the magnetic field will no longer be continuous; there will be a discontinuity, and the amount of discontinuity is given by the surface current. Similarly, the normal component of the electric flux will not be continuous; the discontinuity is caused by the surface charge density.



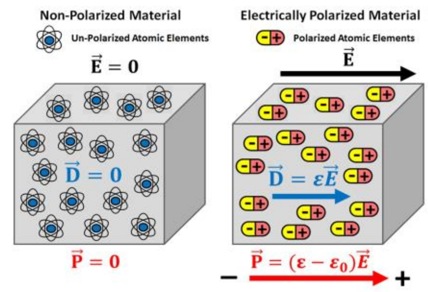
So now let us look into the electromagnetic properties of the material, okay? So now we will see how electromagnetic waves behave differently in different materials while propagating. In other words, you can say that several materials can be classified based on their electromagnetic properties. We can start with the constitutive relations, where we can define the fundamental electromagnetic properties of materials. So, there are three fundamental properties: electric permittivity (ε), magnetic permittivity (μ), and electric conductivity (σ), right. So, these are the constitutive relations.

So, D equals εE. This tells you about the electric response of the material; B equals μH, which tells you about the magnetic response of the material; and J equals σE. It tells you about Ohm's law and the electrical conductivity of the material, right.

So, if you want to understand what happens with electric response, this is basically dielectric permittivity. This epsilon is a diagnostic physical property that characterizes the degree of electrical polarization a material experiences under the influence of an external electric field. Similarly, from the magnetic response, you can say that magnetic permeability is a measure of how well a medium stores magnetic energy. Right, and σ is a measure of how well it conducts the field.

Dielectric permittivity (ε) — constitutive relations

- **Dielectric permittivity** (ε) is a measure of how well a medium stores electric energy. It can be thought of as a measure of how much interaction an electric field has with the medium it resides in.
- The **permittivity** (ε) is defined as the ratio between the electric field ($\boldsymbol{E}$) within a material and the corresponding electric displacement ($\boldsymbol{D}$):
$$\boldsymbol{D} = \varepsilon_0 \boldsymbol{E}$$
- When exposed to an electric field, bounded electrical charges of opposing sign will try to separate from one another. The extent of the separation of the electrical charges within a material is represented by the **electric polarization** ($\boldsymbol{P}$).
- The electric field, electric displacement and electric polarization are related by the following expression:
$$\boldsymbol{D} = \varepsilon_0 \boldsymbol{E} + \boldsymbol{P} = \boldsymbol{\varepsilon} \boldsymbol{E}$$



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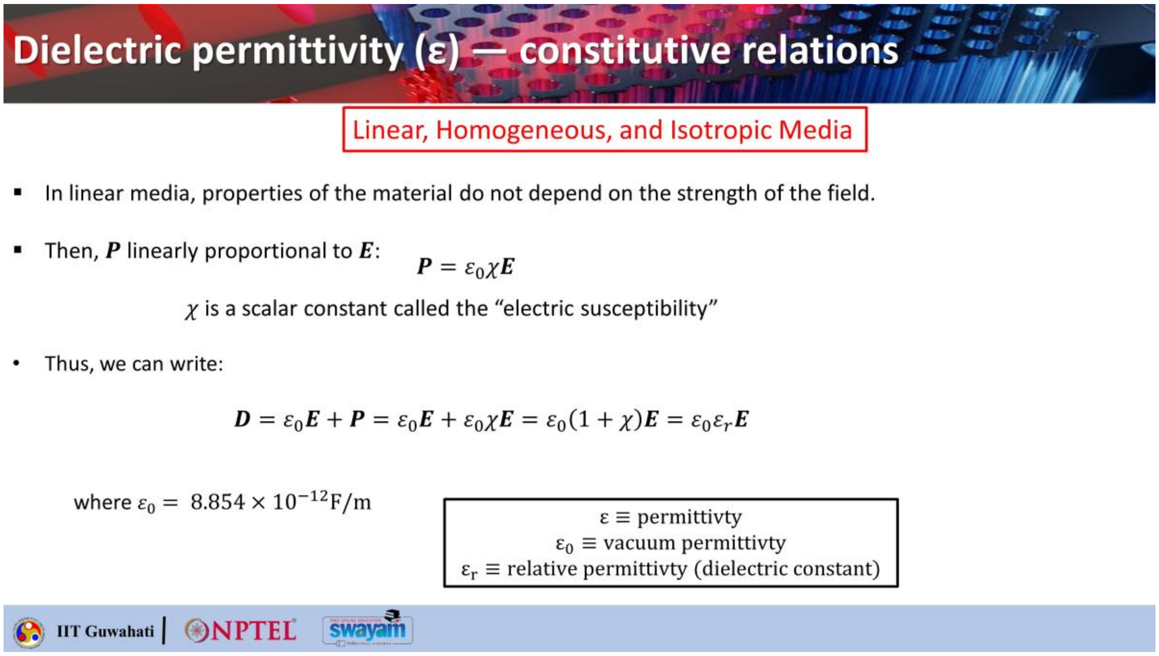
So, at first, we will define the dielectric permittivity ε, and this permittivity is closely related to the capacitance. So, you can see that dielectric permittivity is essentially a measure of how well a medium stores electric energy. It can be thought of as a measure of how much interaction an electric field has with the medium in which it resides. So, the permittivity ε can be defined as the ratio of the electric field E to D. Therefore, you can see that epsilon naught can be expressed as D over E, right. You can say that permittivity is essentially the ratio between the electric field E and the corresponding electric displacement.

So, if you want to understand what displacement looks like, this is a pictorial representation of it. So, what happens? This is a non-polarized material, so you have a positive nucleus, and then you have an electron cloud. So, these basically represent unpolarized atomic elements. However, when you expose this material to an electric field, the bound electrical charges of opposite signs will try to separate from each other, and the extent of the separation of the electrical charges within a material.

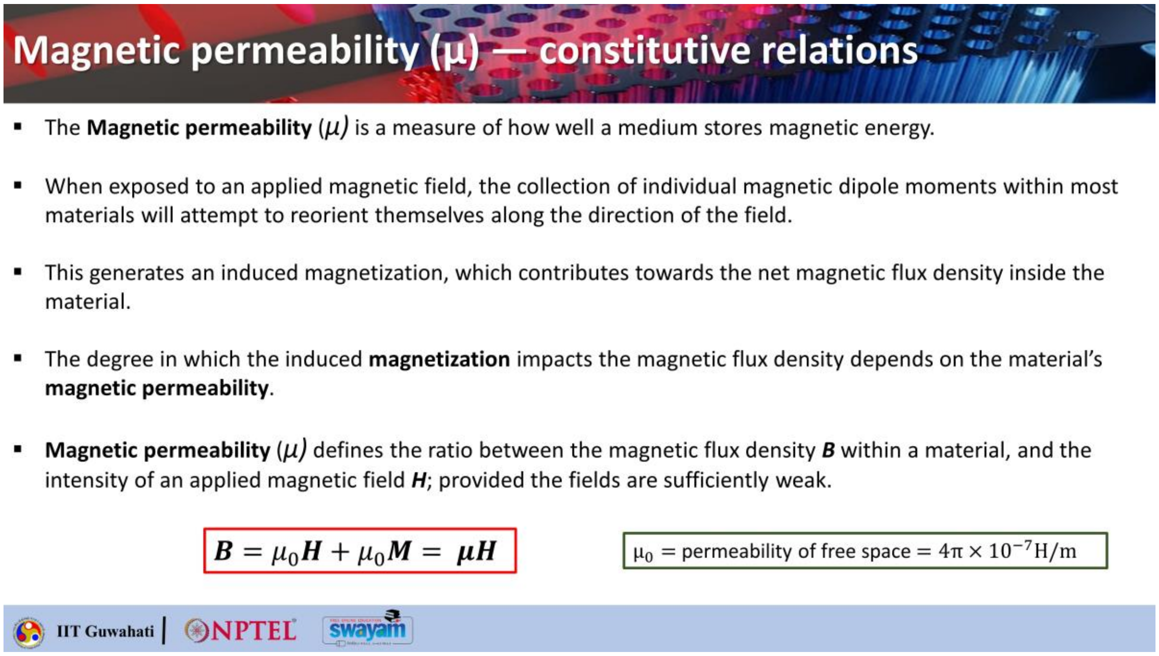
It is represented by the electric polarization. You can say that in this particular material, where there is no electric field present, the displacement is also 0 and the polarization is also 0. Here you can see that the polarization is basically $\varepsilon - \varepsilon_0$ into E. Therefore, there is an effective displacement in the presence of this electric field, and that is D equals εE. Okay. The electric field displacement and polarization are related by this particular expression.

You can write that D equals ε_0E + P, and you can simply express these two together as εE, okay.



Now, we will define the constitutive relations for a linear, homogeneous, and isotropic medium. Now, in linear media, the properties of the material do not depend on the strength of the field. It means that here the polarization P is linearly proportional to E. Therefore, you can simply write P equals $\epsilon_0\chi$E. So, what is chi χ? Chi is essentially a scalar constant known as the electric susceptibility.

Then we can write D equals ϵ_0E + P. So, you can replace P with $\epsilon_0\chi$E. You can take ε as $\varepsilon_0(1+\chi)$, and this $(1+\chi)$ is nothing but your relative permittivity ε_r. Therefore, I believe that ε_0 and ε_r together can be written as ε. So, that is how all these equations are related: D equals εE, or you can write $\varepsilon_0\varepsilon_r$E, or you can write $\varepsilon_0(1+\chi)$E, okay. So, this is how you can correlate the displacement field with vacuum permittivity, relative permittivity, or susceptibility.



Now, when you come to magnetic permeability (μ), okay. Inside a material medium, as we have seen, permittivity is determined by the electrical properties of the medium.

Permeability will be determined by the magnetic properties of the medium, right. So, magnetic permeability is simply a measure of how well a medium stores magnetic energy. So, when exposed to an applied magnetic field. The collection of individual magnetic dipole moments within most materials will attempt to reorient itself along the direction of the field. So, this will generate an induced magnetization, and it will contribute to the net magnetic flux density inside the material. So, the degree to which the induced magnetism or magnetization impacts the magnetic flux density depends on the material's magnetic permeability.

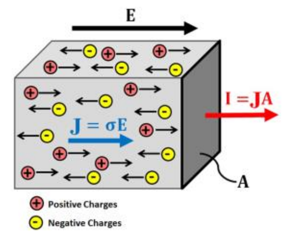
Magnetic permeability can be defined as the ratio of the magnetic flux density B within a material to the intensity of an applied magnetic field H. You can say that μ is nothing but a ratio of B and H. Therefore, μ equals B divided by H, provided that the fields are sufficiently weak. So, if you want to express B in terms of $\mu_0\boldsymbol{H}$, this is the vacuum permeability plus $\mu_0\boldsymbol{M}$, where M is the measure of the magnetization of the material, and these two together can be written as $\boldsymbol{\mu H}$, okay. So, μ_0 is basically the permeability of free space, and μ is the permeability of that particular medium.

Conductivity (σ) — constitutive relations

- The **conductivity** describes the degree to which a material conducts electricity.
- When an electric field is applied to a material, free charges within the material experience an electrical (**Coulomb**) force.
- This force causes the free charges to move through the material along the direction of the applied field (***i.e. electrical current***).
- *The ease at which electrical charges move through a material under the influence of an electric field depends on the **material's electrical conductivity**.*
- Electrical conductivity (σ) can be defined as the ratio between the current density (**J**) within a material and the electric field (**E**). This relationship is known as **Ohm's law** and is given by:

$$J = \sigma \boldsymbol{E},$$

$$\sigma = 1/\rho \quad [\Omega.\mathrm{m}]$$



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Source: https://em.geosci.xyz/content/physical_properties/electrical_conductivity/index.html

The next important parameter is conductivity, which describes the degree to which a material conducts electricity.

So, this is purely associated with the electric field, okay. So, when an electric field is applied to a material, free charges within the material will experience an electrical force, which is also known as the Coulomb force. This force will cause the free charges to move through the material along the direction of the applied field, and that is how you will get a. So, the electrons, which carry negative charges, will move in the opposite direction of the applied field, while the holes will move in the direction of the electric field. So, you are basically getting a current. So, the current will be in the direction of the flow of holes or in the opposite direction of the flow of electrons, right. The ease with which the electrical charges move through a material under the influence of an external electric field depends on the material's electrical conductivity.

You can say that when an electric field is applied, you can obtain a surface current density $J = \sigma \boldsymbol{E}$. Once you know J, if you take a cross-section A, multiplying J by A will give you a current I across this particular cross-section, right. Electrical conductivity can be defined as the ratio of the current density J within a material to the electric field E. This relationship is also known as Ohm's law.

So J equals σE. What is σ? Sigma is nothing but J divided by E. σ is also the reciprocal of resistivity. So, this is conductivity; this is resistivity. So, kind density is basically defined as electrical kind per unit cross-sectional area. In many cases, the electrical properties of a material are characterized by the electrical resistivity ρ.

So, ρ is nothing more than the reciprocal of electrical conductivity. So, you can see that the unit is basically an ohm meter.

Electromagnetic properties of Materials — Important parameters

- **Velocity of the EM waves:**

Compare:

$$\nabla^2 \mathbf{E} = \mu_0 \varepsilon_0 \varepsilon_r \frac{\partial^2 \mathbf{E}}{\partial t^2} \text{ and } \nabla^2 \mathbf{E} = \frac{1}{v^2}\frac{\partial^2 \mathbf{E}}{\partial t^2}$$

$$\Rightarrow v^2 = \frac{1}{\mu_0 \varepsilon_0}\frac{1}{\varepsilon_r} = \frac{c_0^2}{\varepsilon_r}$$

In Free Space (Vacuum):

$$\mu_0 = 4\pi \times 10^{-7} \text{ H/m}$$

$$\varepsilon_0 = 8.854 \times 10^{-12} \text{ F/m}$$

$$c_0 = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 299{,}795{,}638 \text{ m/s}$$

- **Refractive index of a material:**

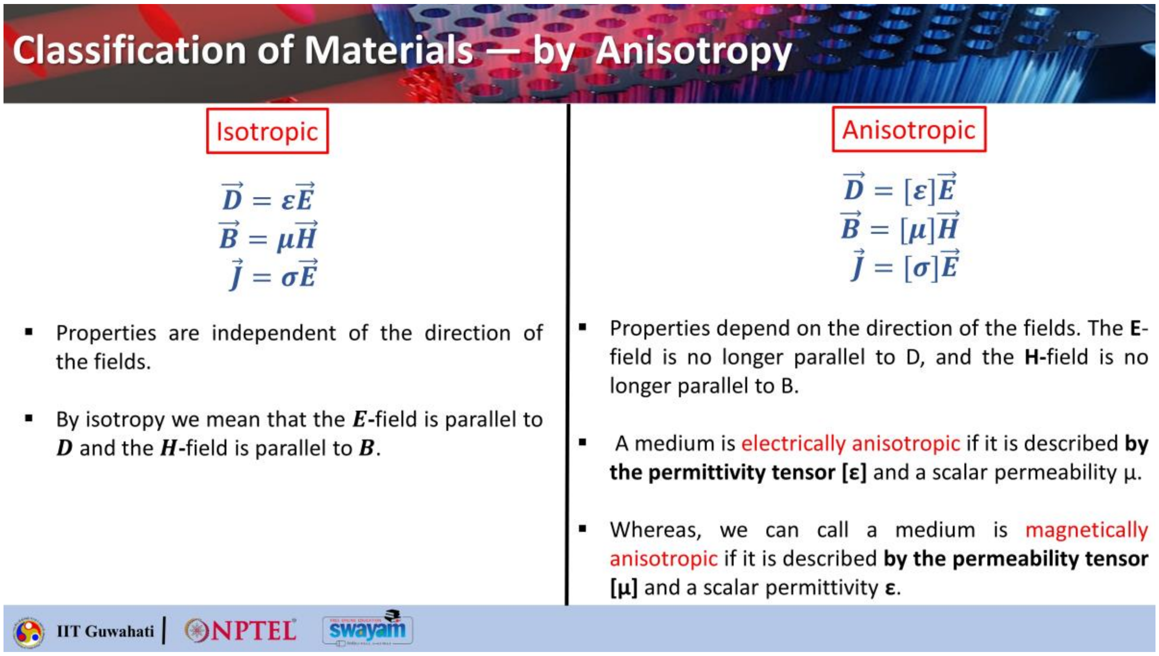
Refractive index is a material property that describes how the material affects the speed of light travelling through it

$$n = \frac{c}{v} = \sqrt{\varepsilon_r} = \sqrt{1 + \chi}$$

Source: https://em.geosci.xyz/content/physical_properties/dielectric_permittivity/index.html

So, how do you obtain the velocity that you have already discussed? So, when you compare the two wave equation forms, you can see that this particular quantity will be equated to this one. You can simply $v^2 = \frac{1}{\mu_0 \varepsilon_0}\frac{1}{\varepsilon_r}$. Essentially, that is c_0^2; c_0 is nothing but the speed of light in a vacuum, and ε_r is the relative permittivity. Now, as I mentioned earlier, there is also a relationship with the refractive index.

So, if you take the square root of this particular equation, you will get v equals c_0 divided by the square root of ε_r, which is nothing but your n in a medium that is non-absorbing. So, you can say that n is nothing but the ratio of c to v, which is the square root of ε_r, and this can also be written as the square root of $1 + \chi$. So, if you know the electrical susceptibility of a material, you can also find out what the refractive index is, and that basically tells you the speed of light in that particular medium.



So, now let us classify the materials by the different electromagnetic properties that we have already seen

So, let us see how we classify materials by anisotropy. Now, when I say anisotropy, I first need to understand what isotropic means. So when I say isotropic, I mean that the properties are independent of the direction of the field. Something like this: D equals $\boldsymbol{\varepsilon}$E, B equals $\boldsymbol{\mu}$H, and J equals $\boldsymbol{\sigma}$E. All these properties, $\boldsymbol{\varepsilon}$, $\boldsymbol{\mu}$, and $\boldsymbol{\sigma}$, are independent of the direction of the field. So, by isotropy, we can say that the E field is basically proportional to the D field, and the H field is proportional to the B field, and so on.

But when you introduce anisotropy, it means the properties now depend on the direction of the field. It means the E field is no longer parallel to the D field, and the H field is no longer parallel to the B field. Therefore, you can actually represent these parameters as tensors. So, a medium is electrically anisotropic if it is described by a permittivity tensor, which is given by this, and it has a scalar permeability. We can call a medium magnetically anisotropic if its permeability is primarily described by a tensor and it has a scalar dielectric permittivity.

Classification of Materials — by Anisotropy

Anisotropic materials

- For anisotropic media, the constitutive relations are usually written as

$$\bar{D} = \bar{\bar{\varepsilon}}.\bar{E} \quad \text{where } \bar{\bar{\varepsilon}} = \text{permittivity tensor}$$

$$\bar{B} = \bar{\bar{\mu}}.\bar{H} \quad \text{where } \bar{\bar{\mu}} = \text{permeability tensor}$$

- Properties are independent of the direction of the fields. Crystals are described in general by symmetric permittivity tensors.
- There always exists a coordinate transformation that transforms a symmetric matrix into a diagonal matrix. In this coordinate system, called the principal system,

$$\bar{\bar{\varepsilon}} = \begin{bmatrix} \varepsilon_x & 0 & 0 \\ 0 & \varepsilon_y & 0 \\ 0 & 0 & \varepsilon_z \end{bmatrix}$$

So, for an anisotropic medium, the constitutive relationships can be written in a general form similar to D equals $\bar{\varepsilon}$, which is another representation of the permeability tensor, and then you have the electric field, right? Similarly, B equals the permeability tensor times the magnetic field. So, these properties are basically independent of the direction of the fields. The crystals are generally described by symmetric permittivity tensors. There always exists a coordinate transformation that would transform that symmetric matrix into a diagonal matrix. In this coordinate system called the principal system, you will see that the tensor basically looks like this, with only the diagonal elements: ε_x, ε_γ, and ε_z.

These represent the permittivities along the x, y, and z directions, respectively. Now let us assume that the principal axis of the crystal looks like this; the permittivity is given as follows. Now, if you take a cubic crystal where x, y, and z are all the same, it becomes isotropic. But if you consider tetragonal, hexagonal, or rhombohedral crystals, two of these parameters will be equal. So, these kinds of crystals are basically called uniaxial crystals.

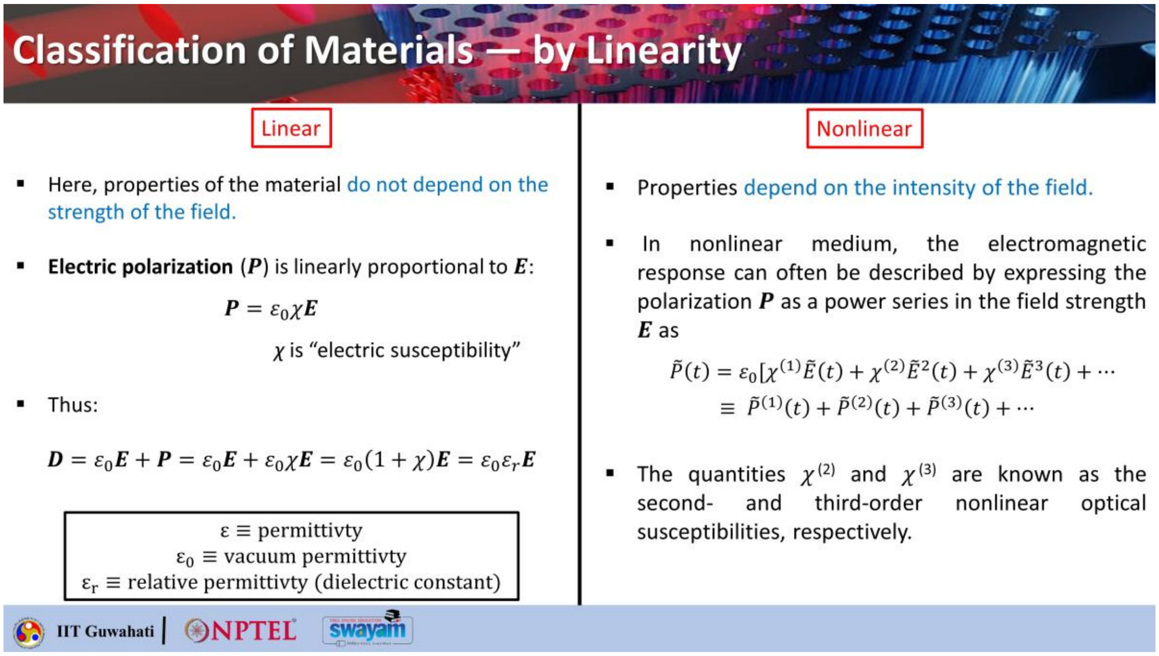
Classification of Materials — by Anisotropy

Anisotropic materials

- Principal axes of the crystal: $\bar{\bar{\varepsilon}} = \begin{bmatrix} \varepsilon_x & 0 & 0 \\ 0 & \varepsilon_y & 0 \\ 0 & 0 & \varepsilon_z \end{bmatrix}$
- For cubic crystals, $x = y = z$ and they are *isotropic*.
- However, in *tetragonal, hexagonal, and rhombohedral crystals*, two of the three parameters are equal. **Such crystals are uniaxial**.
- For a uniaxial crystal, the **permittivity tensor** can be written as: $\bar{\bar{\varepsilon}} = \begin{bmatrix} \varepsilon & 0 & 0 \\ 0 & \varepsilon & 0 \\ 0 & 0 & \varepsilon_z \end{bmatrix}$
- Here, the z axis is the optic axis. The crystal is:
  - positive uniaxial if $\varepsilon_z > \varepsilon$;
  - negative uniaxial if $\varepsilon_z < \varepsilon$.
- **Bi-axial:** In *orthorhombic, monoclinic, and triclinic crystals*, all three crystallographic axes are unequal.
- We have $\varepsilon_x \neq \varepsilon_y \neq \varepsilon_z$, and the medium is **biaxial.**

So, maybe epsilon x and epsilon y are equal, while epsilon z is different. So, in that case, you can simply write epsilon in two instances, and epsilon z will be written differently. So, this is how the permeability sensor of a uniaxial crystal will look. Now, what happens to the permittivity along the z direction if it is greater than the permittivity of this tube? We can say that this is a positive uniaxial crystal. In the other case where epsilon z is smaller than epsilon, we can refer to it as a negative uniaxial crystal. Now there are crystals that are also biaxial; these are basically orthorhombic, monoclinic, and triclinic crystals where all ε_x, ε_y, and ε_z are unequal.

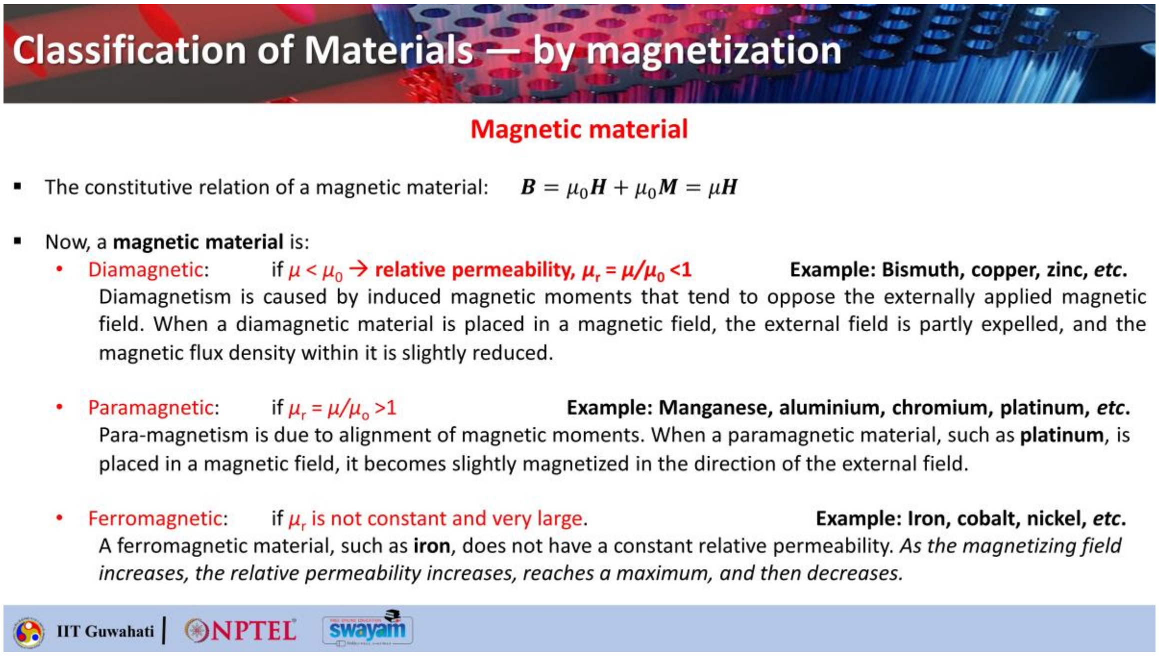
So, when all of them are unequal, the medium is biaxial, which means that every direction will have a different dielectric permittivity.



So, now let us see how the materials can be classified as linear and nonlinear media. So, when you say "linear medium" here, the property of the material does not depend on the strength of the field, okay. The electric polarization P is basically linearly proportional to the electric field E. Therefore, you can simply write P equals $\varepsilon_0\chi$E, where χ is the electric susceptibility.

Thus, you can write D equals ε_0E + P. Since P can be replaced by $\varepsilon_0\chi$E, you can factor this out, and this is what we have seen until now, right? Now, if you go to a non-linear medium, the properties basically depend on the intensity of the field. In a non-linear medium, the electromagnetic response can often be described by expressing the polarization P as a power series in the field strength E. Thus, you can write the polarization as $\varepsilon_0[\chi^{(1)}\tilde{E}(t) + \chi^{(2)}\tilde{E}^2(t) + \chi^{(3)}\tilde{E}^3(t) + \cdots$, and so on. So, you are actually seeing that you are getting higher-order non-linear terms, like this, because of the higher field intensity or strength. This quantity is χ^2 and χ^3; they are known as second and third order non-linear optical susceptibilities, right.

Now, that was about the classification of materials by their electric field properties.



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Let us now classify materials by their magnetization properties. So, we are basically talking about magnetic materials. The constitutive relationship is nothing but $\boldsymbol{B} = \mu_0\boldsymbol{H} + \mu_0\boldsymbol{M} = \mu\boldsymbol{H}$ which can be expressed as $\boldsymbol{B} = \mu\boldsymbol{H}$. Now, a magnetic material is basically diamagnetic if μ is smaller than the vacuum permeability, μ_0. That means the relative permittivity μ_r, which is basically the ratio of μ over μ_0, is less than 1.

So, some examples of this kind of material are bismuth, copper, zinc, etc. So, diamagnetism is essentially caused by induced magnetic moments that tend to oppose the externally applied magnetic field. So, when a diamagnetic material is placed in a magnetic field, the external field is partially expelled, and the magnetic flux density within it is slightly reduced. Another type is called paramagnetic. If your μ_r, which is the ratio of μ and μ_0, is greater than 1, we call it paramagnetic.

Examples include manganese, aluminum, chromium, platinum, etc. So, paramagnetism is due to the alignment of magnetic moments. So, when a paramagnetic material such as platinum is placed in a magnetic field, it becomes slightly magnetized in the direction of the external field. The third category is called ferromagnetic. So, here, if you know that relative permeability is not constant, it is very large.

An example of this kind of material is iron, cobalt, and nickel. So, in a ferromagnetic material such as iron, it does not have a constant relative permeability. As the magnetizing field increases, the relative permeability also increases, reaching a maximum before decreasing. So, these are the three different types of materials that can be classified based on their magnetization properties. So, purified iron and many magnetic alloys have maximum relative permeabilities of 100,000 or more based on this.

Classification of Materials — by Conductivity

- On the basis of the relative values of electrical conductivity (σ) or resistivity ($\rho = 1/\sigma$), the solids are broadly classified as:
  - **Metals:** They possess very low resistivity (or high conductivity or $\sigma >> 1$).
  - **Semiconductors:** They have conductivity intermediate to metals and insulators.
  - **Insulators:** They have high resistivity (or low conductivity or $\sigma << 1$).

Source: https://www.esaral.com/classification-of-solids-in-terms-of-the-forbidden-energy-gap/

Now, let us classify materials based on their conductivity. Now, based on the relative values of electrical conductivity (σ) or resistivity (ρ), which is essentially the reciprocal of sigma, solids can be broadly classified as metals, where they possess very low resistivity or, conversely, very high conductivity. So, sigma is much larger than 1. Then you have semiconductors, which have conductivity that is intermediate between metals and insulators, meaning they possess qualities that are in between those of metals and insulators. So, insulators are basically materials that have very high resistivity, or you can say very low conductivity, meaning that sigma is much less than 1 in the case of insulators. Here is a table that actually describes the different properties of conductors, semiconductors, and insulators.

Classification of Materials — by Conductivity

Properties	Conductor	Semiconductor	Insulator
Resistivity	$10^{-6} - 10^{-8}\ \Omega/m$	$10^{-4} - 0.5\ \Omega/m$	$10^{7} - 10^{16}\ \Omega/m$
Conductivity	$10^{6} - 10^{8}$ mho/m	$10^{4} - 0.5$ mho/m	$10^{-7} - 10^{-16}$ mho/m
Temp. Coeff. Of Resistance (α)	Positive	Negative	Negative
Current	Due to free electrons	Due to electrons and holes	No current
Forbidden Energy Gap	$\cong 0$ eV	$\cong 0 - 1$ eV	≥ 6 eV
Examples	Pt, Al, Cu, Ag	Ge, Si, C, GaAs, $GasF_2$	Wood, Plastic, Diamond, Mica

Source: https://www.esaral.com/classification-of-solids-in-terms-of-the-forbidden-energy-gap/

The resistivity of a conductor is in the range of 10 to the power of minus 6 to 10 to the power of minus 8 ohm-meters. For semiconductors, it is from 10 to the power of -4 to 0.5, and for insulators, it is very, very high, ranging from 10 to the power of 7 to 10 to the power of 16. So, conductivity will be the inverse of it, and you can see that the unit is more per meter. The temperature coefficient of resistance for a conductor is positive, which means that as the temperature increases, the resistance also increases; for a semiconductor, it is negative.

So, it is the other way for insulators as well. In conductors, the current is mainly due to free electrons; in semiconductors, it is due to both electrons and holes. Insulators do not support any current. Conductors do not have any energy gap. Semiconductors have an energy gap ranging from 0 to 1 electron volt, or sometimes a little more.

And then, insulators have more than 6 electron volts. Examples of conductors include platinum, aluminum, copper, and silver. Semiconductors, as you all know, include germanium, silicon, gallium arsenide, and so on, okay? Insulators include materials such as wood, plastic, and others. So, with that, we come to the end of this lecture. We will start the discussion on electromagnetism as an eigenvalue problem in the next lecture.

So, if you have any doubts or queries regarding this lecture or any of the previous lectures, you can send your questions to this email address, mentioning "MOOC" and "photonic crystal" in the subject line.