Electromagnetic Theory
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Lecture - 10
Electrical Potential -I

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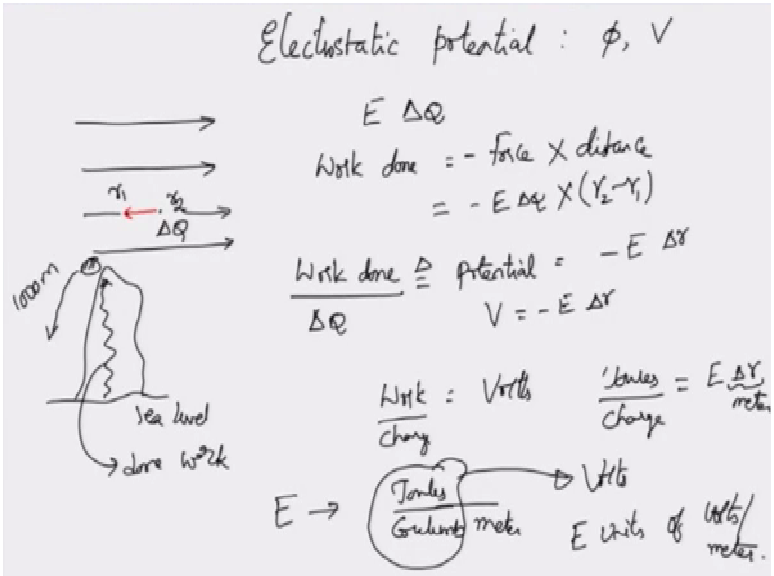

So let me introduce you to this concept. So I am looking at new concept called Potential. Well, since we are still dealing with electrostatic cases I am looking at electrostatic potentials. This electrostatic potential is sometimes denoted by phi or sometimes denoted by v. I will use v for now, but sometime later I might switch to phi. It does not matter which one you use, both terms are equally used in literature.

So if I want to define electrostatic potential let me first talk about the point charge. So back to the point charge over here, we know how to calculate its electric field, so there is an electric field out there, okay. Now I choose a particularly interesting electric field pattern, right. Let say I do not want to choose the point charge I choose a particularly interesting electric field which I somehow have generated in which the electric field is pointing horizontally.

And it is all uniform along this particular direction. Such an electric field we have already discussed how to obtain, you can have a long line of charge and this is essentially the electric filed that it would be producing except that its magnitude would be decreasing but here I am assuming that the magnitude is same, okay. Now, consider two points here consider a point r2 consider point r1 here. Now imagine that I am taking a particular test charge delta Q, okay.

I am taking a test charge delta Q and I am going to move this test charge from r2 to r1 okay. I am going to move the test charge along this red path from r2 to r1 remember that my electric field is uniform which means its magnitude is not changing and its direction is also always along the horizontal to the right, okay. So I am moving from r2 to r1.

Now I know that if I do not move it, right if I just keep the test charge at the position r2 since the electric field is directed to the right the charge would have simply accelerated to the right because if I have a test charge delta Q there will be force acting on this and the force will be E times delta Q, correct? The force acting on the charge delta Q is E times delta Q and if I have to move the charge from one point to another point.

What I would have to do is I have to perform some work. Which way I am performing the work? I am actually moving this charge from against the field right? From point r2 to r1 against the field, field is pointing to the right but I am trying to move to the left. So I have to expand some energy I have to perform some work in order to move this point. How much work should I perform?

Work done must be equal to the force that I apply times the distance I move. So this is again from the mechanical concept of moving a block. So I have block I move it a distance of D and I apply a force F then work done will be force * distance. I will add negative sign here just to indicate that this work is done against the field. Since I am doing the work because I have to move this charge from r2 to r1 I am doing work against the field.

This is a work that is being done against the field. Work done is force * distance. What is the force? Force is E Delta Q which is the test charge times the electric field at that particular point, times the distance. So let's say I am moved a distance of r2-r1 over here okay along this path I moved the distance of r2-r1. If I now define how much work, I need to perform in order to move one-unit charge.

I define this as the potential. That potential will be equal to -E time's delta r, where delta r is the length along which I have moved or the distance I have moved, okay. And this potential can be represented as V. So V = -E delta r or sometimes you can see that the same thing written has –E into delta x because this is a x and y plane you can either do that one. Okay I am going to use E - delta r for now, okay. So this is the potential.

Now what significance is the potential? You remember from your mechanics courses that potential-- for example you have an object which is lying at sea level and we take the sea level as the origin and there is a mountain on the sea you know like adjacent to the sea okay not on the sea of course there will be adjacent to the sea there will be a mountain. Suppose there is an object up here and from that height the object falls down.

Let us say this mountain is about 100 or 1000 meters high from the sea level. So when this object drops down all of the energy that this object possessed by virtue of it being top on the mountain would actually be converted into Kinetic energy as it drops down, right? However, how did this object get up there? How did the object manage to get itself up to the mountain? From one hect to take the object from the sea level up to the mountain.

So someone must have done work and whatever that work that someone has done gets stored as a potential energy or we say that this particular on the mountain top the potential is say 1000, it means that there must be 1000 units of potential energy that would be that one can harness to convert into kinetic energy when you drop the object. Okay, very similar to this one.

Suppose if you take the charge and put it at the point r1 so you move the charge delta Q from point r2 to r1. If you now just leave the charge, if you see just stop, the charge will move on its own because the force is pushing on this one, the charge will move and it will get accelerated, there will be a kinetic energy. However, that kinetic energy when it goes from r1 to r2 will be the kinetic energy-- sorry will be the energy that it has expanded from the potential energy that it has gained by someone moving the object, okay.

So if someone moves against the field we are imparting the energy to the object and in the object can actually convert that energy potential energy into kinetic energy. Okay. So this is the potential, potential is -E into delta r note that the fact that this potential is work done against the field is indicated by a – sign, okay. So this is important we will later see why it is important, okay.

Now here is where that Newton per coulomb and work per meter thing comes from. You have work done per delta Q, what would you think as the units of work -- work will be force, force is work done is Newton's times meter, right. Newton times meter is the work done. Okay. Work done per charge will be Newton per coulomb into meter. Okay. But we have also seen that for the potential, right this is electric field for the potential.

So we have seen that work done per charge is the potential. In SI unit's potential is measured in terms of volts. So work done per charge is potential which is measured in volts and work itself is measured in terms joules, right. So I have the potential this is volts, work is joules per charge but work done was also equal to E times delta r, okay. E is the unit for which we are looking for.

Delta r is the distance that I am measuring so this must be equal to meters. So the units for E might turn out to be joules per coulomb okay charge is Coulomb per meter. Okay. However, we have also seen that potential itself is nothing than work done per charge. So we have seen that the units of electric field will now be written as joules per coulomb into meter. But we have also seen that potential is defined as work done per coulomb, right.

Electrostatic potential is work done per coulomb; work done per Coulomb is nothing but Joules per Coulomb. So I can replace this joule per coulomb into volts that is giving me electric field units of E measured in units of volts per meter. Okay. So this is where the units for a electric field comes from volts per meter.

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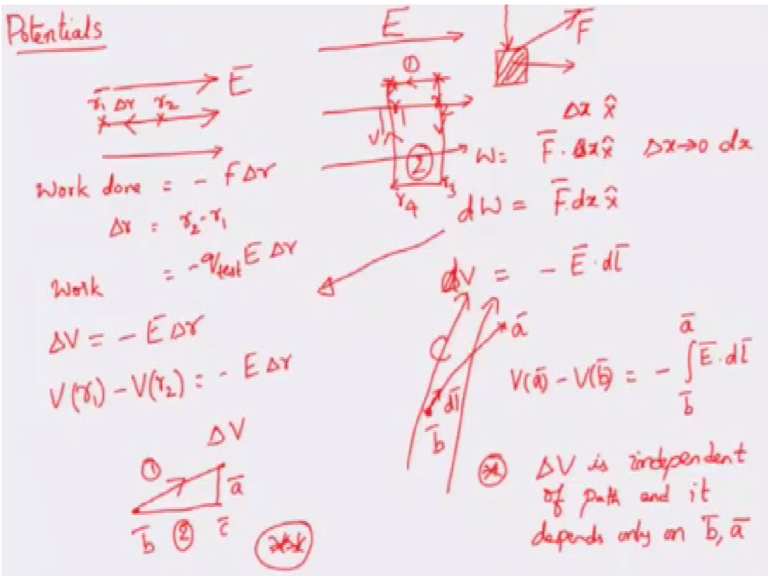

If you recall, we had a uniform electric field in a given region of space. This electric field over a small distance at least we can consider it to be constant. Let the electric field will be directed along the horizontal to the right side, okay. And then I pick two points over here called this points as r2 and r1 if you would like and then I will define or I will find out what is the amount of work that is required to move a charge particle from or a test charge from point r2 to point r1.

Please note that this movement is against the electric field, we will take this against the electric field into account when we write down the expression for the work done on the charge. So work done on the charge is the force that we are applying times the distance that we are moving. So if I call the distance as delta r and delta r from this figure can be easily seen to be r2-r1.

So when I move a distance of delta r along this particular path against the field I will be performing some work. But I know that the force on the test charge is also given by the electric field at that particular point and we are assuming the electric field is constant as we move along from r2 to r1, okay. So what we actually get is not exactly, I mean well though this is work done.

If we divide this work done from the test charge that is to say if you calculate what is the work per unit charge or work per test charge that will give me the potential. Although, here we have shown that this is the work done one should actually think of this as differential amount of work done that is the extra amount of work that we need to do in order to move the charge from r2 to r1, assuming that the charge is already moved from some point to r2.

So in essence, what we are getting from this equation is not the absolute potential but the potential difference between r2 to r1. So with reference to some 0 potential this quantity that we have just written down is actually the potential difference between the points r2 and r1. So you have delta V is equal to - E delta r, because I assume that direction of movement is along a certain r direction. And what is the potential difference here?

It is the potential difference between the points r1 and r2 which is equal to minus E delta r. Now there is no rule or there is no requirement that I should always move along the line that is parallel to the electric field. Okay, sometimes I might choose a path in which the electric field would be making an angle with respect to the path along which I am moving charge. In that case how does this definition change?

Well you know this already from mechanics that if I take a particular object and I apply a force, the force will move the object and if the force in line with the displacement of the object then the force experience by the object will also be maximum. However, if I now pull or push this object, if I pull this object for example in this direction not all of the magnitude of the force is experience by the object.

How much force is actually experience by the object or how much work that is done by the object is given by the dot product between the two vector. So if I call this delta x as a vector or I can call this as a delta x into x hat has the vector element or the direction along which I am moving then the amount of work that is done is actually the dot product between the two vectors F and the displacement vector.

So I know that the amount of work done is F dot delta x and in case we let delta x go to 0 in the limiting process delta x go to 0 this becomes dx so the work done or the differential amount of work done will actually be equal to Fdx, sorry F. there is an x hat here dx x hat, okay. So this is the amount that the work done. Now with the same analogy over here with respect to the electric field.

We can now write down saying that the voltage difference or the differential amount of the voltage between the two points if I move a distance along dx is actually given by –E dot dL, okay. What is this dL? If you remember the coordinate system that we talked about dL was the live element. It is actually the vector in which, which gives us direction of movement. So if I move in an arbitrary path then that arbitrary movement can be characterized by a line segment which is a vector line segment which is dL. Okay.

So now if pick two points b and a let us say I am moving the charge from point b in the space to another point a in the space okay. So I am picking up particular path over here. The path, the total potential difference between the two points or the potential difference between the two points a and b will be obtained by integrating this expression dv is equal to –E dot dL over this entire path.

So if the path is given a name called C then the potential difference between the two paths namely V at a minus V at b this potential difference will be integral from b to a E dot dL. What is E and what is dL, dL is the vector element line element here and then electric field is the electric field of the region of space in which we are moving the charge, okay. So this expression is very crucial for you to note.

The potential difference between the two points is given by the line integral of the electric field which is a vector quantity along the particular direction. So note that both the electric field as well as the line segment both are vectors, so you need to specify the direction of path as well as you need to specify the corresponding values of the electric field at each point in the path that you are taking.

Now there are certain very easy consequences that we can find out. One of the things that has happened over here is that the left hand side is actually a scalar. And for the electrostatic field that we are considering this line integral or the path integral is completely independent of the path that you have chosen and it depends only on the end points of the path. Okay. For example, if I take one path from point b to point a, say I go straight up along this path then the potential difference will be some delta V this delta V is V of a – V of b.

So this is some potential that you are going to get. If you follow the path number one which is a straight line path from b to a. Suppose you do not go straight from b to a. Let’s say you go from b to some other point c and from c you head to point a. Of course you are still going to the same final point but you are now taking the different path. So now let’s call this as path two.

It can be shown that for the electrostatic field that we are considering this line integral or the potential difference will not be a function of the path that I have chosen. All paths with the same initial and final points will lead to the same potential difference. Okay. So delta V is independent of the path. So this is one very important observation, let us mark it down delta V is independent of the path. And it depends only on the endpoints. Okay.

You could choose any path and this will depend only on the endpoint. Now a possible explanation for this particular observation that we have made can be obtained by looking at the point charge fields. Okay. So these are the point charge fields that I have. Now imagine that I am looking at moving from along this particular path from this point let's call this as r2 and this point ad r1. I take this direct path, okay.

Here the electric field is along the direction of along the radial direction which means that along this path one the electric field is opposite to the direction of the line segment but essentially they are parallel to each other right, so parallel but in the opposite direction. So there is some amount of potential rise or potential difference that would happen as you move from r2 to r1. Okay.

So if you take path one you are moving in parallel to the electric field because electric field is also directed in the same line but it is in the opposite direction except. So when you move from r2 to r1 against the field there will be some amount of potential difference. Now this is the x, y plane. Because I am looking at particular plane these are the fields in this particular plane.

We will talk about other different coordinate system in which the electric field are properly given for the point charge later. Okay. Now suppose I take different path. Let us say I go from r2 to r3 perpendicular to this line. Okay. And then I go straight up from r3 to r1. So I go from r3 perpendicular to this r3 actually let us not do it this way let us go again from r3 to r4 and then go back from r4 to r1.

So this long widened path that I have taken from r2 to r3, r3 to r4 and r4 to r1 and please note that the direction is also is very important, right. So I cannot say r2 to r3 and then r4 to r3 it does not make sense. So these paths whatever that we are drawing are directional path that is there is direction associated with the curve. Okay. So we will come to the direction sometime later. Now you have r2 to r3 movement.

Now what is the direction of the electric field, assume that you are moving in this way what will be the electric field? That would be sorry—one second let us not consider the point charge, let us consider the uniform electric field that we are considering, sorry about this for point charge the path that I have chosen is not exactly correct so we will come back to that point charge in a little bit of time okay. Let us go back to the uniform electrical field itself.

So I have the uniform electric field going along horizontal to the right as we have considered so far. Now on this electric field I am moving from r2 to r1 by taking a path from r2 to r3, r4 to r1 okay. So I move from r2 to r3 and r3 to r4 and r4 to r1. While I move from r2 to r3 the electrical field is always perpendicular to the direction of my movement, right. So I am moving vertically downwards from r2 to r3.

But the electric field is directed to the horizontal and to the right. So if you again remember the force and the object analogy if I apply a force perpendicular to the object the object does not move. No work is being done energy is expanded but no work is being done, okay because the object is not moving. So with the same analogy over here I can move the point charge perpendicular to the electric field and I will be doing no work. Okay.

Now once I moved from r2 to r3 without any work I am now going to move from r3 to r4 when I move from r3 to r4 I will be performing some work against the field and this amount of the work will be exactly equal to the work that we have obtained if you have taken from r2 to r1. Because this is parallel to the direction, it has the same length here and the electric field is the same value from this path r3 to r4 and r2 to r1.

So because of this the work done or the potential difference that I have accumulated or the potential rise that we have seen will be the same as r2 to r1. Again when I move from r4 to r1 there is no work done because the object is moving or the charge is moving perpendicular to the electric field. Okay. So this is a possible explanation for this one you can show mathematically that this statement actually holds for electrostatic configuration. Okay.

So electrostatic fields have this property that the potential difference is independent of the path and the potential difference depends only on the initial and final points, okay. Now here is a very important observation that we can immediately make. This is so important that I am going to put two stars to this observation. Previous was one-star observation now will make a two-star observation. What is that?

The observation is that you go from r2 to r3 you get no work, r3 to r4 you did some work against the field, please note against the field. Now you go from r4 to r1 you did no work but what happens if you go from r1 to r2? This is like if you take an object or a man and then carry this object or a man up the hill and then let the man go when you have done work from carrying the man from sea level to the mountain top.

You have done work against the earth gravitational field, so earth gravitational field is actually trying to pull the man but you have taken the man all the way up to the mountain top, okay. So you have expanded that work and that work is the work that we have performed on the charge where we moved from r3 to r4. Okay. You could move on a horizontal scale but then the potential energy would not charge.

So if on the mountain top if you can move horizontally the potential with respect to the sea level would not change. However, now if you imagine that if you take the person to the edge of the mountain and then just drop that particular person, what would happen all of the potential energy that we have given to the person so that the person can drive from sea level to the mountain top will now be converted or will now be given back in the form of kinetic energy.

The man gets accelerated and then losses all the potential energy gets converted to kinetic energy. So if I now ask in the same vain if I move from r3 to r4 I did some work against the field so there is some amount of potential energy sitting in the charge now okay. Now, when the charge moves from r1 to r2 that movement is aided by the field and not against the field.

So if you assign a sign of minus for the work done from r3 to r4 sorry plus for the work done against the field from r3 to r4 movement you should get the same amount of work, you should recover that potential energy when you move from r1 to r2, okay. So the work along r3 to r4 will be exactly equal to the negative of the work from r1 to r2. So once we have done this complete closed path movement we are back to the same point right.

So we started from r2 we went all the way around town and we came back to the point r2. So in this particular case, when we came back we observe that the total work done must be equal to 0. What would it imply in terms of the potential? Well it would simply imply that the potential with respect to that same point the difference will be equal to 0. But crucially there is right hand side sitting in the expressions for potential difference.

And that line integral will go to 0 now. So what we are showing is there if you take a closed path of the line integral so you have E.dL along the closed path this is equal to 0. This is very important and any field which would satisfy this criteria is called a conservative field or a lamellar field, a conservative would use because lamellar and everything would be two fluid mechanics for me to use.

So we have conservative fields which have this property that the closed path line integral would always be equal to 0. Okay. So what is the takeaway from all these? We can define potential through the work done per test charge in moving along. You can choose any path to move but the work done per charge which is the potential or the potential difference with so to speak will be independent of the chosen path.

But it will be dependent only on the end-- initial and the endpoint or the final points of the path. Okay. And the third and very important observation is that if you move in a closed path you start from a point you move around and you come back to the same point the potential difference will be 0 but in terms of the line integral that closed line integral will actually be equal to 0. Any such field that would satisfy this criteria is called a conservative field.

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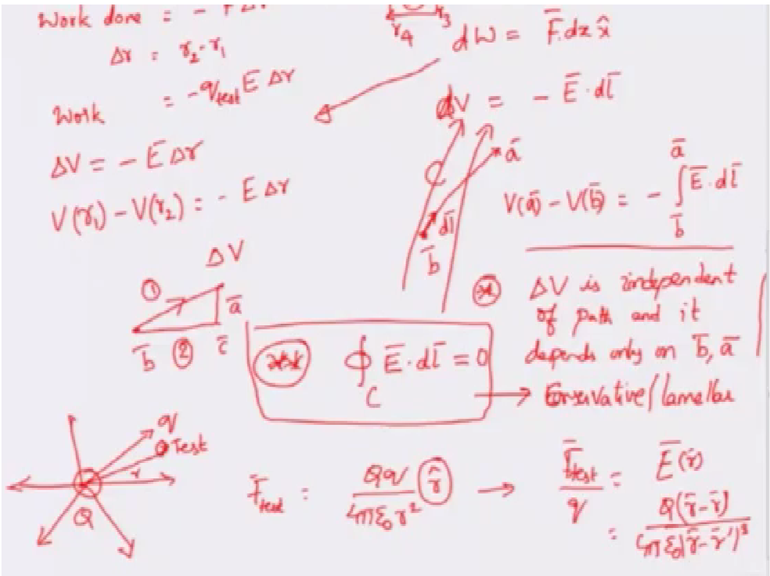

With this now let us go to the point charges that we left of slightly earlier, okay. Have a point charge. So there is a point charge Q here and the electric field for the point charge that we have not actually calculated the electric field; we have calculated electric field for a line charge but we did not calculate the electric field for a point charge. But we know how to calculate the electric field for a point charge.

Because if I place any test charge over here this is my test charge and this is the charge that is whose field I am calculating then the force on the test charge will be equal to Q and let us call this as small Qq by 4 pi epsilon 0 and the charge is kept in air or free space r2 which is the length or the separation between the source charge and the test charge and it will be in the direction that joints the two lines or joints the two charges.

The charge Q and the charge small q – okay so this is where the charge Q is located okay. So this is the force is actually directed along the line that joints the two charges. Now this is from Coulombs law, from Coulombs law you just simply find out what is the force per test charge by dividing it by Q on both sides and remembering all the things that we talked about how Q should be small Q should be large find all those things.

You will find that the electric field at any point in space is given by because of the point charge is given by Q by 4 pi epsilon 0 let us go back to the vector notation of r - r and r and r vector and then have r - r prime. Okay. What is the direction r that we have look here or this direction r. Is the same as the cylindrical direction r? Actually no. For charges which are spherical or you know which are in the form of a sphere.

We have a different coordinate system called a spherical coordinate system which allows us to work which spherical charge distribution much more easily than other coordinate systems such as Cartesian and Cylindrical. So this r that we have written here in the point charge electric field is actually along the direction of the radius in the spherical coordinate system. Now it might be confusing.

Because we are using the same letters and again and again for different coordinate system but that is this kind of the nature for this particular course so do not be alarmed, spend a little bit of time in understanding what these different notations are and you will start following them very easily.