Electromagnetic Theory
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Lecture - 18
Conductor and Dielectric - I

This class is not so much about giving you any new ideas of electrostatics, but rather to use whatever that we have learnt in the last few classes for some application. You will begin before we can start with application. We will begin by discussing a little bit about the nature of matter, a particular type of matter that we are going to consider is called dielectric, a common word for dielectric is an insulator.

The word insulator suits this material because at room temperature there are no free electrons available in this insulator, so that there is no easy way of enforcing a current through this or making the current flow through this material, insulators and that is why they are insulators. In contrast to insulators, we have conductors, conductors have a lot of free electrons that is their valence shells or no often atom are not really filled.

And therefore there is lot of free electrons available, in fact the energy band of conduction and valence band actually sort of overlap with each other with the net result that even a small amount of push by (()) (01:23) electric field is sufficient for the electrons to travel to constitute a current okay. So that is why metals are very good conductors of electricity. Then there is a class of materials, which actually fall between insulators and metals, which are called as semiconductor.

However, we will not discuss semiconductors in this course; it will be very difficult for us to go into the operational point of view of semiconductors. So we will first discuss dielectrics, and then in the next class, we will indicate briefly something about conductors okay. We do not have to discuss conductors also in too much detail. There is not much to discuss ideal conductors, but dielectric is very important.

Why it is important, we will later study some of the devices known as capacitors, inductors and transmission lines. These capacitors, inductors or transmission lines, the value of the capacitance or inductance or the characteristic impedance in case of a transmission line depends on the material that it is made up of okay. So you can actually enhance the capacitance by filling in the material of the capacitor by a dielectric.

So if you do not feel anything, then there is essential air between the two plates of the capacitor. However, if you insert dielectric medium in between that then you can actually increase the amount of capacitance okay. Capacitances also show up in several other places for example, in an integrated circuit that you develop, there will be a capacitance whenever there is any two dissimilar charge configurations okay.

And the value of the capacitance again depends on the insulating layers between the two charge distributions. So we will look at dielectric in slightly more detail than what is usually done. We will introduce quantity called polarisation okay and we will show how the flux density D is related to the polarisation P and then this entire thing is related to the electric field okay. So we did not actually have to use much of the D field in the last class or in the last few classes.

We said D flux density is equal to constant epsilon zero times E where epsilon zero was the permittivity of free space. Now we will assume that we are actually dealing with dielectrics and we will see that epsilon zero has to be replaced by a different quantity altogether okay. To begin with dielectrics, let us so we will not be able to deal with dielectrics again in too much of details, but we will use some simplified assumptions that actually helps us understand qualitatively how a dielectric would behave okay.

A dielectric for example is this one okay, this is a piece of material which under room temperature does not really conduct electricity. So this is a dielectric. So if you actually were able to look inside this piece of material or it could be this piece of material or it could be just this particular paper okay.

You would actually see that this is made up of you know molecules or atoms right. Imagine that there is only one kind of atom and what does an atom consist of, an atom consists of a central nucleus which is positively charged and a surrounding electron shells right. We will consider a simplified model of such an atom.

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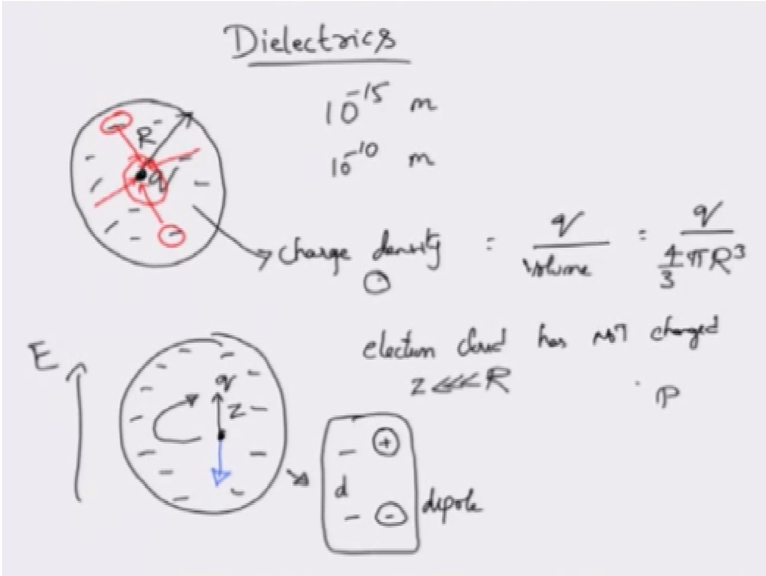

We will assume that atom consists of an electron cloud okay. There is a charge in between, which is the nucleus; however, the extent of the nucleus is actually quite small because in nucleus the diameter of that one is around 10 to the power minus 15 metres okay whereas the radius of the valence electrons or the electron shells that are there is in the order of 10 to the power minus 10 metres okay.

This is in the order of angstroms and this is in the order of 10 to the power minus 15 metres. So you can see that there is an order of 10 to the power 5 magnitude difference between the extent of the nucleus and extent of the electron shell okay. We will assume that the shell of the electron cloud that is around the nucleus is all filled with uniform electron such that the charge density inside is uniform.

What is the uniform charge density is there inside the shell. This is given by the total charge q okay divided by the volume of the shell. Assuming spherical shells simpler to deal with this one, this will be equal to q by 4 by 3 pi r cube where r is the radius of the shell okay and this is the volume, 4 by 3 pi r cube is the volume and this would be the charge density okay. This is the charge density of this particular spherical shell okay.

Well, we will take up from this one. Now in ordinarily matter would consist of many many many many such atoms and each atom can be represented in this crude manner of having a nucleus and electron cloud around that one okay. Ordinarily what happens is that matter is electrically neutral indicating that there is no force acting on any of these atoms and then there is essentially no electric fields inside the matter okay.

The average electric field inside the matter would be zero. Of course, you have to remember that, when the matter is at a certain temperature not equal to absolute temperature, there will be some jiggling, you know the nucleus and electrons have some amount of kinetic energy because of the temperature and there will be some agitations okay. These atoms or molecules will be agitating a little bit okay.

So there is essentially some amount of electric field that is generated, but over the extent of the volume that you are considering of the matter that you are considering, this electric field would all be in some random directions, so that there is no net electric field or the average electric field at on any particular direction will be equal to zero okay.

Now to such an atom, we will assume that the matter that we are considering is made up of only one type of atom okay. There is a nucleus and then there is an electron cloud. To such an atom, what happens if I apply an electric field okay. So I am going to apply an electric field along this direction. Now I know that the nucleus which is positively charged would like to move along the direction of the electric field okay.

So you would expect that the nucleus would move along the direction of electric field, of course because there is a corresponding force that is pulling the nucleus back right. There is a restoring force that tries to pull the nucleus back to its original position. The distance that the nucleus would move would not be too much okay.

In any case, these insulators are such that the charge, the electrons and the nucleus cannot be separated all together that is there are no free electrons, the electrons do not just come out and then just get separated okay. So because of all this, the movement of the nucleus towards the electric field in the direction of the electric field will be very small and let us called that amount as z okay.

So effectively what we have created is that, the nucleus has been pushed from its original position to a new position that is just away from its original position okay. So there is a charge now, which is concentrated at q and there is still electron cloud that is there. We will assume that the movement of this nucleus is so small and there are lot of electrons around this particular atom and this electron shell has not really changed much okay.

So we will make an assumption that electron shell or electron cloud has not changed okay, has not changed, this seems reasonable as long as the movement of the nucleus is very short compared to its original position. So we will assume that z is very very small compared to r okay. So there are now two forces acting, we will see what happens to, what is the value of these two forces and what would happen to this atom when there is an external electric field okay.

Now before we see what happens here, let us look at the first picture over here, where I have an atom and let me assume that there is no electric field. So let us say after application of electric field, the nucleus has moved, but when there is no electric field, the nucleus is at the centre of the electron cloud. What is the force acting on this nucleus. You can imagine a small Gaussian surface around this nucleus okay.

And then try to find out what is the force at this particular centre okay. You can see that since the electrons are all completely surrounding the electric field will all be directed radially okay and they would all be coming in this way. However, because of symmetry if you consider any small portion of the electrons on one surface of the shell, there will be an equal amount of charge on the other shell indicating that there is no net force acting at the centre of the nucleus okay.

So there is no net force experienced by a nucleus, and the nucleus (()) (11:03) put at that point okay. However, once the external field is applied, the nucleus moves to a slight distance z and then, the electron field we are assuming to be unchanged, but which will now exert a different amount of force here right because there are symmetry, which was originally there has now been broken.

So because of the symmetry that was broken, the field that is exerted on the charge q is now going to be different from the previous case okay. What we have created in this process of applying the electric field is some sort of a charge distribution in which the distance between the plus and the minus charges is some amount of distance. So there is essentially this kind of a situation we have created okay where in there is a positive charge and a negative charge.

And these two are separated by a certain distance d okay. Now what we want is what would be the electric field because of such, you know matter at any point in space. So what we want is how would we calculate the electric field at any point in space when there is a distribution of positive and negative charges in this particular way. Please note that this is not exactly equivalent to what I am considering in the sense that the positive charge, you can easily identify to be the nucleus.

However, the negative charge that I am considering is essentially the effective charge of the electron cloud that is setting over here. So there is an effective charge, which I am considering. Situation such as this okay where the two charges plus and minus are you know are considered and you want to find the electric field at some point P, far far away from the two charge configuration is called a dipole okay.

This is the word which tells what we are talking about a pole is an older term for charge. You know, if you studied electricity and magnetism in your high school or in your first year of colleges, you would have seen like poles repel and unlike poles attract right. Instead of telling you like charges repel and unlike charges attract. So this pole terminology used for charges is something that we borrowed from magnetic okay.

Where we have a north pole and a south pole and since there are two poles di stands for two, this is a dipole okay. This is two poles which are of opposite nature okay and then they are separated by a certain distance d and the idea is that we want to find out what is the electric field at any other point. So this calculation is important, if you want to understand what is happening to this matter in this particular case where the electric field was applied okay.

So we will begin by calculating the field of this dipole and then use that result to understand what is happening inside that of a dielectric okay. So what we want is the field okay, of the dipole. In order to obtain that one, let us place the dipole charges at a distance from the origin, equal distance from the origin along the z axis okay.

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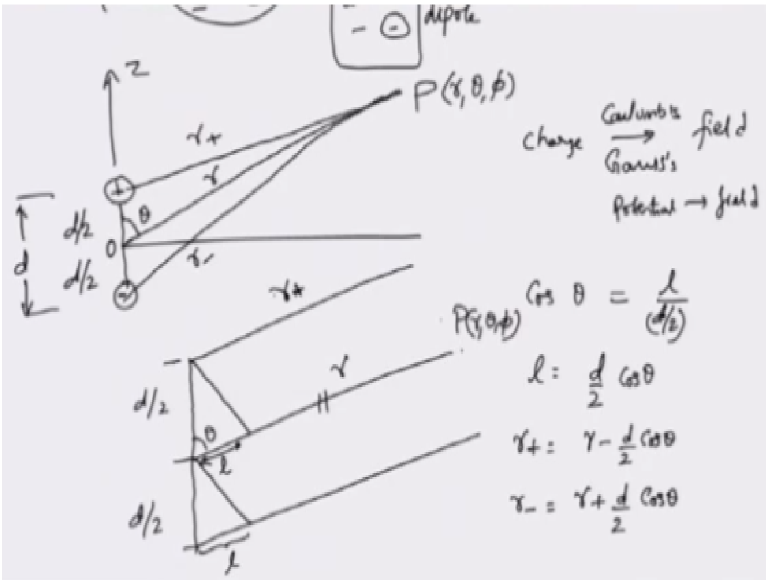

So this is my z axis and this is where I have placed my two charges okay. So this distance is d by 2, this distance is d by 2, so for a total distance of d between the two charges. So I have total distance of 2 between the two charges and now I want to find out the field at a point p, which is in general given by r theta and phi in spherical coordinate system right. So in this case it is natural to use spherical coordinate system, so we will be using that.

So at any point p, I want to find the field okay. Let us see that, I can find out the potential by various means right. Given charge, how do I find out the field, you can apply Coulomb's law okay when there is no symmetry or in a very brute force method you can approach finding the field by applying Coulomb's law okay. If there is some symmetry, you can use Gauss's law to obtain the field.

However, in cases such as this, there is neither symmetry nor application of Coulomb's law is simple. There is another method of calculating the potential and then calculating the field right. So this is how the potential and its gradient operation help us find the field of some configurations where the other two methods do not any work.

So applying Coulomb's law is fairly difficult here, applying Gauss's law is even more difficult; however, going to the potential actually is fairly simple compared to these two. Why because potential is just a scalar. Now what is a potential because of this two charges, well the potential is going to be because of the potential of the plus charge and the potential of the minus charge right.

The plus charge let us say is located at a distance r plus right from the point p whereas the minus charge or the negative charge is located at a distance of r minus from point p okay. Let us also for our purpose, write down the distance from the origin okay. This is the distance from the origin to the point p and call this as r. You will immediately notice that r plus is less than r which is less than r minus right. So you will immediately notice this thing.

What we will assume is that, this r is very large. So you are looking at a very very far away distance from the dipole configuration okay. The angle that r makes is actually given by theta, which is the angle theta in terms of the spherical coordinate system. So this is the angle theta and this is the value of r. Phi fortunately does not really be required to represent over here we will not worry about that phi okay.

Is there a relationship between r plus, r and r minus, that is not this one that is I know that r plus is less than r and r is less than r minus, but is there any other relation that I can find out and it turns out that there is a relationship that I can use, okay? and to obtain that relationship, please note that this is going to be very very important you will see this one in antennas later. So it is very important that you understand this one.

Since the distances are quite far away right. So this is the point p is kept very very far away from the origin okay. I can sort of consider these three lines, r plus, r and r minus to be very nearly parallel to each other okay. I will consider them to be very nearly parallel to each other. This will work only when r is very large okay. I know that this distance is d by 2. I know that this distance is also d by 2.

Now if I draw a perpendicular from the r plus line onto the r line and draw another perpendicular from r line to r minus line okay. I will see that this is the extra distance that separates r plus and r minus right because if you look at this way right, if I look at this one, this length is r plus okay, which is exactly equal to r plus an additional distance right. What is this additional distance?

Well you will have to find out what the additional distance by looking at the angle okay. You can see that this additional distance is going to be, well instead of talking about this angle theta, we can actually also talk about this angle, let us call this as some other angle right. So we need a different name for this one, is it alpha probably something that can be worked over here and I can find out what is this distance.

This distance happens to be d by 2 cos alpha, why because cos alpha is, no actually, we do not have to use this alpha here, we can still make everything work with theta itself because with theta, I have the adjacent side and the hypotenuse. So yes, I can actually represent this extra length. Let us call this as L in terms of d by 2 and theta by writing this as cos theta as L by d by 2, which implies that the extra distance that differentiates between r plus and r is given by d by 2 cos theta okay.

Similarly, this length will also be equal to d by 2 cos theta and in terms of this d by 2 cos theta, you now have r plus as r minus d by 2 cos theta right. R plus is less than r by a factor of d by 2 cos theta and r minus is greater than r by a factor of d by 2 cos theta okay. Now we can use all these relationship, find out what is the potential at this point okay. Potential at the point r theta phi okay and from there evaluate the electric field.

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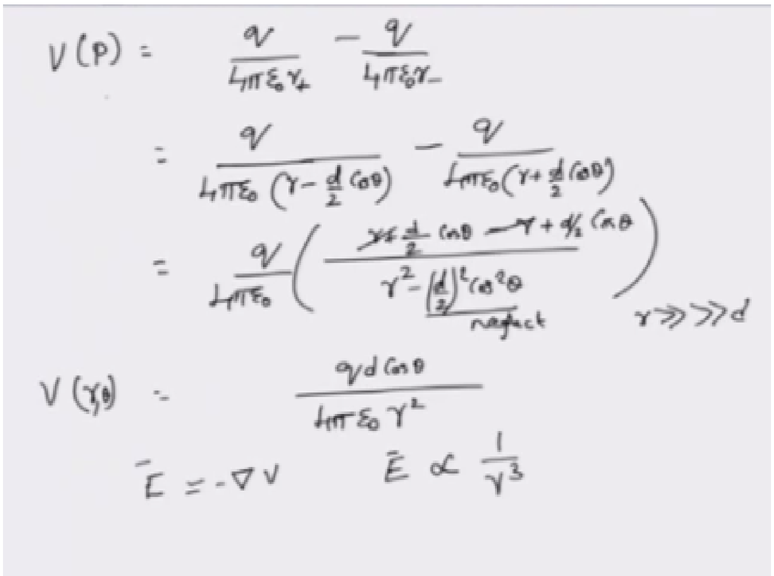

So to do that one, let us first write down the potential at the point p and the potential at this point will be equal to q by 4 pi epsilon zero r plus minus q by 4 pi epsilon zero r minus, I am assuming that this two charges are kept in free space okay. We will consider dielectrics in a movement shortly okay. This is the potential that I have, but I also know what is r plus and r minus, in terms of r and theta.

So I can write this as q by 4 pi epsilon zero r minus d by 2 cos theta and q by 4 pi epsilon zero r plus d by 2 cos theta right because r minus is greater than r and r plus is less than r and r is less than r minus okay. Now I can multiply these two okay by taking this q by 4 pi epsilon common first and then multiplying them I get r square minus d by 2 square cos square theta in the denominator.

And on the numerator I get r plus d by 2 cos theta minus r plus d by 2 cos theta giving me an opportunity to cancel r okay and what I have is q d cos theta right. There is d by 2 Cos theta plus d by 2 cos theta divided by 4 pi epsilon zero okay. Now there is r square minus d by 2 square cos square theta. If I assume that r is very very very very large compared to d okay. That is the dipole is essentially very short distance separation whereas compared to the point where I am evaluating the fields, then I can neglect this okay.

I can neglect this d by 2 square because if d is very small, then d square is going to be even more small. Maximum value of this will be d by 2 whole square that will happen when theta is equal to zero. So even with that, since this assumption is valid okay, I can drop this term and then simply write this as 4 pi epsilon 0 r square. Now this is the potential that is dependent only on the radial distance and theta from the origin.

Now this is actually important result. So far we have seen that potentials of a point charge or a charge distribution was going as 1 by r. In this case, the potential is actually going as 1 by r square, which means that since the electric field is derivative of the potential right since the electric field is minus gradient of v, we expect that electric field would go as okay 1 by r cube and it is true you can actually look at this expression for gradient in spherical coordinate system and you will see that electric field indeed goes as 1 by r cube okay.

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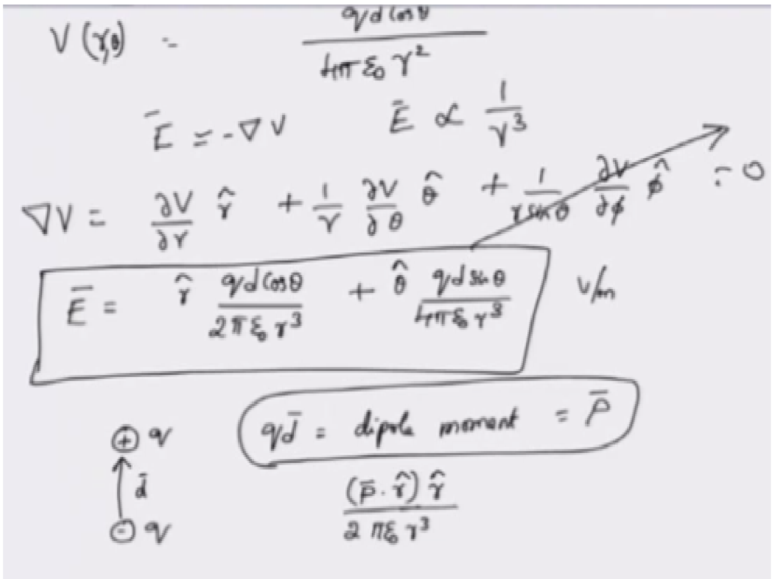

I will write down the expression for gradient in spherical coordinate system, you can confirm this one with the result given in the text book. This is del v by del r, r hat okay, this is the potential differentiated with respect to r and this quantity is going to be non zero here clearly because v is dependent on r plus 1 by r del v by del theta, this is also going to be non zero v depends on theta. There is a cos theta dependence on theta for the potential v.

There is also a term that depends on phi okay. The phi term which is 1 by r sin theta, del v by del phi, phi hat. However, this term is equal to zero because the potential is not dependant on phi okay. So potential is independent of phi therefore this term is equal to zero. Now if you evaluate the other two derivatives and perform this derivation, operations appropriately spent couple of minutes.

You would actually find that the electric field can be written as r hat, q d Cos theta divided by 2 pi epsilon zero r cube. Please note the dependence of 1 by r cube as we just promised plus theta hat q d sin theta divided by 4 pi epsilon zero r cube okay volt per meter. So clearly, electric field has two components r and theta components okay.

This quantity q d which is associated with the distance between the two charges and the amount of charge that is distributed at a distance d or separated at a distance d is actually given by or is thought of as a vector okay. The distance d vector from the charge minus q to plus q and q d, which is now a vector is called a dipole movement where d is a vector from minus q to plus q okay.

In terms of the dipole movement, you can represent for example, this particular r component as q d and this dipole movement is typically denoted by small p okay. This is denoted by small p vector. I can write down the r component as p dot r hat divided by 2 pi epsilon 0 r cube. I can define this one right. P dot r into r hat divided by r pi epsilon zero r cube okay.

So this dipole movement is going to be very important because we will now imagine that that dielectric material that we have is actually filled with such dipole movement okay. Now do you see the relationship between this dipole and the situation that we were considering earlier right do you see this relationship between the two. Here was our realignment of the charges right after application of the electric field.

The charges got realigned, the nucleus moved distance z away and the electron cloud was separated. So this movement caused a dipole to appear. So in case of a matter, we will assume that all of these are composed of these atoms and when I apply an electric field in a particular direction, then there will be this realignment of the charges. The nucleus would move slightly towards the direction of the electric field if the electric field is not very strong okay.

And then the electron cloud will just shift a little bit, although in the approximation that we are going to make, the electron cloud remains the same, the nucleus is moving, but in certain cases, the electrons which can move and then the nucleus essentially remains the same, but these assumptions essentially give rise to a situation where I have some positive charge and some negative charge which are now distributed at a certain distance right.

So this for example would be a dipole with some charge here and some charge here and this dipole can be thought of as a vector with a magnitude of q into d, q being the charge on the dipole on one of the charges on the dipole and d being the distance between the two and the vector dipole is given by a vector from minus q to plus q okay. We will now assume that the matter or the dielectric is now composed of such many many such dipoles.

And then we try to obtain what is the electric field. Now before we obtain the electric field, we also want to ask one question like how easy is it to realign the two charges. Consider paper, consider wood okay or consider some other insulator at room temperature. If I apply the same electric field to all these different materials, would all the materials shift by the same amount z.

Obviously no, it actually depends on the material that you are considering right. So this strength in under which the movement can happen like you know if you apply an electric field, how easily the nucleus would move a certain distance z is captured in terms of what we call as polarizability and polarizability for an atom is given by how much distance does the nucleus move when you apply an electric field or how easy does the nucleus would move when you apply an electric field okay.

So this is called as polarizability and we will look at the relationship between polarizability and electric field and try to see if there is a simple relation that we can find out okay. Alright, so let us look at the relation between polarizability and electric field okay.