Electromagnetic Theory
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Lecture - 02
Coulomb's Law

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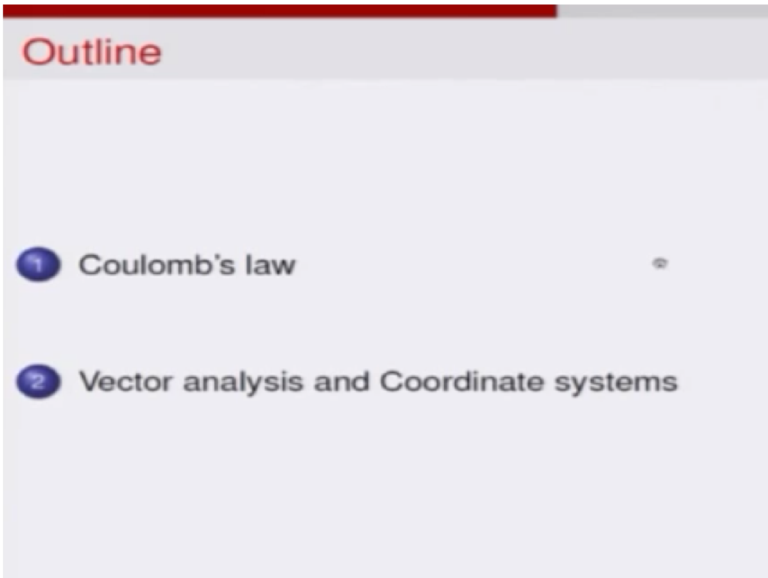

So our first lecture is titled Coulomb's law and coordinate systems in the course electromagnetic theory. So we will begin by first looking at Coulomb's law, and then study little bit of vector analysis and coordinate systems.

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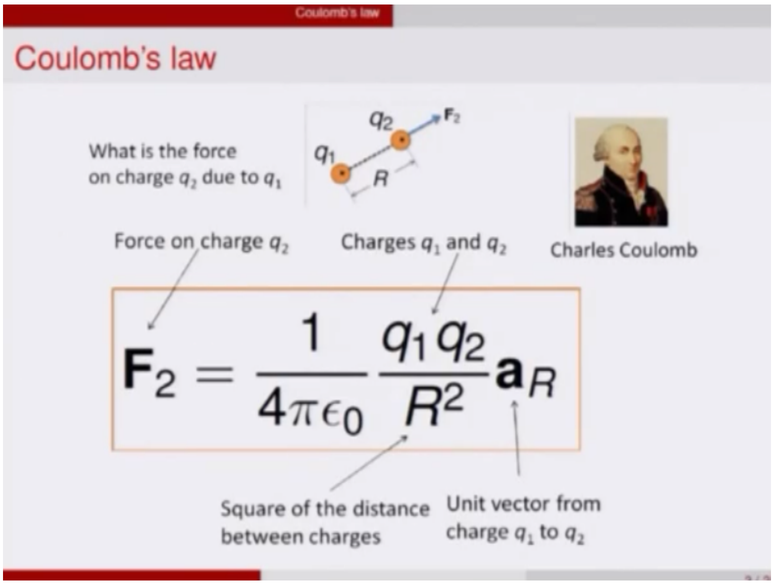

So let me first give you Coulomb's law in all its mathematical query and then we will discuss each of these terms of coulomb's law one by one, okay. So here is a physical scenario in which Coulomb's law is applied. You have two charges, we call them as point charges. We assume that the separation between the charges is very large compared to the size of these charges, okay.

So we label these charges as q with a subscript one, and second charge q with a subscript two. And the goal is what is the force of charge q1 that exerts on the charge q2, which we label as F2, okay. So the question is what is the force that the second charge experiences because of the presence of the first charge. And an answer to this was sort by many people. The first published version of the law is by the French Engineer, Charles Coulomb.

And his law states that, in the modern notation states that the force on the second charge is equal to one by four pi epsilon zero q1q2 by R square aR. Please note that this is in the modern vector notation that we are talking about. Whereas Coulomb's original way of giving law was slightly different, but mathematically equivalent to this boxed equation. So what are the terms that are sitting here, q1 and q2 are the charges.

q1 is what one could call as a source charge, q2 is what one could call as a test charge, okay, upon which we are trying to find the force. R is the separation between the two charges, so you can see here that R is the separation between the two charges q1 and q2. Although I am not shown it properly here, we assume that this R is very large compared to the dimension of the charges. These charges we are going to discuss everything.

This aR is a unit vector, which is along the line, that is directed from charge q1 to charge q2. So along this dotted line, that you can see this is the unit vector from charge q1 to q2, which I am labelling as aR. I will use a different notation later on, but for now this is the simple notation that I have used. And this left hand side is the force on charge q2, okay. This R square tells you that it is the square of the distance between charges.

So if you think of everything else as some constant, you can see that the magnitude of the force is inversely proportional to square of the distance, okay, so keep this in mind.

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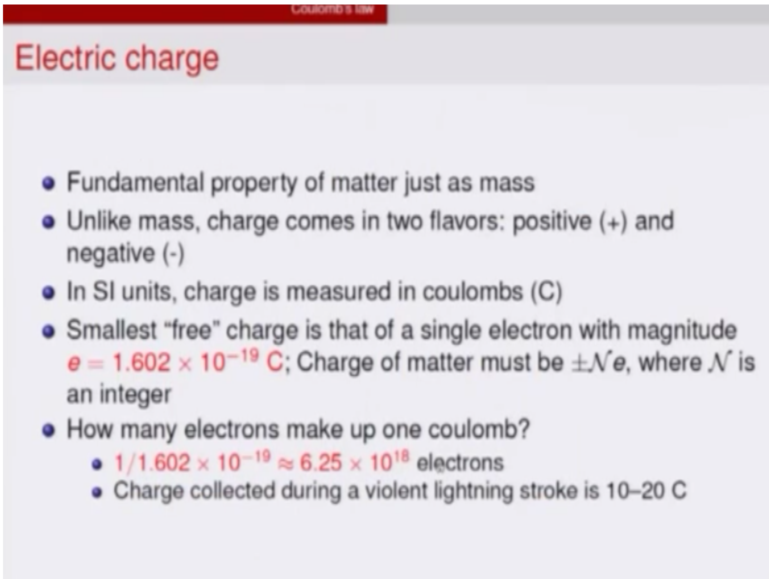

The first item on the right hand side that we had, up on which the force was dependent, was electric charge. Now what is an electric charge. An electric charge is a fundamental property of matter just as mass. Now when you talk of mass, you can have only one type of mass, right. Whereas charge comes in two flavours, it could be positive or it could be negative. And these are the only two types of charge that one can have.

Matter is made up of atoms, atoms are made up of protons and neutrons forming nucleus, and electrons forming electrons, right. So you have I am sorry, so you have nucleus and electrons, and are what is charged negatively, whereas protons are what is charged positively. Of course for a neutral atom the amount of positive charge must be equal to the amount of negative charge. Otherwise they would not exist.

How do we measure charge, we measure charge in the SI units of Coulombs, okay. This is the same Coulomb who gave us the force law between the two charges, and we have honoured him by measuring charge in terms of Coulombs, okay. Now I just now told you that matter is made up of atoms and atoms are made up of electrons and nucleus, okay. Electrons are the smallest free charges that are available to us.

And the magnitude of these charges is 1.602 into ten to the power minus 19 Coulombs. So please note here the order of magnitude, this is ten to the power minus 19, a very, very, very, very small number. Any observed charge that you might measure, should always be a multiple of this number e, okay. You can have a charge equivalent of ten times e, or you can have a negative charge of minus 500 e.

But you will always have to have an integer multiple of this charges, okay. These are the free charges that are available to us and this is the smallest such free charge. Just to give you an order of estimate, how many electrons does actually require to make up one Coulomb. You can see that one electric, I mean one electron charge or one smallest magnitude free electron charge is 1.602 into ten to the power minus 19 coulomb.

So to make up one coulomb you need to have one by the value of e. So note that here I am only looking at magnitude, because electrons are, you know, particles so we do not have to consider their number as negative. The charge on electron is negative, but the electron itself is a number out there for us. So one by 1.602 into ten to the power minus 19, which turns out to be 6.25 into ten to the power 18 electrons.

This is a huge amount of electrons that is required just to make up one coulomb of charge. Now to put it another perspective if you collect charge during a violent lightning stroke, you will be able to collect only up to ten or twenty coulombs. Now you can see that one coulomb of charge is really a huge number, because it requires a huge amount of electrons to make up one coulomb.

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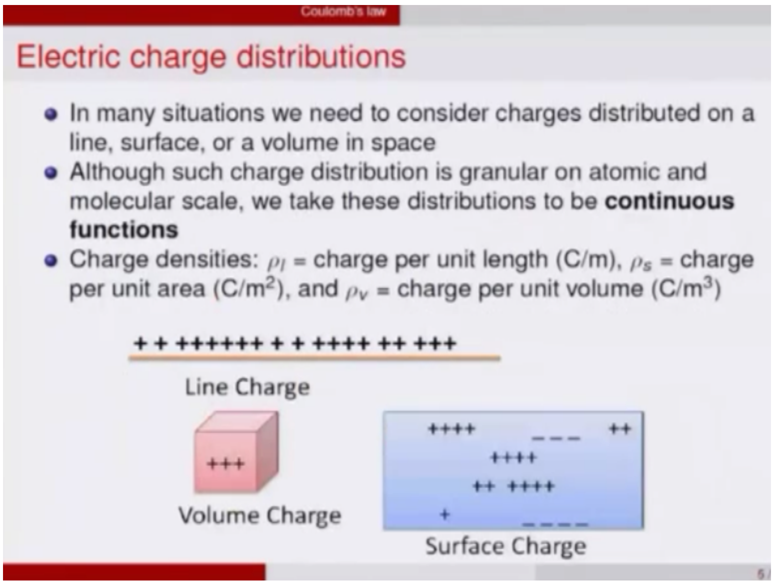

Now in many situations, we will not be restricting ourselves to a point charge. Remember what is a point charge? A point charge is one whose dimensions are so small compared to the distances that are involved. In this aspect if you for example want to consider sun as a charged body, and earth as a charge body, one can looking at the distance between sun and earth, for people on earth, sun simply appears as a point charge, okay.

Such is a point charge, and this is something that you would have been familiar with gravitational law also for example. So when you are calculating the gravitational force between sun and earth, you do not think of the distribution of the mass of a sun or distribution of the mass of an earth. But rather you think of them at the first, I mean initially with not great amount of error as these two as point masses, right.

Point masses are those in which they have mass, they have well defined special location but they do not have any mass distributed. Their mass is distributed as a point in other words, they do not have any size or the dimension of the masses. Similarly, if you are considering sun as a charged particle and earth as a charged particle, for the distance that are involved you can think of them as point charges.

However, if you are calculating the force between two protons within each other located inside a nucleus, you will not able to make this approximation, because the distance between these two is very small compared to the dimension. So point charge is an idealization, in which we neglect the dimension or the size of the charges, or the particles that are carrying the charge, when compared to the enormous distance that is involved, which is actually true.

So any matter that would take is consists of atoms, atoms together form molecules, so you can say that matter consist of molecules or atoms. Now if you see the distance between and electron and a proton, this distance in the atomic scale is very large. So you have this well big nucleus here, and an electron sitting at a considerably far distance so that you can approximate the nucleus as the point charge, you can approximate the electron as a point charge.

Now if for some reason this charges are distributed over a line or a surface or a volume, and if you want to study, how the forces are there on this line or a surface or a volume, you can go to one extreme and start looking at matter as a collection of atoms and start applying Coulombs law to the entire atomic distances, right. You can apply that to an intermolecular distance.

So at the fundamental level because of the discrete nature of the atoms or molecules, you might want to apply Coulomb's law individually. But remember, go back to one slide and you can see that a total of ten to the power 18 electrons are making up only one coulomb of charge, which means that if you want to sit and apply Coulomb's law you will be applying Coulomb's law ten to the power 18 times just to make up of one coulomb of force, right.

These are located, these charges are distributed across as in a matter. If you start applying Coulomb's law to individual atoms, you will be dealing with an enormous number of such atoms, that this application would simply be meaningless, right. This is the same situation as you would encounter in a fluid. For example, you will model water as a continuous fluid, although you know that water is at ultimate level composed of molecules.

And molecules are not coming as continuous units, but they are discrete units. There is a molecule, there is another molecule and there is a distance between the two molecules. However, because of a large number of electrons or large number of molecules that are involved, right, large number of atoms that are involved, one can forget this discrete nature and then talk about a continuous distribution of charges.

Just like water flows, charge flows, right, or charge is distributed on the line or a surface or a volume. These are called as charge distributions and these distributions come up often in practice. So we would better be able to describe this distribution, so you have line charge in which the density of charges could be varying along its position. So here you can see that there are about two charges over here over this distance.

And the distribution gets denser over here again it gets little rarer over here, gets denser, rarer, denser. Although I am showing one plus another plus, you know I am showing just about a few of the plus signs. It does not really mean that there was only one plus. You remember, there are about 10 to the power 18 electrons for one Coulomb. Therefore, there are large number of electrons in each of this plus signs that I have shown.

But the important point is that I can take this line charge and say, well, how many charges are there per unit length. So that, the total length that I want to obtain can be obtained by looking at the distribution and integrating it over the entire line. So we define a charge density, which is line charge density, given by charge per unit length and we measure this by Coulomb per meter.

Why Coulomb per meter because charges are measured in Coulombs and length is measured in meters. So the SI unit for line charge density or linear charge density is Coulomb per meter. Similarly, you might have charges distributed all over on a surface, such as a PN junction. So in that case, you will be talking about charge per unit area. Okay, you have surface charge density.

It is denoted by the Greek rho with a subscript s, s standing for surface, so you have surface charge distribution that is measured in the SI units of Coulombs per meter square. Finally, you can have charges distributed in a region of space, so that forms charge per unit volume. So you have Coulombs per meter cube.

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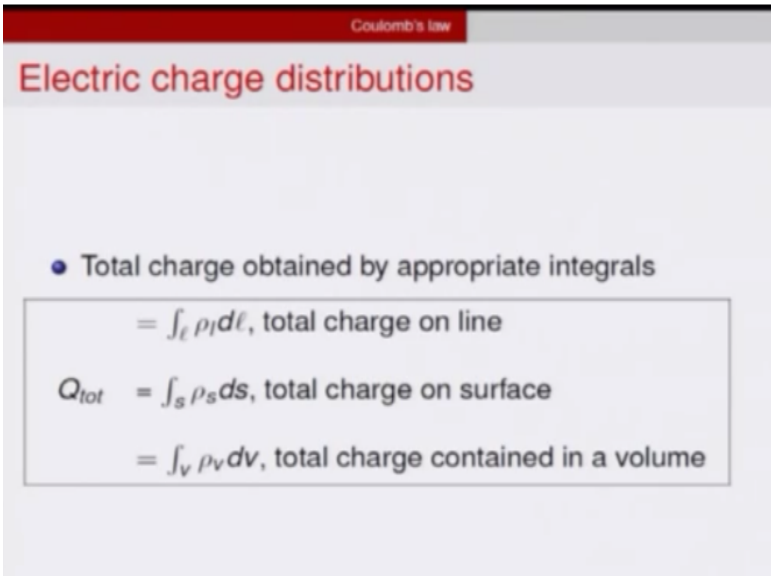

So how do I obtain the full charge or a total charge? The total charge on the line can be obtained by integrating it over the line, along which the charge is distributed. If you integrate the surface charge density over the surface, on which the charge is distributed, this becomes the total charge, which is a surface integral. You can do a volume integral to find out the total charge that is contained inside a volume.

So this is actually more or less a one-dimensional integral. This is a two-dimensional integral. This is a three-dimensional integral, but these integrals are differentiated by their subscripts. So this integral l stands for a line integral, integral s stands for surface integral, integral v stands for a volume integral. Sometimes we will also find them to be triple integral signs to indicate clearly that this is a volume integral.

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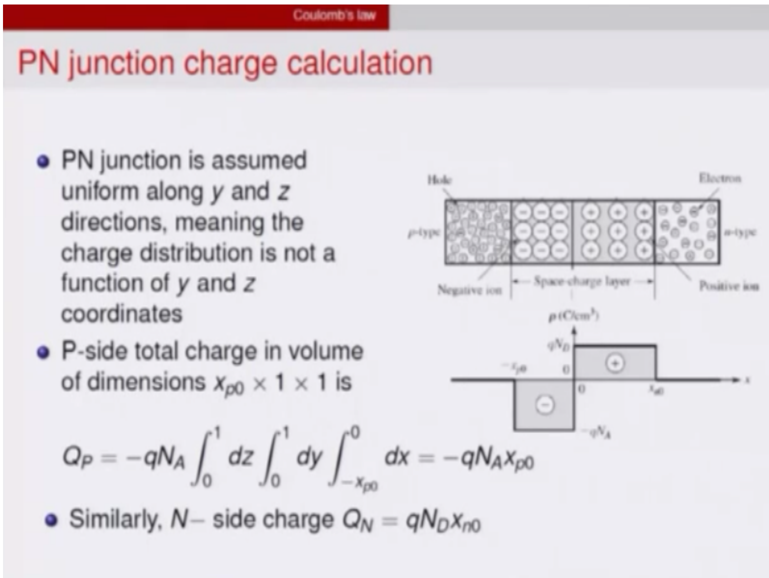

Let me just give you a brief example of how do we calculate the charges, okay. Here is a PN junction, something that you might have studied in your electronic courses. A PN junction is an essential component for making up a diode or mosfet or a transistor and in a typical PN junction, you will have doping donors on one side and acceptors on the other side and when they are made into contact with each other, charges from one type migrates to the charges on to the other type leaving behind what is called as a space charge layer.

To the space charge layer, on to the right of here, which is the N type region, the charge density in Coulomb per centimeter cube here is qNd where q is the same as E earlier what we have seen. So this qNd is the charge density that is charge per unit volume measured in cubic centimeter cube on the N side space charge region and similarly on the P side region, the charge density is minus qNa.

Why this should be plus sign and why this should be a minus sign is something that you must be familiar with the electronic courses because charges move leaving behind immobile ions, which are positively charged here and charges diffuse on to the other side leaving behind immobile ions, which are negatively charged here. So if we now ask what is the total charge on the P side and what is the total charge on the N side.

You can do that by doing an integration over the volume. Now, before we do that we want to tell you that this is a one-dimensional charge distribution in a sense. Because we are assuming that this PN junction is uniform along the y and the z directions, so whatever the charge density that is there is varying only along the x direction, so as the charge density varies only along the x direction, the integrals become very simple.

So if you consider a volume, which is defined by the dimensions, Xp0 1 and 1, you just need to integrate over this region, y and z integrations do not really affect because they are just integrals over 0 to 1 and they are constant integrals that would come out. The charge density also is constant, therefore it can be put out of the integral and you can see that the total charge on the P side is given by minus qNAXp0.

Similarly, you can carry out an integration to find out what is the total charge on the N side of this junction and we will see that the total charge on this side is qNDXn0. Okay, and because of this charge conservation, the magnitude of the charges on the P side must be equal to the magnitude of the charges on the N side. Okay, we will be talking about PN junctions a lot later.

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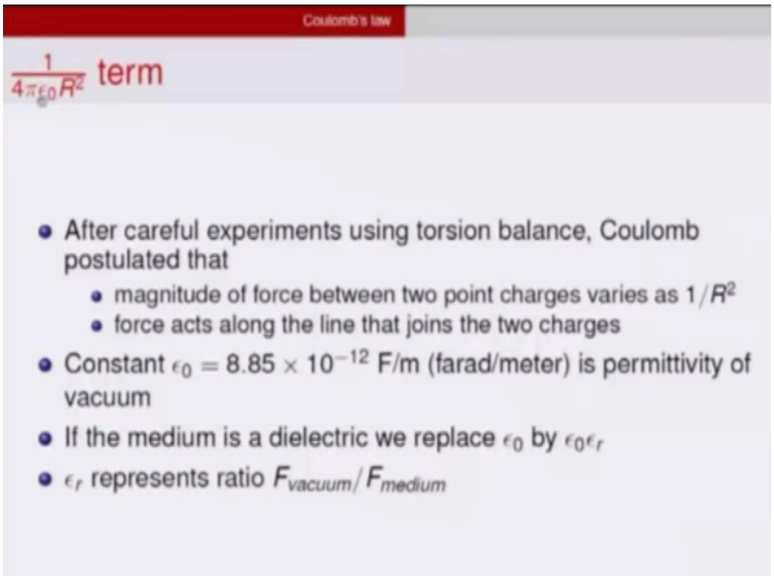

So let us come to another term, which is 1 by 4 pi, epsilon 0 r square. Okay, here you can actually split this up into two terms, 1 by 4 pi epsilon 0 and 1 by r square. 1 by r square was a major factor that was obtained experimentally or that was postulated experimentally after Charles Coulomb performed a number of careful experiments using what is called as torsion balance, which he himself discovered and he postulated that the magnitude of the force between two point charges varies as 1 by r square.

This is the universe square law, very, very similar to gravitational law. Unlike gravitational force law, when the force is attractive in the gravitational law, here you can have attraction as well as repulsion. In colloquial terms, we know that like charges repel and unlike charges attract. Both these repulsions as well as attraction happens with a force that varies as 1 by r square, where r is a separation between the two charges.

Okay, now force is a very peculiar quantity. You cannot just say I applied 10 Newtons of force, Newton being the measure of force in SI unit. I cannot say that oh, I applied 10 Newtons of force. The question that immediately someone asks is where did you apply the force, along which line did you apply a force, right? This is very simple. I can take this object and if I apply force from the top, right, I would not be moving the object.

Assume that the object is sitting rigidly on the support, if I apply force here, say 10 Newton, I would not be able to move the object. On the other hand, if I apply a 10 Newton of force here, I would be able to move the object. So clearly, the direction along which the force is acting is very, very important and that direction, which Coulomb found out was that the force acts along a line that joints the two charges.

So you have two charges over here, you draw a line. So the force, if you assume that both of them have same sign, this is a positive charge, this is a positive charge, then this charge, the source charge will push the positive charge, the test charge along the line that joins the two charges, so you can push it away. If it was an electron, it would attract it along the same line.

If it was a negatively charged particle, it would be attracted along the same line as the source charge. So Coulomb's law actually is of two sub laws in the sense that magnitude of the force is given as varying as 1 by r square that is the first part of the statement and the second part is that the force acts along the line that joins these two charges, okay.

So if you leave out the 4pi terms, which 4 is a number and pi is another number that you, there is another unfamiliar constant sitting here in this term. This unfamiliar constant is epsilon 0 or epsilon naught. What is this epsilon 0 or epsilon naught? This is in SI units given by 8.85 x 10 to the power minus 12 Farad per meter. Farad if you remember is the way in which we define capacitance, sorry we measure capacitance.

So you might suspect that epsilon 0 or epsilon naught is something to do with capacitance and you will be right and this is known as permittivity of vacuum. Now, instead of vacuum, you take the two charges and put them with the material, okay. that material is called as a di-electric material, then we need to replace this epsilon 0 by epsilon 0 into epsilon r, where epsilon r now represents the ratio of the force in vacuum.

That is if the two charges were placed in vacuum, what is the force between the two and now the two charges placed inside a medium, what is the force between the two? The ratio of these two is given by epsilon r. Clearly, because this is ration of the same dimensional quantities. This has no dimensions, okay and epsilon r will be equal to 1 for free space and it will be greater than 1 for any other material.

We will be talking about what a di-electric medium is and we will also look at what epsilon r is in more detail as we go along.

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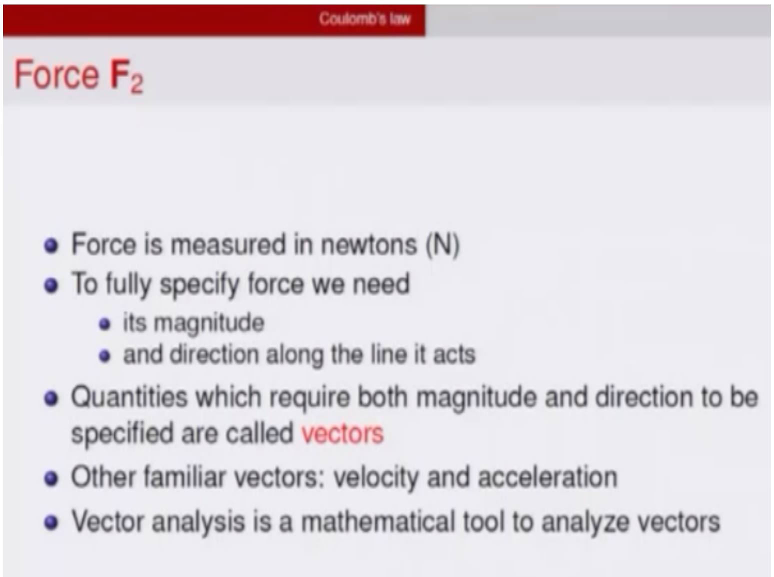

Finally, we have the left hand side of the equation, which is the force F2. The subscript 2 indicates that this is the force experienced by the second charge because of the presence of the first charge and force as we have said is a vector quantity and it is something that is measured in terms of Newtons, okay and the symbol for Newton is N. To fully specify force as we have said requires magnitude as well as the direction along which it acts.

These quantities which require both magnitude as well as the specified direction along which they are acting are known as vectors. You might be familiar with other forms of vector such as acceleration and velocity and how do we understand these vectors. There is a mathematical tool to analyze and understand vectors called as vector analysis. Before we talk of vector analysis, we also need to know how to represent these vectors numerically and that is the subject of coordinate system.

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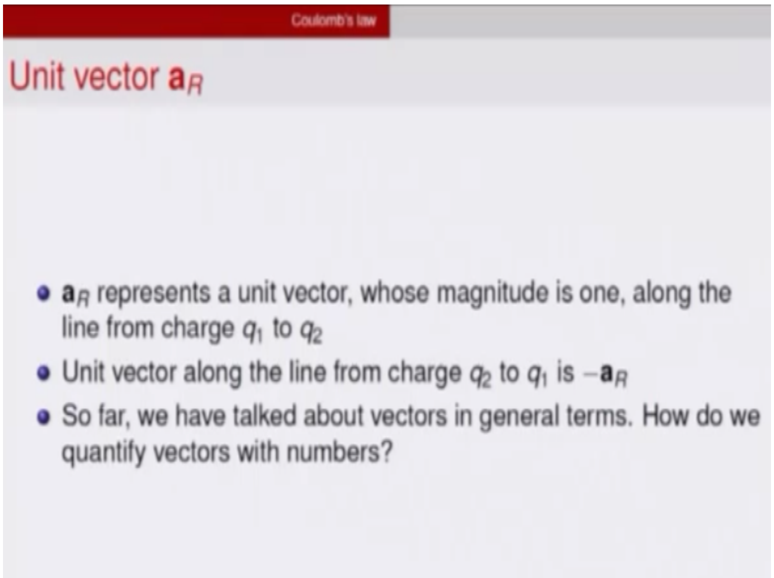

This is again a vector quantity ar represents the unit vector, whose magnitude is 1 along the line from charge q1 to q2, okay. If you want to specify a unit vector along the line that joins q2 to q1 that is a vector that originates from q2 back to q1 that unit vector will simply be minus of ar, so it will simply be negative of ar and this part is hopefully very familiar to you. So far, we have talked about vectors in their general terms.

We now quantify vectors with numbers using coordinate system or we will quantify vectors using vector analysis.