Lecture - 21
Polarization – II (Contd.)

We have shown something using mathematics, you know we did some theorem, we did another theorem, but what is physically, where is this physically volume charge and surface charge coming from. Well to do that go back to the dielectric, okay. So, let us say this is a dielectric boundary that I have and because of an applied externals electric field, there are dipoles in this, okay. So, now there is lot of dipoles in this, I am showing only 5 or 10.

But you have to assume that there is a lot of these dipoles, okay. So, I have dipole here, I have a dipole here, sorry this is not correct, okay. Then, I have a dipole at the boundary, okay. Now, it turns out that if I choose any volume here, okay. Typically, this volume would have dipoles, I am indicating the fact that there are these dipoles which are actually no accounting for all the charges inside. In this volume, there would essentially be a neutralization of the charges, okay.

Each dipole will be paired by another dipole and these charges would be all neutralized in the sense that there would be paired with something else. However, on the surface, there would be these unpaired charges and these are the equivalent surface charge, a polarization bound charges that we are going to get. So, this is the surface charge distribution rho s p. So, surface charges are actually created.

Because of the insufficient cancellation of the charges inside the dielectric medium, okay. So inside the volume, they have been neutralized. Otherwise, these charges are present on the surface and these surface charges are the once that are actually responsible for equivalent field distribution at some other point, okay. So, this is the surface charge and if there are volume charges, they would actually be interior to this dielectric. They will not be on the surface.

They would all be interior to the dielectric. So, rho s p the surface charge density is defined as p dot s because we have to understand that the surface itself could be irregular in shape. So, at

every point on the surface, I have to find out what is the surface normal and then on that surface normal, I have to find out what is the contribution of p on that surface, okay. So, p is the density vector, okay. So, it has certain orientation, remember that and when you apply a potential.

There would be a component of p along the surface normal and that would form the surface charge distribution. Similarly, the volume charge distribution because of the polarization is minus gradient of p. So, if at all you have a nonuniform dielectric, right, then this quantity will be not zero, okay. There would be certain volumes, in which there is del prime dot p that is nonzero and that would create the volume charge density of the volume, okay.

So, you could as I said absorb rho s p inside rho v p by defining the appropriate limits of the volume charge.

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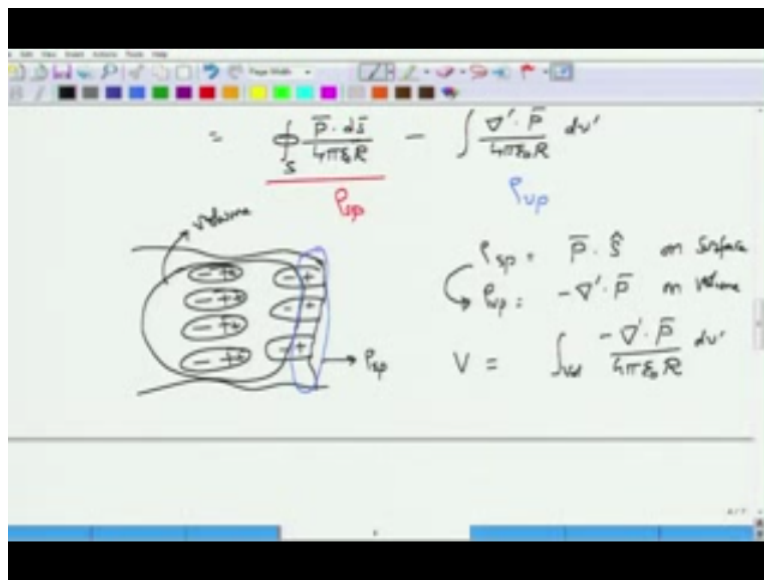
Remember we did this; the most general distribution is the volume charge. Then, you could convert a volume charge distribution into a surface charge, surface becomes line, line becomes a point, right. An appropriate integration limits if you apply, you can convert a volume to surface, surface to line, line to a point charge.

So, you could accommodate this rho s p inside rho v p and you can write v, the potential as the volume charge integral of minus del prime dot p divided by 4 pi epsilon r integrated over the

volume of the dielectric. If I have observed rho s p into rho v p, okay. Now, this is the potential, you could find out what is the electric field, right. So, you could find out what is the electric field.

Now, we do not really want to find the electric field at this point because this next argument tells us what is the relationship between the potential here and the potential due to the free charges, okay. So, let us assume that there is arbitrary charge distribution, okay. This arbitrary charge distribution is responsible for the external electric fields, right. So, this is responsible for the external electric field and the potential at the field point.

Now, I have placed a dielectric here and the dielectric is described by the equivalent volume charge density rho v p, okay and rho v p is given by minus gradient prime dot p. So what would be the potential at this point. The potential will be because of the charge distribution. Let us called this as rho f, f indicating free charges, okay. These are the charges which we have placed and these are the charges which are actually responsible for external field and polarizing the dielectric.

So, I have free charges here and I have charges rho v p. Therefore, the potential at the field point actually will be because of both free as well as the volume charges, right. So, that would be rho f minus gradient dot p, this is the volume charge divided by 4 pi epsilon 0 R d v prime, right. Again, we are actually evaluating the fields at a very far away distance from the dielectric, okay. Now, if this is the field, right? if this is the field that I have, I could take it one step further and right down this as rho f plus rho v p divided by 4 pi epsilon 0 R d v prime, right?

If I define a rho total as rho f plus rho v p as far as the potential at this point P is concern, where which I am very far away from the dielectric, there is absolutely no difference between rho f and rho v p. Therefore, from the field point of u that is from the point of u at point p, where I am calculating the potential, both these charges add to the potential, right?

Therefore, Gauss' law has to be replaced from del dot D equal to rho f which would be valid, if there is no dielectric in between to modify this Gauss' law to del dot D equals rho f plus rho v p,

but what is rho v p? Rho v p is del dot p, minus del dot p, okay. At this point, it really does not matter whether I am looking a del prime dot P or del dot v. The del operation would essentially enclose an one big volume (()) (06:54).

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This del operation that we are writing would actually be a volume around the field point that I am considering. Therefore, it does not matter whether this is del prime. This del prime has to be done at the dielectric, whereas the del dot d that I am doing would be at the field point, okay. So, I can simply write this as del dot P and then pull this minus del dot p to the left hand side and I get D, sorry I actually made a small mistake here, this is del dot D is equal to rho f is correct.

However, the field that I have is actually because of see this del dot D equal rho f actually becomes del dot epsilon 0 E because I am assuming that outside of the dielectric, this D field is actually epsilon 0 into E, correct that is the field outside the dielectric. This would be equal to rho f plus rho v p now, right. So, with this I can move this rho v p back into the left hand side because rho v p is still minus del dot p and if I do that I will get epsilon E plus P, okay.

And this would be equal to the free charge density rho f, okay. So, this epsilon 0 E plus P is given a name D which is called as flux density, okay. So, in terms of this flux density vector D, which is measured in Coulomb per meter square. I now have full relation of D and rho, which is del dot

D equals rho f, okay. Where D itself is equal to epsilon 0 E plus P, okay and you can clearly see that this D vector will be different in different materials.

Why would they be different in different materials. Because the P vector would be different in different materials, right. Some material will have a larger value of alpha E, some materials will have a larger value of alpha orientation, some will have a smaller value, some will depend on certain, you know if them dielectric itself is composed of many species of molecules, then there will be dependency on the position itself.

Some clusters would be at a higher value of polarization, some clusters would have a lower value of polarization, right? So, you could have inhomogeneous. You could also have anisotropic media in which this polarization vector depends on the direction of the applied field. So, if I apply a field along say horizontal axis, then it would in due the certain polarization and if apply an electric field.

You know at an angle theta with respect to the horizontal, then that would in due the different kind of polarization. So, you can actually have directional dependence, dependence on the space and this one based on this polarization vector P. In fact, the polarization vector P need not even be dependent, I mean it need not be linearly proportional to electric field. It could in most cases when the electric field is very strong or very intense.

Then the polarization vector P becomes proportional to E square, E cube, E to the power 5 and so on, okay. So, such materials are called as nonlinear materials and they are especially useful for nonlinear optics situations, okay. We can do lot of interesting stuff with nonlinear optics materials. However, the handling of this nonlinear optical materials which are basically P depending on E square, E cube and so on, it slightly, well slight is an exaggeration.

It is actually quite difficult and we will not be considering them over here. We will also not consider the inhomogenous condition that is we will not consider the dielectric to be composed of different different type of materials, so that epsilon itself is the function of space coordinates,

okay. So, we will assume uniform dielectrics, for which we can characterize them by a simple number, okay. We will also consider anisotropic materials very briefly, okay.

But anisotropic materials will make that appearance quiet after sometime, okay. We have just obtained a relationship between a new vector quantity D, which is flux density. Of course, we have already used flux density or vector D when we used Gauss law to describe or to obtain the electric field when using Gauss law. However, this is the formal definition of flux density D, okay. You can think of D as defined in terms of polarization.

Earlier, I made a statement saying that chi E for free space is equal to 1, but that is not correct. Chi E for free space is equal to 0, but for any other dielectric, it would be greater than 1, okay. So chi E for free space is equal to 0, sorry for my earlier mistake. Now, we have seen that this vector P in most cases, it would be proportional to the electric field. The first power of the electric field E via the susceptibility parameter chi E, right.

So I can introduce this P into the expression for D and write D as epsilon 0 1 plus chi E electric field E. Now, we have given a name for 1 plus chi E as the relative permittivity. So, relative permittivity is 1 plus chi E, chi E stands for susceptibility and for materials, which is not free space, this value will be greater than 1, epsilon r will be greater than 1 for materials that are dielectrics and not free space, okay.
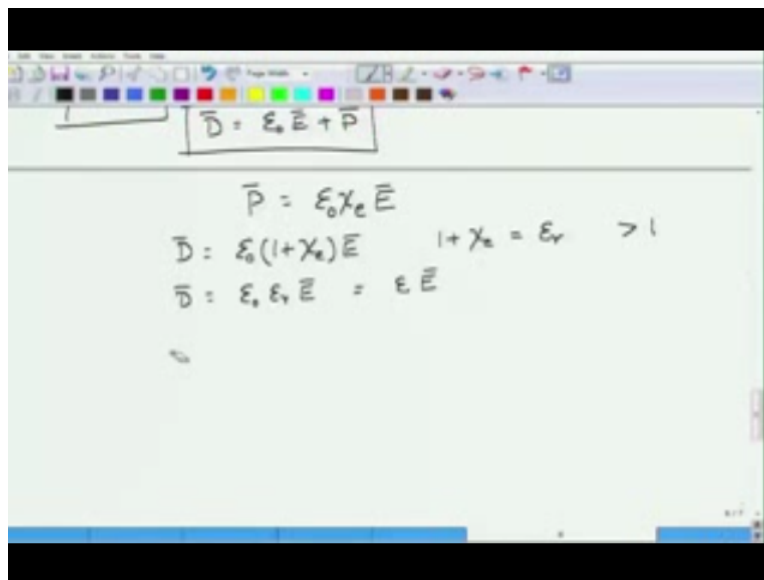
For free space epsilon r is equal to 1, which again indicates that chi E will be equal to 0. For any other material, epsilon r is greater than 1, okay. So, in terms of that I can write down D as epsilon 0, epsilon r electric field E. Sometime this is shortened further and simply written as epsilon E, okay. You have to note from the context that epsilon could be different in different materials because the polarization vector P would be different in different materials.

Some materials will have a smaller value of the polarization and for some materials will have a larger value of polarization. Some materials would be at a different temperature. Therefore, the oriental polarization will be different and for some materials, it would be different value of oriental polarization, making the value of D to be different from electric field. Of course, this

epsilon r is not a microscopic quantity that is - This is not actually defined for a single atom or a single molecule or a single ion. This is actually a macroscopic quantity.

You take a big piece of dielectric material, okay and then you find out this susceptibility and then defined this epsilon r and when you are doing all this, you should remember that you are actually doing some sort of averaging. Because these materials are defined in terms of the polarization vector P and remember what polarization is, polarization is net dipole moment per unit volume. What was that unit volume there? That is the volume inside that of the dielectric, okay.

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It would typically be a few angstroms wide. So, that there are sufficient dipoles inside, okay. But it should not be too large. For example, if I am taking 1 meter by 1-meter dielectric slab surface, I should not choose the delta V, the unit volume to calculate the dipole as 1 meter by 1 meter. It should be small, okay. Such that on the level of where we are considering, we can consider the vector P as a continuous function, okay.

However, if you try to make it too small, then you are dealing with individual atom dipole polarization, right. In that case, it would be very erratic. From atom to atom, it would keep changing and that is not a very good thing to handle for us. So you need to reduce the volume delta V, such that on a level that we are considering on the macroscopic level, P can be thought of as a continuous function of the space.

However, you would not consider delta V to be so small that you are looking at irregularities on the atomic scale unless that is your interest of in the problem, okay. Similarly, epsilon r can also be considered as a function of space. In addition to epsilon r being function of space, it could also be a function of direction. For example, there are certain materials which would be polarized in one direction with the certain value.

And if you apply an electric field in the different direction, then they would be polarized with the different value, okay. So this directional dependent can also be considered macroscopically and attributed to the susceptibility chi E or the permittivity epsilon r or its shortened version epsilon. So, we say that epsilon which varies with respect to space or the spatial coordinates as in homogeneous dielectric.

If epsilon is the function of the angle of the applied electric field that is direction of the applied electric field, then such materials are called as anisotropic materials. In the most cases, when the electric field inside the dielectric medium becomes very strong, then P will start to become a function of second power of electric field, third power of electric field and so on. Such materials in which the polarization vector P depends on not on the first power.

But on the higher powers of electric field are called as nonlinear materials and nonlinear materials are particularly useful for nonlinear optical application, okay. You can actually take two wavelengths and create a new wave length, you can take a one wavelength and create half a wavelength, okay. You can do all sort of things when you have nonlinear materials. In this course, we will not be looking at nonlinear materials.

We will be looking at linear anisotropic materials. Anisotropy of course makes it entrance quiet late in the program or in the course, okay. Alright, so we have D is equal to epsilon times E. Now, let us ask this following question. Suppose, I have a dielectric slab, okay. Let us consider this to be a Teflon slab, okay and this Teflon is given by a susceptibility chi E of 1.1, okay. At some temperature, this chi E is 1.1. What can be say about the fields inside and outside.

Let us assume that outside, the fields are all uniform, okay. So, the electric field outside, let us called this as E out, is given by u hat and has a constant intensity E 0 measured of course in volt per meter. U hat simply indicates upward directed electric field, okay. I did not want to specify this as x or y. So, it is upward directed electric field. Outside of the Teflon slab, you know on the other side.

Again I have the electric field outside of the slab, as upward directed uniform electric field of magnitude E 0 volt per meter. Now, what can I say about the fields inside the dielectric?
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Well, all I can say about the fields inside the dielectric is E in stands for the field inside the dielectric, then P in, the polarization vector are the dipole moment density, right, that would be equal to epsilon 0 chi E times electric field in which is 1.1 times epsilon 0 inside electric field E in. What would be D in, the D vector, D vector would be 1 plus chi E times epsilon 0 E in, 1 plus chi E is 2.1. So, this fellow will be 2.1 epsilon 0 E in, right with appropriate measurement units.

Now, if you ask, well you are given what is the electric field outside the material and what stopped you from finding the electric field inside the material. The answer is I know how electric field inside is related to polarization and D vectors. I know how the electric field is varying outside, but I do not know how to relate these 2 things, right. I do not know how to relate electric

field outside to electric field inside and to obtain that relationship, we need to understand boundary conditions, okay.