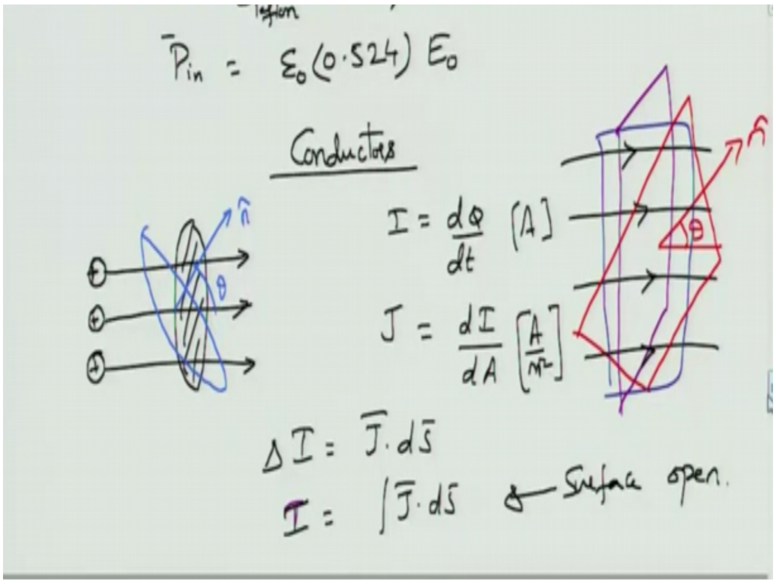
Electromagnetic Theory
Prof. Pradeep Kumar K
Department of Electrical Engineering
Indian Institute of Technology – Kanpur

Lecture 23
Continuity Equation & Conductors - III

(Refer Slide Time: 00:13)



So we have, couple of classes ago we began saying that materials can be classified based on their energy band gap and energy band gap for a dielectric medium will be very large. So at room temperature they would not normally conduct any electricity. Of course, if the temperature is raised and if the band gap is lower then they would be conducting electricity. However, in normal temperature and dielectric material or an insulator would not conduct electricity.

However, for a conducting material the energy band gap is almost, I mean that is so close that valence band overlaps with conduction band. So there is lot of free electrons in the conduction band that are available because the outer shells are having free charge and they would have very easily contribute to this free charges and these charges are able to move through the lattice, okay?

They would move from one atom or one lattice to another lattice, they would displace the electron sitting there. That electron would move and then it would displace the other one. It is like, on a long traffic, on a long road, you have cars, you can imagine cars and one car moving to the next car's position, the next car moving to the next car's position and so on. What you have to realize is that even though the cars themselves might be moving at a very small speed but overall traffic movement could be much faster.

So even though electrons might be jumping from one lattice to another lattice, one lattice to another lattice at a very small speed and indeed that is what would happen. The overall electron distribution, the distribution would be changing much rapidly. So that is how electricity is conducted inside a conductor and conductors are those materials which have a large number of free electrons.

So we will be looking at that conductors and we will be asking the same kind of questions that we ask for dielectric. What would happen if there is a conductor and if there is a free space or some other dielectric, what would happen to the electric fields at these two regions. That is all that is sufficient for me. But before I go there, I want to ask one very important question, in a conduction at least on a static case, can there be electric field.

Imagine this, take a conductor, some solid conductor, put some charges in between, would you actually see the charges remain there? Turns out that those charges do not remain inside the interior of the conductor. What would happen is, the charges which are kept inside the conductor would develop an electric field and since these charges are all like charges that we have kept, they will start to repel each other.

So the electrons would fly away from the interior point and they would all land at the surface, okay Now it could have happened that they would simply leave the surface and go away but we are surrounding this conductor by a dielectric medium. Even air is a dielectric, so if you surround by a dielectric medium dielectric does not conduct electricity. Therefore, the charges which originates from the interior would end up as surface charges on the dielectric.

We have in fact solved a problem that of the infinite plane charge and we found that the electric field produced by this infinite plane charge, had electric field which was normal to the surface. The charge was placed like this, there is an infinite surface charge here, which you can think of as having a metal. You have a metal that is completely filling the space below the plane of the surface changes and there is a nice dielectric above here.

And the electric field was always normal to all these charged surface, electric field was normal to the charged surface and D field would also be normal to the charged surface. But the important point is that inside there are no electric fields because if there was electric period inside that would accelerate the electrons, constitute the current and that is something that is not happening in the static conditions.

Unless there is means to generate and substation current, this would not happen. So inside, the most important point of a conductor is that, under the static conditions there are no fields and no charges inside the electric field. It does not matter whether that is normal or tangential. It could be normal electric field, but that would also exert some acceleration, some force on the charges and that is again strictly forbidden, okay?

You can ask how much time does it actually require for the charge distribution that is kept inside the material to become a surface charge. How much time would it take It turns out that time which is called as relaxation time is just about 10 to the power minus 19 multiplied by 10 to the power minus 20 seconds. This is much below attosecond, attosecond is 10 to the power minus 18, if my memory serves right.

Then this time is even less than that. For a good conductor such as copper, silver, aluminium, this time is very very small. So therefore one can assume that just after about a couple of attoseconds, one can assume that there is essentially no charges or electric field inside the conductor. All the charges that are inside would be converted into surface charge distribution. They would nicely associate themselves to the surface and they would form a certain distribution.

There are very interesting questions about that. We will take up some of them later. But before we go there, we might want to ask, I mean we want to answer or we want to introduce small quantity called current. We have already introduced voltage. Voltage between any two points is the amount of work that is required to move a charge and we have obtained an expression. So let us introduce current now.

Discuss that a little bit and then go to conductors because one of the properties of conductors is that they would conduct electricity. So we would like to know what is electric current? Electric current is because of the motion of charges. In a conductor these charges are electrons. Electrons are negatively charged. So when you apply an electric field electron would move in the direction opposite to the electric field.

However, conventional current means that the direction should be in the direction of electric field because in conventional current we assume that the charge carriers are positively charged, okay? However, we do know that current in cases is actually because of the electrons in conductors because of the electrons. So the conventional direction of the current is opposite to the direction of the electron flow, okay?

Electron flows pass from right to left. Conventional current flows from left to right. Of course if the electron flew from right to left, the conventional current will flow in the other direction. But what is current? Well, current as you know is essentially rate of change of the charge. That is if you were to consider a particular plane and then look at how many charges are flowing past this plane.

Actually a point they could consider, how many charges are flowing past this plane or a point that would give me the current charges flowing past per unit time, that would give me the current. Now there is no rule that I have to keep this plane in this way. Assume that charges are all moving in this way, okay, to calculate the current you just need how much of charge has moved past a particular plane.

However, I can also consider a plane in this direction. Notice that charges are still moving in that original direction. Just because I decided to change the plane orientation, does not mean that charges also have to move in that plane. However, what would happen is, the strength of the electric field across this plane would be reduced because now this original direction of motion of the charges, would not be the same as the direction of the normal to the plane.

If there is an angle between the two, cal this angle as theta, then the current would be reduced by a factor of cos theta. To take into account this orientation of the current we introduced and to deal with these planes, we introduced a quantity called as current density J, with the idea that current density J gives you how much of the current is flowing past a given oriented plane.

An oriented plane or an oriented surface is simply the one in which we can describe the, or define the surface normal. To get an appreciation of this current and current density, imagine these are the lines of J field. We could easily see that, for example we take a wave guide. Okay, you take a wave guide which we will be discussing towards the end of this course and then (()) (09:19) it would generate some wall currents.

And these currents would all have their different magnitudes. Along the top surface they would have different magnitudes and J lines will all be flowing in different directions, okay? Now imagine placing a plane. If you place a plane, that is parallel to this, there would essentially be no current because as much J lines are entering, as many J lines are coming out.

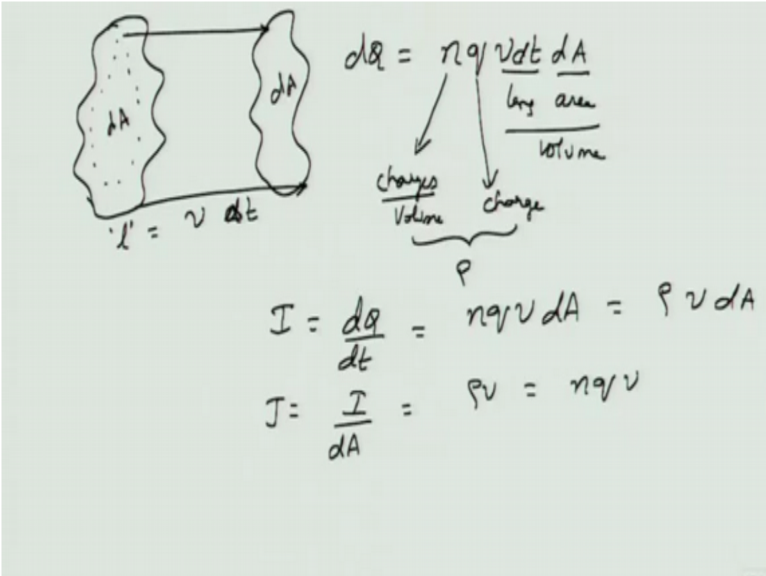
When I say J line, I actually mean current density vector. So current density vector would not be changing here. I mean, it would be as much entering, as much living, therefore there would be no current through this plane. If I keep a plane that would look like this, there would be some current but that amount of current will be dependent on the angle of the normal of the surface that makes with respect to the J line.

So the amount of current that would be there would be dependent on the angle between the normal and the normal to the plane as well as the J line. So you can capture that by defining the current as J dot ds. This would be the total current delta I and that would be J dot ds. I am just for the sake of this one assuming that J field is all uniform. Of course, it is possible and in a wave guide example that I gave you, it is possible, it is in fact happens that J will be non uniform.

That it will be different at different points on the wave guide plane. When will I get maximum current, I will get maximum current when I chose a plane that will actually be completely perpendicular to the J lines, so if the J field angle between the J lines and the normal to the surface would be equal to zero, only then I will get the maximum current.

So I know that the incremental current that I get out of a surface which I have kept as ds would be J dot ds. The total current I would be equal to the integral of J, over the surface. The surface could be irregular and J itself could be non uniform. Should it be closed or open? Are interesting thing, right? Should it be closed or open? I will leave it as an exercise for you to think about this and this surface must be open, okay?

(Refer Slide Time: 11:48)



So this is the current. Is there a relationship between current and velocity of charges? Yes, there is. Imagine that I have a surface here. I have some surface called this a dA, in a time delta t or dt, if you imagine that there are charges on each of these surface, these charges would have moved after certain distance. How much distance they would have moved past, if each of these charges are in uniform motion, uniform velocity V, then they would all move a length of V dt. So this is some length, okay?

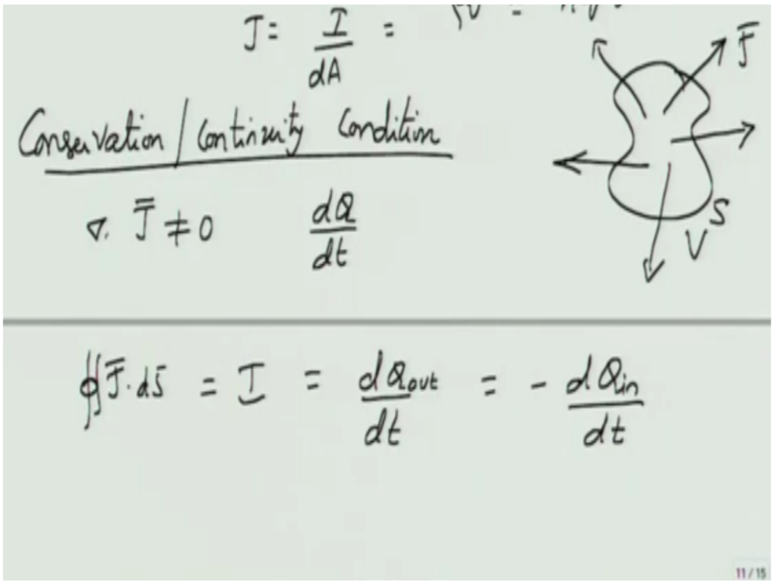
And there would be position in the space. Their surface area is a the same dA which I am considering. So how many charges have actually gone past per second. Well, you can calculate this, right? See, first of all the total charges that have passed here would be n q v dt dA. This is the length and this is the area, or this can be the height and the area. So together you are going to get a volume here. n is the number of charges, per unit volume.

So number of charges per unit volume multiplied by something per unit volume would be this one multiplied by volume will cancel the volume and q is the charge on each charge. q is the amount of charge that is carried by each charge carrier. So this n multiplied by q will be the charge density. So this would essentially be the charge density, in a general way represent this as, if this is point form of this one, it would be rho.

This is the amount of charges that have gone past me or gone past this particular plane in time dt. So the current that must have gone will be dQ by dt. And that is given by n q v multiplied by dA or rho times v dA. This is the current that has gone past. However, I am interested in the current density. Current density is current per unit area, so that is given by rho multiplied V. Rho is the volume charge density, V is the velocity of the charge carriers.

If these charge carriers can be counted, like the charges per volume that we wrote this would be n q v. This would be the current and what do I measure current and current density in? Current is measured in ampere and current density is measured in ampere per meter square. Because this is the density of the current.

(Refer Slide Time: 14:37)



Now, we will discuss very briefly what is called as conservation equation or sometimes called as continuity equation of the current, okay? You have to realize that this conservation and continuity condition has nothing to do with Maxwell's equations. Okay, this is just a fact that the total number of charges inside this universe cannot be changed, that is a global conservation that must happen.

But locally what it means is that if there is some current then there must be the amount of charges inside a particularly small volume must be going down. So at a point if there are J lines coming out, then it means that charges must be flowing along the J line which means the original charge distribution must be going down at that particular point. That is all the conservation of charges will fail.

Charges are being transported. Therefore, the amount of charges in fact would be going down. It is like bus, you imagine and people are coming out. If you imagine this people as J lines, the current density vector lines, then the only way this J would be non zero, would be when the number of people inside are reducing. This is nothing to do with electromagnetic, this is just the conservation of people.

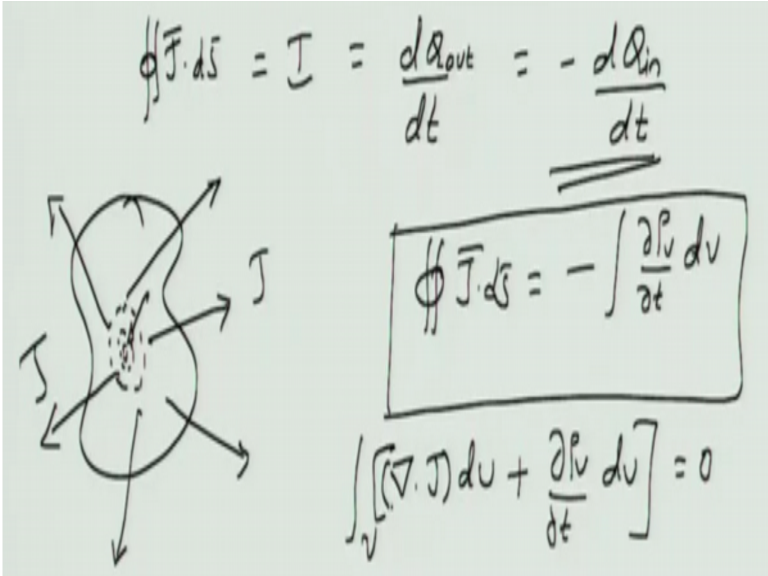
So instead of conservation of people you have conservation of charge carriers. So whenever there is a non zero value of J in a closed surface, not the open surface, in a closed surface, the only way you can have the J lines to be non zero or the vector field J to be non zero is when the number of charges inside the volume enclose by the surface is reducing. So there must be a balance between the two.

So people inside the bus are reducing and hence people outside the bus are increasing. That is as simple as that. So imagine that I have a closed surface. This time it is closed surface and this surface is bounding a certain volume and let us say over this closed surface I will evaluate the divergence of the J vector, so I will evaluate the divergence of the current density or essentially picture how the J field lines will be coming out and they would be all be emanating outside this closed surface.

So I know that as long as J is non zero or more precisely as long as divergence of J is non zero inside this particularly closed surface, there must be charges which are getting reduced, correct? So what I have is that, the J field lines of course over the entire surface will give me the total current. If I take the J vector and integrate J over ds over the closed surface in this particular case it is close because I have to consider the region of the surface.

So this should be equal to the total current I and this should also be equal to the amount of charges that are coming out. Please note that this Q out indicates that charges are coming out of this closed surface. However, if there are charges coming out and equal amount of charges must be getting reduced inside, correct? Charges are getting reduced inside.

(Refer Slide Time: 17:54)

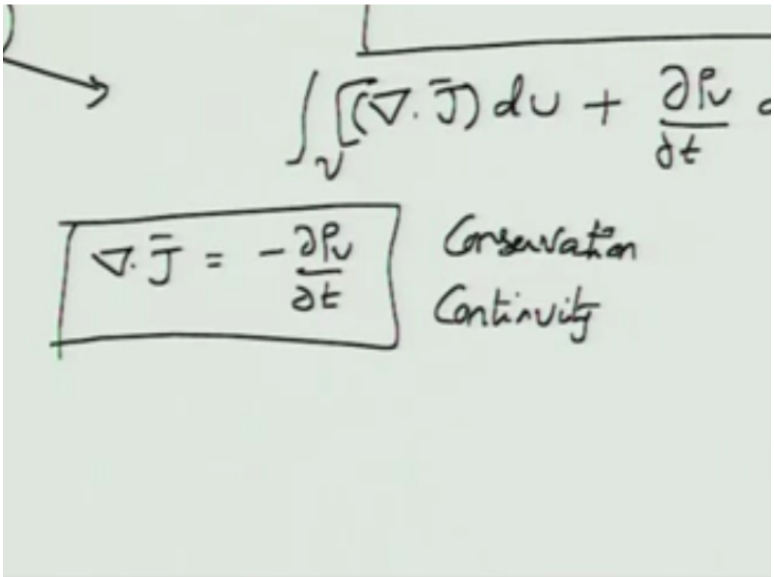


Now I know how the charge enclosed by a volume is related. If I know the volume charge density rho V, then the charge inside can be written as minus integral of v, dv correct? So there is a del by del t o that because now rho is a function of this space coordinates. So I need to replace this expression minus dQ in by dt as minus integral del rho v by del t by dv. Okay, this is a crucial relationship.

This surface integral of J over this closed surface is actually equal to this change of volume charge density. Charges inside are getting reduced. So this is the surface that we were considering, the current density outside is actually non zero indicating there is a divergence of J that is non zero, okay? That could only happen when the charge density inside is actually reducing.

So as the charge distribution reduces, the charges are essentially flowing outside and you are getting J vector to be non zero, or more precisely the divergence of J to be non zero. I can apply divergence theorem to this left hand side. Remember, I can apply divergence theorem to the left hand side and convert this into a volume integral. So J dot ds becomes delta J which is a divergence over the volumes plus del rho v by del t over the volume is equal to zero, after bringing the right hand side to the left hand side.

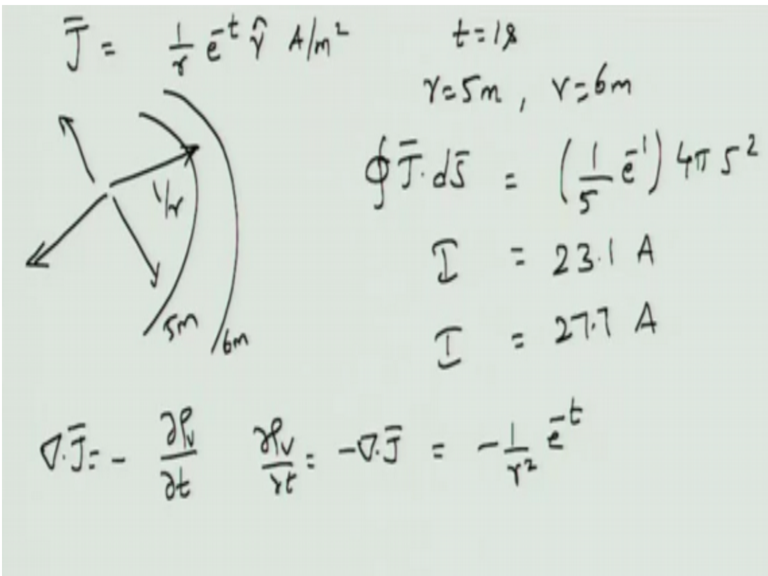
(Refer Slide Time: 19:37)



Now if there is a volume integral that is going to zero, the only way that can happen is when del dot J is equal to del rho v by del t indicating that the charge distribution, changing charge distribution is the source for divergence of J, okay? This is called as the conservation or the continuity equation, okay? And this is very very important.

You will see that this equation or this conservation law plays a very important role when you have to leave electric field and the magnetic field which are varying with time. The only way you can bring about a relationship between the two is when you impose the conservation equation or the conservation law.

(Refer Slide Time: 20:21)



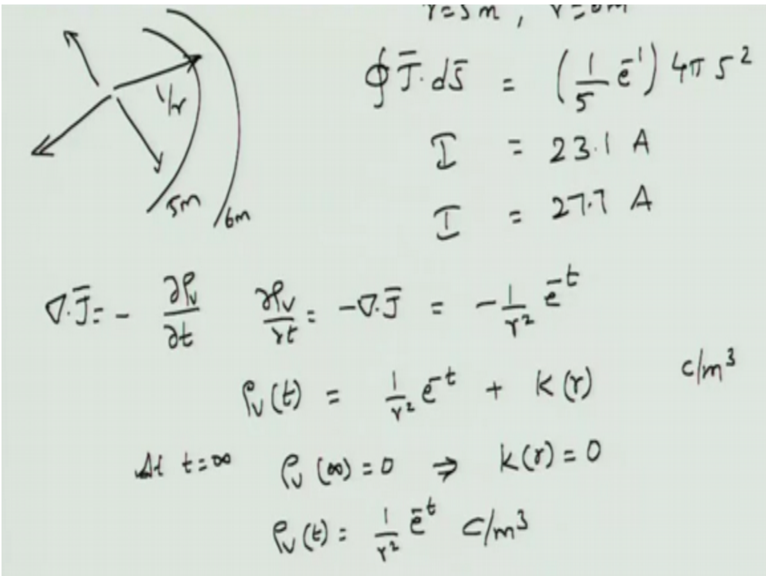
Just for a sake, let us do a small example. Let us assume that J is equal to 1 by r e power minus t ampere per meter square along r. So it is radially going away as 1 by r. The J vectors are going as 1 by r along radial direction and they are changing with time by an exponential value, e power minus t. Let us say, t is equal to one second, I want to find out what is the current at radius r equal to 5 meter.

This is 5 meter and something at 6 meters at radius of 6 meters. Let us do this problem, this is simply integration of J over the closed surface, in this case surface is a sphere of radius 5 meters and 6 meters, so this becomes 1 by 5 e to the power minus 1, this is the J vector and multiplied by 4 pi 5 square because 4 pi r square is the surface area of the sphere, correct? So if you do this one you will see that at 5 meters the current is 23.1 amps, okay?

If you do the same thing at r equal to 6 meters, the current will be 27.1 ampere. So the current is actually increasing as we go away and away. Well, what could be the reason for that? This is 2.7 ampere. Now we will see the reason, but before that let us look at how the volume charge density is changing.

I know that del dot J must be equal to minus del rho v by del t, so clearly del rho v by del t must be equal to minus del dot J and you can evaluate the divergence of this J. If you do that one you are going to get minus 1 by r square, e power minus t, this is the scalar of course. So this is the charge density as varying with respect to time.

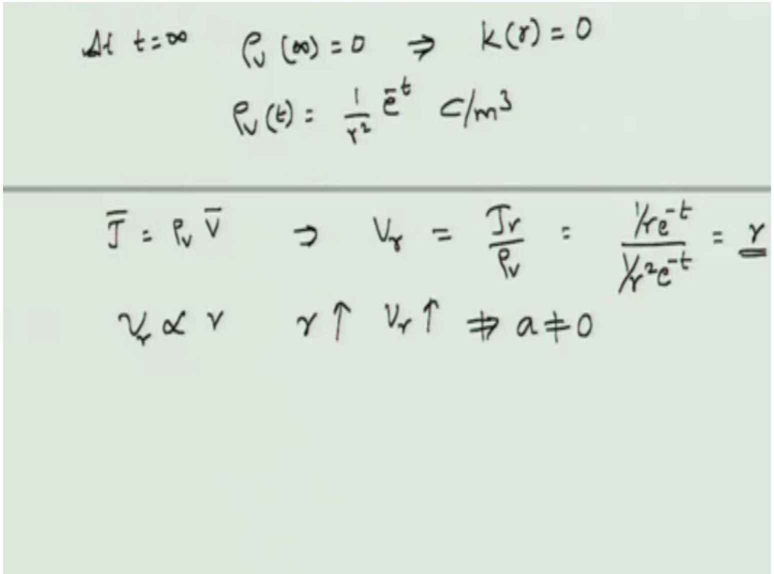
(Refer Slide Time: 22:17)



If you look at what is the overall charge variation with respect to time, you have to do that, you have to obtain that by integrating this one. Okay, integrate this thing and you will get integrating this fellow you will get 1 by r square, integrating with respect to time of course, 1 by r square e power minus t plus some constant of integration k of r measured in coulombs per meter cube of course, right?

At t is equal to infinity if I assume that all the charge has been depleted which means rho v at infinity is equal to zero I can show that k of r will also be equal to zero. So the volume charge density inside this material is given by or inside this sphere at any radius r is given by 1 by r square e power minus t, coulomb per meter cube. What would be the velocity of this charge carriers?

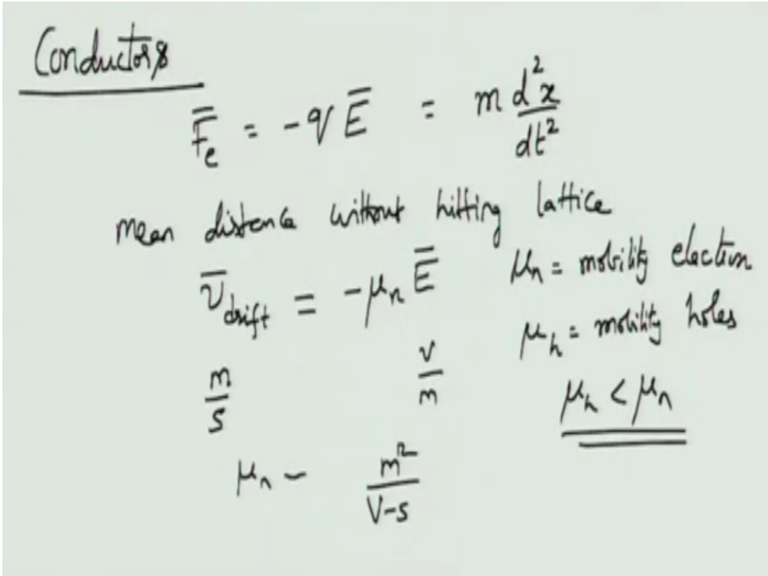
(Refer Slide Time: 23:15)



I know that J is equal to rho v times velocity v, v is the velocity. Clearly J is along the radial direction, so v must also be along the radial direction. I get the radial velocity vr is equal to J along the radial direction divided by the charge distribution that would be equal to 1 by r e power minus t divided by 1 by r square e power minus t which is equal to r.

See, velocity is directly proportion to r, indicating that if r increases velocity vr increases and this implies that the acceleration will be equal to not zero. S which means that the charges are getting continuously accelerated. There is some force which is accelerating, that force we have not calculated and this is the reason why the current is increasing as we keep increasing the radial distance. So we have looked at enough about electric current.

(Refer Slide Time: 24:21)



We will now resume with conductors. So we will go back to conductors and we will start talking about the conductors. Well, conductors as we have already discussed has lot of free charges and these charges experience the force, when you apply an electric field, this charges experience a force which is given by minus q times E, E charge experiences this particular force and you might think that they would start getting accelerated.

So if I now equate this one to m d square x by dt square where x is the position of the electron from its equilibrium position. I am measuring the displacement x, then according to this equation the acceleration must now begin because of the force that is being applied. However, what happens is that, in a metal there are these lattices, right? And these lattices all consists of positive charges.

So the electron begins to accelerate leaves about in a direction that is given by the electric field and then gets attracted or gets hit by the lattice and then slows down. Again it begins to move, hit the lattice and slows down. It keeps on doing this. There is actually a mean distance that the electron would actually travel without hitting the lattice. Without hitting the lattice it would actually move a certain amount of distance.

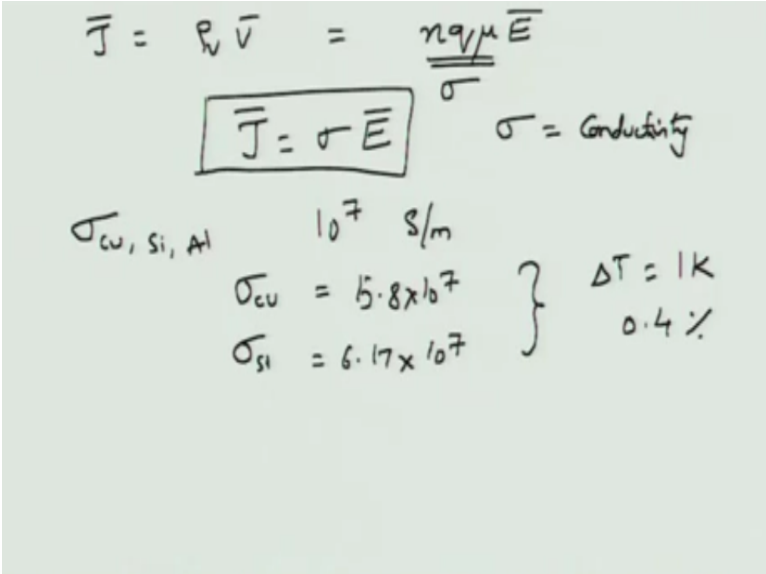
This is so called mean free path of the electrons. So electrons begin to accelerate, hit the lattice, slow down, begin to accelerate hit, slow down and so on. So I essence what happens is that there is an average velocity that is developed which would not be the same velocity if the electrons were in free space. If the electrons were in free space and if the electrons were to be accelerated by the application of electric field it would simply begin to accelerate and you can obtain that by following this expression, the Newton's law.

However, inside a lattice, inside a material medium, it begins to move, hits and then slows down, giving you an average drift velocity. This is still large but it is not as large as the velocity in free space and this average drift velocity for an electro is given by, interesting that it is actually proportional to electric field and that proportionality constant is called as the mobility of the electron.

Mu n is the mobility of the electron. Although we are not going to discuss semi conductors in this course, in semi conductors there is actually what is called as holes, for all practical purposes holes behave as positively charged particles. They would also begin to accelerate except that their acceleration would be in the direction of electric field, whereas the electrons would move in the direction opposite to the vector field and the whole mobility is also important, especially when you are studying optoelectronic or VLSI microelectronic circuits, you will see this mobility of the holes and there is a corresponding hole velocity.

It turns out generally that mu h is much less than mu n and this is the factor which actually reduces the speed of most of this ICs, okay? It is the fact that holes cannot move as fast as electrons that is responsible for the clock speeds, in some sense, it is not that that is the only factor but in some ultimate sense that is the reason. So we are only looking at electrons inside a conductor, we are not looking at this one.

So what would be the units of mobility of the electron? Drift velocity is in meters per second, electric field is in volt per meter, so mobility must be measured in meter square per volt second. So this is the mobility units and for copper and other this mobility is quite high. Now we have taken the minus sign here. This is indicating that we are considering electrons if it was positively charged particles you could drop the minus sign. The force would be along the direction of the electric field and the mobility would be that of the appropriate charge carrier.
(Refer Slide Time: 28:38)



Now we also know that the current density J is given by the volume charge density at any point times v, okay? We also have seen that this rho v is given by n q mu e, right? The volume charge density rho v itself is given by number density n multiplied by q and v is the drift velocity which is mu multiplied by E. I have dropped the negative signs here, I am considering only the positive signs because conventional current is defined in terms of the positive current.

So this quantity n q mu is called as the conductivity of the material sigma and is denoted by sigma. So we have this so called ohm's law. This is not really a law. This is just an empirical observation and this conductivity is the parameter that determines how much current is produced in response to the applied electric field, okay? For copper or silver or aluminium, these qualities are all in the order of 10 to the power 7 Siemens per meter, okay?

Copper is slightly less, this is about 5.8 multiplied by 10 to the power 7 and then for silver this is around 6.17 multiplied by 10 to the power 7. You might think that these two are constants but they are actually not constants. They would actually increase with temperature. So if you increase the temperature by 1 Kelvin then the corresponding copper and silver conductivities would also increase by 0.4% for every 1 Kelvin rise in the temperature.