Electromagnetic Theory
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Lecture – 26
Capacitor - II

So in the previous class we started discussing capacitors, we calculated the capacitance of a spherical capacitor of concentric spherical capacitor by using two methods. First method was Q-method, in which we assume a certain charge distribution, a reasonable charge distribution on the conductors that constitute the capacitor. And in the second method, we called as V-method, we solve the Laplace's equation and find the electric field.

Find the total charge that is contained on the conductor and the ratio of those will give us the capacitor value, the capacitance of the capacitor.

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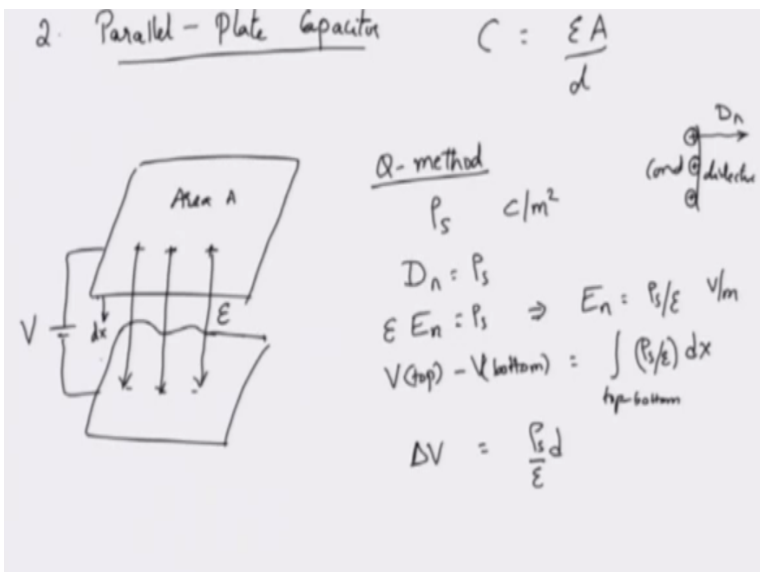

Here we will continue to find capacitance of some practical structures. The capacitance that we are going to consider now is, that is something very familiar to you guys this is called parallel plate capacitor. Parallel plate capacitor is something that you are familiar with whenever you talk of capacitor even those who don't necessarily do this course or will never do this course in their lifetime will still proudly have an idea of what parallel plate capacitance would be.

The formula for that is fairly simple that you can remember it, it is Epsilon A / d, where A is the area of the plate of the capacitor and d is the separation between the two plates. So let us see where this famous formula comes from and also see whether this is actually valid or not valid or if it is valid, how much is this valid? So parallel plate capacitor simply consists of two parallel plates.

So, it consists of two parallel plates of same area and make both of them of the plates are composed of conductors which conductivity Sigma, which can be thought of to be infinity and then you apply a potential by a battery by connecting the two plates. Now because of this potential that is applied we know that there will be charges induced on the top and bottom plates. The charges are induced from the top and bottom plates.

And there will be an electric field from the top plate to the bottom plate. So, if we start looking the expression for capacitor, the capacitance of this structure, let us first consider the Q-method. In the Q-method, we will have to assume a certain charge distribution. So reasonable charge distribution that we can assume is that of a surface charge density Rho S. So we assume a surface charge density of Rho S Coulombs per metre square on the upper as well as the lower plates of the capacitor.

Now, from boundary condition that you have seen, we know that there will only be normal component of the electric field from the top surface or the top plate. So, if you recall what the boundary condition was you have a conductor and you have free space or dielectric, of course we will assume that there is a dielectric Epsilon sitting in between these two. So conductor and dielectric we have already seen from the boundary condition that there will only be the normal d component.

And this normal d component will actually be equal to the surface charge density on the surface of the conductor. So, we have seen this from the boundary condition. Now, we also know that D n will be related to the electric field, normal electric field. D n is equal to Epsilon E n. So this will be equal to Rho S which gives me a uniform electric field, which is all normal to the plate and this is given by Rho S / Epsilon volt per metre.

Now, what is the potential difference between the top and the bottom plate? So potential difference between, say the top and the bottom plate is obtained by the line integral and this line integral is that of the electric field Rho S / Epsilon times some dx. If you call this as x is equal to zero and this as some x is equal to d, then the line integral, which you are taking will be dx and an appropriate path is what you are considering so say from d to zero.

And the potential difference will turn out to be as Rho s / Epsilon * d. I am not distinguishing between whether this is zero to d or d to zero, because simply I am interested in the potential difference not in the exact potential of the top and the bottom plate. So, for the configuration that we have written in this way, the potential difference will be from the top to bottom plate will be equal to Rho S d / Epsilon.

So, if you are connected the positive supply to the upper potential, then the upper plate will be at the potential of Rho S d / Epsilon with respect to the bottom plate. So, this is the potential difference. Now, you already know what is the charge density? Right?

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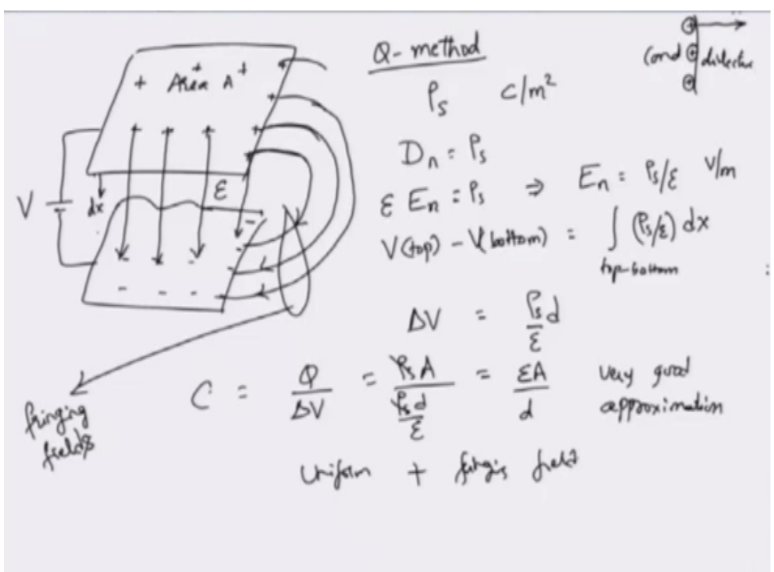

The ratio of these two capacitors can now be obtained by Q / Delta V, where Q is the charge that is contained. So, how much charge is actually contained? If the plate has an area of A, the total charge contained will be equal to Rho S * A. So, if there is a uniform surface charge density, here on the upper plate as well as on the lower plate, the total charge contained will be equal to the surface charge density multiplied by the area.

So, this will be equal to the Rho S into A. Now divide this one by the potential difference that exists between the plates which Rho S d / Epsilon. Rho S clearly cancels on both numerator and denominator and you will be left with Epsilon A / d. This is the formula for the capacitors that we were looking for. Unfortunately, this formula is wrong. Now, we actually don't really say that it is wrong.

It is a very good approximation. This is very very good approximation and something that is very handy to use to the actual capacitance of a parallel plate capacitor. To see why this is not exact but this is approximate; you have to see what is happening at the edges of the plate. For reference I am showing you what is happening at the edges of one of the top plate and the other edge of the bottom plate.

But you have to remember that these edges are, there are 4 edges. So at all edges this is precisely what is happening? So the field is uniform in the centre. No doubt, the field line start from the top plate and they would drop down to the bottom plate. This is perfectly alright. However, the charges that are there at the edge, they are accumulated. So these charges not only give you the electric field in the direction downwards.

But there would also be the electric field in the tangential or in the horizontal direction. So the electric field lines would also start looking to have some non-zero tangential component and there would sort of – You know, go from the upper plate to the lower plate by following a curved path in addition to the uniform downwards path that we have shown earlier, which is valid in the centre of the capacitor. Now at the edges of the capacitor you have this curved path.

These curved paths or these curved electric fields are called as fringing fields. And any capacitance or any capacitor which has this finite area or you know edges, wherever there are edges there would always be this fringing fields. It is important that there is fringing fields are there only in the edges. So, if there are capacitors which have edges there would be these fringing fields. And fringing fields also contribute to the capacitance.

So, the one capacitance that you have seen is the capacitance in which you have place the uniform dielectric down here. But, there would also be a capacitor that would exist between these top and bottom places because of the fringing field. So, the capacitor is actually because of the uniform electric field plus the fringing fields that exists because of the edges, charges associated with the edges.

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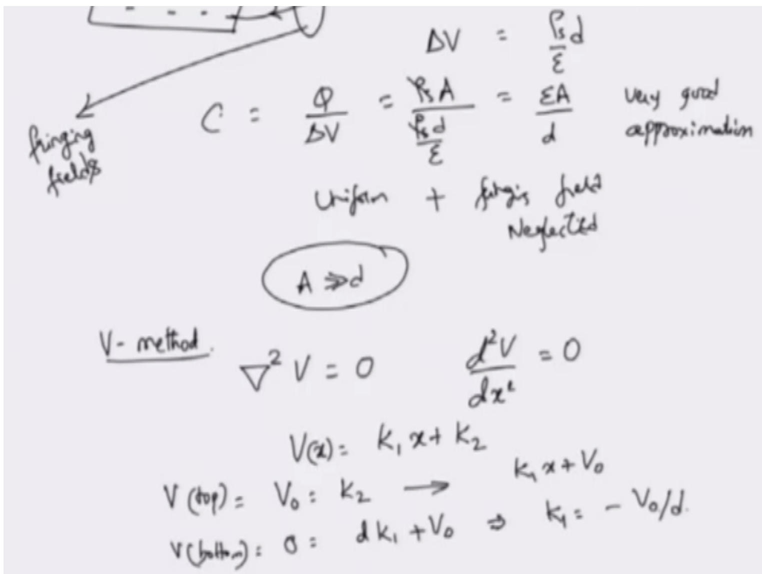

And in calculating the capacitance of the parallel plate capacitor we have completely neglected this fringing field. We are able to neglect this fringing field only when the area of the plate is very large compared to the plate separation. So, only when A is much higher compared to the plate separation d, then we can say that the capacitance is Epsilon A / d and that actually forms the very good approximation.

Otherwise this is not really true because fringing fields have been neglected. It turns out that to include fringing fields in a close form expression is very difficult. Therefore, what we normally do is, we actually use a numerical method such as method of moments, a popular method to find the fringing fields. We calculate the actual charge distribution that happens by taking into account the edges.

And from there calculate the charge enclosed, calculate the potential difference and calculate the capacitance. And it can be shown that those calculations that we do actually much more, I mean, correspond much more closely to the experimental values. This Epsilon A / d is valid as long as A is much larger than d. Say A is about 10 times the separation, then this expression is very good. Ok, this was for the Q- method.

How do we actually use another method which was the V- method to calculate the capacitance? To obtain V- method we need to solve Laplace's equation. So, again going back to the assumption of neglecting fringing fields and assuming everything to be uniform the Laplace's equation solution, in we can look at the solution in Cartesian co-ordinate system you know the plates are assumed to be in the Cartesian co-ordinate system described by the Cartesian co-ordinate system.

And since the field's lines are all uniform and going only in a particular direction, I can take this as with only one variable. So, I have d square V / dx square is equal to zero. And that is only one variable that is necessary for me. How do I solve this? Well integrate this one twice, so V of x will be equal to first integral will give you some constant K 1. So that will be dV /dx is equal to K 1. Second time you integrate; you will get K 1 x + K 2.

Again you have V at the top, which is valid at X = 0, at the top plate is the applied potential difference or the assumed potential of V 0. So, V 0 will be equal to K 2 and V and at the bottom, since K2 is equal to V 0, then it becomes K 1 x + V 0. And at the bottom surface x is equal to d. So the potential is zero there and this would be dK 1 + V 0 which implies that K 1 is equal to -V 0 / d. Is that correct?

So dK 1 + V 0 is equal to zero. So, dK 1 into d is equal to -V 0. So, therefore K 1 is equal to -V 0 by d.

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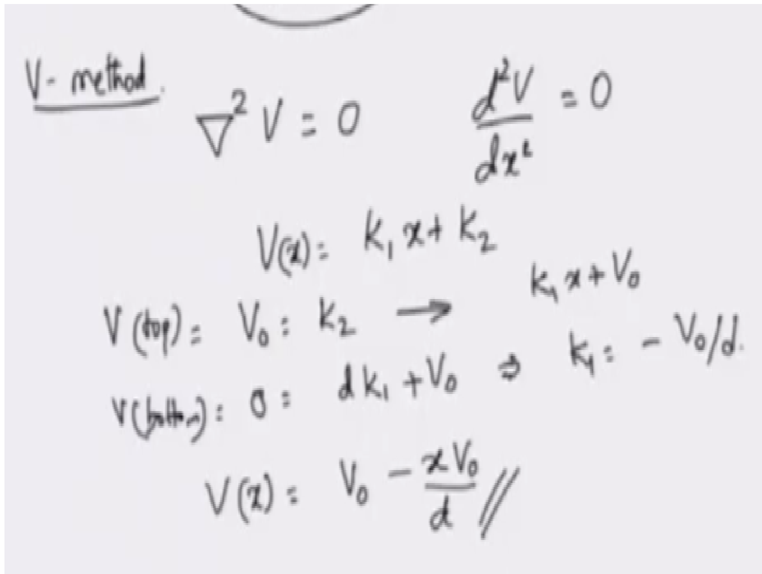

So if you can put these two into the solution, so you will get V of x as V 0 - x V 0 / d. So, at x equal to 0 these are the potential V 0 at x equal to d, these are the potential of 0. So, this is the potential function between the two parallel plates and you can see that the potential is linear. You will see the linear potential whenever there is uniform electric field, whenever electric field is not uniform only then you will see some different potential that is not a linear potential.

In this case the electric filed assume to be uniform therefore potential is linear and it is given V 0 - x V 0 / d. What is the electric field?

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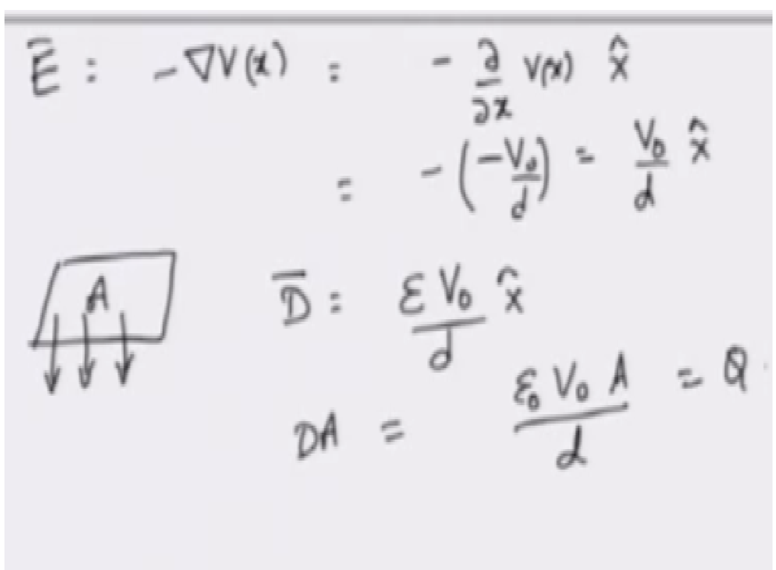

Electric field, you did not have to do anything. It is actually uniform. And what is the electric field? Electric field is – Del V of x that is gradient of x. Gradient will give you - Del / Del x of V of x along X direction. And if you differentiate this quantity with respect to x you will get - V 0 / d, there is already another minus because of this minus condition, so it actually becomes V 0 / d into x hat.

So, this is a uniform electric field directed downwards along the X axis and that is given by V 0 by d. Next what we should do? In order to compute the potential, I need to know the charge distribution. So, to obtain the charge distribution I can consider the plate of the area A. I know what the electric field that is coming down here is, so I also know what is D field is now. D field will also be along X direction.

And it would be given by Epsilon V 0 / d, because d is equal to Epsilon E. So, this will be D equals Epsilon V 0 / d along X direction itself. The total charge enclosed will be given by the magnitude of D into A, because D field is everywhere perpendicular to the surface area of the top or the bottom plates. So this will be equal to Epsilon 0 V 0 A / d. This is the total charge that is enclosed.

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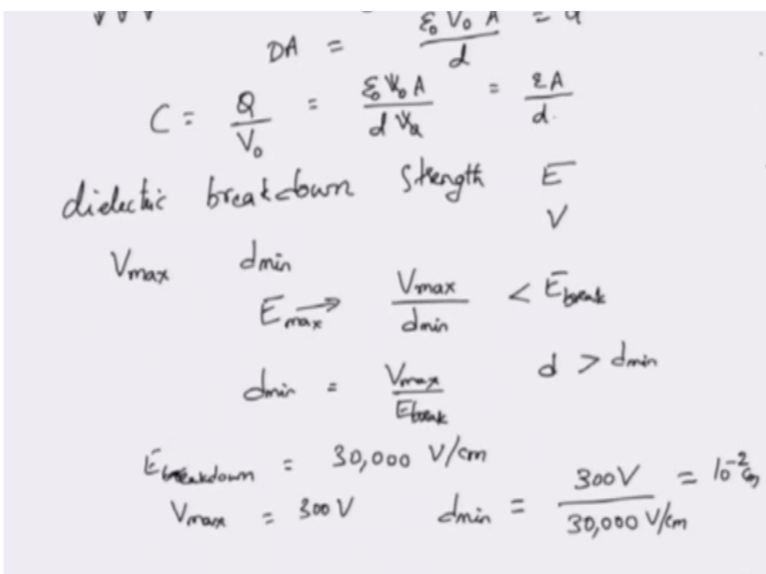

Now capacitance is given by, charge divided by the potential. The potential applied is V 0. So, clearly this will be equal to Epsilon 0 V 0 A / d divided by V 0. V 0 cancels on both sides and you are left with Epsilon A / d. So, this is the capacitance that we have looking for earlier in the

Q-method and we actually obtain the same result using V-method. Now it is interesting that when you look at the ratings of a capacitor, capacitors are given a certain rating, in terms of a voltage.

That is a maximum voltage that you can apply across a capacitor. If you apply more than the voltage that is rated, then there will be arcing between the capacitor and the capacitor simply gets damaged. Why is the capacitor getting damaged? What happens is that, for every dielectric that you consider there is a certain breakdown electric field.

A breakdown electric field is one if you apply more than this electric field, which is more than the breakdown electric field what happens is that the electric field becomes so strong, that it actually rips apart the molecules of the dielectric. In the sense that it ionises the dielectric and dielectric will not be a dielectric anymore. So, it becomes into a different kind of a matter. So, there is always a certain maximum electric field that you will have to apply to any given dielectric.

And this maximum electric field is called as the dielectric breakdown. Because beyond this, the dielectric simply breaks down. So, there is a dielectric breakdown strength. And since electric field is related to the voltage you can either quote the breakdown strength in terms of the electric field E or in terms of the maximum voltage that you can apply. So, what is that got to do with this parallel plate capacitor? Well, there is a certain maximum voltage that you can apply.

So let us call this is as the maximum voltage. The maximum voltage will be reached and assuming that you want to fabricate a capacitor with a small value of d. Why do I want to have a small value of d? So that, I can fabricate larger values of capacitance. See capacitance is universally proportional to d for a parallel plate capacitor. So as d becomes smaller the capacitance increases.

So, I can make it smaller but I cannot keep making it smaller because as d becomes smaller V 0 / d which is the electric field that becomes larger. So, if d is at its minimum value then the electric field associated with that which will be the maximum is given by whatever the applied field, let us say the applied is also at its maximum and this is given by V max / d min. And this quantity has to be less than that breakdown electric field.

This actually puts down the limit and how much d min you can use. The d min should be equal to V max / E break. Indicating that d, the separation should always be greater than this minimum value. For example, consider the breakdown strength of a given dielectric as about 30,000 volt per centimetre. And let us say that the capacitor is rated to work at 300 volts. So the capacitor is rated to work at 300 volts.

And it actually has a breakdown electric strength whatever the material that fills has a breakdown electric strength of 30000 Volt per centimetre. Then, what should be the minimum value of the separation. The minimum separation should be the maximum voltage that the capacitor can withstand which is 300 Volt divided by the breakdown electric field strength of that dielectric, which is 30, 000 Volt per centimetre giving you 10 to the power -2 centimetre.

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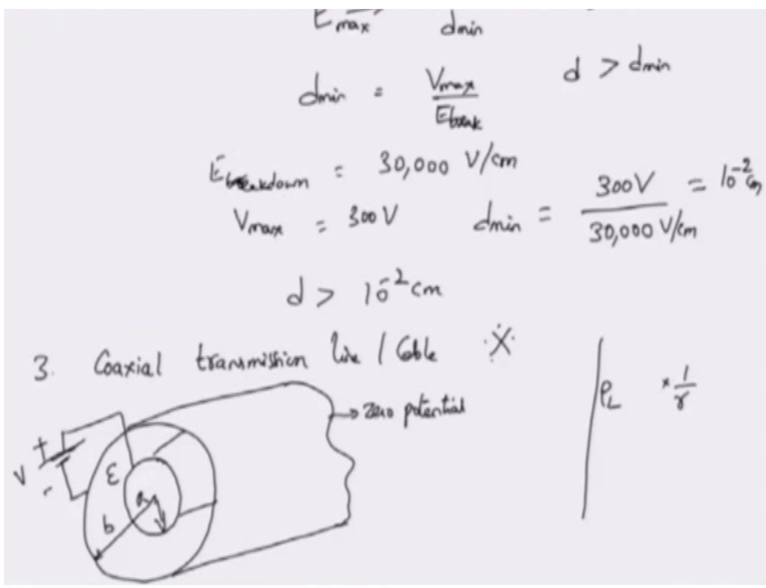

So your separation d must be greater than 10 to the power -2 centimetre so, as to avoid breaking down the dielectric and damaging your capacitor. Let us look at third and one of the most widely used transmission line or a transmission cable called coaxial transmission line or a coaxial cable. This is very important because for up to say few hundreds of megahertz one can actually use coaxial tables.

There are axial cables which can be used up to a few hundreds of megahertz and these are ubiquitous, you can see them in every lab whether you are connecting a signal generator to your circuit board, you are connecting 2 signal generators, you are connecting an oscilloscope to your circuit board, you would normally use a coaxial cable. At least at one end it would be a coaxial cable.

So, how do we calculate the capacitance of a coaxial cable? First let us look at the, what is the structure of a coaxial cable. A coaxial cable is composed of two cylinders one having a radius a and the other having a radius b. So, it is composed of two cylinders one having a radius a and the other having a radius b and the material in between is filled with a dielectric of permittivity Epsilon.

So, this is the coaxial cable as you can see, the cable is uniform along the length that we are going to consider and our objective would be to calculate the electric field, when I hold the inner potential at some value be zero and take the outer potential as the reference. So, outer is zero potential and the inner one is at certain applied potential. If you recall our discussion on electric field of a line charge, you would see that we actually solved this problem.

But we will not do that one. We will derive the capacitance fresh by calculating it using the Q-method or the V-method. The reason why we have, I wanted you to remind you of that we have already solved this problem is because when you consider a uniform line charge of some line charge density Rho L we found that the field at any point because of this line charge would be going as 1 / r, it would be proportional to 1/r.

And when you calculate the potential, you found that you could not choose infinity as the point of reference. For a point charge, we could choose infinity as the point of reference and calculate the absolute potential. For this line charge of certain line charge density, you could not use infinity as the point of reference. So if you remember the exact calculations that we did, we actually took the ratios of this some r1 to r2, we said, okay at 16.5cms I have a potential of zero potential.

And then at 10cms I have a potential of 5volts and then gradually I looked at all the equipotential surfaces. You know, if you remember that you would remember that, that was essentially a coaxial cable that we discussed. And the point that I am trying to remind you is that, I have to take one of those potential points from a certain distance from the inner charged cylinder as the point of zero potential.

And calculate all the potentials with respect to that point, not the infinity point as the point of reference.

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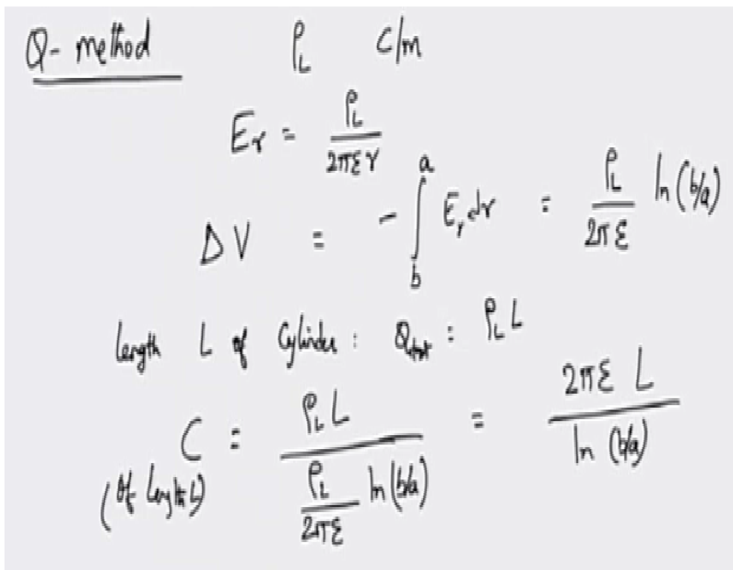

With that remembering thing in your mind, let us use the Q method to calculate the capacitance. So, we don't completely forget the fields of a line charge. We will assume that the inner cylinder is now induced by a line charge Rho L Coulomb per meter. A similar minus Rho L Coulomb per meter would appear on the outer cylinder as well. So, with this Rho L Coulomb per meter, what could be the electric field?

I am not going to derive this, the electric field will be Rho L/2pi epsilon r, hopefully you know how this electric field is obtained, this is radial electric field that we just described. What could be the potential V? The potential V between the two points is also given by, let us say since I want the potential difference between the two points, delta V is given by integrating this Er from outer surface to inner surface with a minus.

So what you get here is Rho L/2pi epsilon log of b/a. This also we saw in the last few classes ago. So, I seem to have everything. Now consider a length L of the cylinder. In this length L of the cylinder, what is the total charge that is enclosed by the cylinder? This could be Rho L into L. I made a small mistake, the limit of the integrally from b to a, I wanted to put the b negative sign in front of the integral, but I by mistake put the negative sign in the limit itself.

So, this is integral b2a and you can calculate this and you will see that this could be Rho L/2 pi Epsilon log of b/a. Now coming back to this length L of cylinder, how much charge is contained in the length L of cylinder, it would be Rho L into L. This is the line charge density Rho L. At times Rho L will be the total charge that is contained. So the capacitance will be equal to Rho L into L of length L.

So capacitance of length L will be equal to Rho L into L divided by Rho L/2pi epsilon log of b/a. So, 2pi epsilon goes to the numerator, Rho L cancels with both numerator and denominator, Rho L cancels there. And we get 2pi epsilon L/log of b/a.

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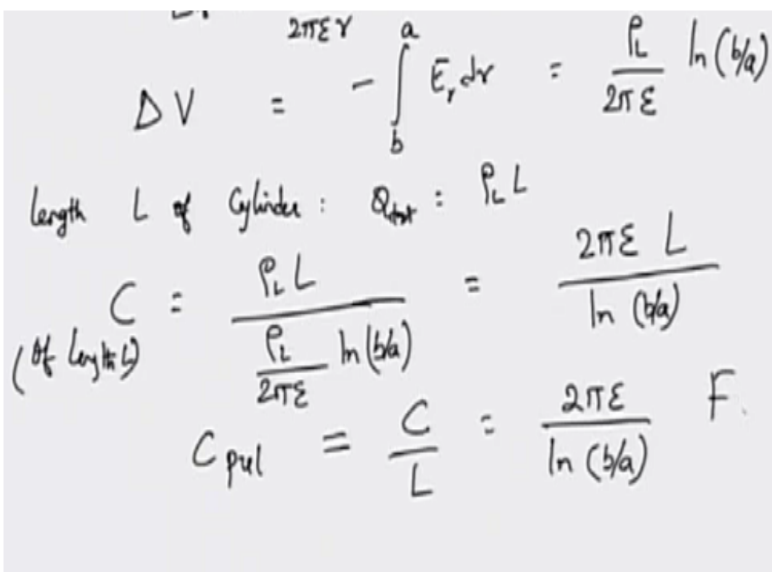

However, in such structures, which are mainly used for transmission lines, you are not interested in that capacitance of a given length, you are interested in capacitance per unit length. Because these capacitances per unit length, inductance per unit length, resistance per unit length and conductance per unit length are the parameters that we used to describe a transmission line. So, capacitance per unit length is capacitance divided by L that is capacitance of length L divided by L and this is given by 2pi epsilon/ log of b/a farads.

A fairly simple formula again to remember. Of course, here again the formula is not completely correct because we assume that all of the charge distribution what essentially a line charge distribution of zero extent and it was all sitting nicely in the center of the cylinder. And that the cylinder was going all the way from zero to infinity or from minus infinity to infinity.

And that is how we say the capacitance per unit length is constant and it is independent of the length of the coaxial cable, which is again strictly speaking not true, because you do have to terminate the coaxial cable at some end. However as long as the length L is large and you are not looking at the edges of the terminating ends, then this capacitance per unit length expression is quite accurate. If not, well you will always have to use a numerical method to solve this problem.

So we will not look at the V method for this. You will get the same condition and it will turn out to be solving Laplace's equation in cylindrical coordinates. I will leave that as an exercise to you guys. Okay? Now, we will consider a situation that is slightly more difficult than what we have been discussing, but at the same that is very, very important practically.

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This is that of a two parallel wires.