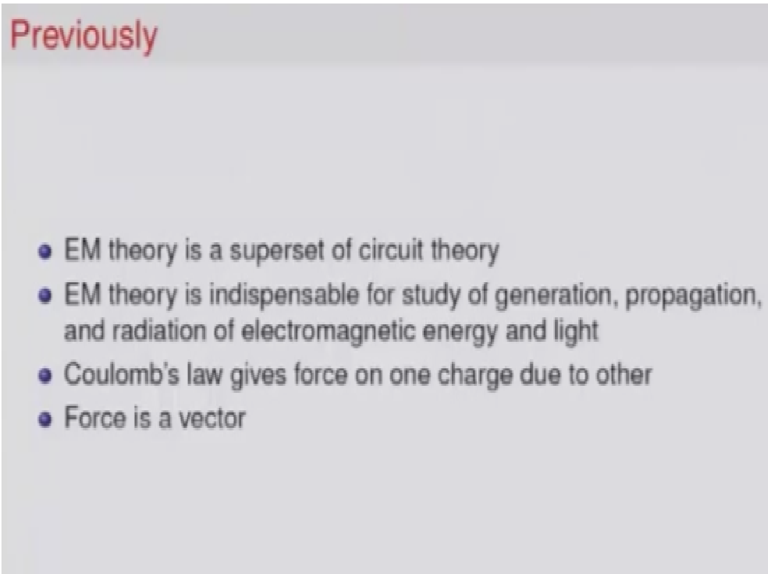
Electromagnetic Theory
Prof. Pradeep Kumar K
Department of Electrical Engineering
Indian Institute of Technology – Kanpur

Lecture-03
Vector Analysis-I and Introduction to Co-ordinate System

So previously we talked about the requirement of electromagnetic theory. We said that there are certain physical phenomenon, which electromagnetic theory will explain and circuit theory will be inadequate for these purposes. So one can think of electromagnetic theory as some sort of a super set of circuits, okay and electromagnetic theory is also indispensable, if you are going to study microwaves, radar, antennas, fibre optics, optical communications and so on.

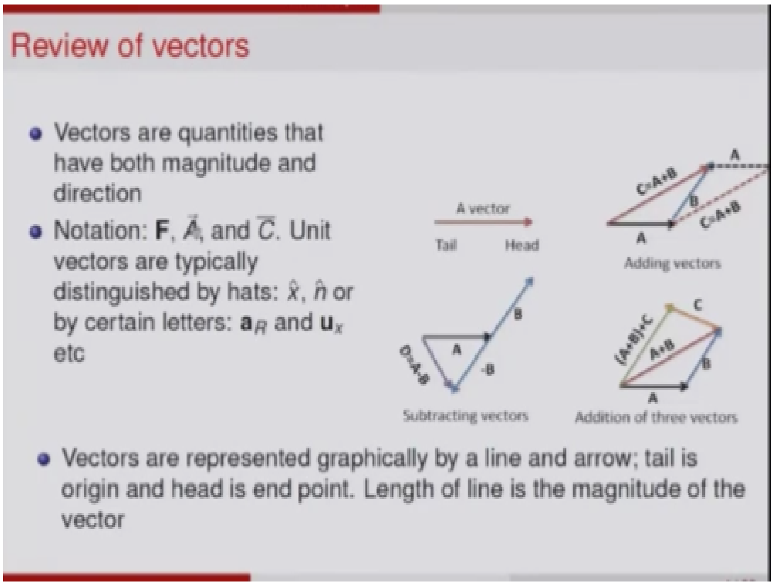
The primary goal of all these different areas is to somehow harness electromagnetic energy and put that one into better use. So electromagnetics in one sense deals with generation, propagation and radiation of electromagnetic energy as well as light. We will see later that, light and electromagnetic waves are one and the same and this union of electromagnetic theory and light was achieved by Maxwell's equations by Maxwell.

(Refer Slide Time: 01:19)



We also studied Coulomb's law, which gives us the force exerted by one charged particle and onto other charged particle and we have seen the forum of this force, it requires two parameters to specify; one is its magnitude, which is the amount of the force that is exerted by the particle on another particle as well as, we have to specify the direction along which this force acts. So force is the quantity which is slightly different or radically different from charge.

(Refer Slide Time: 02:01)



In terms of the fact that force requires, both magnitude as well as direction to specify, whereas charge can be specified only by giving its value, number. Okay. So quantity is that require both magnitude as well as direction are called vectors and we denote a vector in our hand written note by using a capital letter, okay to denote a vector and then we place a small arrow to distinguish that this quantity, the capital letter A, with its arrow is different from just a capital letter.

If you don't write an arrow, what it means is that? It is a scalar. It can be specified by the number okay, whereas if you write an arrow that becomes a vector. This is the hand written notation, sometimes a different notation is used, in which you take the capital letter and instead of putting an arrow, you put an over bar. Sometimes you also see people putting an under bar, but that usage is quiet rare.

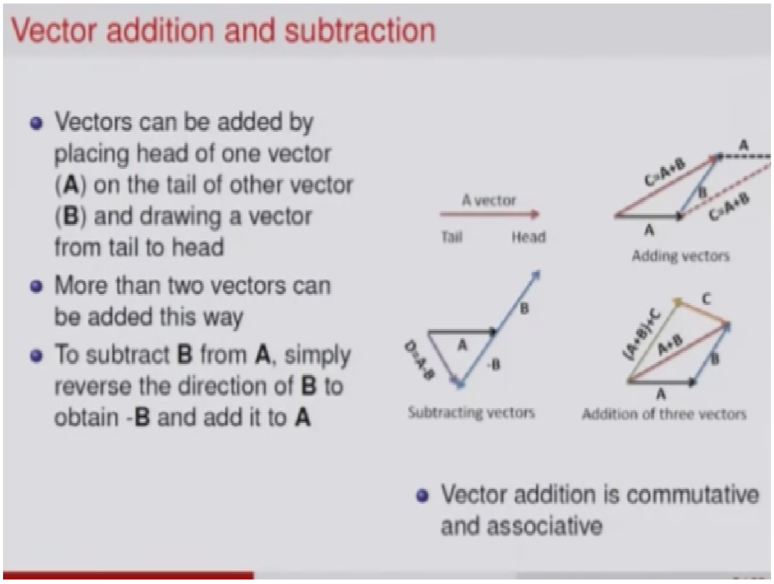
In text and printed materials, you would find vectors denoted by a bold phase or a bold print with a capital letter. So here you can see that this f, vector f which could represent a source or some other vector quantity is printed in bolt. We also denote unit vectors; we will discuss what unit vectors are later. So we denote unit vectors by putting a hat or a caret on top of the letter. These letters could be small letters, for example, here x hat indicates the unit vector in the direction of x coordinate.

n hat indicates a unit vector in the direction of n or you sometimes find notation by putting up a bold phase small letter and then putting a subscribed to indicate the direction of a component. So for example, aR, a, being the small letter but bold phase, indicates that this is the unit vector and this R subscribed to this letter a, indicates that this is the unit vector in the direction of R. Similarly, you will also find sometimes ux, written with u standing for unit vector.

Again a small case letter bold and x standing for the direction of the vector along which this vector is acting. Now a vector is defined graphically also by giving 2 points, one is the tail or the origin of the vector and the other is the end point or the final point of the vector or the arrow head of the vector. So this, for example is a vector. You have to distinguish between a vector, which is directed in this way.

Now if I draw an another vector that would be pointing in a different direction, these 2 vectors will be different from each other, okay. Now you also can see that we can perform algebra with these vectors before we can go to the algebra, let me just point out that in the graphical representation of a vector, the length of the line stands for the magnitude of the vector, okay. So if I have a shorter line with an arrow, that would indicate a smaller magnitude vector.

(Refer Slide Time: 04:59)



This would indicate a larger magnitude vector in relation to the smaller one, okay. Now we want to consider addition, subtraction and other operations on vectors, for now let us focus only on addition and subtraction. We will come to different operations with vectors later, so we can add 2 vectors, something that is familiar to you from your undergraduate studies, so you have 2 vectors and how do we add them.

So this is the recipe for adding, okay. I will not go into the justification of how you obtain this one. I hope that you will be reading up on the reference book or you can recall from your earlier studies. You start with the vector; 2 vectors that need to be added. So you have a vector A, which is pointing horizontally in this way and have the certain magnitude, okay. You also have a vector B, which is pointing in a direction that is given by this blue line, okay.

It also has the certain magnitude, which is given by the length of this line. Now what is the resultant? Or what is the sum of these 2 vectors? Now before we go to the sum, you have to understand that how did I get this vector B? okay. Vectors, when you are to perform the sum or the difference of these vectors, have to be defined from a common origin point, okay. A common origin point for the 2 vectors A and B is for example, this tail. Okay.

So, tail at this point will be the common point for A as well as B. So, which means that the blue line vector B should actually be defined parallelly over here, okay. This is the 2 vectors which are acting and the vector C, the red coloured arrow is the resultant of A and B, but to calculate the resultant or to calculate the sum of the 2 vectors, what I have done is; to take these vector B and translate it parallelly to get this vector B, which is the same vector.

But this vector has its tail at the end point of A. So this is the first recipe, you can actually translate a vector parallel in a plane that you want. The 2 vectors are mathematically equivalent, however physically they may not be equivalent. For example, I have this object here, okay, now I apply a force along this direction, so now you can see that this is the starting point of the force and this is the end point of the force, let us assume that.

This vector is making contact with this object at this point. So this is the vector that is acting at this particular point. Now mathematically you can represent this vector by a line with this as the origin and this as the end point and that could be a vector, that would look like this. Now you can translate it parallelly, mathematically a vector, that is, that I am showing here and a vector here, both are equivalent, however physically these 2 are not at all equivalent.

This vector is not all acting on these object or you could translate it down, okay in the same plane, again the 2 vectors are mathematically equivalent, but the 2 vectors are not physically equivalent, okay. The reason why I wanted to specify this is because, we sometimes get carried away by mathematical operations, okay, you can actually translate a vector parallelly along a plane.

But then you have to always constantly remember that these vectors are not physically equivalent, however mathematically you can translate them, you can add, you can subtract and the resultant vector will always be correct, okay. So with that aspect in mind, you have a vector B, which was originally defined from the common point of the 2 vectors A and B having been parallelly translated over here, okay.

So we have parallelly translated the vector B. Now I want to find the resultant of A and B, this is the second step of the recipe. The first step is to parallelly translate a vector, the second step is to now add the 2 vectors, how do we add the 2 vectors? The sum of the 2 vectors, which in this case, sum of A and B is C and this is given by a vector, which has its origin at the tail of A and at the head of B. So you can see that the red arrow goes from the tail of A and all the way up to the head of A, okay.

I have said that, you can translate a vector parallelly and they will be mathematically equivalent and you can see that over here as well. So I can take the vector A, I can leave the vector B as it is. Now I can take the vector A and translate parallelly to get a vector, which is this dashed vector, this is also labelled A, because mathematically this vector and this vector are equivalent, so I have translated this vector parallelly such that the tail of A is at the head of B.

Now I have one tail here and one head here, but you can actually now joint the tail of B to tail of A to get the resultant vector A+B, which is the sum of the 2 vectors A and B, but please note carefully that this vector is the same as this vector, because you can see that these 2 vectors are parallel to each other, okay. This is the law of triangular addition for 2 vectors and this is something that you will have to practice a little bit to get familiar with this sum or addition of the 2 vectors.

Because we will be looking at not just 2 vectors, we will be looking to add more than 2 vectors. Now how do I add more than 2 vectors? You do the recipe pairwise, that is; if I have 4 vectors to begin with, call A, B, C and D. First I find the sum of A and B, then I find the resultant of the sum with C and then I will add the resultant of these 3 vectors to the next vector D and I can keep doing this one until I get tired, right.

So let’s do that one for 3 vectors and you can actually see that how the process works, so you have a black line, which is representing a vector A, here is its tail, here is its end or the head of the vector A. To this vector, I want to add the vector B, okay, B is this blue line vector, which is now added so as to get the resultant A+B, so this A+B is the red vector over here, okay. So I hope this is all right, you have a vector A, you place the tail of the vector B to the head of vector A.

Then you draw a line from the tail of A to the head of B, so as to get the resultant A+B, okay. Once you have the resultant vector A+B, now I want to add to this A+B vector, a different vector called C, okay. So now C is the orange vector over here, I am going to add A+B to C, how do I do that? I have one vector A+B, if C is not originally at this place, you can translate C somehow, if c was a vector that was defined elsewhere on the plane, you parallelly translate the vector until the C tail reaches the head of A+B.

Now you can forget about A and B and you can simply concentrate on the 2 vectors A+B and C. I have A+B vector and C vector, you now draw a new line which goes from the tail of A+B to the head of C, okay. So this way you can add 3 vectors and this green line or the green vector is the resultant or the sum of all the 3 vectors, okay. So I hope this process is convincing, you can show that vector addition is commutative as well as associative by looking at A+B or B+A.

So you can show that A+B will also lead to the same vector C or B+A, see the blue line and the dashed black line will also lead to the same vector C, which is just a parallel translated version of this solid red line, okay. So vector addition is commutative, to show that vector addition is associative that is whether you start with A+B+C, if this is a sum of the vectors that you want. You can first find the resultant of A and B and then find the resultant of A, B; that is A+B and C vector.

You can do that one by going first to A and B to get A+B, to that you add C to get A+B+C. Alternatively you can start with B+C here, that would give you a vector from the tail of B to the head of C, okay and from here, you can then add a vector A, you will again get back to the same vector, you can try this an exercise to convince yourselves that vector addition is commutative as well as associative.

So far we have talked about addition of vectors, we will also need subtraction of vectors, we also need to subtract one vector from another vector, so how do we do? This operation, how do I subtract B from A, remember that subtraction of vector is equivalent to addition to the vector A, the negative of B, that is A-B is equivalent of A+(-B), okay. To get to that one, you need to know, what is –B? –B is a vector which has the same magnitude as vector B.

But it will be pointing in the direction opposite to that, it will be pointing exactly opposite to the direction of B and has the same magnitude, remember magnitude is the positive quantity, okay. So vector B, which is defined from this point along this direction, has the same length as the vector –B, except that vector B is pointing in this direction, whereas vector –B is pointing exactly in the opposite direction.

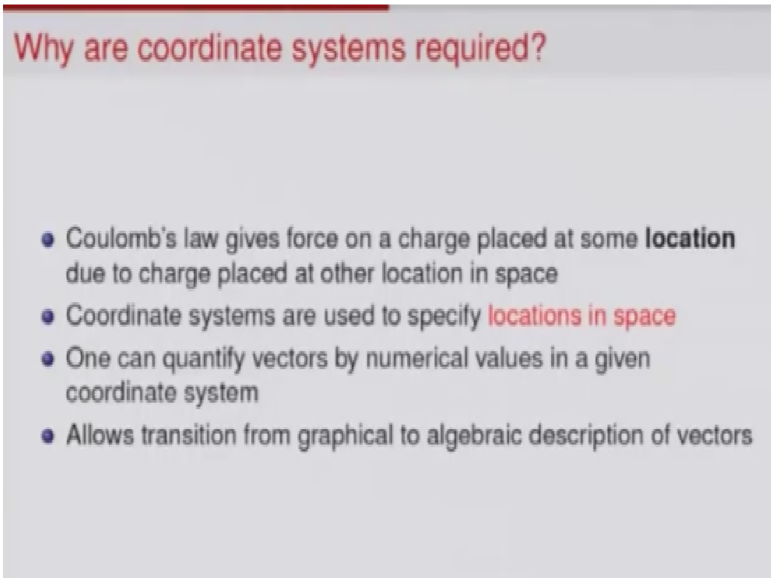
Now to add 2 vectors A and –B, is very simple, you take a vector A, you place the vector –B, the tail of vector –B at the head of vector A and then you draw another line from tail of vector A to head of vector –B, so you get A – B, which is you can call as a different vector D. I hope that you are convinced now that the vectors can be added and subtracted, now if you are asking or thinking about, how do I multiply the 2 vectors?

We will have to wait for that for some time, okay. We don't really need the multiplication at this point, we will wait for some time, before we tackle the subject of multiplication, okay. So far we have talked about vectors in a very very general way, okay. We drew some lines, then we added lines to another line and we found the resultant line, so we did all these things, however, this was all the graphical method of doing vector analysis or vector algebra, okay.

You might keep doing graphical methods of vector analysis, but then you will soon tired yourself out if the number of vectors involved is more than 3, okay. For example, in certain problems, you will be seeing that there will be charges which are say 10 or 20 placed around in a plane and if you want to add all these resultant force because of all the individual charges, so you will be adding 20 such forces may be all of different magnitude and different direction.

So it will become tiresome if you want carry out the graphical analysis, though graphical analysis helps you in visualising, how the vector would look? Okay. So we need an algebraic method of handling this, this algebraic method of handling vectors can be accomplished when we introduce a coordinate system, okay. So what coordinate systems do is that they help you specify the locations in space by numbers as well as describe a method in which a vector can be equivalently described by a set of numbers.

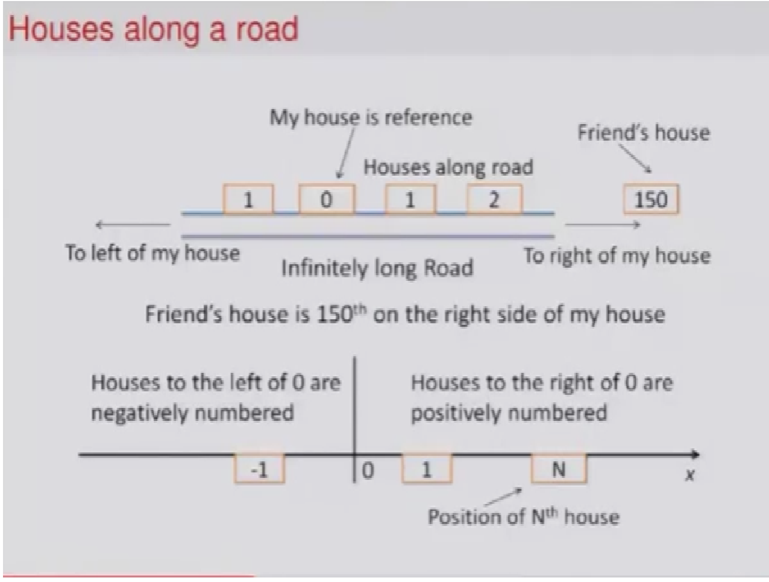
(Refer Slide Time: 16:24)



Now this is the great advantage, a vector which could be oriented in a random direction in the space, is now being described by a set of numbers and if you also find out how to add or subtract vectors with just these numbers, then we can not only do this as the algebraic exercise, we can also put in onto a computer, okay, input this to a computer, so that the entire thing can be computationally performed.

So you will actually save a lot of time, if you are able to associate numbers to vectors and coordinate systems are the ones which give you give you those numbers. Now before we proceed further, I want to clarify one very important aspect. Coordinate systems are independent of vectors in the sense that, if I change the coordinate system, the corresponding vector, the vector will not change, however the numbers that we associate with the vectors will change, okay.

(Refer Slide Time: 17:31)



This soon become apparent to you, we will go with some examples as you can see, but please keep in mind that coordinate systems and vector themselves are independent. The vector does not depend on the coordinate systems for its existence, however the numbers we attached to the vectors are dependent on the particular coordinate system that I have chosen or you have chosen, okay. But normally we choose certain standard coordinate systems and work with that, so that the minimal, there is a minimal amount of confusion.