Electromagnetic Theory

Prof. Pradeep Kumar K
Department of Electrical Engineering
Indian Institute of Technology – Kanpur

Lecture - 33
Introduction to Magnetic Field

So in the previous classes, we have seen electrostatic fields. We have looked at the different equations that govern how to obtain electrostatic fields. We have seen that if there are two charges, then one charge exerts the force on other charge. Of course, this charge is, this force is mutual in the sense that if you have two charges, both charges exerts forces on each other. We know how to calculate this force.

We have seen that coulomb's law allows us to calculate the force on one charge to the other. Instead of dealing with forces, we actually introduced a concept called a field, which we define as the force experienced by a test charge in the presence of another charge, in the sense that you assume that even when there is no test charge, there is some sort of a field of region or some sort of an influence around a given charge or a charged distribution.

And if you want to, of course this distribution by itself, even though there is an electric field, you will not be able to sense that electric field unless you place a charge in the region, where that the field may be appreciably stronger. So what we have replaced is this direct action at a distance between two charges or more than two charges by an intermediary agent called as electric field or more precisely electrostatic field because the charge configuration was not really moving with time or changing with time.

And we also introduce another vector to deal with the complications that would arrive in matter. So, when you want to consider matter, then you model matter in terms of induced dipoles because of the external electric fields. And these dipoles themselves will produce an electric field of their own and inside a material, whether the material is conductor or dielectric, we saw how the dipoles would interact in order to give a certain field configuration.

And we introduced two laws, one law was Del dot d equals rho, where d was the electric flux density vector. And in this point form what we have just said, it means that whenever the d field is not zero around a closed surface, then it means that there is some charge inside that surface. So this is the divergence, the physical significance of divergence. We also saw another law relating to the electric field that curl of electric field was equal to zero.

Because curl of electric field was defined in terms of the closed line integral of the electrostatic field around that loop, you would see that that would actually correspond to a potential difference of a given point with respective itself, which means that that particular quantity will be equal to zero. It will be zero unless while you have moved the closed path the electric field has not really changed in between.

That is, it does not really, this lot is not applied to electro dynamic case, where things are changing with time. But so far we have not considered things changing with time. Therefore, everything was static. And we will still continue to assume the static case except we introduce the concept of current. Now, current was charges in motion. So, we could define current in a thin filament of conductor as the net charges that would have moved past a given point.

So, if you see some positive charges moving to the from left to right and some negative charges moving to the right to left and so on, you have to find out what is the net charge that has been transferred in that conductor across a given point and that would correspond to the current. And we introduced, instead of dealing with just filamentary current, we could deal with currents in a region of space by going to current density.

So, we introduced current density to deal with the situations, where current could be specified on a for example a flat piece of conductor, the current could be specified as current per unit width. So, we have seen all these and strictly speaking when we talk of current, we are no longer in the static situation. The fields that are generated would not be in the static situation. And we actually never use that thing to find out what is the electrostatic field that would be generated.

We used current for a various for a different reason. One effect, we did not mention about the current, which is very intimately connected to what we are going to discuss now is this. Imagine that this is a current carrying conductor. So this current is along, let us say, Z direction. And this is the current that is and this current is being carried around in this long thin conductor. So if you are actually look at this one, there would, you can also replace this current by one charge moving past with a certain velocity, where the velocity will be directed along the Z axis.

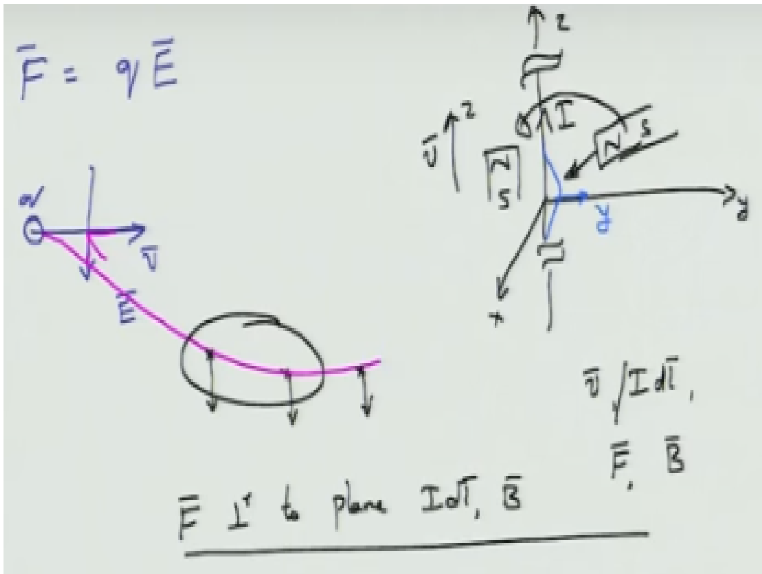
Now I imagine, to this current I imagine placing a magnet. Now, this is obviously not a magnet. But you should imagine that this is a magnet, with this phase as the North Pole and this phase as the south pole of the magnet. So, you have this current going along the Z direction, which I have taken along the Z direction and now I am going to introduce a magnetic field in such a way that the north pole points along the X axis.

So the magnet is introduced in such a way that the North Pole points along the X axis and you keep it at a certain distance over here. Now what would happen? When you have such a magnet introduced into this current carrying conductor, the wire would actually get or this conductor conducting wire would actually get deflected and it would just experience a small amount of deflection, depending upon how strong the magnet is, it would experience a deflection.

But if you do not really put in a large magnet, you would actually see a very small amount of deflection along the Y axis. Now this is something that is very new to us. We have seen earlier that you could have case, where a charge was initially may be at rest and then you apply an electric field and the charge begins to move. So, if the charge is this and then you have applied an electric field going from left to right in my view, then this charge will begin to move from left to right.

You know, it gets accelerated like this. It just moves from left to right. If the charge was already moving in the left to right direction and if I place an external field, you know I introduce an external field here then it would continue to move in the left to right direction, as long as the electric field is in the left to right direction. And we know that at any, for any charge that is there in an electric field E, what is the force that is acting on that charge?

(Refer Slide Time: 07:23)



So, the force acting on the charge, we know, is given by F is equal to 'q' times electric field, where electric field is the field that we have introduced by some means. So, if I have a charge and then in the presence of an electric field, it tends to get aligned in the direction of electric field. If the charge was initially at rest, then it begins to move along the electric field, in the direction of the electric field.

If the charge was actually coming in with a different direction of velocity and your electric field is not coinciding with that one, then there will be a small amount of deflection, small or large depending on how strong your electric field is, but there would be a deflection of the electric field. So the path would be different. For example, if the charge is coming in from left to right and the electric field is applied from top to bottom.

Then what happens is that, at this point for example, imagine this is the velocity vector and this is the electric field that you are applying and what would happen is, this charge that you have will develop a certain component along electric field. There will be now two forces, one force because of the kinetic energy, this is the field that I have applied, so there will be one force because of the electric field that I have.

And then there is another force because of the charge velocity itself. So, there are two forces and essentially it would get deflected and if you keep looking at that the path would actually go and get deflected. So, this would be the path that an electric field would take, if the velocity vector is different in the direction from the electric field. But it is very important to know that at any point along this path the force acting on the charge because of the electric field would always be in the direction of the electric field.

You can, in the exercise do a small calculation to actually predict the path and you will be able to see that this path depends on the velocity that is coming in and the electric field that would be coming in. But, it is important to note that at any point on the path, the force on the charge would always be along the direction of the electric field itself. Now, let us get back to the magnet and the current carrying wire that we were talking about.

Now I have a wire, which is carrying current in the ‘Z’ direction. And then I introduce a magnet in the X direction. So in the ‘X’ direction, I would introduce the magnet. And I have told you that, when I do this, the wire would just get deflected a little bit towards the ‘Y’ axis. So you can, and there is no electric field introduced here. So there must be something that is happening from the magnet that is causing the wire to deflect.

Now will this deflection will be always along ‘Y’? Turns out that it would not be always along ‘Y’. For example, if I change the magnet orientation such that the North Pole would actually point along the ‘Z’ direction that is parallel to the current ‘I’, then there would not be any deflection. This are not complete experiments, but these are some qualitative ideas based on complete experiments done by people over the last two thousand years in studying magnetism.

And the idea is that if the North Pole is directed along the ‘X’ axis, in this case, there will be a deflection of the wire along the ‘Y’ axis. If the north pole of the magnet directs along itself along the ‘Z’ axis, then there won’t be any deflection. So, in any intermediate cases, that is, you could move around your magnet and direction and then the amount of deflection would also change. So, there is some source acting on the current carrying wire, which is actually pulling the charge in such a way that, there are three vectors now.

So there is one vector, which is the current direction, which you can think of as a charge. For example, if you don't want to think of current, you can think of a charge moving along the 'Z' axis with a certain velocity 'v'. So you have the charge moving with the velocity 'v' or equivalent to the current direction or the current element direction, which is along the 'Z' axis in this case. You have this vector, this or either of the two vectors.

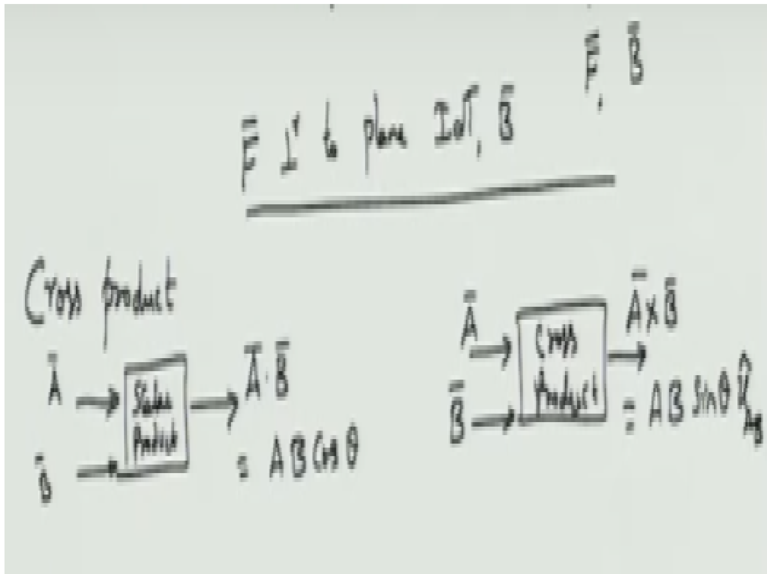
The effect would be essentially the same because 'q' times 'v' in some sense, the current. And then you have a force, which is causing the wire to be deflected. And finally there is something that is generated by the magnet that is actually the north pole of the magnet is something that would also be considered as a vector because you could place the North Pole along 'X' direction, 'Y' direction, in any arbitrary direction it could be placed along the 'Z' direction.

So there is something that is associated with the magnet. So, there is some vector associated with the magnet. Let us call that vector as the vector 'B'. Then, it turns out that the source must be perpendicular to plane that contains both, the current element or the moving charge, as well as this vector 'B'. This is interesting because we have not seen such vectors. We have not seen an action, where the force on a charge would result in a direction that is completely perpendicular to the charge motion.

So, if you go back to the electrostatic case, you would not see that. In the electrostatic case that you have, you would not see that the force is in a direction perpendicular to velocity and electric filed. It would always lie in the same plane as we (()) (13:13) in fact, it would lie in the same direction as the electric field. So, we have not had an opportunity to discuss conceptually the case, where the force would be perpendicular to two vectors.

And to deal with that, we need to expand our vector analysis to consider what is called as cross product of vectors. We will come back to all these force acting on that current element and this vector 'B' and talk more about what vector 'B' is. But, first we need to introduce the concept of cross product.

(Refer Slide Time: 13:46)



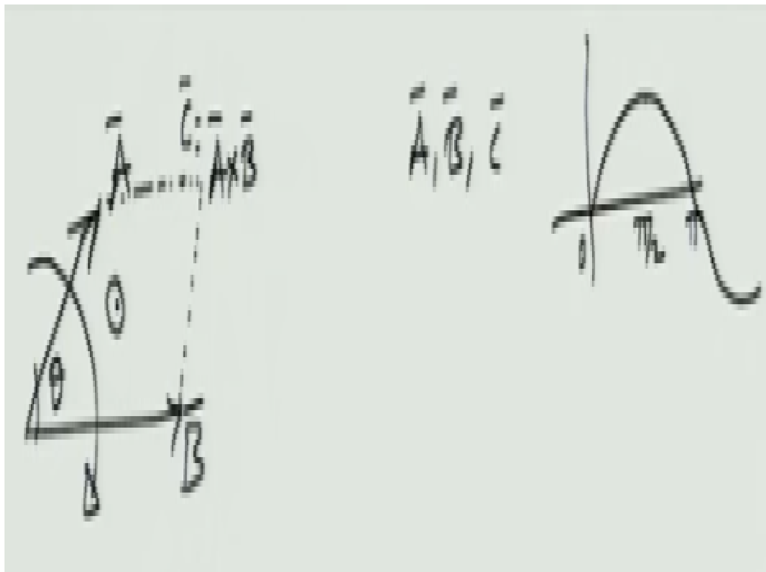
So, what is cross product of two vectors? We have already seen one type of product. We have already seen one type of product called the scalar or the dot product, in which you took two vectors and then produced a scalar. So, you could think of this scalar product as a machine that would take two vectors as inputs and would generate a scalar at the output. And this scalar was given by, was defined as a dot product of the two vectors.

And it was given by magnitude of 'A' vector, magnitude of 'B' vector and the angle between the two. Now, a cross product is a different beast altogether. The output of a cross product is not a scalar. It is a vector. So, you still take two vectors as inputs, they may not be along, they may be oriented in any direction, there would be a certain angle between the two orientation. And then the result of cross product will be a vector.

Now, you can see that this is something that you would want because you can take one vector as IDL, which is the current element along the vector direction DL and the other vector 'B' as the 'B' field or the 'B' vector of the magnet itself. Sorry, I gave away the game, it is the magnetic field 'B'. So you have vector 'A' and vector 'B' and the result can be that force on the current carrying conductor, which is another vector such that it could be perpendicular to the plane that contains both the vectors.

So, cross product is denoted by ‘A cross B’ and is defined as magnitude of ‘A’ into magnitude of ‘B’ sin theta and in the direction, which is perpendicular to the plane that contains both, both vectors ‘A’ and ‘B’.

(Refer Slide Time: 15:40)



So pictorially, cross product looks like this. So you have a vector ‘A’, you have another vector ‘B’, these two have a certain angle theta and then the corresponding vector, the cross product of these two vectors will be in the direction that would be perpendicular to the plane that contains both these vectors. So, you expand this ‘A’ and ‘B’ and you can think of this as a plane and then the vector would be the cross product of the vector ‘C’ is equal to ‘A cross B’ will be in the direction that is perpendicular to this plane.

So, for example I have this plane and if I take this as vector ‘A’ and this as vector ‘B’, then the resulting vector, the cross product of ‘A cross B’ will be perpendicular to this one. So I have ‘A’ and ‘B’ and it would be perpendicular to the plane that contains this paper. Now you could argue, this vector cross product could be going into the paper or it could be coming out of the paper.

Now, how do I distinguish between the two? Tells out that there is no distinction mathematically. However, you have a certain convention that you follow and that convention is that ‘A’, ‘B’ and ‘C’, the vector ‘A’, vector ‘B’ and the cross product vector ‘C’ will actually form the right angle triangle, sorry, right handed motion. In the sense that, is given the right handed rule, in the sense that you have a vector ‘A’, you orient one direction to vector ‘A’.

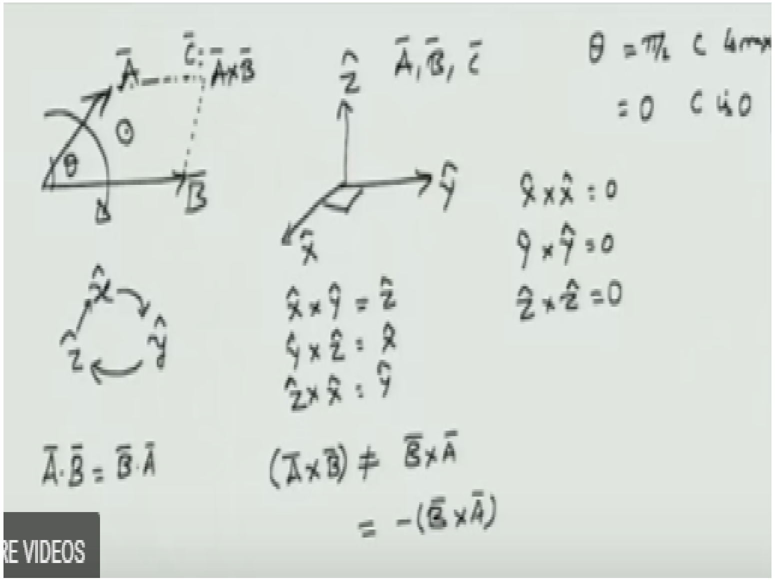
And then you curl along the direction, your fingers along the direction for ‘B’ and wherever the thumb points, that would be the direction of ‘A cross B’. So in this particular case, you have a vector ‘A’ horizontally, vector ‘B’ coming towards me and therefore you need to curl from ‘A’ to ‘B’ and you will see that the resultant vector would actually be going into the plane. So, you think of this as the vector ‘A’.

So the vector ‘A’ is coming out like this and then you need to curl your fingers, it is slightly difficult for me to show it over here. But you need to curl your finger from direction of ‘A’ to ‘B’. So you can think of a screw which is being rotated from ‘A’ to ‘B’ and the screw would advance in the direction that would be perpendicular to this plane. So this is the right handed rule, which we have already seen being used, when we talked about curl.

But this is where that rule is coming from. So, you curl your finger from ‘A’ to ‘B’ and then the direction of the cross product would be perpendicular to that plane. And what is the magnitude of that one? If you look at the expression for the magnitude, the magnitude depends on the angle, sine of the angle between the two vectors. Now the sine function as you know is zero at zero and then it maximum at pi by two.

And then of course it goes down to zero at pi and it goes negative. So it goes negative, but you don’t really have to consider the negative case because then you know the directions would be switched.

(Refer Slide Time: 18:55)



There would be maximum value, the cross product will go to its maximum when the angle theta between the two vectors is equal to pi by two. Your magnitude of ‘C’ is maximum, where ‘C’ is the cross product of ‘A’ and ‘B’. When theta is equal to zero, the magnitude of ‘C’ is zero. So when theta is equal to zero, it simply implies that ‘A’ and ‘B’ are parallel to each other. They could be either parallel, in the sense, that they would be in the same direction.

Or they could be anti-parallel, in that case, the angle will be pi and sin pi is again zero. So, whenever you have two vectors, which are parallel to each other and you try to find out the cross product of them that would be equal to zero. So, if you go back to this Cartesian coordinate system, you have defined three unit vectors along the directions ‘X’, ‘Y’ and ‘Z’. We have defined three vectors ‘X’, ‘Y’ and ‘Z’, unit vectors along these appropriate directions.

You can see that, if you try to find the cross product of the unit vector ‘X’ with itself that would be equal to zero, cross product of ‘Y’ with itself will be zero, cross product of ‘Z’ with ‘Z’ itself is equal to zero. However, if you take one vector, ‘A’ vector as ‘X’ and ‘B’ vector as ‘Y’, then cross product of ‘X’ and ‘Y’ will actually point in the direction of ‘Z’ and because the angle between these two is ninety degrees, sin theta will be equal to one.

And the magnitude of ‘X’ and ‘Y’ is also equal to one. Therefore ‘X’ cross ‘Y’ is equal to ‘Z hat’ that is a vector in the ‘Z’ direction or the unit vector ‘Z’. Similarly, you can see that if you go from ‘Y’ to ‘Z’, you know imagine turning a screw from ‘Y’ to ‘Z’, that screw will then point towards ‘X’ direction. So, you have Y cross Z being equal to ‘X’. And last one would be to take the vector from ‘Z’ to ‘X’, so you rotate from ‘Z’ to ‘X’.

And then you will see that the screw actually moves along ‘Y’ direction, therefore giving you ‘Z’ cross ‘X’ is equal to ‘Y hat’.

Now there is a mnemonic, which I used to remember in which direction you have to go in order to get this particular vector and that would be to write then like this, in terms of some circle okay, in the sense that now ‘X’ cross ‘Y’ would point along ‘Z’, ‘Y’ cross ‘Z’ would point along ‘X’ and ‘Z’ cross ‘X’ will point along ‘Y’. Here is an interesting thing. Dot product did not care whether you took ‘A dot B’ or ‘B dot A’.

It did not care because the physical meaning of dot product is that you take one vector and then you try to find out what is the component of this vector, this flat pen on one of this, on my finger, you know the finger direction in which my fingers are pointing. So, this component would be the same, whether you put this vector and try to find out what is the component on this hand or you put the vector in the original position, but then you move my hand.

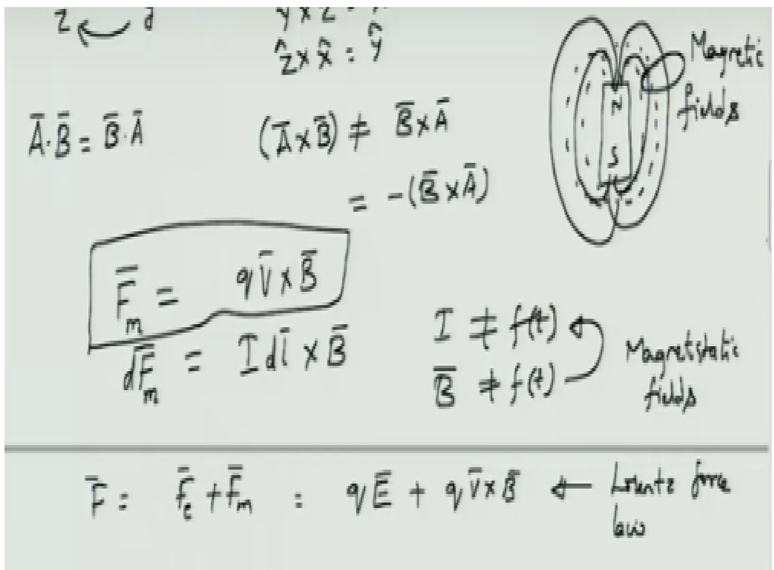
So you see that it is essentially the same length. It would not matter to me, whether you took ‘A dot B’ or ‘B dot A’. Both give you the same value and this is captured in mathematics, mathematical language saying that the dot product of two vectors is a commutative operation. It does not care whether you go from ‘A dot B’ or ‘B dot A’. Evidently that is not the case in cross product. It is quite obvious. See you take a screw. So, now you imagine going from ‘X’ to ‘Y’.

So it is, imagine going from ‘X’ to ‘Y’ and the screw must advance in the direction of ‘Z’. So you imagine this one, now the screw is pointing and the screw would just come out along ‘Z’ direction. If you go from ‘Y’ to ‘X’, so you turn the screw from ‘Y’ to ‘X’, in the sense that now ‘Y cross X’ you are considering, then the screw would actually go into the material, into the plane, in such a way that it would be pointing along ‘minus Z’ direction.

So you have a screw, which is coming out like this, it would be along the plus side direction. If it go this way, it could be along the 'minus Z' direction. So we have 'A cross B' not equal to 'B cross A'. Instead 'A cross B' is actually equal to 'minus B cross A'. That is the vector would point in the direction that is opposite to 'A cross B'. So we don't really go any more into details about the cross product, I think we have said enough that we wanted to say.

So, why did we actually go to cross product? If you remember the experiment that we were thinking of was a wire going along the 'Z' axis, a magnet oriented along the 'X' axis and the deflection that we were seeing or the force that we were seeing was along the 'Y' axis.

(Refer Slide Time: 23:53)



And we can capture all that by saying that the force experienced because of the presence of the magnet, which we are going to call as 'F m' is equal to 'qV cross B', where V is the velocity vector and B is the direction in which the north pole of the magnet is pointing. And it would presumably give you some sort of a field.

This probably is much more easier for us to imagine. You know whenever you take a magnet and you look at some high school text books, you would see that, if you bring some iron filings around this one, small iron, pieces of iron, bits of iron, which you can actually purchase from somewhere, you will see that these irons would actually stick around this wire. So they would actually stick around something like this.

So, you have seen this kind of pictures earlier and this orientation or the fact that these irons would arrange themselves indicates that there could be some sort of lines coming out. Some sort of lines that are coming out of this magnet. And these lines are pretty much their way in which the magnet would influence the motion of these iron particles. And these lines are called as, because these lines vary with respect to the positions.

These lines are actually fields and these are called as magnetic fields. The idea of magnetic fields or in general the idea of field was actually introduced by Faraday, where he formulated certain rules corresponding to these field lines. But this idea of field, in some sense of having a region of influence existed before Faraday as well. So going back to this, now we have to introduce or we have introduced a different agent, which is responsible for the deflection of the current carrying wires in a given direction.

Here we have written this in terms of a single charge moving with a velocity ‘V’. So, for the force that is experienced by that single charge in the presence of magnet or magnetic fields generated by magnet is given by ‘qV cross B’ or in terms of the current carrying wire, it would be not the full magnetic field, but the infinity symbol amount of the magnetic force ‘dF m’ is equal to ‘IdL cross B’, assuming that ‘B’ is essentially constant.

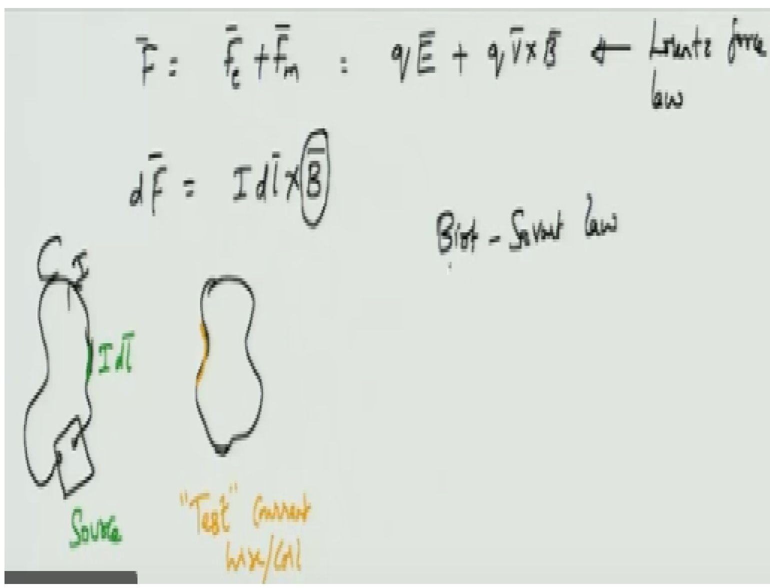
This force on the current carrying wire, the small infinity symbol amount of the force on the current carrying wire is equal to IdL, the line element or the differential line element times ‘B’, assuming that the current itself is not varying with time. In fact, because current is not a function of time, the fields that are generated, see this was a different case, so we assume that the current carrying wire and there is a vector field ‘B’.

However, it turns out that these currents capable of generating a field of their own. So, this field is the field that they generating and this fields will not change with respect to time, as long as the current is not changing with respect to time. So these fields are called as magnetostatic fields. So, this static ness is only in terms of the fact that the magnetic field will not change with time, current is also not changing with time.

But to form that current, the charges are actually in motion. So, there is some time dependent activity going on, but the net effect of the time dependent activity is such that, the current is not changing with time and the fields that are generating are also not changing with time, giving you the magnetostatic fields, static with respect to time. So we have this law, if you want to consider, this equation 'F m' is equal to 'qV cross B, now actually completes the total force on a charge.

So, if you have a charge moving with a velocity 'V' in a certain direction and then you have both electric as well as magnetic fields, then the total force acting on the charge will be due to the electric field and due to the magnetic field. The force because of the electric field will be 'qE' and the force because of magnetic field is 'qV cross B'. And, this force law is known as Lorentz force law.

(Refer Slide Time: 28:23)



So get back to this 'dF m' and I am going to remove this 'm' here because in the next few modules, they will be considering only the magnetic fields and the forces because of those magnetic fields. So the force 'dF', the infinity symbol force on the current carrying wire is actually given by 'Idl cross B'. Of course you would immediately realize that you cannot really have an infinity symbol current.

So you cannot have a small amount of current that would be present. But rather, the current actually has to form a loop, because you cannot just have an isolated piece of current. It means that charges are getting generated and getting accumulated at the nodes and we have just seen that such an accumulation is not possible, at least not happening. So, connected to certain generator or a battery and then it would complete the circuit.

So, there is actually a closed path for the current to be flowing and we are assuming that this current is steady current or direct DC current, that is, it is not changing with time and we have considered the force because of only a small portion of this current carrying wire. And this portion is Idl. And then we found out what is the force. If there is one more current carrying wire here, what is the force of this one, on to the other current carrying wire?

It has to be another current carrying wire and in order to experience this force. So what we are actually looking at is, this as the source, so this is the source and this is your test current or a test current wire or a coil. And this is the force that we have seen. This is the force law that we have. However, we are interested in finding what this vector field ‘B’ will be. What this vector field ‘B’ will be? And to get this vector field ‘B’, we need to import what is called as Biot-Savart Law.