Electromagnetic Theory
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Lecture - 38
Magnetic force, torque & dipole (continued)

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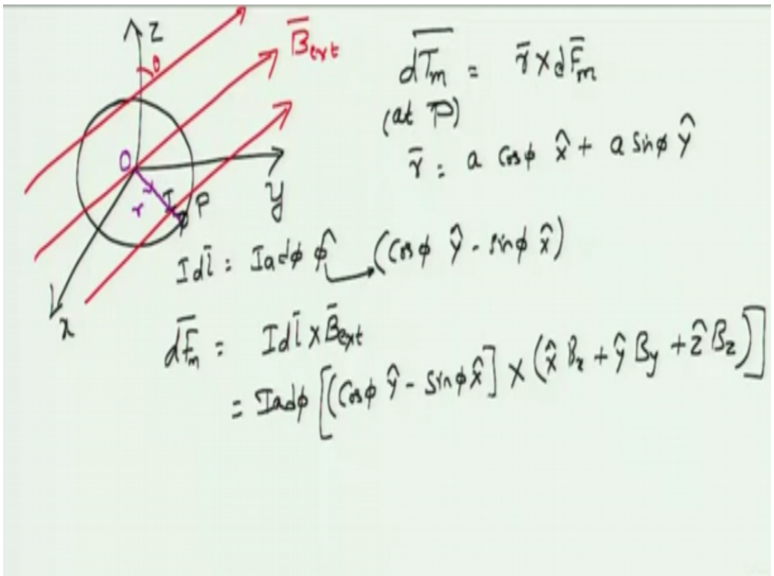

This is the field of a dipole. Now what we are interested is if this field of a dipole or if this dipole exists and we apply an external magnetic field, will there be a force on the dipole, okay? It turns out that there will be a force on the dipole and we want to calculate that force. So assume that we have a small dipole in the form of a loop carrying a current I. Now I am going to apply a magnetic field at a uniform magnetic field of value B external, okay?

And this angle that it would make with respect to z axis would be theta. What happens physically is that, the loops starts to align itself along the direction of the magnetic field and we want to show what would happen there or what would be the reason for that. Consider a point, any point over here, okay? So there is a current that is going through this. So there is going to be a current element, I dl, right?

The current element is Ia d phi, phi hat. Of course, I will also replace this phi hat by cos phi, y hat minus sin phi x hat in the xyz coordinate system and then I ask for what is the magnetic torque at this point, okay? So if I call this point as P, I want to find out what is the torque at that point P. Torque I know is given by r cross F. Where r is the vector which is drawn from the origin the fixed point to the point where I am calculating the torque.

So if this is the origin, this is the vector r and you know it is like I am applying the force at here. What is the force at point P, multiplied that by the cross product of that with respect to r because only the force that would be perpendicular to this r direction would result in a movement of rotation. Otherwise, it would not rotate and torque actually measures how much rotation is possible.

So the torque d Tm at the point P is given by r cross d F and this force is because of the magnetic field that we have applied because of the external magnetic field that we have applied. So this become d Fm. So now let me write down what is r here. r is a cos phi x hat plus a sin phi y hat. What is the magnetic force d Fm at point p. The magnetic force is I dl cross B.

B is the external uniform magnetic field that we have applied and dl, I dl we have already written as I a cos phi y hat minus sin phi x hat, okay, cross d phi will be constant, so that I am going to write down later. So let me write down this. I a cos phi y hat minus sin phi x hat, cross with B external. B external field can be written in terms of its x y and z components. So I have x hat Bx plus y hat By plus z hat Bz, okay? This entire thing multiplied by d phi.

So we are going to write this as I a d phi and this is the expression. Now you have to recall the cross product formulas or you can see that one, y cross x will give you minus z, y cross y will give you zero and y cross z will give you x component. X cross x will give you zero. X cross y will give you z, x cross z will give you minus y.

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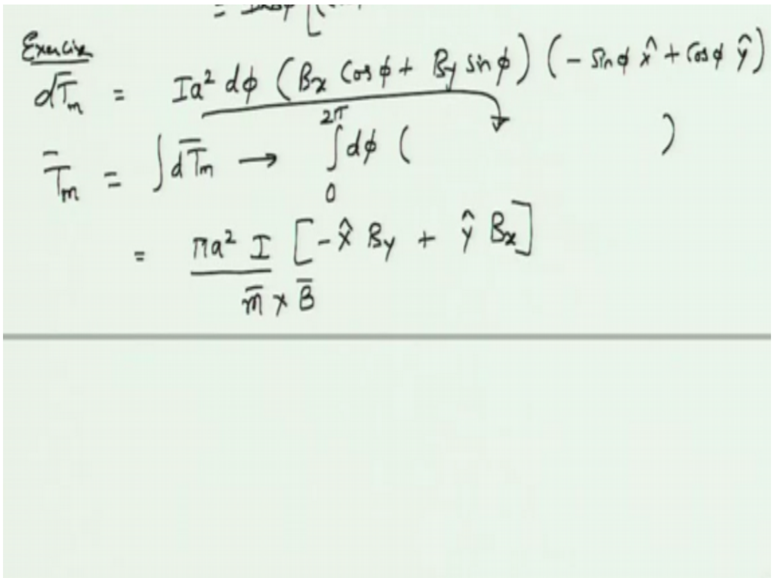

So if you do all this cross product you can see that the torque, if you substitute all these expressions into torque you will see that the torque becomes I a square d phi times Bx cos phi plus By sin phi multiplied by minus sin phi x hat plus cos phi y hat. I will leave this as an exercise to you to show that this is the expression for torque that you are going to obtain. This involves lot of cross products.

It is not difficult, it is very easy to do this except that it is tedious and to show all the steps would mean we are going to be distracted a lot. So I will leave this as an exercise, you can show that this is what it is and you are not interested in the differential torque, you are actually interested in the total torque, so in order to obtain the total torque you need to integrate this one.

Implies integrating over d phi from zero to 2 pi of this expression. So you can put this expression and integrate over 2 pi, again there are 4 terms here, cos phi sin phi, cos phi cos phi, sin phi sin phi and sin phi cos phi. The cos terms cos phi sin phi and sin phi cos phi integrate to zero leaving behind only two other values which is given by pi a square I, okay, minus x hat By plus y hat Bx.

Again if you recognize pi a square as the area of the loop, that we have considered, the area of the loop S, then this quantity I multiplied by S is the magnetic moment m and the magnetic moment m is directed along z direction, so that would be the vector magnetic moment m, right? And if you do this vector magnetic times, the B field, the B external field you are going to get this expression in the top.

And instead of this writing this as m z hat, it is customary to write this as m cross B. So this is m cross B because the loop can have any direction, because the magnetic moment of the loop can be directed in anyway but as long as the field is uniform over the loop and this is important. You are not considering loops of radius kilometers. You are considering groups of very tiny radius over which the magnetic field is going to be uniform and then the torque that is produced because of the magnetic force is given by m cross B.

So this expression is very important because we are going to use this expression later. So this torque because of the magnetic moment m which could be directed anywhere in the space will be m cross B. So the torque will be in the plant that will be perpendicular to both m and B, If you take a compass needle which could be considered as a dipole, and you place it anywhere, it would point in a certain direction.

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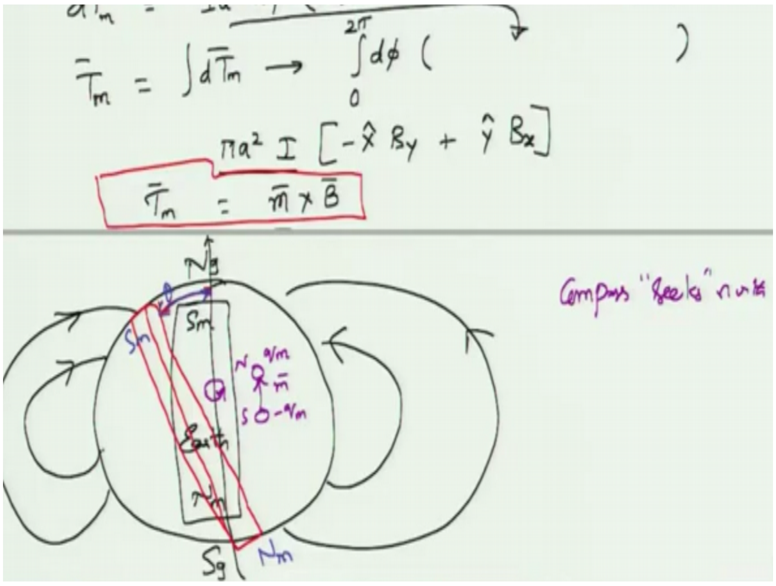

This is your earth, or this is my earth as well. Earth has magnetic field which can be thought of as a big bar magnet of magnetic poles South and North except that the geographic North is located at magnetic South and geographic South is located as magnetic North. Of course this magnetic pole itself actually reverses direction every few thousands of years.

So for the current era in which we are living, the South pole of the earth magnet coincide with the geographic North of the earth and similarly North pole of the magnet of the earth, coincides with the geographic South pole. So because this is a dipole essentially there will be field lines around it and the field lines would be going from North to South poles, right? So this would be the field line at a far away distance.

So the field lines actually goes through somewhere over here and then there are field lines surrounding this. Now because there is a field here and if I now take a compass and place the compass here, the compass would actually point to the North, okay because the magnetic moment would affect it in such a way that the compass would actually point to the geographic North, because it would be aligning in the opposite direction to the magnetic field B.

So it would point in the direction of geographic North, okay and that would be the magnetic field and of course dipole itself is taken from South to North. So in our notation, the dipole was taken from South to North and for the compass it points to the geographical North. So this with a fictitious charge of qm and minus qm, if you can think of, or you can think of a small ba magnet, you could of course also think of this as a small loop of a certain radius and carrying the current I m, okay?

But the important point is that, this compass always points to the geographical North. Why it is pointing the geographical North because that is where the South pole of the magnet is located. So it gets attracted to the South pole of the magnet. But because South magnetic pole is actually North geographic pole, we say that compass seeks North, okay? So if you take a compass which can be freely rotated, if you place this one, it will always try to point to the direction of South magnetic pole.

After a few thousand years, our next generation people would actually see that the magnetic would seek South. This is a slightly simplified result because the actual poles are not aligned directly to the North or the South geographic pole but there is actually a small amount of declination over here. So there is a small amount of declination over here. So it is a small amount of declination which is about 12 degrees.

The South pole is around 12 degrees declined with respect to the North geographic pole and the North magnetic pole is 12 degrees declined to South geography poles. So when you decide compasses, you will have to consider this declination and then compensate for that declination. It would actually point to this 12 degrees, then you have to apply an additional force that it would actually point to the proper geographic North pole. So this was the force on the dipole.

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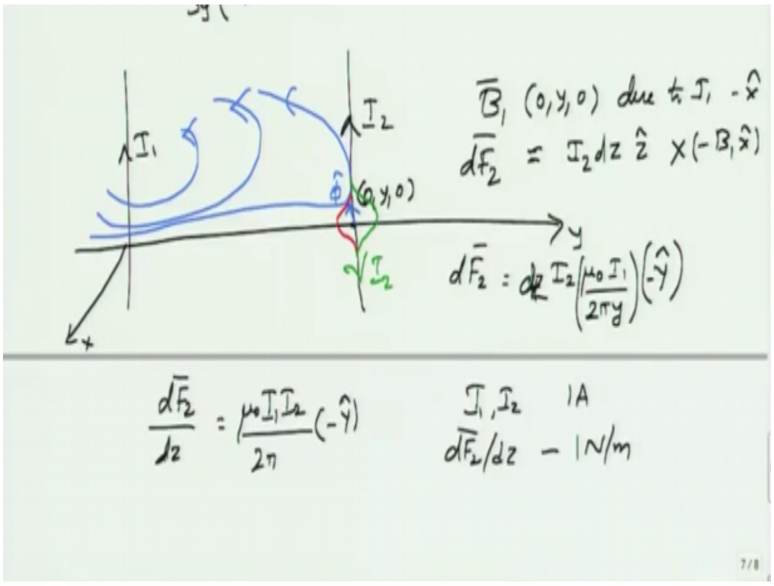

Before leaving this forces, there is one last thing which I would like to talk about and that is force on two wires, okay? Let us assume that there are two wires I1 and I2 and these wires are carrying currents in the same direction, so this is y and this is x, I want to find out the force on this wire I2, at the point here. At this point, on the y axis, I want to find out what is the force on the second wire because of the force due to the first wire, because of the current in the first wire.

Clearly there will be magnetic field lines around this. So there would be magnetic field lines and there would be a magnetic field that would be passing through this zero y zero point, if I find the magnetic field there it would be along the direction minus x, you have to imagine this slightly. So the magnetic field here would actually be pointing in the phi direction, but it is phi for y axis would actually be pointing in the minus y direction.

So what would be the magnetic force there. The magnetic field at this point called this as B1, at zero y zero, due to I1 points along minus x direction, which is actually the phi direction for that, but on the y axis phi is directed along minus x direction, right? So go back to cylindrical co-ordinates to really understand this statement.

So there will be a certain force that this point experiences or this wire experiences and that force is given by approximately given y I2 dz which is the current element located at that point I2 dz we have a very small hat z and the current element is pointing around the z axis cross minus d1 x hat, B1 being the magnitude of the current field because of the current I1 and it is directed along minus x hat.

So this would be the Infinitesimal amount of force that this gets experience, right? So you can write this out as d F2 is equal to I2 B1 would come out, what is the magnetic field B1, B1 is mu zero I1 by 2 pie y. This is the magnetic field because of the current in the wire I1. So this would be I2 mu zero I1 by 2 pi y. What would be the direction of this force? This is z hat cross x hat. z hat cross x hat will be along y direction, but there is a minus sign here.

So this would be along minus y hat. So let me just emphasize that the direction is along y hat and the magnitude of the infinitesimal force is I2 mu zero I1 by 2 pi y. Sorry, there is a dz over here. So let us write down that dz. So if you evaluate what is force per unit length kind ofa think. Then the force per unit length is I1 I2 mu zero divided by 2 pi, the force perimeter length I want it at one meter, so this would be 2 pi itself at y is equal to one and it would be directed along minus y.

If further I1 and I2 carry a current of one Ampere, then this would actually be the expression. That is if the force per unit length is one Newton per meter, then I1 and I2 carry and if they are equal amounts of current then that would be the definition of ampere and the direction is minus y indicating that this wire actually bends or experiences a force towards this wire I1, right? So wire carrying the current I1.

So there is essentially a force of attraction between two wires and instead of current I in the direction which is same as I1, if you had a current which was in the opposite direction, then there will be force of repulsion. So two wires carrying current in the same direction get attracted to each other, two wires carrying currents in the different direction will repel each other. So this is the force and the definition of ampere which is the internationally adopted definition for current, okay?

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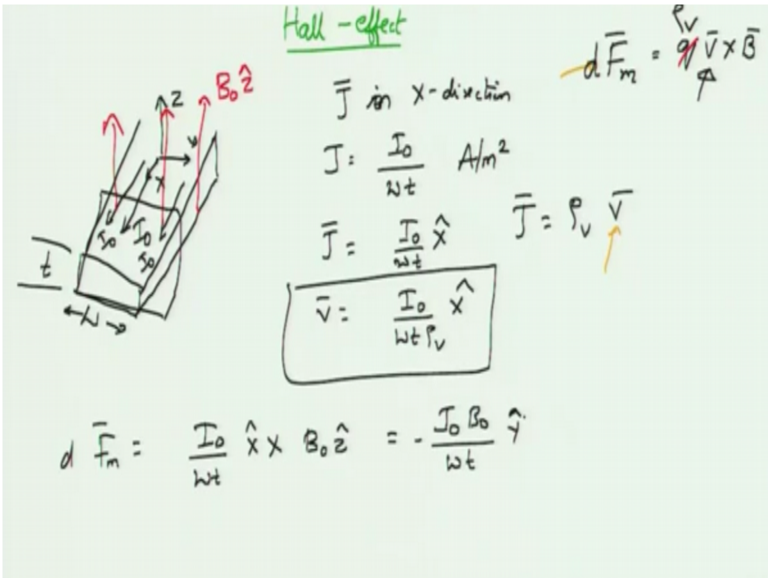

So we discussed force, there is one last thing, although I said last but I would like to talk about this effect, which is important in semi conductor devices. This is called as Hall effect. What is Hall effect? It is simple, you take a conducting material, even if it is not perfectly conducting, don’t bother it is alright. You take a material, there must be some way of, some amount of current it must sustain.

So it could be for example a semi conductor bar, that a thickness t and having a width w and it is uniform in the z direction if I am going to assume that one. And I can force the current through this. Let us say, I have forced a bias current of I zero through this one, which means there would be charges which would be flowing. If the semi conductor bar has holes then the holes will be flowing along this direction I zero, the electrons would be flowing opposite to I zero, okay?

Now let us also put down the axis, the axis to the top is z, this side of the axis is y and the direction of the current is x. Now what I am going to do is that, I am going to apply a magnetic field which is uniform throughout and having a value of B zero and directed along z axis. That is I am going to apply a magnetic field perpendicular to the bar and I want to find out what happens to this.

It turns out that because of this magnetic field there will be a charge separation leading to a voltage difference between the two ends, two phases of the bar that we are considering, a conducting bar that we are considering and this would produce an electric field which in turn will produce a voltage between the two phases or the two ends of the bar and this voltage is called as the Hall voltage.

And we are going to discuss what is this Hall voltage and this element itself is sometimes called as a Hall sensor. So we are going to discuss that one. So we have applied a magnetic field B, it would be pointing in the direction z which is coming out of the bar here in the normal direction. So what will be, first let us look at what will be J, the current density J will be in the x direction, correct?

The current density will be in the x direction because the current is actually going along x, so you should actually imagine current lines I zero, uniform bias current lines like this and therefore J vector will be along x direction. What is the current density there? The magnitude of the current density is, whatever the total current I zero is divided by the area and what is the area, the cross sectional area through which these lines are coming out is w multiplied by t, right?

So this is the cross sectional area that they are coming out and this has w and t, so width of w and thickness of t and this is the magnitude of the current density and the vector current will be equal to I zero divided by w t multiplied by x hat. Now the idea will be this. There are this charges which are constituting the current and they would be flowing in this direction I zero or opposite to I zero depending on the type of the charge.

But the presence of the magnetic field will now exert an additional force on them and we want to calculate that additional force. We already know what is the force exerted because of the magnetic field, that will be q multiplied by v cross B. I know B is directed along z direction, I do not know what is v, I want to find out what is v, and to find to v, recall the relationship between J and v.

J is given by the volume charge density rho v multiplied by the velocity of the charge carriers which is v bar. So if I know J I know v, if I know v I know Fm. If I know Fm, then I can find out what is the resultant force. From the resultant force I can know what will happen to the charges. This is the goal. So first find out, so this is my first step, this is my second step and third step will be to evaluate the total force on the charge carriers.

So using this expression I know what is J, I can easily find out what is v, v is given by I zero divided by w t rho v going along x direction. So I have completed the first step. So what will be the magnetic force. Magnetic force will be the charge carriers experience. Since this is not a single charge but rather charged distribution I need to modify this equation slightly. This becomes d Fm the infinitesimal force that is experienced by the charged carrier that would be in place of q that becomes rho v.

So the magnetic force, the differential magnetic force, or the magnetic force in that sense that the charge distribution experiences is given by I zero divided by w t, rho v cancels out, because rho v is in the denominator, rho v is in the numerator that goes away. So I have x hat cross B zero z hat, gives me I zero B zero, divided by w t, okay and x cross z will be along minus y direction.

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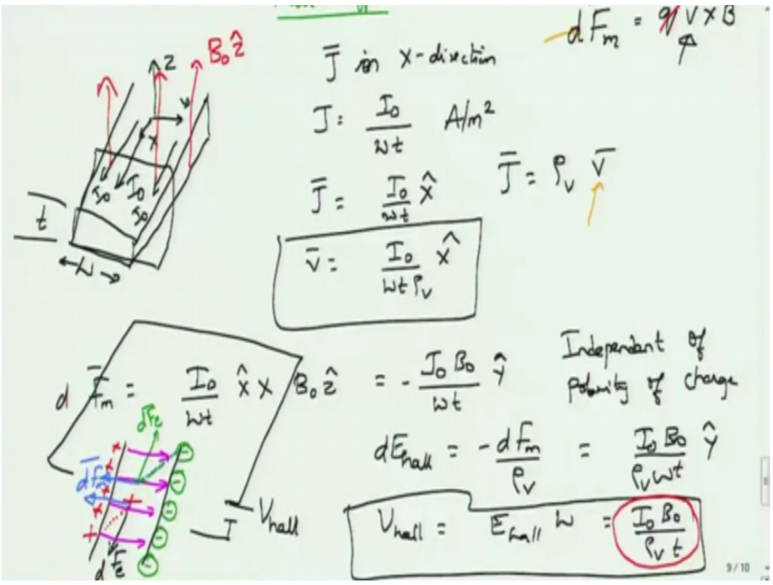

What is interesting to note is that, this force that we have obtained is completely independent of the type of charge carriers. Independent, I am going to write this one to emphasize this, independent of polarity of the charge, both positive charges as well as negative charges, that is positive charges in the form of holes and negative charges in the form of electrons, will experience the force along minus y direction itself.

So if you look at the total force this is experiencing of the bar, you see that this is the bar that I have. Let us imagine for a momentum there is a positive charge carrier over here. There is already a force because of the electric field, because of the bias current there is already an electric force due to this. The electric force is directed along x direction or the differential force if you want, this would be the differential electric force that is experienced, but there is also a force because of the magnetic field and that will be directed along minus y direction.

This would be the magnetic force d Fm. What would be the resultant direction? Resultant direction for this charge carrier will be along midway between d Fe and d Fm and it would actually push the charge to the edge of this bar. So you can actually have charges anywhere over here and these charges would actually be pushed to the edges and essentially line up on this left edge, okay?

If instead of this charge, I had a negative charge here, now the negative charge experiences an electric field in this direction. This is the electric field for the negative charge carriers or the electrons, because electrons move in the opposite direction to the field, where as magnetic force is still in the same direction. So what will happen to the resultant field. The resultant force will be along this direction which means that the charge actually moves to this edge.

So there is charge here which gets lined up. So what you have done is to essentially create a charge separation. All the positive charges are aligned to the left, all the negative charges have been moved to the right, because of this there will actually be an electric field inside. So there will be an electric field inside which would be directed in this way and this electric field will create a certain potential difference if you apply, okay?

If you connect two conducting wires to it, you would see there will be potential that is developed. So you will see that there is a potential developed and this potential is called the Hall potential. What is the potential? First we need to look at what is the electric field that is developed. The electric field that is developed must be such a way that it actually opposes whatever the magnetic forces because it has to create a force balance, so it will actually be enough to oppose the magnetic field.

So electric field is force per charge. So this would be minus d Fm divided by rho v, d Fm being the magnetic force divided by rho v and this minus sign because the electric field because of the Hall effect has to oppose the magnetic field. So if you substitute expression for d Fm and carry out the simplification you are going to get the electric field as I zero B zero divided by w t, rho v is there, okay, times y hat.

This is correct because electric field is directed from the positive to negative charges and getting the Hall electric field also directed along the positive y axis and this is alright. So what would be the Hall voltage, voltage will be electric field times the width or the integration of the electric field from point here on to the left to the point on the right.

So this would be the Hall voltage and that would be equal to the electric field fortunately is uniform here or in the case that we have considered this would be the Hall voltage multiplied by w that is given by I zero B zero divided by rho v, thickness t. This is the Hall voltage that is very important.

Why if you look at the Hall voltage, it actually depends on rho v and we know that if the bar is made up of entirely positively charge carriers, then the Hall voltage will be positive and if the bar is made up of negative charge carriers such as electrons then the Hall voltage will be negative. So if I want to test a bar whether it is a p type bar or an n type bar then it is very easy for me to do so.

I establish certain bias current and then I connect two conducting wires and put a volt meter over here and then apply a magnetic field which is uniform in normal direction to the bar and I will see the polarity of the voltage. If the polarity is positive then I know that the charge carriers are all positive, otherwise I know that the charge carriers are negative. In fact, this was used to establish the fact that there are holes and holes are positively charged by early semi conductor people who conducted these experiments.

And these Hall sensors are actually available which do not actually require an external magnetic field. Earth has a magnetic field, so you take a Hall sensor place it on earth, and then that earth magnetic field itself will do the charge separation and then generate a Hall voltage. That can be used in fact to measure the magnetic fields as well.

So this was to kind of find the charge distribution but you can turn it around and say I know what the charges are, let us say I just take a piece of metal, then I place it in the magnetic field, then the charges, the Hall voltage would be of a certain polarity and the magnitude of the Hall voltage depends on the magnitude of the magnetic field. So assuming that this magnetic field is uniform I can actually find out the magnetic field itself. I can measure the magnetic field, okay? So this was what we want to discuss about force and torque on the dipoles because of the magnetic fields.