Electromagnetic Theory
Prof. Pradeep Kumar K
Department of Electrical Engineering
Indian Institute of Technology – Kanpur

Lecture – 42
Inductor and calculation of inductance for different shapes

In this module, we will discuss inductance. This is one of the other circuit element that you would encounter apart from resistance and resistor and capacitor. We have already discussed capacitors and resistors earlier. Now we are going to discuss inductance and inductance of an inductor. Now, unlike a capacitor or capacitance calculation, you know inductance normally comes in two flavours.

One is Fells inductance and the other is mutual inductance. Now, there is Fells capacitance as well, but that concept is not widely used compared to the capacitance between two or more conductors. In contrast, inductance of a single circuit or a single conductor is as much as important as that of inductance between two conductors. To properly understand inductance, you need to understand Faraday's law, but that is something that we are going to cover in the next module.

So for now, we will not introduce complications because of Faraday's law. We will simply look at the definition of inductance and then we try to find out inductance of several practical geometries and leave the kind of theoretical justification behind that for a different module. So that said, let us begin by defining inductance.

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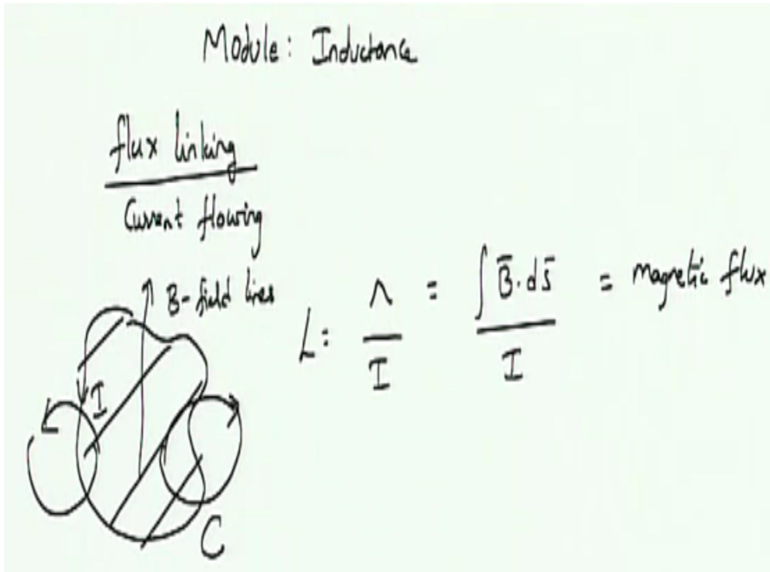

Inductance is just like capacitance. Inductance is also a function of geometry. The way the wires are arranged, the way the wires are moving that defines the inductance. And in that respect, inductance and capacitance are essentially the same. If you recall capacitance was basically how much flux or electric flux that was contained between two conductors.

And similarly, inductance actually tells us how much flux or magnetic flux is contained between a certain conductor and a certain surface that we are going to discuss. Now, we did not really talk about electric flux containing between two surfaces in the capacitance, although one can actually take that approach, rather there we talked about how much charge was stored on a conductor, to what is the potential difference that we applied.

Now, for inductance, we actually talk of flux that is linking particular circuit. So, if you have to understand that the capacitance and inductance are essentially dual elements and one can actually talk about aspects of capacitance in a manner that would be exactly same for the inductance and vice versa. But traditionally it has been the way the discussion has been that capacitance is discussed by talking about the charge storage to the voltage that is applied.

And inductance is talked about in terms of how much flux a particular circuit gets linked for a certain current that is flowing in the circuit. So, inductance actually is defined as ratio of flux linking to the circuit to the current that is flowing. Now, what do we mean by flux linking or flux linkage. This turns out to be notoriously difficult concept to state in its generality. Because there are cases, where the flux linkage is not so obvious, it is very ambiguous kind of a thing.

For simple geometries, one can actually define, what is the amount of flux that is linking a particular circuit? But for geometries that are contrived, it is a kind of difficult to speak very clearly in terms of flux linking. For us flux linking would simply mean that the magnetic flux that is linking to particular circuit. To appreciate that, consider a certain circuit. So this is my circuit and there is some current that is flowing through the circuit.

Obviously, when there is current flowing in this particular circuit, the circuit in the form of a wire that we have considered, there will be magnetic fields. So there will be magnetic fields that would be circulating around this current carrying wire. However, not all the B lines that I am drawing over here, these are the magnetic field lines. And magnetic field lines, not all of them will actually be linking the circuit C here.

See, you can see that the flux that is at the center would not really link, but the flux that would be here, field lines at the edge would actually be linking. So, this is qualitatively the flux that is getting linked. In other words, if lambda denotes the amount of flux that is getting linked, then this is given by integral of B dot ds, which is essentially the magnetic flux. In this simple loop example that we have chosen, even the central field lines would actually link to the circuit or would contribute to the flux linkage.

There are certain situations, where only partial B fields would actually linked the circuit and in that case, we call it as partial flux linkage. So in this loop case that we have shown, that we have shown here, the entire B field would actually link. So what is the surface area that we are considering? This is the surface that the circuit contour actually encloses. So in, for our example, this would be the surface area that we are looking at.

So this is the amount of flux that is linking. In fact, this is the amount of magnetic flux. So if you divide this one by the current that is carried by the circuit, you will get the inductance. So, this is essentially the definition of inductance and we will study some simple examples first to understand and appreciate how to go about calculating inductance and then later return to some of the difficult aspects of inductance.

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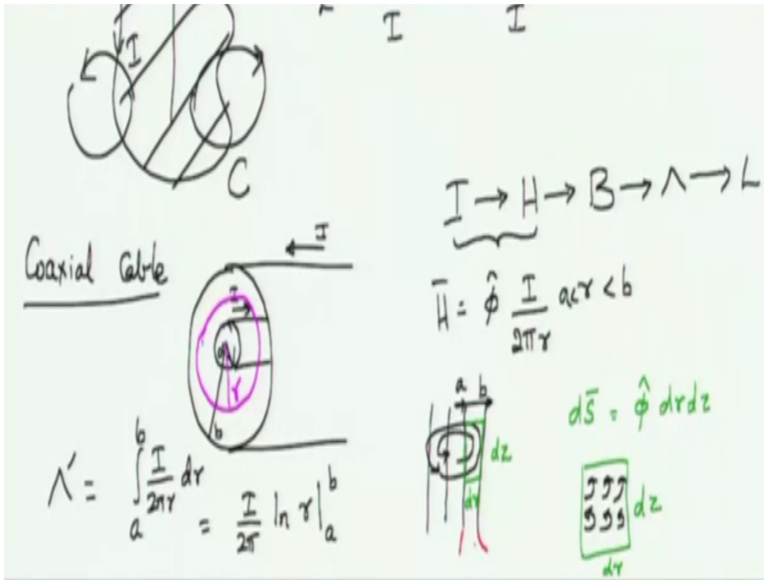

To do that, let us consider as first example, a coaxial cable. We will later consider a different version of the coaxial cable. For now, let us assume that this coaxial cable has a central conductor of certain radius ‘a’ and there is an outer conductor of radius ‘b’. The inner conductor carries a current ‘I’, whereas the outer conductor carries a current of minus ‘I’ because this forms the return path to the conductor.

The outer conductor actually forms the return path to the current. So inner conductor carries a current ‘I’ and outer conductor carries a current minus of ‘I’, that is ‘I’ in the reverse region. We already know how to calculate the magnetic field for this. And we will be interested in calculating the flux that is linking to this coaxial cable. So the approach that we are going to follow would be to start by assuming a certain current ‘I’.

And then finding out the magnetic field ‘H’ and if the medium is different, then you need to find the relationship between ‘H’ and ‘B’ to find out ‘B’. From ‘B’, you have to find out what is the flux that is linking. And divide by ‘I’ to get the inductance. This is the path that we are going to take to calculate inductance. This should remind you of the path that we used for capacitance.

We assumed a certain charge, from there we assumed, we obtained the electric field, then we obtained the potential difference and then took the ratio of charge to potential difference, to get the capacitance. So similarly here we are going to start with the current and hopefully end up with inductance. We already know the result for current and ‘H’. So here we are considering a coaxial cable.

So if you assume an amperian loop, if you assume an amperian loop of some radius ‘r’, such that ‘r’ is between ‘a’ and ‘b’. Then you know that the total current will be ‘I’. And then the magnetic field will actually be along the phi direction. So ‘H’ is along the phi direction and it is given by I/2pi r, where ‘r’ is less than ‘b’. That is in between the inner and outer conductor, the magnetic field is along phi direction. And this is the magnetic field that you are getting.

Now, what do we make of this flux linkage? To get the flux linkage, I need to find out what circuit I am really considering. And this circuit must have conductor to conductor. So the circuit that I am actually interested is, if you actually look at this way for the conductor so there is an inner conductor of radius ‘a’ and an outer conductor of radius ‘b’. Now around this will be the magnetic field lines.

The magnetic field lines are all around this way. If you focus your attention on to these two conductors, that is conductor this one and this conductor, you will see that there are some magnetic field lines that are going through this region. Now, if you take this particular area, which I have shown here, this area has a certain length dz, along the ‘z’ direction in which the current is being carried.

And it has a certain width dr. So where will this incremental surface area point to. It would point along phi direction. So, the surface area that is pointed by this one will be along the phi direction. And this value will be dr into dz. Now, I can actually take this surface area and keep moving around the ‘z’ axis. So imagine now, this is your coaxial cable, this is the inner conductor that we are talking about this is essentially carrying current.

Now I am going to consider a surface area that would look like this. So you have this top surface, right, bottom and left surface. Now to this side is my outer conductor, so you have to imagine, I am not properly showing this one, but you have to imagine that there is an inner conductor and there is an outer conductor over here and I am considering the circuits between these points. So between this and this point, in this region there will be magnetic field and the magnetic field is are all going around phi direction.

So for this conductor, the magnetic field lines are all crossing around or curling around and then if you actually cut open that coaxial cable you will see that in this region the magnetic field will be, so let me redraw that region for you. So in this region which is around dz and dr the magnetic field lines will all be going in, so this would be the magnetic field lines. So, these magnetic fields will all be along phi direction.

And therefore they are sort of cutting through this surface. They are all at perpendicular to the surface and therefore they are sort of cutting through the surface. Now, I can actually take this piece of cross section (()) (11:24) and actually move it around ‘z’ direction and if this coaxial cable is infinitely long or at least very very long, then no matter where I place the surface, I am actually going to end up with same amount of flux that is cutting through this surface.

So, if you actually let ‘z’ go completely to infinity then it’ll not really be okay because now you are considering an infinite surface area and the amount of magnetic flux that is linking to an infinite surface area will be infinity and you will not get any meaningful results. Rather than that we take the integration over say 1m along the z axis and whatever the ‘r’ values that needs to be there and then the result will actually be the inductance.

And if you divide that by 1m you are going to get inductance per unit length. This was precisely the thing that we did for capacitance per unit length. See, when we had a coaxial cable we assumed a certain charge distribution, this charge distribution was the line charge rho L coulombs per meter that is what we assumed and when you take charge per unit length and divide that one by the potential difference then you would end up with capacitance per unit length.

So, similar to that we are going to talk about inductance per unit length by removing the integration over ‘z’ and then taking the integration over only ‘r’. So with that in mind, what would be the flux that is linking per unit length? The flux linkage per unit length, may be you can call this as some gamma prime or lambda prime, that would be ‘a’ to ‘b’ because your dr can go at the most from ‘a’ to ‘b’. So, ‘a’ to ‘b’, I by 2pi r.

And then what would be the surface area? This is dr. The integration with respect to dz has gone. So this integral can be evaluated and this integration is given by I by 2pi and log of r between a to b.

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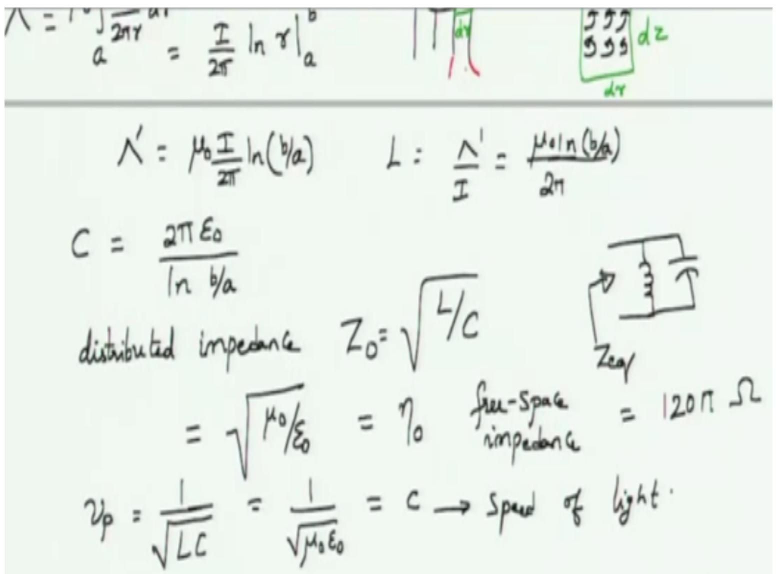

So clearly the flux linkage per unit length lambda prime is given by I/2pi log (b/a). Now if I divide lambda prime by ‘I’ then I end up with inductance of this coaxial cable, which is 1/2pi log (b/a). Is that alright? In between we forgot to put mu 0 because this flux is not related to h but related to b. So, the flux lambda prime is actually b dot ds and b will have the mu 0 factor and h is I/2pi r, whereas b is mu 0 I/2pi r.

So inserting mu 0 everywhere which I forgot to insert earlier, I’ll get the inductance per unit length as mu 0 log(b/a) divided by 2 pi. If you recall what was the capacitance per unit length, capacitance per unit length was 2pi epsilon 0 divided by log of b/a for the same coaxial cable with outer conductor radius of ‘b’ the capacitance was 2pi epsilon 0 divided by log b/a. Now, we are ready to actually calculate a very interesting circuit property called as distributed impedance.

Distributed impedance is completely opposite to that of the localized impedance that you have seen. What is localized impedance? Localized impedance is one in which I am considering a particular L and a particular capacitance, let us say, and then I am calculating the impedance of this particular circuit. In this localized impedance as the name localize suggest, the circuit dimensions, the inductance and capacitance circuit dimensions, are taken to be very very small, whereas for a coaxial cable, the coaxial cable is not a small quantity.

It is actually this long cavity; I mean long cable. And therefore if you want to actually calculate you have to calculate things in terms of capacitance per unit length and inductance per unit length. These quantities which are measured in per unit length are called as distributed quantities. They are not associated at a particular point in a coaxial cable. It would be completely foolish to ask, what is the capacitance at 10m for this particular cable?

Because at that particular point, the capacitance actually will not exist. However, you have to ask what is the capacitance per unit length for these structures? So, because of that these impedances are, I mean if you can say what is capacitance per unit length and inductance per unit length, you can also talk about their ratios or their combinations in terms of the general impedance and this impedance will also be a quantity that would be per unit length.

And we will later see that in transmission lines, which are very important for applications this distributed impedance Z0 for lossless transmission line, for a lossless coaxial cable for example is given by L/C, where L is inductance per unit length and C is capacitance per unit length and the value for this coaxial cable would be square root of mu 0 by epsilon 0. We can say that the log b/a factors cancel out and 2 pi factors also cancel out.

And in the numerator you are left with mu 0 and in the denominator you are left with epsilon 0. This value of mu 0/ epsilon 0 actually has a special denotion. I mean we denote this one specifically with some quantity eta 0 or eta and we call this as free space impedance. So, we call this as free space impedance and this turns out to be approximately 120pi ohms or 377 ohms. You can actually see this one by plugging in the values of mu 0 and epsilon 0.

Mu 0 is 4pi*10 to the power -7 Henry per meter and epsilon 0 is 8.85*10 to the power -12 farad per meter. So if you plug in those values, you will see that this free space impedance is around 120pi ohms or 377 ohms. Associated with this coaxial cable, there is another quantity which will become very important. This quantity is called as the phase velocity. Phase velocity, you will later see that.

It is given by for this lossless coaxial cable as 1 by square root and C and plugging in the values of L and C, we will see that this is actually equal to 1 by square root of mu 0 epsilon 0. And this quantity will actually be equal to the value of c, which is speed of light. Hopefully this might come as a surprise to you because you started out with some circuit, we were talking about capacitance and we talked about inductance.

Capacitance had some electric field and inductance was defined in terms of magnetic field, but somehow we have formed a quantity, which is actually giving me the value of speed of light. So does this actually mean that there is some relationship between light and electromagnetic fields? Yes, there is and that is what we are going to discuss after we have completed Faraday's law. So, this was for one simple geometry that we calculated the inductance, more precisely what we calculated was self-inductance.

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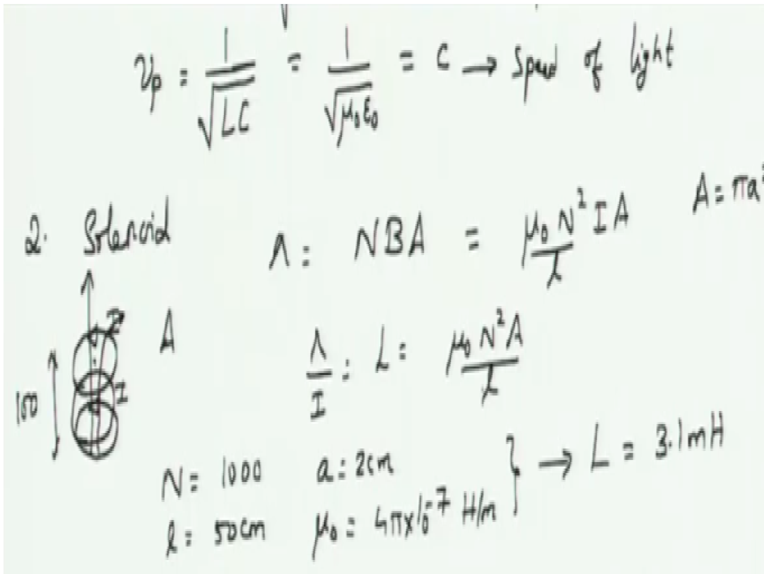

Consider a second example. This example is of a solenoid. If you consider a long long long solenoid you know that the magnetic field is actually, entirely along the axis and completely confined within the solenoid. And because the solenoid can also be thought of as consisting of circles of certain cross sections say A. These solenoids actually are wires which are wound on a rod.

So, if you take a cylindrical rod then that cylindrical rod has a cross section A and each of these turns is actually carrying a current ‘I’. So the magnetic field can be thought of as linking this entire infinite number of circular loops that we have taken because from one loop to another loop there is a magnetic field that is getting linked. So you can actually define inductance for this one by first finding out what is the magnetic flux that is linking.

Magnetic flux turns out to be NBA, B is the magnetic field of the solenoid, A is the cross section of the solenoid and N is the number of turns. For example, if you go to a solenoid which has about 100 turns with certain turns per meter ratio, then this magnetic flux through that solenoid will be NB*A. This is of course an approximation because if you have a finite length solenoid, then the fields are exactly not confined and there is some amount of leakage fields outside.

But for us this would be alright. I also know what is a magnetic field? Magnetic field is simply NI/l that is if I am considering ‘l’ as the length of the solenoid then NI/l will be the magnetic field multiplied by mu 0. So, I can actually rewrite this as mu 0 N square by l I*A. You can say that A is pi ‘a’ square assuming that this circles or the wires actually have a radius of ‘a’. So the cross sectional area is pi a square and then, lambda over I, which gives you the inductance is given by mu 0 N square A / l.

As a simple example take a case of solenoid, which has N of 1000 and it has a length of about 50cm. Clearly this 50cm solenoid for which we are talking about the inductance will be completely different from the inductance that you find in practice or maybe the better way of saying this one is to actually talk about the capacitance of this solenoid or capacitance of a coaxial cable, which can easily run into 50cm and then capacitance of this small structure that you would have used in our laboratory.

Those two concepts would indicate the difference between the localized impedances and localized capacitances, inductances versus distributed capacitances and inductances. So, with N equals 1000 turns and ‘l’ as 50cm and assuming ‘a’ as 2cm and that the solenoid is filled with air that is there is no ferrite rod kept in between. So, mu 0 is 4pi* 10 to the power -7 Henry per meter. If you plug in these values and calculate what would be the inductance of the solenoid?

You will get that this is would be around 3.1 milli Henry. So, this is clearly not a very good way of forming an inductor.

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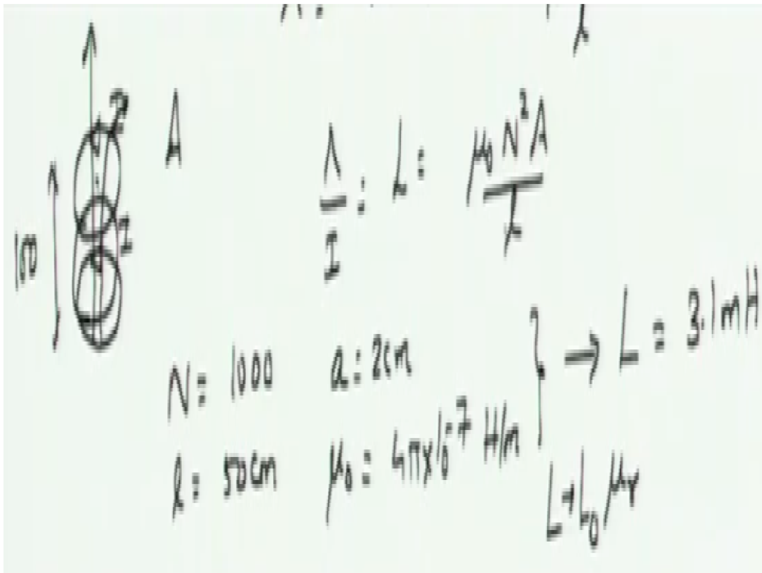

If you want to form an inductor with the larger value of L you need to somehow introduce a magnetic material, so that the inductance basically becomes inductance without any magnetic material, which we can call as L0. In this case it would be 3.1mH multiplied by mu r which would be the permeability of the magnetic rod that you are going to insert.