Electromagnetic Theory

Prof. Pradeep Kumar
Department of Mechanical Engineering
Indian Institute of Technology – Kanpur

Lecture - 46
Displacement Current

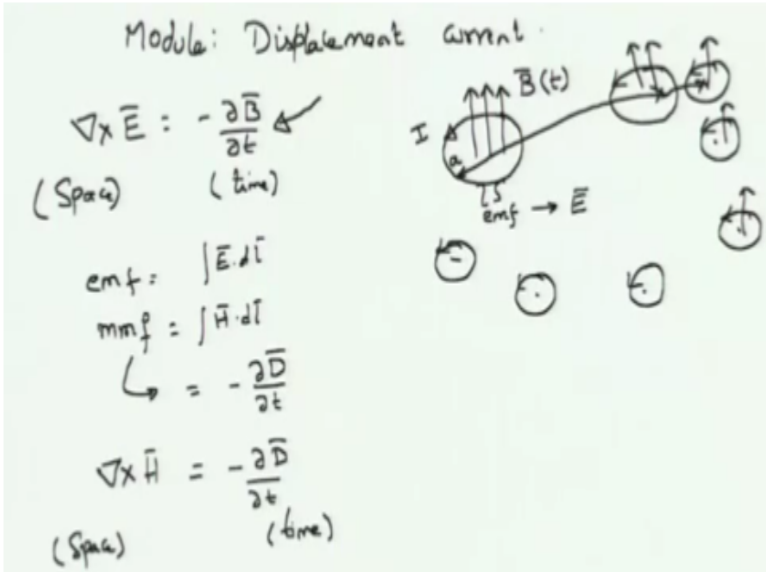
In this module we will discuss displacement current. This is a very important part of Maxwell's equations and this completes our discussion of the last, that govern classical electromagnetic theory. If you have viewed the previous modules, you might have observed that, in the beginning we did not let charge distribution vary with time, we did not let current vary with time and the fields that were generated.

Because the charge distribution was static with respect to time and current distribution was static with respect to time. We got the fields, which were also static with respect to time, the electro static field was because of the static charge configuration and when the allowed charges to move in time creating a current right, that current was steady current or sometimes called as DC. And the corresponding magnetic field that was produced was also static with respect to time.

There was no time variation to the fields and we obtain certain laws which were express both in the integral form as well as in the differential form. But when you lo okay at module about Faraday's law, then we will see that we relax this assumption that, the fields are time in variant. We let time the fields vary with time. The magnetic field, if you allowed it to vary with time it would create a time varying magnetic flux.

And if there was a complete circuit or closed circuit it would induce, the changing magnetic flux that is linking to the circuit would induce an EMF right, and this EMF is a quantity that is related to the line integral of the electrical field. So, clearly if the magnetic field were to be changing with time, it would create a time varying EMF, which would imply that the electric field now has to be time varying right.

(Refer Slide Time: 02:15)



And we expressed this Faraday's law in a compact form by writing this as curl of electric field would be equal to minus del B by del t. This was the point form for Faraday's law. What is the physical meaning for this one? Physically it means that, you can consider a loop okay, almost a closed loop is what we are considering and when you let the magnetic field passing through this loop change with time right.

So, if this B field were to change with time, then there would be an EMF induced, a current would flow in this circuit and an EMF would be induced and EMF is something that depends on the electric field. So, what we have, and you can do this one at different points in space right, so with curl operation what we mean is that, this radius of the loops keeps on shrinking, so that you are actually evaluating this at particular point in space and at any point in space.

You can actually keep doing this, you can you imagine loops around those points of vanishing radius and if you lo okay at the EMF induced, the EMF induced will actually depend upon the amount of magnetic flux linking that is changing with respect to time. So, if the flux that was changing with respect to time all over this place, then there will be an electric field that was induced, that would also been changing with space right.

So, the electric field induced here, would be probably different from the electric field that would exist here and that field would be different here depending on the strength of the magnetic field variation. So, if the magnetic field variation were to be uniform in the entire region, the electric field would be same, but it would also be varying with time right.

So, you lo okay at this equation of curl express in the point form for electric field or for Faraday's law, you would see that if B field is varying with space and it is changing with respect to time and at any point around space if the B field is changing with respect to time, it would create a changing magnetic field which would create a space varying electric field correct, So the field around this would actually be varying with respect to, varying depending on how much the magnetic field is changing right.

So, here you have a space variation of the electric field related to the time variation of the magnetic field. Of course, if you evaluate this curl operation at different points you will actually see that electric field itself could be varying in space, because the time variation of the magnetic field would be different at different points okay. So, you have all this combinations, so have inhomogeneous magnetic field varying with time.

You have inhomogeneous magnetic field not varying with time, homogeneous field varying with time and homogeneous field not varying with time. In only the cases where there is time variation will there be a EMF that would be induced and this EMF induced will change the electric field itself. So, the electric field is no longer actually a static field, it is actually changing and more importantly.

If you take a hypothetical point charge around a region where the magnetic fields is changing and you just go around the magnetic field in whatever direction that you want, there will be, So in the earlier case for the static one, you would go around a loop, but the potential difference would actually have been zero. The EMF around a closed loop was equal to 0. But clearly that would not hold in this case.

Because as you move around, by the time you have move the charge around one revolution, in that time the magnetic field would have changed and it would have resulted in a different EMF value. So, when the hypothetical charge comes back to the same point, it would no longer be at the same potential. The immediate effect of this rule is that, or this law is that, previously what was considered as an equipotential surface, such as the conductor is no longer an equipotential surface correct.

You can imagine a conducting surface, and then you see that by the time you have taken the loop around the conductor right, or inside the conductor, if the magnetic field were to be changing in that time, then there will be an electric field induced right. So, if you do this from point to point or a conductor, you will see that the electric field will not have the same potential at different points.

It actually will, the potential of the conductor at different points will actually change okay. And you can show later that this change has to happen I mean, you have to have the potential difference to be changing from point to point. If you have to convey a time varying energy or time varying signal from one point to another point and it precisely this rule that conductors not equipotential when time varying fields are present is used to construct transmission lines, wave guides, antennas so on and so forth okay.

So, the causality for Faraday's law seems to be that, we have lost the curl of electric filed equal to 0 relation. Now the curl of electric field is no longer 0. In fact, it depends on how much the magnetic field is changing there right. So, if the magnetic field is changing, electric field will have to change at least with respect to space. This is what Faradays law was about.

So, if you want to kind of summarize what Faradays law is it simply says that, if there is a changing magnetic field in a point of space, there will be a space variation in electric field. Now from symmetry, you might question that, is there a way in which changing electric field or changing electric flux density to be more precisely the changing electric flux density or flux it does not matter, if the electric flux is changing with respect to time, will there be magnetic fields induced right.

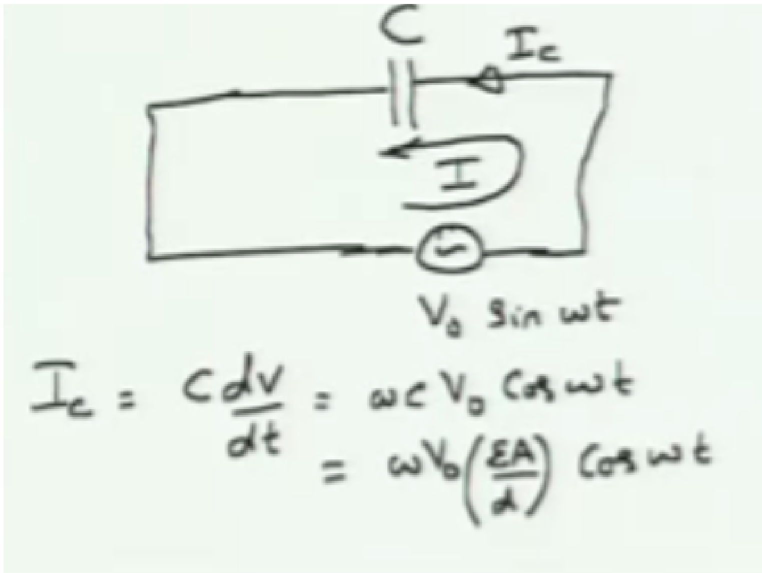
Am I looking at some sort of a relation which would tell me that there is a quantity which you can call as magneto motive force. Just as you add a quantity called electro motive force, which was the line integral of, EMF was the line integral of electric field right, between any two points. You can similarly talk about H dot d l and call this as magneto motive force okay, in analogy with the electro motive force.

So, the question that we are asking is, will there be any EMF induced, will there be any MMF induced, if there is change in electric flux density right, will this happen will there be a situation where electric flux density associated with a conductor or a circuit would change and it would introduce magneto motive force which would mean there will be a changing magnetic field correct.

Now, in point form you might be expecting for us to write curl H is equal to minus del D by del t. why have I written del D and not del E? The reason is clear. In this Faraday's law experiment, or in the Faraday's law expression, the quantity that we were interested was the magnetic flux and magnetic flux is dependent on B. So by the analogy here, we have used electric flux and we have also kept a minus sign but as you see sign is just a convention to determine whether EMF is positive in a given sense or not.

So the sign is not very important for us, but the important point is that, if there is time variation of the electric flux density, will it induce space variation of the magnetic field, would it be possible? We will explore whether such a situation is possible by considering one of the classic examples of displacement current density okay.

(Refer Slide Time: 10:05)



We will see how this classic example tells us that this relation which we have written, you know this is our relation which we can say hopeful relation, that is, we hope that this relation would exist and this relation, we will so that actually exists, not in the form that we have written but in a slightly different form, but nevertheless this expression is true. We do have changing electric flux changing the magneto motive force inducing the magneto motive force.

And hence inducing a magnetic field okay. To show that, this is what would happen, consider the classic case of a parallel plate capacitor okay, connected to this picture may not be very nicely written but, I have a parallel plate capacitor inside the parallel plate capacitor, I can put a perfect dielectric, I will also assume that the plates are quit large compared to their separation right. So the plate cross section is quite large compare to their separations.

So, I can actually do not have to worry about the fringing fields right, fringing fields would exist where ever there is cross sectional, which is actually similar to that of separation is equal to the cross section. So, this is my ideal capacitor okay, so the capacitor is producing a voltage, lets a V 0 sin omega t some frequency omega not really important what frequency that is and then, we ask several questions.

One of the first questions that we would ask is whether there will be a current through this loop right. This is forming a close loop, except for this capacitor gap okay, except where the capacitor is located. So, will they be a current in the loop. What would circuit theory tell us? Answer is yes. There will be a current, and let us called that current as some Ic okay. Let us call this current as Ic, which is the charging current for the capacitor.

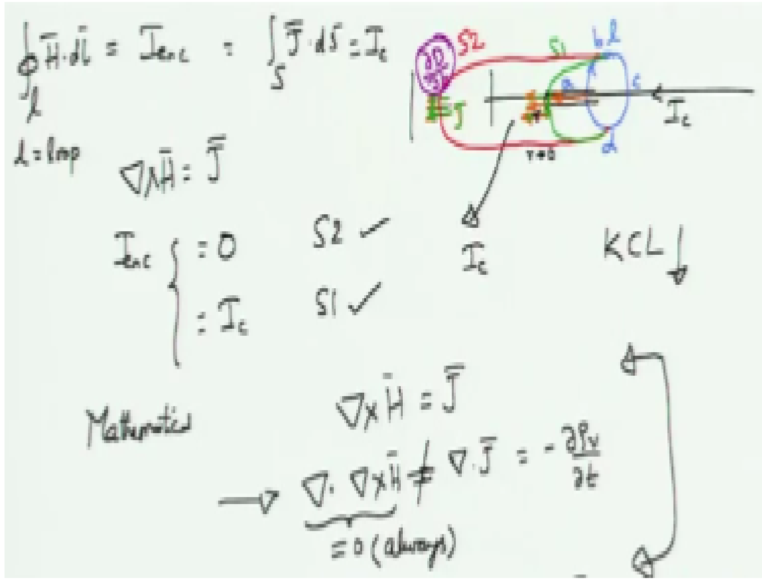
If the capacitor has a value of C capacitance, then the charging current would be C d v by d t which would be C or omega C V 0 cos omega t if you differentiate V 0 sin omega t you will get omega V 0 cos omega t and then, you can use the classic expression that we developed for the capacitor right. For a parallel plate capacitor, which is filled with some perfect dielectric of dielectric primitively epsilon.

This becomes epsilon A by d, where A is the area of the capacitor the cross section area of the capacitor and d is the separation between the two and there is cos omega t. This is the charging current that we would expect. Now if there is a current inside here right will there will be a magnetic field associated and will that magnetic field be changing with respect to time. To answer that, we need to see what happens when there is a current carrying conductor.

What does Ampere's law tell us? Ampere's law tells as that, whenever there is a current carrying filament, for example, if this is my filament, and this current is passing through this filament right, there will be a magnetic field surrounding it right. This is the magnetic field that would surround, the magnetic field lines would be surrounding it like this correct. We have calculated what is the magnetic field around this current carrying wire.

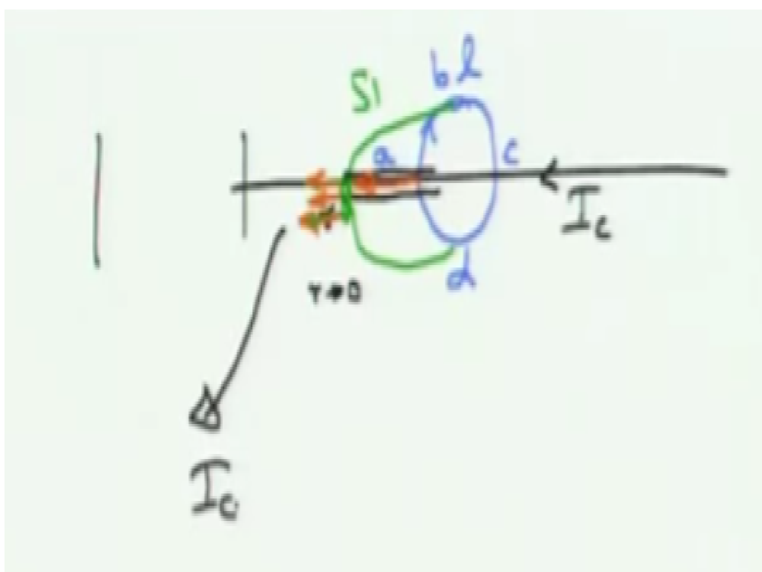
And if there is a current I, the magnetic field will be obtained if you lo okay at a closed loop integral of H right. This is what Ampere law tells us.

(Refer Slide time: 13:30)



Whatever the current that is enclosed by that closed contour so let us call this contour as some L okay, to indicate that this is actually a loop. I could have used C, but I have already used C for capacitor. So, I do not want to confuse two C's. So, I am using this L to stand for loop okay.

(Refer Slide Time: 14:00)



So, to this circuit I will consider a loop okay call this as some a b c & d and let us say the direction is chosen in such a way that you go from a b c d to a okay. So, this is the loop that I am considering and then to this loop, I will say whether they will be a magnetic field and the answer would definitely be yes. Because, this current enclosed can actually be written as integral of J dot d s where s would be the surface that is encapsulated by or enclosed by the loop correct.

Surface is enclosed by the loop. So, let me consider a surface okay. Let me consider a surface over here that lo okays like this. Remember there is no specific way and which I have to consider the surface. This is what this laws tells me. It does not matter what surface that is, as long as this surface is completely enclosed or some surface is actually bounded by this loop L. Then the current enclosed must be given by the current density times or the surface integral of the current density through that particular surface.

So, let us call this surface as S 1. I understand that, there are things which are getting very crowed over here. So, let me rewrite by expanding that region around which where we are interested. So, there is a charging current here, this is the current carrying filament, this is where the capacitor is sitting okay, I am just exaggerating the figure, so that you can keep track of loops and surfaces. So, this is the loop a b c d a.

So, this is the loop and the surface that we considered S 1 was the one that was in this fashion right. So, this surface S 1 actually meets the filament at one point and the current density. Sorry, the current density here would be, current is actually coming out in this fashion. So the sense of rotation, I have taken is such a way that, current and this loop would form the right hand rule okay, the fingers would curl around the loop direction.

What is the current density here? Well technically, the current density here goes to infinity but when you are multiplying that with a varnishing surface area the result will be a finite value okay, if you are not very happy with this infinite current density you can actually assume that, the filament is having a short radius of say r and in the limit of r going to 0. So, the current density can be assumed to be uniform out okay.

And if you look at this surface, the current density vectors are all uniform and coming out. And if you integrate the current density over the surface area, which is this surface area over which the current density is non-zero, then you will see that this current should exactly be equal to I C correct.

So, this current enclosed must exactly be equal to I C and everything seems to be fine. So, the point form of Ampere’s law seems to be fine even when there is time variation of the current. What happens if the current is not varying with time or more precisely what happens if this EMF source that I have connected is DC voltage? Then the capacitor would have actually act as a open circuit right, it would charge to whatever, the voltage that you have connected and once that charging process is over.

And in this case where there are no resistances in the conductor the charging process is infinitely fast. The moment you turn on the DC voltage the capacitor charges then it becomes open circuit then the current through any element will be equal to 0 because the potential is exactly equal so there is no possibility of a current. So for the DC case, we any way get 0 current and that 0 current is what is predicted by this Ampere’s law as well.

Now, it also seems to have predicted the correct value of the charging current even when the current, charging current is actually varying with respect to time correct. I enclosed is actually turning out to be the charging current, even when the charging current is varying with time. Why is the charging current varying with time? Because, I have created a time varying voltage right. So, I have applied a time varying voltage resulting in time varying current which is correctly seem to predicted by Ampere’s law.

So, where is the question of electric flux here? What is wrong with this result that we have obtained, or are we wrong? To answer this question, lets imagine going to a different surface remember there is no restriction on the surface, that am going to consider to apply the Right hand side for Ampere’s law. All that I am required to do is that, that surface also is bounded by the same loop a, b, c, d, a. So, let me now consider a surface which would go in this way okay.

This surface let us call as S 2 and now I want to find out what is the current density vector here right. Assume, that this current density vector is uniform and it has to be very very concentrated out there know, just turn a point that is coming out, where the surface is cutting through the current density vector J right. So, this is my current density J. Now what is your prediction. What is your current density here? Well, this is a capacitor, there is the gap inside.

You can actually instead of putting it with a dielectric you can replace the dielectric by air and there will not be any charges inside so there is no possibility for me to actually have any current through this gap. Even, with a perfect dielectric, there will not be any current through this. Because there are no free charges. As long as there are no free charges there is no charge motion there is no current.

So, if I apply the right hand side of Ampere's law to this, I will actually see that this should be equal to 0 right. The I enclosed would be equal to 0 on surface S 2 if, I consider surface S 2 and it be equal to the charging current, If I consider surface S 1. Now for the same left hand side I am getting two different values of Right hand side and I can pick and choose whatever value I want. If I pick surface S 1 then no change in Ampere's law is needed okay.

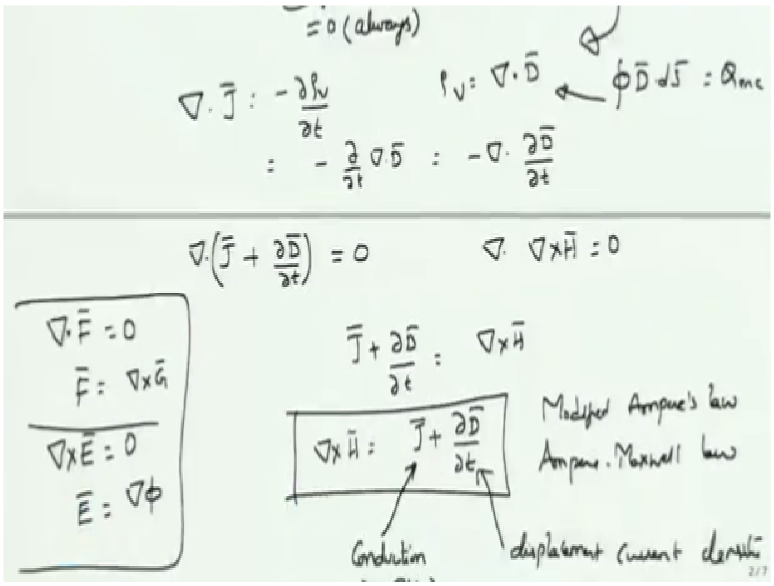
But if I pick surface S 2, then I am actually predicting completely different more over if the charging current is here I C, Until the point it reaches the gap and in the gap in the current is 0 one of the other lose also breaking down KCL is actually breaking down right. KCL is not working here because current incoming at a node is not equal to the current that is out going. So what is really happening here, what is going on that is wrong?

First of all, we agree that this is wrong, because the left hand side has not changed it is still integral of H over the loop L right. The right hand side seem to give me two different values depending on which surface i have used and i cannot have two different values for just because, i decided to choose two different surfaces. Today I might choose surface S 1 tomorrow, i may choose surface S 2 and that cannot be determined.

The answer should not be determined by our arbitrary way of choosing the surface. There has to be some consistency and this is precisely what Ampere's law does not have okay. Ampere's law is said to be inconsistent when there are time varying fields involved okay. The text book way of showing you that there is inconsistence is very interesting. So you start with Ampere's law okay, so this is what Ampere's law would tell you in the point form of course.

Then you apply a vector operation called divergence on both sides right. So you apply the divergence operation on both sides i immediately recognize that del dot J must be equal to minus del rho v by del t in general, correct.

(Refer Slide Time: 24:30)



Of course, if you are going to apply this one in a case where there are no charge variations, then del dot J will be equal to 0 and everything seems to be fine. Why because, this expression divergence of curl will always be equal to 0 always. It will be equal 0, today it will be equal to 0, tomorrow and it will be equal to 0 as long as you are alive we are alive. So because this is 0, clearly that would force del rho v by d t equal to 0 everywhere.

Now, if I have some chargers and if there are actually chargers which are vanishing in a given region of space, I cannot always say that rho B is 0 there. There is some charge and this charge configuration is changing with time. So because of that, I cannot all that time say that del dot J is equal 0. So clearly something is wrong, so this would not be equal to del dot J for all the time.

What should we do? Well, this is point where Maxwell recognized that Ampere's law was initially stated only for currents or the currents switch was steady currents and when there are time varying currents involved, the Ampere's law would not really work there okay because, we have forgotten that electric flux which would be changing would also start to impact on the magnetic field.

Of course, Maxwell arrived at that this very profound way of observing this after a lot of thinking and one of the things that i have to mention here is that Maxwell did not have the advantage of using vector analysis I mean true the way I have shown you that there is mathematical inconsistency because dell divergence of curl of H equal to 0 right. However, when Maxwell formulated electromagnetic theory, vector analysis was not yet invented.

So, he did not certainly arrive at inconsistency in this fashion. He arrived at inconsistency in various other fashions in this particular way which a capacitor kind of a situation and then recognize that he has to modify the Ampere's law and he modified it. This modification hided by a very clever idea okay. We have del dot J is equal to minus del rho v by del t. Of course, again I have to emphasize that, I am working everything in vector domain.

But Maxwell had not seen vector analysis. So, he used the different approach, but the results are essentially same okay. So, in our mind so if Maxwell knew vector analysis, this how he probably would have proceeded. So he would have started by del dot J equal minus del rho V by del t and he would have recognized that rho V can be written in terms of D field right. Divergence of the electric flux density gives me the charge distribution at that particular point correct.

This is coming from Gauss's law which states that, if you integrate the electric flux density over the close surface, this would give you the total charge enclosed right. The point form of this is, this expression. So, he simply replaced or he would have replaced rho V by del dot d giving you minus del by del t del dot D and he would have also interchange the divergence and time operations, so that he would have obtained minus del dot del D by del t right.

Now I have two divergence operations on left and right. So, I can actually pull them together and write del dot J plus del D by del t must be equal to 0 right. Now, this is lo okaying exactly like some del dot del cross H equal to 0. In fact, this is one of the two laws of vector analysis. Given any field which has zero divergence, then that field can be expressed as curl of some other vector field. If any vector has zero curl.

Then that vector can be expressed, or that vector field can be expressed as gradient of some other scalar field okay. So, invoking these ideas from vector analysis, we can proceed by saying that J plus del D by del t must be equal to curl of H okay, or the traditional way of writing this one is to say that curl of H is equal to j plus del D by del t and this is what is called as modified Ampere’s law. What has been modified here?

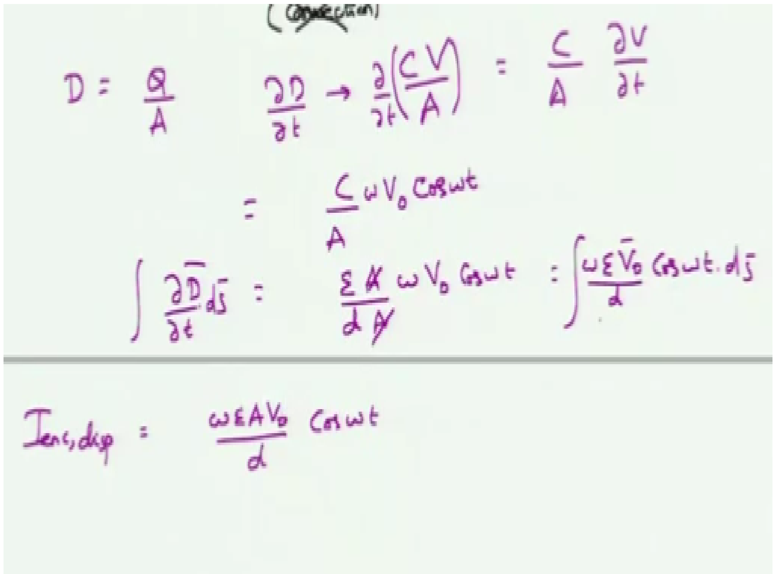
What has been modified is this addition of a term called del d by del t and this is called as Displacement current. The reason why this is called as displacement is because D is called displacement vector in earlier you know in Maxwell’s time D was called displacement vector, and this del D by del t performs exactly the same function as the regular current density function would do right.

J in order to distinguish between this placement current, we times called J as conduction current okay, or it could also sometimes be equal to convection current, although we are not really considered convection currents in our course okay. So, we have two currents now, one current is because physically chargers are moving okay and this movement of the charges inside a conductor constitutes conduction current.

Now there is other term which is actually the variation of the electric flux density and that also performs the same role as that of the conduction current and therefore, this is called as Displacement current. Well this is density again, because to actually get the displacement current you need to integrate over a closed line and over closed surface right. This is the point form of modifier Ampere’s law or sometimes called as Ampere Maxwell law.

When there is a possibility of no conduction current, as is would happen inside that of the capacitor there are no conduction current, there are no free charges for us to carry the current. So, in this case J will be equal to 0 but, what will be non 0 is del D by del t so del D by d t will pick up whatever the charging current Ic is, and in that process give you the continuity of current okay otherwise, charges would have piled up right.

(Refer Slide Time: 29.09)

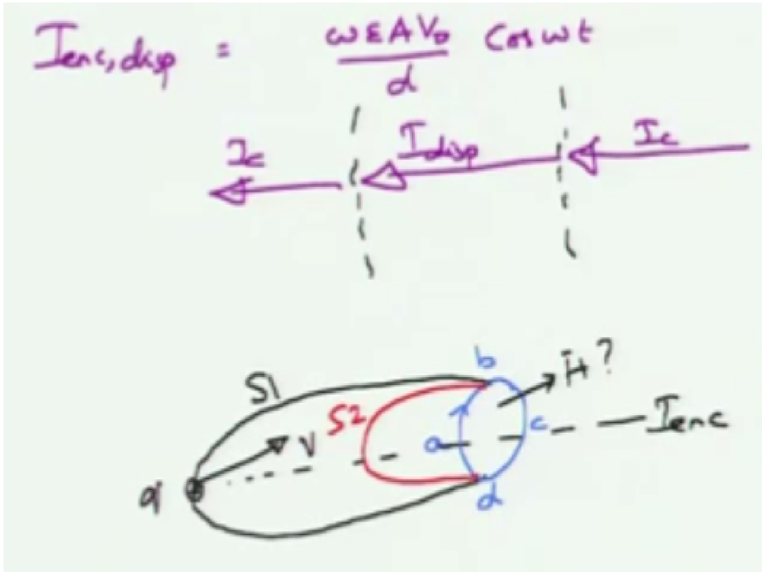


But this is a situation where you cannot really have that one, so the role of the conduction current is now being played by displacement current okay. To give you an idea of how this displacement current will be inside capacitor, remember that D is given by Q by A and del D by del t will be and what is the charge that is there? That would be C into V, for a capacitor charge is C into V divided by A right.

So, del D by del t will be del by del t of this quantity, which I can obtain by removing this C and A outside here and then say that, del V by del t, now C by A, what is del V by del t? This was V 0 omega V 0 cos omega t correct. So, there is C here, C can be expressed as epsilon A by D omega V 0 cos omega t.

There is A in the numerator, A in the denominator, that cancels out with respect to each other okay and then you get omega epsilon V 0 by D cos omega t this the current density right, this the displacement current density To actually get the total current, i need to multiply this one by, or I need to integrate this one by the appropriate. So, this is the one of the appropriate vectors V 0, then I need to integrate this one over the surface area right.

(Refer Slide Time: 30:38)



When i do that i see that this is simply since this is perpendicular to the surface, this would corresponding to simply multiplying this one by the total area a right. So this expression the total current, which would be because of the displacement would be equal to omega epsilon A V 0 by d cos omega t and if you see what is that we have return earlier, this is exactly the same current the charging current IC that we have obtain here.

So the charging current is omega V 0 epsilon A cos omega t divided by d. So, this is precisely the same current that we have obtained. So, you have a charging current IC, which is then converted in to displacement current inside the capacitor okay. So, this is how the current continuity is ensured and we again at the outside, there is charging current only right. So, outside there is charging current so that, there is the continuity of the current ensured okay.

Well, this analysis should have convinced you that, the addition of this displacement current was a very very important step if this has not convinced you, let us lo okay at one other simple example okay. I will not carry out the analysis for this one but, I will just give you the flavor of what is involved. Suppose I consider a charge okay, let us, consider of the positive charge which is moving with a certain velocity okay.

Let us, say the charge is moving in this particular direction with some velocity v, charge moving with a velocity obviously constitutes a current right okay. If I consider a loop, this loop again you know it could be in this direction I am not very regressly considering the direction of the loop. For this loop, will there be a magnetic field? Well there will be a magnetic field provided, I take surface which would go through this point right.

So, at this particular time, I have to consider a surface S 1 okay, which would just connect to the charge and this surface as to change as the charge moves along this particular direction. But, the point is that, at this instant of time when the charge is located at this point there, if i consider a surface that is just touching or just imaginarily passing through the charge, there will be current density.

Because the current is moving V is there so, this is the direction for the current density and this current density the component of current density along normal to the surface will give me the current enclosed and there would be an essentially a magnetic field correct. Now, if I did not have displacement current, this is no actually no conduction current I mean there is no wire which is actually carrying this charge q along the direction for I enclosed.

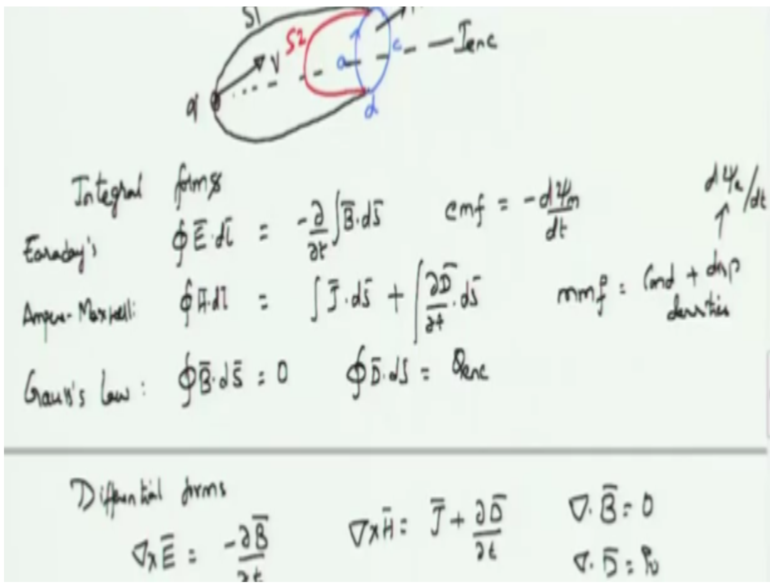
There is no wire is here so whatever, that is there, that must have been only the electric flux density changing through this loop or there is a displacement current, because I could very well imaging this surface S 2, which would not enclose the charge or it would not pass through the charge and hence would not give any current right and would not give you any magnetic field. So, this content enclosed, for the surface S 1 is simply because of the displacement current.

Okay and not because of the conduction current and it is true why because, a charge in motion which is changing its velocity would not introduce static electric field, it would actually introduce a time varying electric field right, if the charge is moving with a certain velocity and its velocity its changing, I mean its velocity is moving, then there will be time varying electric fields introduce, the electric field will not be static anymore.

So, this addition of displacement current actually completes Maxwell's equations and they form the bases for all of the classical electromagnetic theory from transmission lines, wave guides optics, antennas, microwave circuits, low frequency generators, AC generators, DC generators, machines and any kind of electromagnetic application that you have in mind which utilizes charges, currents in motions and the corresponding fields which are interacting.

There are only certain set of equations called Maxwell's equations which are sufficient to describe all these phenomena okay. So, let us write down these equations once and for all, to show you where we are at this point and from now onwards, we will consider some ramifications of this equations okay. So, let me write down the equations in the integral form here, okay and then write down the equations in the differential form.

(Refer Slide Time: 35:13)



So, we have Faraday's law, what is Faraday's law? The EMF induced right, which is given by the integral of E over a certain loop, must be equal to minus del by del t integral of B dot d s. Remember Faraday's law, actually has two types of relation right, one where the path is varying and one where the field is varying but both are the essentially equivalent right, in words this would be EMF induced is equal to minus d psi m by d t, where psi m stands for magnetic flux okay.

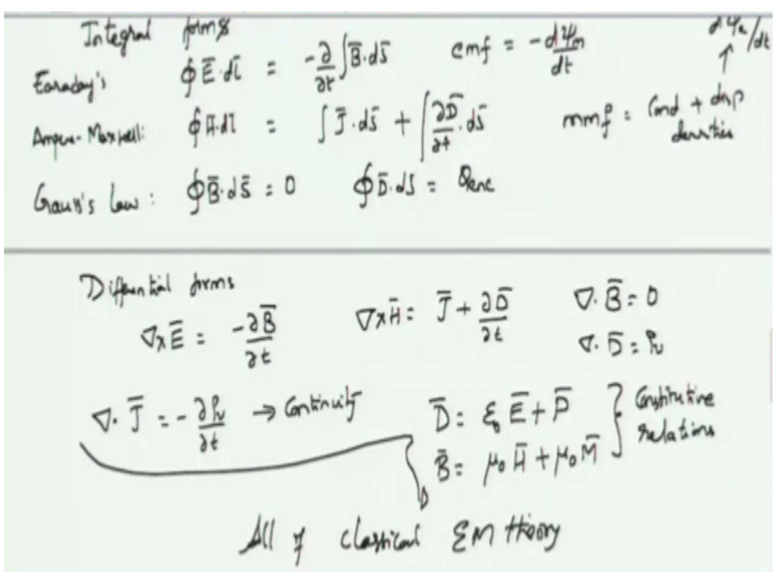
Next law is Ampere Maxwell law okay, Ampere Maxwell law states that, the magneto motive force which is given by the line integral of H is given by integral of J dot d s, which is the conduction current plus the displacement current del D by d t dot d s okay, again I am not assuming that the paths are all moving with respect to time.

In words, this is magneto motive force is equal to conduction current density plus displacement current densities okay and displacement current density is because of the change in electric flux density okay. We have two laws which are divergence based, and these are called as Gauss's law okay. One for electric fields and one for magnetic fields. So for the magnetic field, del dot B is equal to 0 and for electric field del dot D is equal to charge distribution rho V right.

Of course, the conventional way of thinking about del dot B equal to 0 is that, there cannot be any single magnetic charges there are no magnetic charges found anywhere in universe. So, if there was some magnetic charges that were found, you have to replace them with magnetic charge density but, this is highly unlikely to be found and even it is found it has really no bearing on the electromagnetic phenomenon that we have already studied okay because, these are not found everywhere.

They are not supposed to be very extremely rare if at all they are formed. However, charges are found in abundance and charge density is related to the divergence of oh, am sorry I was supposed to be writing the integral form. So let me write down the integral form for this. Integral form is B dot d s is equal to 0 integral of d dot d s is equal to charge enclosed right and the corresponding differential forms are, and in both cases whatever I was saying essential the same.

(Refer Slide Time: 38:04)



So, for the differential form Faradays law becomes curl of the electric field equals minus del B by del t and Ampere Maxwell law becomes curl of H is equal to J plus del D by del t. Please note sign over here and two Gauss's law become del dot B equal to 0 for magnetic fields and del dot D equal to charge density for electric fields. These, are four Maxwell's equation.

Now, in addition to these equations, you also need to consider three other equations, one equation is del dot J is equal minus del rho V by del t. This is the current continuity equation which states that in any given region there will be current only when the charge distribution there is actually changing decreasing okay. Then, we have Maxwell's equation written in terms of E, B, D and H okay, but we have to relate them.

How do we relate them? D will be epsilon 0 electric field plus P, where P is the polarization. When, there is matter involved, P kind of relates the electric field to the matter whatever that is happening in the matter right. So, this is the relation for the D and E this P for simple media will be proportional to electric field and for complicated media it could be non-linearly dependent on electric field, it could be dependent on electric field applied some time ago, it could be dependent on electric applied somewhere in a different direction okay.

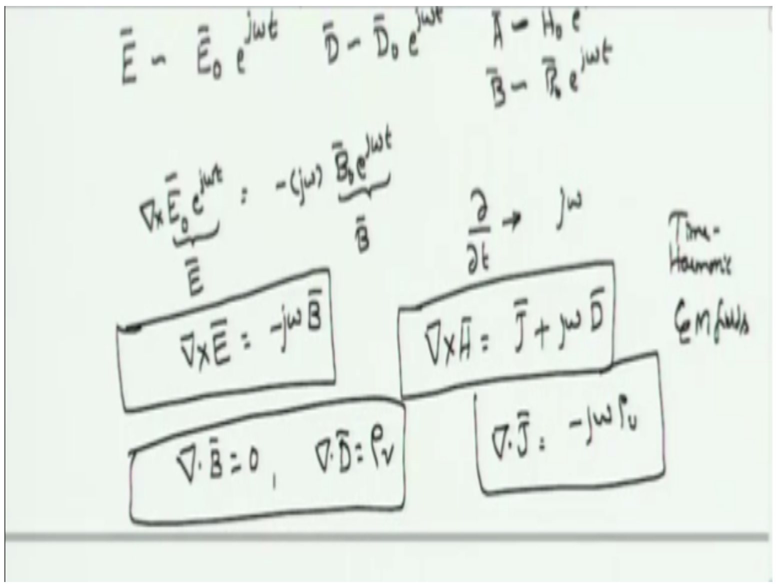
All these complicate cases are possible and they actually lead to very interesting electrical properties of materials. There is a corresponding relation for B and H, which is even more exotic, this is mu 0 H plus mu 0 m, where m is the magnetization and that describes how the materials have to be modeled diamagnets, ferromagnets, ferrimagnets, anti-ferromagnets.

You know all paramagnets, all these magnetic materials are characterized by different forms of magnetization m compare to the electric relations or electric materials, magnetic materials are mostly non-linear and exhibit hysteresis okay. So, they are not that easy to work with compared to electric materials okay. And magnetic materials are also strongly an isotropic compare most electric materials. These equations that relate D, E, P, B,H, M are called as constitutive relations.

Constitutive relations are those which tell us how the electric field would be hive inside a matter and how the magnetic field would behave inside matter okay. So, the integral form of the equations are the differential form of the equations which ever we use, these set of equations plus the addition of three equations and in fact in the constitutive relations, you are again going back Maxwell's equation to analyze what is happening.

So, in a sense that, constitutive relationships, I have already been covered in Maxwell's equation so in addition to these four Maxwell's equations the continuity equation is a conservation equation that is sufficient to describe all of classical electromagnetic theory okay.

(Refer Slide Time: 41:30)



There is one additional form of these equations which are useful for us okay and this additional form results when we assume the time variations of the electric fields in the form of a facer. What do I mean by that if the electric field let us say has certain value is E 0 right, it is pointing a certain direction E 0 but with respect to time, it is changing has some sin or cosine or in general changing has e to the power J omega t right?

So it is actually sinusoidal wave which is pointing in a certain direction but with respect to time it is changing as a sin or cosine wave okay or in general a complex exponential wave okay. We assume that all the fields are changing like this so d will also be pointing in a direction, we know, but it would be varying with the same frequency, H and E would also be varying as H 0 E power J omega T and B would also be varying as B 0 e power J omega t right.

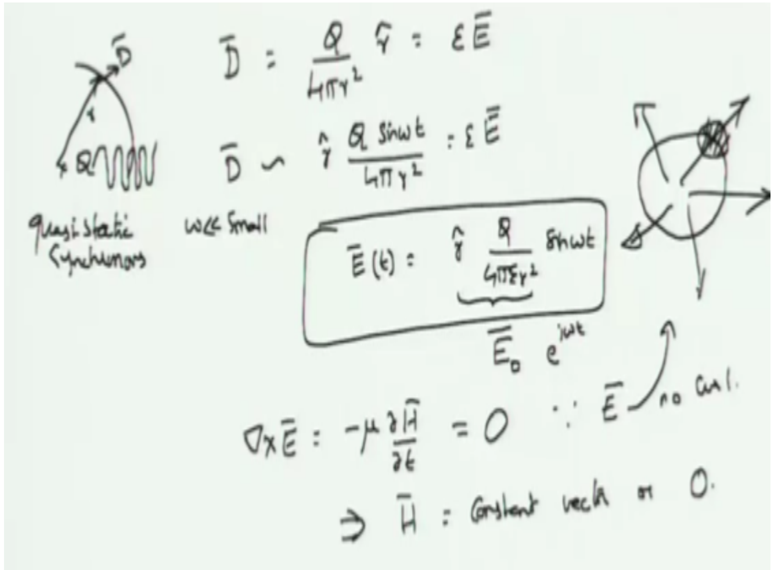
In this case, what will happen to the differential form equations? Well, if you take the curl of this electric field, you will see that this would be curl of E 0, e power J omega t must be equal to minus del B by del t and B is B 0 and time variation is happening on only on e power j omega t so I can pull that I mean I can differentiate e power J omega t to get J omega and then I get B 0 E power J omega t right.

So, you see here that this is E 0 e power J omega t nothing has happen because I have applied the curl to this one that is presence of E J omega t has not really done anything to this and this E 0 J omega t is nothing but electric field itself and B 0 J omega t is nothing but magnetic field B itself right. So, the curl equation actually becomes minus J omega B with the understanding that electric filed is pointing in a direction given by the vector E 0.

And it is varying with respect to time has a sinusoidal or a cosinesoidal function as of frequency omega right. So what really has happened is, we have replaced this del by d t operator by multiplication by a factor of J omega okay. So with this, I can actually write down all the other forms I will leave them as an exercise to you. I have curl of H is equal to J there is no time variation there, but there is a del D by del t there.

So replace that del by d t by J omega and this becomes D okay. Then, we have del dot B equal to 0 and del dot D equal to rho V no change in these two equations. The continuity equation becomes del dot J equals minus J omega rho V okay. So, these equations are known as harmonic form of equations or some times to emphasize that is harmonic motion is happening in time, they are called as time harmonic electromagnetic fields okay. This is the time harmonic field expressions for electric and magnetic fields.

(Refer Slide Time: 44:53)



So we have seen these different equations let us actually see whether there is some arbitrariness included when we are writing the solutions okay for the Maxwell's equations. Let us consider an ideal dielectric okay. And for this dielectric, if there is a charge sitting somewhere, then the D field for the static case will be Q by 4 pi r square in the direction of r. Of course this D field would also have been equal to epsilon times E, where the epsilon is the permittivity of the dielectric that you are considering.

So, you have a charge here which is q and am lo okaying for the D field around distance r at this point and this D field is give by this expression. What is this charge is actually varying slowly with time in the form of say some sin omega t right? If that happens, then one might reasonably expect that D would also start varying slowly the keyword here is that it is actually varying very slowly, okay.

In technically, this term is called as quasistatic or sometimes this is called as synchronous variation okay. Assume that the frequency omega is very very small okay, in that case, in the charge quantity of charge increases slightly because when it is getting multiplied by Q sin omega t when it increases slightly d field would also increase slightly but, in phase with whatever the charge variation okay.

So we are neglecting all the other propagation effects for now so D might be expected, you might expect that this would also be equal to Q sin omega t divided by 4 pi r square okay. This would be equal to epsilon E so clearly E for the time varying case, will be equal to r hat q by 4 pi epsilon r square sin omega t. Now, the question is whether such an electric field will be the solution of Maxwell's equation? We will try to find this out okay.

The point to note here is that electric field is radially outward okay. It might be changing with respect to time that is the magnitude of electric field might be changing with respect to time but the direction of the electric field is all along radial direction. In fact, this r hat q by 4 pi r square can be considered as some E 0 right, the direction in which the electric field is pointing and this sin omega t is nothing but imaginary part of e power g omega t.

So this is precisely the form that we actually used earlier to write down the time harmonic solutions okay. In these earlier expressions, you could have used H 0 cos omega t or H 0 sin omega t as well to get the same relationships okay or similar relationships okay. So here I am, this is an example of electric field varying sinusoidally okay and pointing a particular direction which is radial here. What is the curl of this electric field?

We know that, this sort of an electric field does not really have any curl, because you consider a small region over here okay, over which you are trying to evaluate the curl right. In this region, as many lines are going in and as many lines are coming out, there is really no curl for this electric field right. So because of that, you have this other equation which is curl of electric field right is equal to minus mu del H by del t assuming that the material medium has a called constant permeability mu.

So, this seems to indicate that curl of electric field must be equal to minus mu del H by del t, but this much also be equal to 0, because E field has no curl. Electric field is pointing all the time along radial direction therefore this equation would mean minus mu del H by del t must be equal to 0. The implication of this is that H must be either a constant vector okay or must be 0. Now, is this consistent with the other curl equation?

Unfortunately, No. Remember we are considering a perfect dielectric, therefore the gi term, the conduction current term will be equal to 0 and since we are considering an ideal dielectric, displacement current can be return as epsilon del E by del t correct. And because curl H is constant or 0 curl of H will also be equal to 0. This implies that this quantity much be equal to 0. Since epsilon is not 0 the solution is that E must be either constant or 0, neither of which is combatable with the assumed solution.

So the bottom line here is that quasistatic solutions okay are not always solutions in general situations.