Electromagnetic Theory
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Lecture 51
Polarization and Poynting Vector

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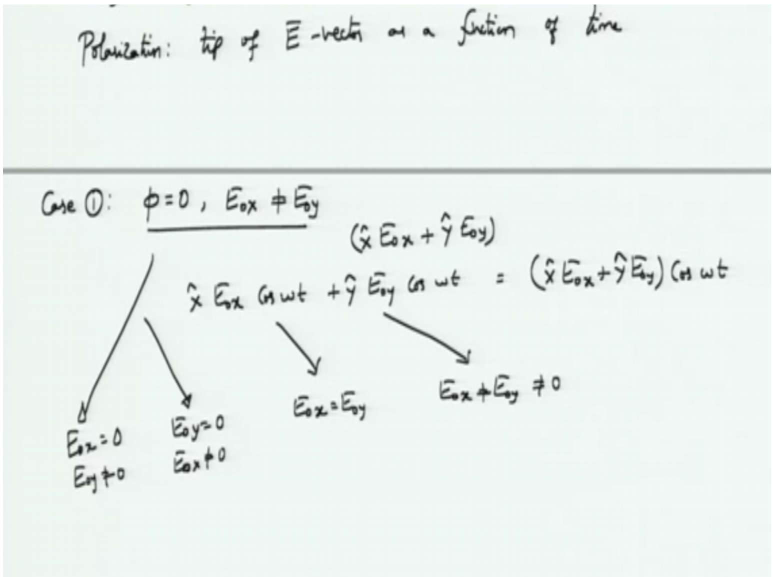

Polarization can be defined in many ways. One most common way is to actually look at this as tip of the electric field vector as a function of time, okay? This is called polarization. So what we were saying was, the electric field have two components, x and y and there would be a relative phase difference between the two which is captured in the form of this angle phi and then fixing up a particular plane.

For simplicity of the equations we can take that plane to be kz is equal to zero. Now we will have different cases to consider. Let us consider case one in which we assume that there is no relative phase difference between x and y component. We will however, assume that Eox need not be equal to Eoy. That is the amplitude of Eox, the x component need not be equal to Eoy.

So what will happen to the phasor of the electric field, the electric field phasor is x hat Eox plus y hat Eoy, right and what will happen to the electric field itself, the electric field is x hat Eox cos omega t, remember kz is equal to zero plus y hat Eoy cos omega t, remember again phi is equal to zero. So this is simply equal to x hat Eox plus y hat Eoy multiplied by cos omega t, okay?

Now you actually look at what the tip of the electric field would vary as a function of omega t. As we have said, Eox need not be equal to Eoy, however, you can subdivide this into further classes suppose Eox is equal to zero, Eoy is not equal to zero, you have the next class which is Eoy is equal to zero and then Eox not equal to zero and you have the third case in which Eox is equal to Eoy and the fourth case what you have is Eox not equal to Eoy and not equal to zero, right?

So you have all these four different combinations. But in each of these combinations if you look, the tip of the electric field vector will point only in a particular direction, what direction that is when Eox is equal to zero, this component the electric field will have no x component, it will only be pointing in the y direction, right?

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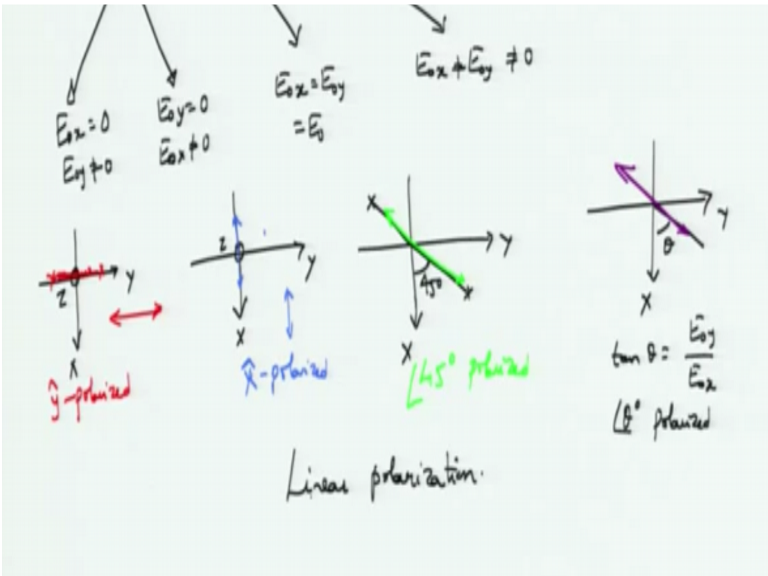

So if you draw the x and y axis in this way, there is a reason why I am drawing it in this way, I will tell you. You are looking from the top, right, you are looking from the top along the z axis, this is the direction in which the wave is approaching you and you are looking from how the wave is actually approaching you, okay?

So here if you look at this expression with Eox equal to zero, electric field component will always be along y, its amplitude can change because this is cos omega t, so at omega t equal to zero, the amplitude will be along Eoy right, then the amplitude starts to decrease slowly, decrease, decrease, decrease, decrease reaches zero at omega t equal to pi by 2 then continues in the other region, I mean in the negative side for y right and then again it continues to decrease, decrease and eventually go to zero.

So in any case the electric field component is oscillating along y axis, okay? So such a way is called as y polarized wave and is symbolically or graphically shown to be with an arrow with these two arrow marks, okay they indicating the direction of wave propagation. The second case is obvious to you now. With Eoy equal to zero, the electric field will have only x component and therefore it would be oscillating along the direction of x, so you have electric field oscillating in the direction of x, right?

This is the z axis, the electric field will go up and down in x axis. So therefore this particular thing is actually called as x polarized wave, okay? The tip of the electric field will always be along x, it could be along positive x and it could be along minus x, but it will be along x. How about Eox is equal to Eoy, what will be the direction of this one? Well, you can easily show that when Eox is equal to Eoy which is equal to some constant Eo, that constant Eo can be taken out here and then you have x hat plus y hat, right?

So the tip of the electric field vector must point along 45 degrees with respect to x axis and here it would start off with a maximum value of E0 and then go to a zero and then go back to a minimum or negative maximum value of minus E0. In any case this fellow is oscillating in this straight line with a 45 degree angle with respect to x and therefore this is called as 45 degree angle polarized, okay?

In general, of course if none of these values are equal, then you will see that it can be at an angle theta, okay? So the electric field would still be along a particular direction, but it would be oscillating at an angle theta and what is the angle theta? Angle theta is given by, the expression for tan theta is given by Eoy by Eox. Based on this, you know what is the angle of oscillation. So this is called some theta degree or theta radian polarized, okay?

So this is the most general case. In all these cases, what you have seen here, right, when the phase difference between the electric field component x and y is equal to zero, the electric field tip of the vectors was oscillating only in a particular direction, it was in a linear direction. Therefore, this entire thing is called as linear polarization okay, this is called as linear polarization, okay?

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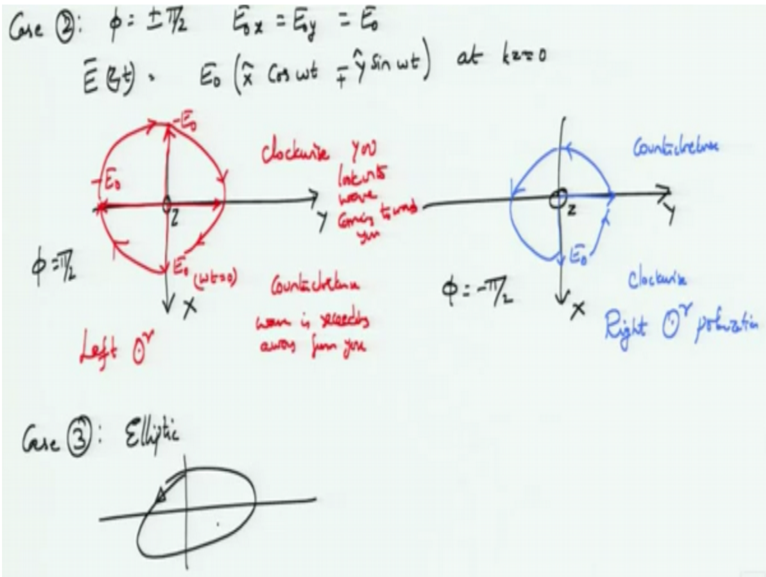

Now let us consider a different case. In this case, we will assume that phi is equal to plus or minus pi by 2. I have two cases out there, plus or minus pi by 2 and then I will assume that Eox is equal to Eoy, the amplitudes of x and y components are equal, okay? So for this, if you write down the expression for the electric field, you will see that electric field can be written as, so let us say the amplitude is equal to sum E0, right, so I have Eo x hat cos omega t minus or plus y hat sin omega t, okay?

So this is the expression for electric field at z is equal to or at kz is equal to zero, okay? Alright, what will the direction of the tip of the electric field vector be in for this particular case? Again let us go back and in this case, it might be interesting to consider or maybe it would be easier to consider plus pi by 2 once and minus pi by 2 once. So we consider phi is equal to pi by 2 in this diagram, we will consider phi is equal to minus pi by 2 in this diagram.

So this minus sign goes for pi by 2 and this plus sign goes for minus pi by 2, alright. So we have everything else that is required for us. So write this as x and y, remember you are looking into the wave that is approaching you okay? So if you are looking along the z axis or minus z axis, the wave is actually seen to approach you, the wave is propagating along plus z direction and you are basically looking at that such that the wave is coming towards you.

With phi is equal to pi by 2, your expression will be E0 x hat cos omega t minus y hat sin omega t. At omega t is equal to zero, which direction the electric field would point? It would point along x direction, right? So it would point along x direction with an amplitude of E0. This would happen at omega t is equal to zero. What would happen at omega t is equal to pi by 2. At omega t is equal to pi by 2, cos of pi by 2 is zero whereas sin of omega 2 is 1, right?

But you have a minus sign up here, therefore the electric field would point along minus y direction with an amplitude of E0, right? So obviously in the intermediate stage, it must have happened that this vector must have rotated right from x to minus y direction and at omega t is equal to 3 pi by 2, again the electric field would have rotated such that it would fall along minus E0 direction, but this time along x and then finally it would have rotated onto the y axis, okay, this would have rotated onto the y axis and then back to x axis.

So as you can see, your wave is rotating clockwise. So this is the rotation of the electric field vector along clockwise direction, but if you look from the other direction that is if you look from the way in which the wave is receding from you or is it going away from you, then it would look as counterclockwise, okay? This way is when you look into wave coming towards you, okay?

And in this case, it would be wave is receding or going away from you, okay? And this kind of a wave in which it is counterclockwise when you see from the way that wave is going out or clockwise as you see when the wave is coming towards you is called as left circular polarization. It is circular because the electric field vector actually traces out a circle, although I have not shown it very nicely over here, but it would actually trace out a circle of radius E0 and therefore this is called as left circular polarization, okay?

This concept of left and the soon to be discussed right circular polarization can be quite confusing. In most cases, it is actually better to write down in terms of the relative phase and the corresponding phasor expression. That way it is much more easier to manage because people have different notations, some people actually use magnetic fields to define the polarization, some people use E to the power minus J omega t.

Therefore, the notations could mix up, some people use the notation that electric field and magnetic field should come towards you as you should look from the wave, some people use a notation, IAAA notation that wave has to recede away from you, okay. So there is a lot of confusion here, okay.

The point to note is that the electric field vector will trace a circle okay and it would be clockwise if you see as the wave is coming towards you or it would be anticlockwise as you see if the wave is receding from you, okay and depending on which one is called as which, you can actually use that particular notation, be sure to check your textbook. So in our book, this is called left circular polarization.

So this was the case for phi equals pi by 2. What will happen if phi equals minus pi by 2, clearly for that case you will have a plus sign over here. So as before you would actually begin with an amplitude at E0 along x axis at omega t is equal to zero and then at omega t equal to pi by 2, the amplitude would have gone to plus phi direction. Therefore, this must be rotating in the anticlockwise or counterclockwise direction, right?

So this is counterclockwise direction. As you see from the wave approaching towards you or it is clockwise if the wave is receding away from you, right, and this kind of polarization is called as right circular polarization, okay? So in general, the polarization can be linear polarization which means that the electric field is oscillating along a particular direction or it could be circular polarization in which the electric field vector is rotating.

Even more general case than these two will be what is called as the elliptic polarization. In the elliptic polarization, what we have is we have a phase difference of some arbitrary value, but electric field in addition to x component and y component of electric field vectors will not even be equal, therefore the entire thing would actually be tracing out an ellipse. So this would be the general direction.

You could again have left elliptic polarization or right elliptic polarization, but that is really of not much of an interest.

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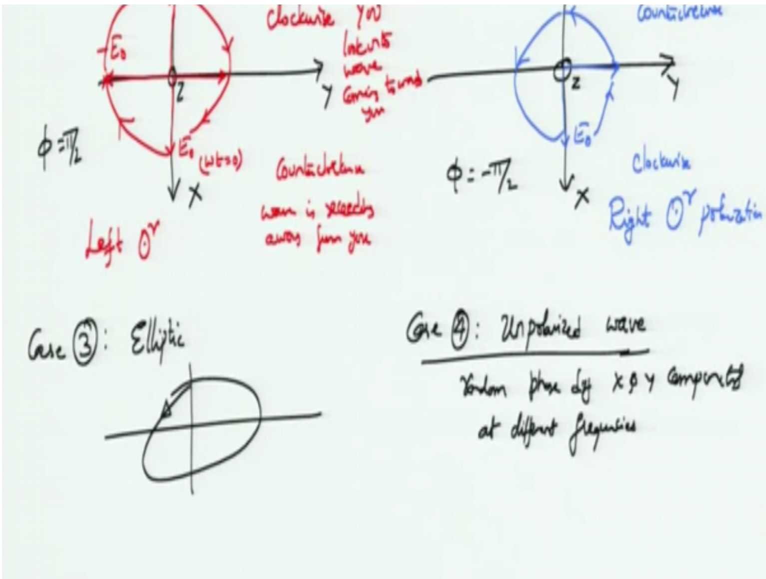

Finally, you have sometimes very important case of unpolarized wave, okay? Unpolarized wave when someone speaks of it, you should know that this is not one of the first three cases that we talked about, this can happen when there is random phase difference between x and y components. Moreover, they need not be usually at different frequencies or it will actually have a combination of this different frequencies.

So they are all at different frequencies and there is a random phase between them with the resultant that electric field vector would point at a particular direction at a particular time and its direction will change with time, okay? Such a thing change with time in a completely unknown manner. So usually we do not talk of unpolarized wave when it is there for a particular single frequency wave, okay? For that, you have only these cases.

You have either linear polarization, circular polarization or elliptic polarization. Unpolarized normally implies that you are considering waves of different frequencies added together, so this was all about polarization. It is not to be said about that, but we will not go into so much of detail, okay?

What we are interested now is to look at why did this electric field vector or what was the significance of the fact that electric field vector was perpendicular to magnetic field vector which was again perpendicular to the direction of propagation and whether electromagnetic waves themselves can carry some energy, okay?

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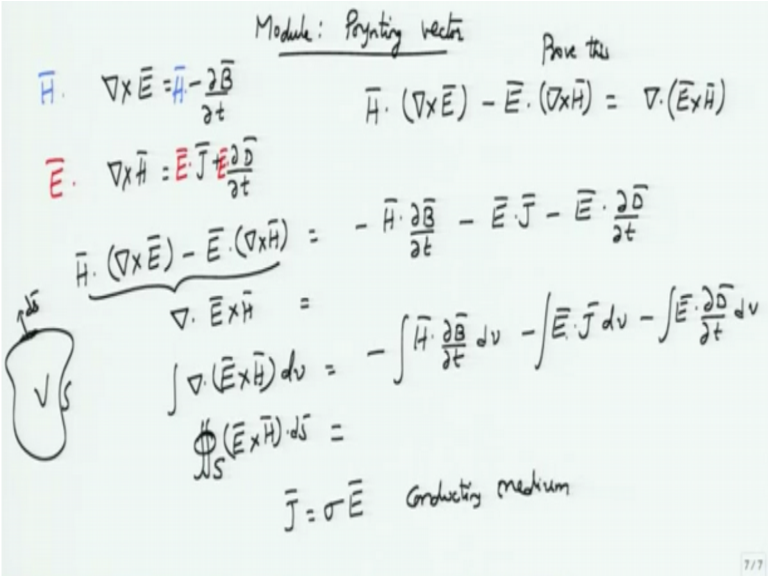

So in order to do that one, we need to introduce what is called as pointing vector. So far we have not talked about any case of energy considerations right for the electrostatic case or the magnetostatic case. We kept that one because those are not really interesting in the static cases. What is interesting is how these energies actually come into picture during the dynamic case or during the time variation case and therefore in this module we are going to study about pointing vector, okay?

This pointing vector actually tells you the direction of energy that is carried by and also gives us an expression for the average power that is carried by the electromagnetic wave. To develop pointing vector, we need to do little back off, okay, start from some Faraday's law and Ampere's law and then employ certain vector identities.

I will not prove those vector identities, but I will assume that you guys know how to prove that or you can actually go back and prove that one, okay? So to derive pointing vector, let us write down Faraday's law and Ampere's law. I know that curl of E is given by minus del v by del t, right? I am not going to invoke the phasor form here. I need the full time dependent forms here. So, I have curl of H being equal to J plus del D by del t.

Of course, I have already told you that J can be zero, but in this derivation of pointing vector, let us keep J to be nonzero, okay? This is an important reason why I am going to do that one as you would see in the other models, okay? Now there is a vector identity which you can actually prove which says that H. del cross E minus E dot del cross H when E and H are two different vector fields, this would be equal to del dot E cross H, okay?

This is a very important vector identity and the proof of this is tedious, it is not very difficult but it is tedious, therefore I am not going to derive this one. So I will say this as an exercise to you, okay? So you can prove that H dot del cross E minus E dot del cross H is given by del dot E cross H.

Accordingly, if I take H dot here that is if I dot from H on both sides, I get H dot del cross E and if I dot with E on both sides here, I get E dot J and then I get E dot del D by del t and then I use this vector identity by subtracting the second expression from the first expression, right? So when I do that one, what I get is the left hand side will be H dot del cross E minus E dot del cross H, right which will be equal to minus H dot del B by delta t.

I have removed minus sign outside of the dot product, okay, minus E dot J because I have dotted the second expression with E and then minus E dot del D by del t, okay? The left hand side of the expression can be replaced by del dot E cross H, so del dot E cross H must be equal to minus H dot del B by del t minus E dot J minus E dot del D by del t, okay. Now with this, let me actually invoke the divergence theorem and then integrate over a certain volume of region.

So in this divergence theorem, I know that have a volume V and the surface S, right, and then integrating this first with respect to volume and then converting using divergence theorem, this expression from volume integral towards surface integral will give me, so let me write down both the steps for you. It might be slightly confusing otherwise.

So this is the integration over the volume, I am getting minus H dot del B by del t integrated over the volume minus E dot J integral over the volume minus integral of E dot delta D by delta t over the volume. The left one I can replace this as integral over the closed surface of the quantity E cross H dot ds where ds is the normal to this surface, okay?

So this is the surface I am considering which is enclosed by this volume V or the surface is enclosing the volume V and then this integral of a quantity called E cross H over this particular closed surface is giving me some 3 terms on the right hand side. Let us look at this term. E dot J. Remember that J and E are related by Ohm's law for conducting medium, right? For conducting medium, J and E are related by this expression in which sigma is the conductivity right?

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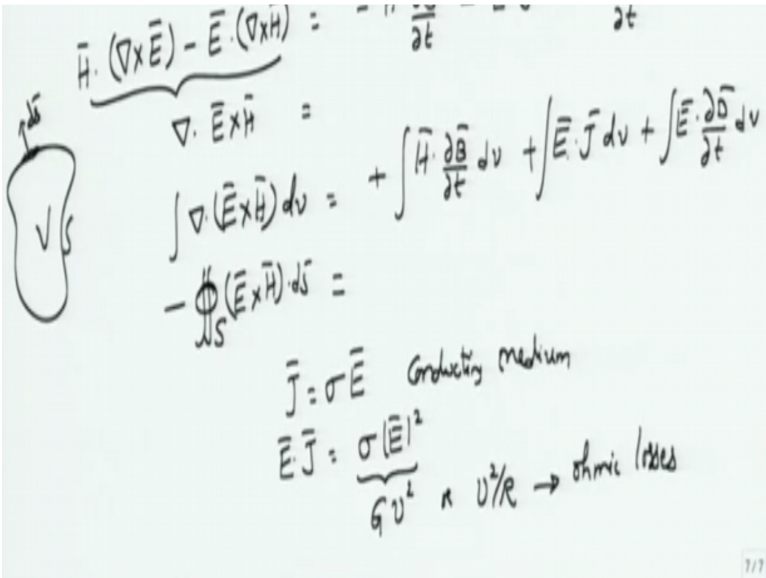

So E dot J would actually be something like sigma magnitude E square, okay? In electric field E is proportional to or is it actually the way in which we normally define for the potential V, right, and this expression should turn out to be something very similar to conductivity times V square, right? If you go to the circuit theory, this would be similar to conductivity times V square or V square by R which actually represents ohmic losses, right?

This should represent ohmic losses. If you want, you can actually put down the minus sign to the left hand side and retain all the plus signs over here to represent the loss or the stored energy and this quantity therefore represents ohmic losses, okay?

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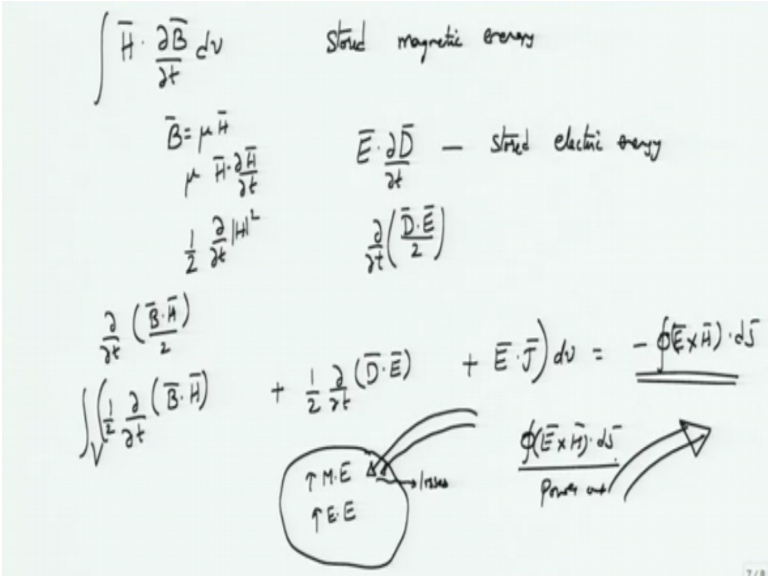

What about the term that you have as H dot del B by delta t integrated over the volume. This term is actually the stored magnetic energy, okay? Why again go back to the circuit kind of an idea, B is equal to mu H, right? So this H dot del B by del t would give you something like H dot mu can come out delta H by del t, right, or you can replace this one by del by del t of magnitude H square half, so half of del by del t magnitude of H square.

So when you do this thing, right, you can see that this H is the way in which you define the current and then this mu is related to the inductance, therefore this is kind of half L I square which would be the stored magnetic energy. Similarly, E dot del D by del t can also be interpreted as stored electric energy. So you can think of this as stored electric energy and this can also be written as D dot E by 2, okay? Of course, del by del t, right?

So this del by del t of D dot E by 2. This factor of half will actually take care of the fact that there is a differentiation of D dot E. So with this and again writing this H dot del v by del t also as del by del t of B dot H by 2, I can actually rewrite this entire thing by interchanging the left hand and right hand thing and say that del by del t of B dot H half or you can push this half outside if you want.

So you have del by del t of B dot H integrated over v plus half del by del t of D dot E, integrated over the value plus E dot J which represents the ohmic losses again integrated over the volume since everything is integrated over the volume let me take that as a common factor, would be equal to minus integral of E cross H dot d s, right? So the way in which we interpret this is now, you have a certain volume.

There is some amount of energy that that is getting lost. There is some increase in the stored magnetic energy. There is some increase in the stored electric energy. So clearly in the region of space I am having some losses and if the magnetic energy and the electric field energy is actually increasing, then that must be supplied by someone else, right?

So someone must be supplying energy so that some amount of energy is lost in the ohmic losses, in the heating of the material or the heating of the region and the remaining of that energy going into the increase of magnetic and electric energies. So with that who is this someone who is giving you this power and that is precisely this particular quantity?

Therefore, this term of minus integral of E dot H dot d s must increase the amount of power that is generated or that is being given by us or integral of E cross H dot d s must tell us the power that is going out. So if you have minus integral of E cross H dot d s as the power that is coming in, therefore integral of E cross H dot d s must be the power that is going out. This is not just the power actually.

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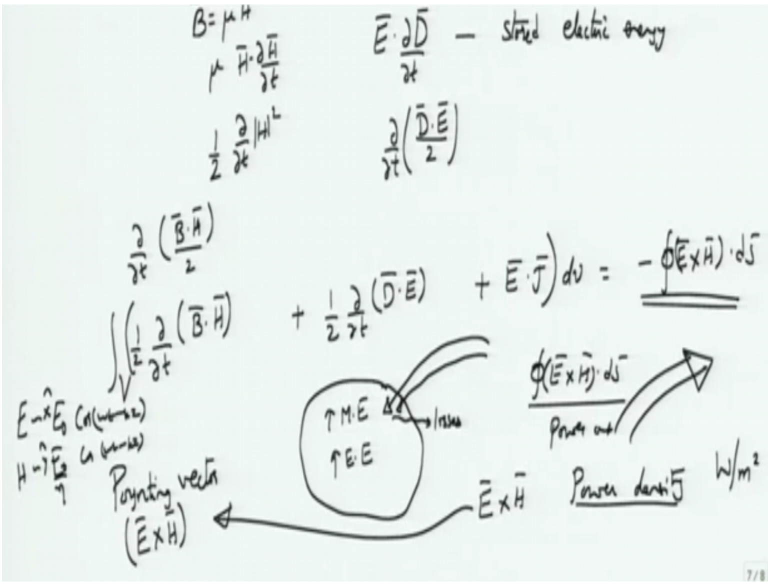

So if this entire thing is the power that is correct. But this power is power times area, right? So therefore the quantity that is inside E cross H must represent power density. This must represent power density and must be measured in terms of watts per meter square in SI units and this power density is actually called as the poynting vector or rather this E cross H is called as the poynting vector but sometimes even as poynting vector density or poynting power density just to confuse you guys.

But the most important point is that this poynting vector E cross H will tell you the power density which is actually going out of a particular closed region, okay? This is the poynting vector. And for the case of the plane wave that we considered, well, E was along having an amplitude sum E0 minus j kz, H had E0 by eta E power minus j kz.

So if you actually take the cross product not in terms of E power minus j kz, but rather in terms of cos omega t minus kz and so on, you will see that this poynting density would actually point in the z direction because E is along x, H is along y, therefore the power carried by the electromagnetic wave will be along z direction. There are some changes that we need to make to poynting vector in order to consider them to be fit for phasors.

A discussion of that, we will postpone further because there are certain other subtle aspects of poynting vector that I would like to point out, okay? So for now in this module we close with poynting vector expression. Some changes to poynting vector expression will be taken up after we consider wave reflection.