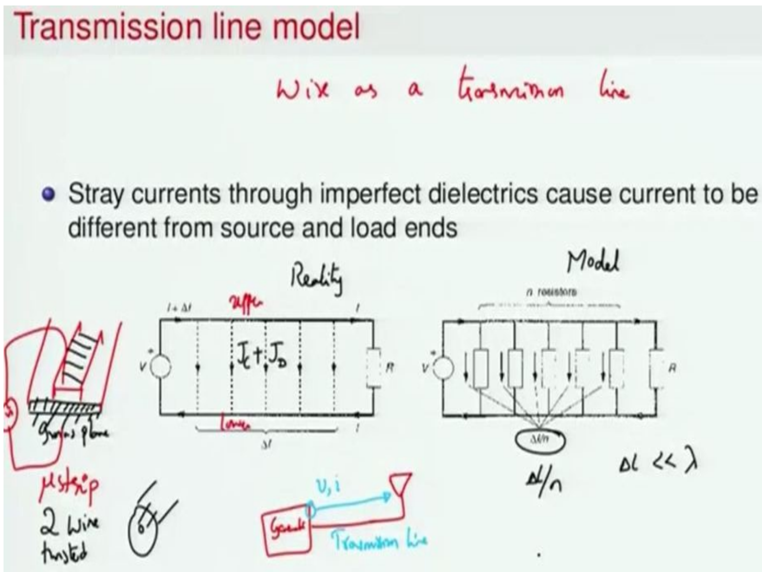
Electromagnetic Theory
Prof. Pradeep Kumar K
Department of Electrical Engineering
Indian Institute of Technology -- Kanpur

Lecture - 58
Transmission Line Model

(Refer Slide Time: 00:14)



By now we have understood that a wire is more than a wire, a wire being physical system or a physical physically made up of with conductors or dielectrics right there are to be treated as transmission line whenever their lengths of you know the wire lengths become appreciably close to the wave length of the interest the wave length of the interest so when that happens you have to treat wire as a transmission line.

So what is this transmission line? If you have obtained an impression that wire being treated as a transmission line is a bad thing you might be right and you might be wrong. In cases where you do not want the wire to act like a transmission line but the wire is actually acting like a transmission line then it is a problem because you do not want the wire to act like a transmission line. You want no delay, do not want any imperfections caused by the wire itself.

However, in those cases if you have to take it as a transmission line then it is a nuisance, it is bad. There are situations however for example you have an antenna okay. And to this antenna, so you have this antenna up here and you want to transmit something to this antenna okay. Then you need to actually have a cable which would be running from antenna terminals to the function generator or the generator of your signal which you are connecting.

In this case you want to treat this as a transmission line because this is precisely what the job of this wire is. It is actually transmitting energy from generator to antenna. So in this case you want to know how better I can make this transmission line. What are the different types of transmission line that are available and how do I analyze this transmission line? That is what we are going to occupy ourselves with in this next part of the module.

We will be talking about, how to model a transmission line and also derive some expressions for voltage and currents on the transmission line. So that is the objective for this particular module okay. First of all, let us learn to recognize what a transmission line is? Although I have drawn it with only one wire right a typical transmission line actually is made up of two conductors so you have an upper conductor over here so this is the upper conductor.

And then you have a lower conductor up here right. These conductors are pieces of metals which mean that they are not always perfect and these conductors are filled with some insulating material they could be filled with nothing but air. Air is also a form of imperfect dielectric or an insulator so it could be filled with air. You might also have a situation where you have ground plane so you have some copper in the form of a PCB and on top of it.

You have a small region or a small thickness of some insulating material and on top of this one you might actually have a metal piece and then you apply a voltage around to this metal piece and the ground plane and then if you look at how the waves would propagate on this top metal surface you will actually be able to the model this propagation in terms of this canonical transmission line model.

Although, we show this type of transmission line it is not necessary that transmission line itself looks like these two wires. This is one example of a transmission line. This is another example of a transmission line. This is known as microstrip transmission line. We will see some of these transmission lines later but all these transmission lines can be modeled by considering them to be made up of two wires right.

And these wires are conductors and that is actually clear if you look at the microstrip also. The microstrip has one piece of conductor which is the top surface and another piece of wire which is the ground plane and any voltage that you are connecting is between this top surface conductor and the bottom conductor right. So you can model microstrip line or two wire line or a twisted pair of line or a coaxial cable right.

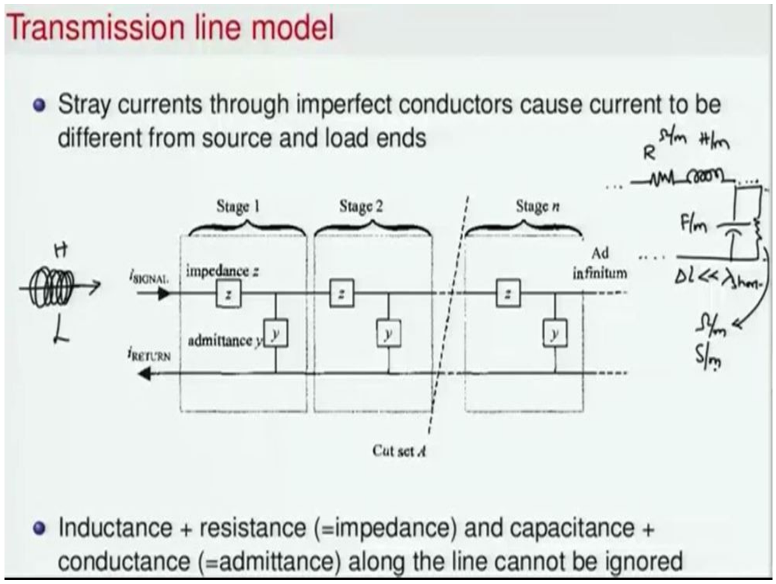
So all these things as this canonical two wire transmission line or two conductor transmission line. Two things you have to recognize, one the dielectric that is filling the region between the two wires may imperfect right. If it is imperfect, then it will carry conduction current plus some displacement current and we know that in a perfect dielectric there should not be any conducting current.

The presences of conducting current imply that there is some leakage conduction current and there will be dissipation because of this conduction current right because conduction current is i and i square R losses will be there. There is a second thing and this behavior of conduction current giving losses can be captured by assuming a set of resistors connected here. Now these resistors are not physically connected.

We are only imagining that these are connected and we assume that these resistors are placed at around delta L by n. So if you have n sections and n resistors then there are placed at a length of delta L by n. Please note that this is the reality. Right this is a two wire model that we have or a two wire physical system that we have. And this right hand side is actually a model that is trying to capture this reality.

And so in this way we are capturing the model is that we imagine that there are n resistors connected from top surface to bottom surface or top conductor to bottom conductor and these resistors are placed at delta L/n. This delta L will be taken to be much smaller than lambda. Therefore, each of these resistors can be thought of as lumped circuits.

(Refer Slide Time: 05:55)



Now that is not the only thing that happens. There was current down okay you modeled that current as consisting of resistors or equivalently some conductor right and along with that one there was also capacitance associated with this one. You have a top surface; you have a bottom surface there will be some capacitance associated. Again this capacitance will not be at a particular point but this will be distributed right.

So it will be distributed in the same way as the resistors are distributed. You have to remember that this model that we are trying to capture. So together the resistor and the capacitor will form an admittance because you have one capacitive reactance as well as when resistive reactant together they would form an admittance. Similarly, on the upper surface and you know just one conductor when there is some current i flowing there will be some magnetic field around it right.

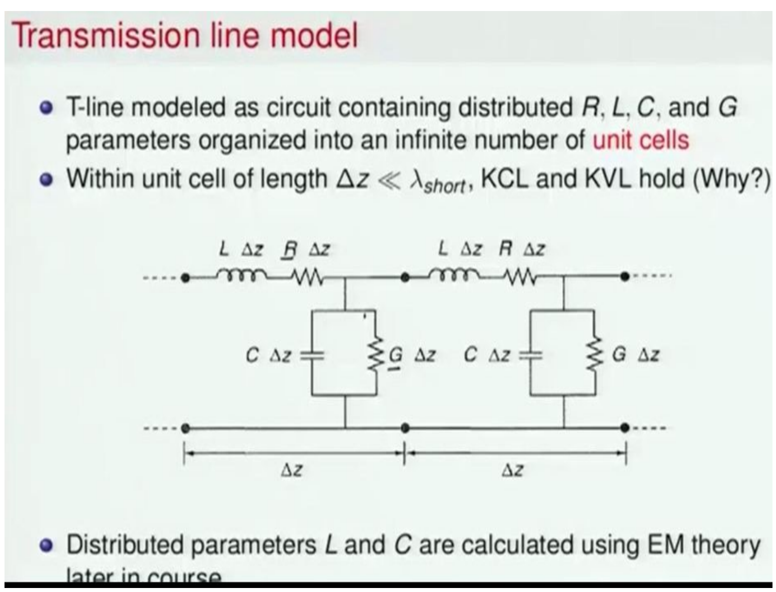
So whenever there is a current flowing and then there is a magnetic field. We have seen that there will be an equivalent inductance so there will be an equivalent inductance L and that inductance needs to be considered. A second thing is that these conductors may not be perfect right so if there are not perfect then there will be some leakage current through this conductor in the form of i square R losses.

These things will be incorporated by considering each of these conductors as composite of sections of resistance and inductance. So this again gets distributed on the susceptance side you have a capacity of susceptance and a resistive susceptance. Again these sections would also be distributed. Each of these sections should have a length delta L which is much smaller than lambda short. So that we can treat each section as lumped circuit.

However, a collection of such infinite sections will form an overall transmission line. In order to deal with the fact that this are distributed we will measure these resistances in terms of ohms per metre. This inductance in terms of henry per metre and then measure these capacitors in terms of farad per metre and this conductance in terms of again ohms per metre or you can measure them in terms of siemens per metre because they are connected in parallel.

Essentially a two wire canonical transmission line can be thought of as composed of an infinite number of stages. Each stage is so small that we can treat them as lumped circuit and each stage will have impedance as well as admittance.

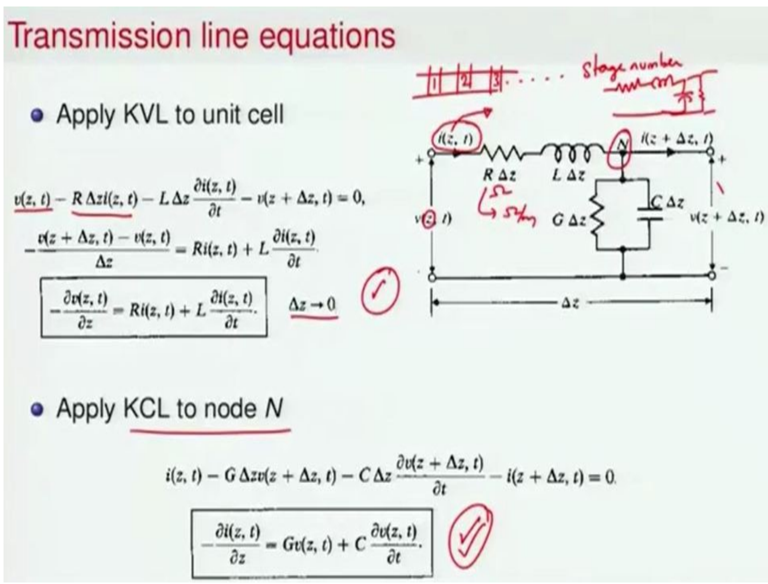
(Refer Slide Time: 08:34)



That is what the model that we are going concern and you can actually see that model over here. You have each section of length delta Z we are assuming that the transmission line is located along with delta Z. So each section has a length delta z which is much smaller than the wave length okay. And in each section you have KCL and KVL being true. We distinguish this parallel resistors connection by writing this as G and series resistors as writing it as R.

As I said these elements L, R, C, and G are called as distributed parameters of this transmission line. This is inductance per metre and resistance per metre, conductance per metre and capacitance per metre. So this is the model that we are going to considered for a two wire transmission line. So with this model let us try to see what happens to the voltage and the current okay.

(Refer Slide Time: 09:26)



In big change from a lumped circuit the voltage here will not be independent of Z why? Because we actually have a lot of stages so you have stages, stages, stages so on. You can imagine this is stage 1, stage 2, stage 3 and so on. Now when you are looking at transmission line section in a particular stage you have to give stage number okay and you find that stage number you have R, L, C and G right. So this is want each stage looks like.

If I want to specify the current, I have to specify the current in the form of stage number. Now instead of specifying the stage number because you are going consider this stages to be of infinity and distributed point wise. We replace that stage number by a continuous variable so at any point Z on the transmission line what is the current? At any point Z on the transmission line or across the transmission line what will be the voltage? This is how the fact that the voltages and currents at points on the transmission line are different enters in to picture.

In addition to the specifying how the currents and voltages vary with respect to time we also have to specify what stage or at what point on the transmission line you are and that would be captured by writing this as the function of both Z and T. This is like thinking of a wave right. So a wave we have already seen in the previous modules have to be specified both of the function of Z as well as the function of time and this is precisely what we are doing here for voltages and currents.

We are assuming that voltage and currents because we need to consider the delay in to account will not behave as simple voltage and current which are just time dependent but they will behave as though they are waves which are dependent on both Z and time. However, within each unit cell or within each stage these voltages and currents are essentially such that lump parameter loss KCL and KVL can be employed.

Consider this nth stage here which is located at a point Z on the transmission line and then you try to apply KVL to this loop. Now the current entering this stage i of Zt and the voltage across the inputs of terminals of the voltage cell is v of Zt. Now once we have passed a stage the current would have changed by a small amount and that current must be evaluated at the space Z plus delta Z that is, it is the next stage current and this is a next stage voltage.

So we need to apply KVL around this loop and when you apply, we will see that this would be v of Zt voltage along this one, -i into R so that is -R because that is not just R this is R into delta Z. This R into delta Z is in ohms but R itself is in ohms per metre. So Ri- what is the current through an indicator? Ldi by dt right so it is L delta Z, delta i by delta t- this voltage v of Z plus delta Z of t equal to zero.

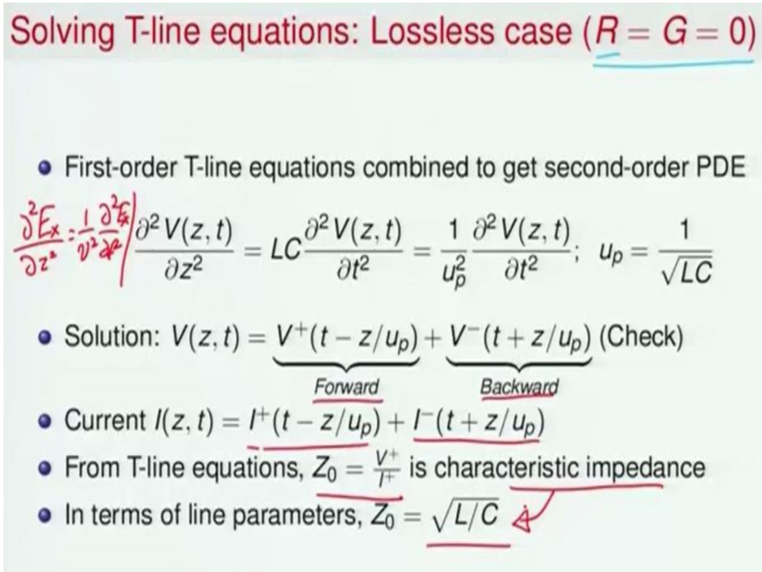
You can rearrange these equations by writing this v of Z + delta Z, v of Z and then pushing out this R delta Z i + L delta Z del i by del t on to the right hand side and then divide both sides by del Z okay. And then take the limit of delta Z going to zero so when you apply limit of delta Z going to zero you end up with this equation which states that the first order partial derivative of the voltage is given by Ri drop + Ldi by delta t or L delta i by delta t drop here.

Similarly, you can apply KCL to node N which is this particular node you can equate what are the currents going and coming out here the current coming in i, current going is i of Z + delta Z. then there are two currents which are following through this. The voltage here is v of Z + delta Z divided by C into del v by del Z and current through this conductor is C into G right so current through this one is C into G and that is what we have written over here.

We can again rearrange the equation to obtain an expression for the partial derivative of current. We will see that this is given by del i by del Z minus of del i by del Z is equal to Gv plus C into del v by Del t. So you have one equation which describes how the voltage is changing. We have another equation which describes how the current is changing; these two are first order partial derivatives. They are actually very very similar to the expression for electric field and magnetic field right.

So you can actually compare them with the uniform preliminary expressions we will do that comparison later but when you do that one, we will see that these two expression are quiet similar to electric field and magnetic field expressions and you can actually combine the first order equations in to second order PD.

(Refer Slide Time: 14:17)



How do you do that? You differentiate one equation with respect to Z and then substitute the other one and then we will consider first a case where we are considering no losses. That is we will assume that the dielectric filling the region between the two wires as being perfect with no G, no conducting current and similarly the conductors themselves which make up the transmission lines are perfect.

Therefore, R is also equal to zero. So you can combine these first order transmission line equations to get a second order partial differential equation and you will see that the equation is del square v by del Z square is equal to LC into del square v by del t square. Now does not it remind you of del square Ex by delta Z square is equal to one by v square delta square Ex by del t square.

I do hope that when you look at this equation you are reminded of that one and this LC we will denote it by a special symbol called one by U p square okay. This is the velocity of what? Velocity of the wave. Which wave? Voltage wave. Voltage is actually related to the electric field therefore what we have really written down is the wave equation for electric field. And this U p is called the phase velocity and it is given by one by square root LC.

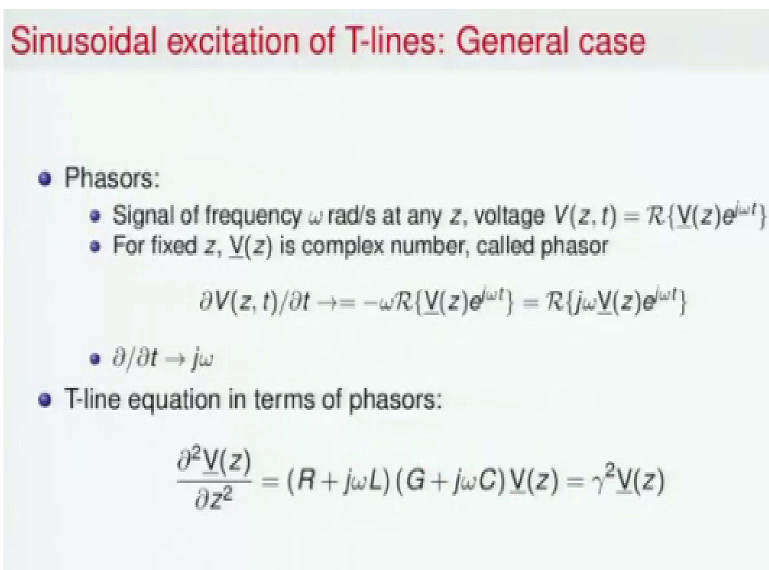
We already know what the solutions for this type of equations are? They must be propagating solutions. You have two types of solutions one is forward wave and one is backward wave.

Backward waves are generated whenever transmission line is terminated and some termination is not perfect so that a voltage wave reaching the termination will get reflected back. Similarly, one can derive an equation for current and then you will see that the current equations are also in the form of wave equation which will have solutions in the form of forward current as well as backward current.

Please note that this backward current is not the return current okay even the return current will have forward and backward components. This current is the current that we are considered in the top surface okay and that will also be made up of forward and backward currents. Now if you actually assume that only forward going waves exists and then try to take the ratio of the forward voltage to forward current amplitudes.

We will see that ratio turns out to be a number Z zero given by square root of LC and this has unit of ohms and this is called as characteristic impedance. For a lossless line characteristic impedance is completely real and is given by square root of L by C.

(Refer Slide Time: 17:02)



To summarize what we have done is that we assumed or we went through. So we will come back to this generalized expression sometime later. So what we have done is we have assumed a transmission line in the form of an infinite number of stages each stage consisting of a resistance, inductance, conductance and a capacitance. Conductance modeling the imperfectness of the dielectric, resistance modeling the imperfectness of the conductor, L modeling the fact that there is magnetic field associated with current and C indicating that the region between the two wires actually makes up a capacitor.

These values are described in per unit length terms and therefore these are called as distributed constants or distributed parameters of a transmission line. To obtain the expression for the voltage and current phase we actually ended up using KVL and KCL. We were able to do that one because we had assumed that the section length delta Z is much smaller than lambda short, the shortest wavelength.

And when we route down KVL and KCL we arrived at two equations which are partial first order and partial differential equations. And we solve those equations to show that voltage and currents are actually waves and they will consist of forward and backward waves. So we will now stop this discussion and then in the next module take up what will be the general excitation of the transmission line and then try to solve for those cases. Thank you.