

**Electromagnetic Theory**  
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**Lecture - 61**  
**Steady State Sinusoidal response of T-line-II & Smith chart**

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Title as circuit element

### Properties of standing waves

- Standing waves formed on terminated T-lines when  $\Gamma_L \neq 0$
- $V_{max,min}$  on line is  $1 \pm |\Gamma_L|$  and  $SWR = V_{max}/V_{min}$
- Voltage maxima and minima repeat every  $\lambda/2$  and distance b/w maxima and minima is  $\lambda/4$
- $V_{max}$  occur at  $2\beta z + \phi_L = 2n\pi$  and  $V_{min}$  at  $2\beta z + \phi_L = (2n+1)\pi$
- Impedance at voltage maxima  $R_{max} = SWR \times Z_0$  and at voltage minima  $R_{min} = Z_0/SWR$  (Why?)

$$\text{Impedance at } z \ Z(z) = Z_0 \frac{1 + |\Gamma_L| e^{j(\phi_L + 2\beta z)}}{1 - |\Gamma_L| e^{j(\phi_L + 2\beta z)}}$$

In this module, we will discuss the steady state sinusoidal response of the transmission lines. We introduced standing waves in the last module and we will continue with a brief summary of what the standing waves properties are. Standing waves are formed whenever a transmission line is terminated with gamma L that is not equal to 0, with gamma L equal to 0 we see that there is only incident wave.

Whenever gamma L is not equal to 0 standing waves are formed on this terminated transmission line. We also found out what is the standing waves ratio in the previous class. We found that it is actually defined as the ratio of maximum phasor voltage to minimum phasor voltage on the transmission line and maximum voltage is  $1 + \text{mod gamma L}$  when the voltage actually reaches its maximum the phase term goes to 0 or 2 pi.

And the minimum voltage on the transmission line is one minus not gamma L and this occurs when the voltage on the transmission line reaches its minima. And the ratio of these two is the

standing wave ratio and we actually also saw the relationship between SWR and gamma L that is given SWR you can find out what is gamma L or magnitude of gamma L or given magnitude of gamma L one can find out what is SWR.

At this point, probably one of the things that I want to specify is what is the range of this SWR? The range of standing wave ratio is between one to infinity. You can see that very clearly when gamma L is equal to 0 standing wave ratio will be equal to 1. When magnitude of gamma L becomes 1 that is when it could be either open circuited load or a short circuited load, the magnitude of gamma/L becomes 1 and standing wave ratio goes up to infinity.

In the same way, magnitude of reflection coefficient for passive transmission line has to be between 0 to 1, 0 corresponding to no reflection, 1 corresponding to complete reflection. We also saw that on a transmission line there is voltage maxima and minima and this maxima or minima repeat every lambda by 2.

And the distance between the maxima and minima whether you are considering an open circuited load, short circuited load or with gamma L something that is not equal to 1 the distance between maxima and minima is always equal to lambda by 4 where lambda is the operating wave length related to frequency. The maximum magnitude will occur whenever the phase term goes to n pi so you have  $2\beta z + \phi_L = 2n\pi$ .

Here please remember that n has to be negative because your z is equal to 0 is located at the load point and the input terminals of the transmission line is located at z equal to -n, so the line itself is there from z=0 to z=-n. Minimize located wherever the phase term goes to  $2n + 1$  into pi. And this is something that I will leave as an exercise which is just one step for you guys to show this.

What is the impedance at voltage maxima? At the voltage maxima, we know that voltage itself is some  $V_0$  plus into  $1 + \text{mod } \gamma_L$  at that point the current actually will be equal to  $V_0$  plus by z not into  $1 - \text{mod } \gamma_L$  the ratio of this two will be seen to be as standing wave ratio into  $Z_0$ . So the impedance at the voltage maxima is real and is given by  $\text{SWR} \times Z_0$  and at voltage

minima the impedance is equal to again it is real but it is given by  $Z_0$  by SWR. I am assuming that  $Z_0$  itself is real.

You can show that this two as a simple exercise. Now this is something that we will also be able to easily show this. At any point on the transmission line the impedance is given by  $Z_0$  into  $1 + \Gamma_L e^{-2\beta z}$  plus magnitude of gamma L into e to the power j phi IL + 2 beta z, so this bracket had to be there for entire phi IL + 2 beta z so let me just right down that bracket for you, okay.

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**Example**

- A T-line is terminated in pure inductance  $jZ_0$ . Find distance from the load to the nearest voltage maximum. The frequency is 1 GHz and phase velocity  $u_p = 0.67c$
- $|\Gamma_L| = 1$  (obvious) and  $\phi_L = \pi/2$  (show this)

$$z_{max} = \frac{\pi}{\beta} \left( n - \frac{1}{4} \right) = \frac{u_p}{2f} \left( n - \frac{1}{4} \right)$$

- First maximum occurs when  $n = 0$ ;  $z_{max} = -u_p/(8f) = -2.5 \text{ cm}$
- **Exercise:** Repeat above calculation when line is terminated in pure capacitance

Here is the simple example for you guys to workout. A transmission line is terminated in pure inductance and a pure inductance let us consider the value to be equal to  $jZ_0$ , this is pure inductance not because it  $Z_0$  it is because of this plus j thing because a impedance of an inductor is j omega L, so for a give value of omega we have chosen L such that term product omega L will be equal to  $Z_0$ .

If your line is terminated with this pure inductance find the distance from the load to the nearest voltage maximum. The frequency is given to be one GHz and the phase velocity is given to be point 7c. What is the phase velocity in term of c? It simply means that, the medium that is fining up the conducting surfaces is not free space, if it was free space then phase velocity u p would have been equal to c otherwise this is less than c, okay.

“Professor to Student conversation starts” Do not worry too much about what this particular think means we will come back to this time later okay in a different module okay. “Professor to Student conversations ends”. The way to solve this problem would be to first find what is magnitude of  $\gamma/l$ . Actually you are most interested in finding what is  $\phi IL$  also, so magnitude of  $\gamma/L$  will be equal to one.

I hope that this is obvious because this is  $jZ_0$  you can get  $\gamma/L$  as  $jZ_0 - Z_0$  by  $jZ_0$  by  $jZ_0 +$  one  $Z_0$  which is nothing but  $j - 1$  by  $j + 1$  one. The magnitude of that is equal to one but the phase is equal to  $\pi$  by 2. “Professor to student conversation starts” If you are not convinced I invite you to show this, okay. “Professor to Student conversations ends”. And then now you substitute this into the expression for maxima.

So you want to find out where the maxima occurs, so you want to find this fellow  $\phi L$  is  $\pi$  by 2 so you transfer this  $\pi$  by 2 on to the right hand side so you have  $z$  equal to  $1$  by 2  $\beta$  into  $2n\pi - \phi L$  substituting for all these results you will see that maxima is located at  $\pi$  by  $\beta$  into  $n$  minus  $1$  by 4, where  $n$  is an integer to be chosen such that the  $z$  max is always negative, okay. Now we also know relationship between  $\beta$  and  $\omega$ .

So  $\omega$  by  $\beta$  was equal to the phase velocity  $u_p$  and  $\omega$  itself is  $2\pi$  into  $s$  in terms of frequency. So you substitute that relationship into this one; you see that maximum can be written in terms of frequency and phase velocity  $u_p$  and is given by  $u_p$  by  $2f$  into  $n - 1$  by 4 four. The first maxima occurs when is equal to 0 at which point the location is minus  $u_p$  by  $8f$  substituting for  $f$  if  $b$  1GHz and  $u_p$  given here as .67 times the speed of light, you will get maximum at -2.5 cm.

Now here is a simple exercise. If you repeat this above calculation except that the line is now terminated in a pure capacitance. For a pure capacitance this would be  $-jZ_0$  and tell me what calculate for yourself when the first maximum occurs. Also calculate when the next maximum occurs. You can either calculate it by this expression putting  $n$  is equal to -1 or you can use the fact that maxima to maxima distance will always be equal to  $\lambda$  by 2.

And therefore you can use that one right so maxima are separated by lambda by 2. You can also find when the first minima occurs by again going to this lambda by 4 thing right maxima to minima is lambda by 4.

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### Example: Complete circuit

- A lossless T-line ( $Z_0 = 50 \Omega$ ,  $\lambda/4$ ) connects source ( $V_g = 10 \cos(\omega t + 30^\circ)$ ,  $Z_g = 25 \Omega$ ) to  $100 \Omega$  load. Find  $V_L$ .
- Start with  $V_g = 10e^{j30^\circ}$  V; Find  $Z_i = 25 \Omega$  (line is QWT) and  $V_i = 5e^{j30^\circ}$  V. Find  $\Gamma_L = 1/3$
- $V_0^+ (e^{j\pi/2} + (1/3)e^{-j\pi/2}) = 5e^{j30^\circ} \implies V_0^+ = 7.5e^{-60^\circ}$  V
- Load voltage  $V_L = V_0^+ (1 + \Gamma_L) = 10e^{-60^\circ}$  V
- **Exercise:** Interchange source and load impedances and find load voltage.

Now that we have seen enough about the circuit let us actually look at the complete circuit analysis, okay. “Professor to student conversation starts” This is important, here we are going to consider the entire transmission line circuit including the source and the load, okay. “Professor to Student conversations ends” So this is the complete circuit you have a transmission line which has a certain length L and then terminated by the load ZL.

So if it terminated by load ZL the load voltage is v l the load current is IL. Now the idea is that what should be the load voltage. What should be this V0+. The answer is actually quite simple. The first thing you have to understand is that the generator at this terminal this is called generator in a older terminology or sometimes called as source, okay. This generator has an internal impedance Z g.

But to the input terminals of the transmission lines the voltage is V i and the current is I i. So this is the voltage and at the current input terminal of the transmission line. Now here is a quick question, what is the impedance seen at this point at the input terminals what is the impedance seen at this point? The impedance will not be equal to the ZL that is obvious because Z L

transformed through a length  $L$  of the transmission line must be the input impedance that you see right.

You have seen the expression for impedance on the transmission line. So the impedance seen here will be  $Z_i$  and that  $Z_i$  will depend on  $Z_L$  and the length. So if you remember the formula it was something like  $Z_0 \rightarrow Z_0 + jZ_L \tan \beta L$  assuming real transmission line or it would be  $\tanh \gamma L$  divided by  $Z_L + jZ_0$  not so that formula you need to remember substitute to that formula the corresponding values of  $l$  something right.

So that formula you need to remember substitute into that formula the corresponding values of  $L$  and  $Z$  not and then see that  $Z_L$  gets transformed into  $Z_i$  so the generator only sees  $Z_i$ , okay. And if you want to find out what is the line voltage you know everything else you can find out what is  $\gamma/L$  at the load because you know  $Z_L$  you know  $Z_0$ . You can find out  $\gamma/L$  you know,  $\gamma$  itself.

Therefore, you can find out the phase factor  $e^{+j\gamma L}$  and  $e^{-j\gamma L}$ . What you do not know is  $V_0^+$  which is the input incident voltage or the incident voltage amplitude  $V_0^+$ , okay. To calculate that look at the equivalent circuit the circuit will consist of a voltage divider with a generator and its impedance seeing  $Z_i$  the voltage at the input terminals of the transmission line is nothing but  $Z_i$  by  $Z_g + Z_i$  into  $V_g$ .

But this voltage must be also the line voltage evaluated at  $Z = -L$ . Please look at this expression very carefully what we are saying here is that this voltage is nothing but the voltage on the transmission line as you come towards the source side or the generator side and at  $Z = -L$  is where the transmission line is located that voltage must exactly be equal to the input voltage on the transmission line right.

Now you can substitute for  $Z = -L$  on that line voltage of the transmission line and you see that this is the expression. If you now combine these two expressions right and call this a  $V_i$  you can immediately see that  $V_0^+$  is given by  $V_i$  by  $e^{-\gamma L} + \gamma/L$  into  $e^{+\gamma L}$

$\gamma/L$ , okay. If the line is lossless then you replace  $\gamma$  by  $j\beta$  and then write down what is  $V_0^+$ .  $V_0^+$  in this particular case can turn out to be complex, okay.

Now if I know what is  $V_0^+$  it is simple matter to find out what is lower voltage. Why? Because load voltage has to be evaluated at  $Z=0$  when you put  $Z=0$  you will see that this term will be 1; this term will also be equal to 1 and this is  $1 + \gamma/L$  times  $V_0$ , this is the load voltage. Let us actually look at a one example here. Suppose we have a lossless transmission line with characteristic impedance of 50 ohms okay and the length  $\lambda$  by 4 collecting the source which is give by  $10 \cos \omega t + 30$  degrees, okay.

And having a generator impedance of 25 ohms and this is connected to a 100 ohms' load, so the load is real, the generator impedance is real, the characteristic impedance is also real, the length of the transmission line is given by  $\lambda$  by 4. Find out what is the load voltage? Well, first convert this  $10 \cos \omega t + 30$  degree into a phasor form, so if you convert that phasor form you get  $V_g = 10 e^{j30}$  degrees, okay. So this would be the generator voltage. Find out  $Z_i$ .

If you remember this case that we considered when  $\beta L$  was equal to  $\pi/2$  which is essentially means that  $L$  should be  $\lambda/4$  we saw that input the impedance as a simple form which is given by  $Z_0^2/Z_L$ . What is  $Z_0^2$  here? 2500. What is  $Z_L$ ? 100, so the ratio of will be 25 ohms. So this is a input impedance. If you are not following this one you can substitute the expressions for  $L/Z$  not into the impedance formula.

And find out that this is actually 25 ohms okay and therefore the input voltage will be a voltage divider between 25 and 50 ohms so  $25/(50+25)$  which is  $25/75$  -- this is  $Z_g$  is 25 so one half of the generator voltage appears at the input terminals of the transmission line which is  $5 e^{j30}$  volts. Now you can also find out  $\gamma/L$ . How will you find  $\gamma/L$ ? You know the load voltage which is 100; you know the characteristic impedance 50.

So  $100 - 50/100 + 50/150$  which is nothing but  $1/3$ ;  $\gamma/L$  is also real. Now you substitute all these expressions into this one right. So  $V_0^+$  is nothing but  $V_i$  you already have calculated what

is  $V_i$ , you know what is  $\gamma$ ,  $\gamma$  is  $j$  into  $\beta$ , right so you substitute that  $\gamma/L$  is  $1/3$  you can substitute find out all these values and you will see that  $V_0 +$  is 7.5 volts, okay.

But lagging by a phase angle of 60 degrees whereas the input voltage is 5 volts leading by 30 degrees, the phase difference between input voltage and  $V_0 +$  plus itself can be found out and this is around 90 degrees, same thing the voltage will also be equal to 10 in to e power  $-j 60$  degrees, so you can actually see that the load voltage magnitude is the same as the generator voltage except that these two are now lagging by 90 degrees, right.

So one voltage the generator is leading by 30 this fellow is lagging by -50 so the phase difference between them 90 degrees. Here is a simple exercise I would suggest that you do this exercise you interchange the source and load impedances, okay and then find out what is the load voltage.

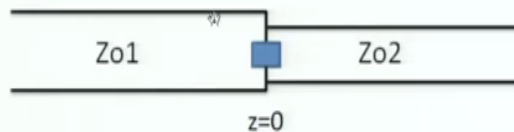
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### Transmission coefficient

- From left  $V_{o1}^+(z=0) + V_{o1}^-(z=0)$  and from right  $V_{o2}^+$
- KCL gives  $I_{o1}^+(z=0) + I_{o1}^-(z=0) = I_L + I_{o2}^+(z=0)$ ;  $I_L = \frac{V_{o2}^+(z=0)}{Z_L}$
- Substituting for  $I_{o1,2}^{\pm}$  in terms of  $V_{o1,2}^{\pm}$  and simplifying

$$\tau = \frac{V_{o2}^+}{V_{o1}^+} = \frac{2Z_{||}}{Z_{||} + Z_{o1}}; \quad \Gamma_L = \frac{V_{o1}^-}{V_{o1}^+} = \frac{Z_{||} - Z_{o1}}{Z_{||} + Z_{o1}}$$

- $Z_{||} = Z_{o2} || Z_L$  is equivalent load impedance seen by first T-line



We will consider more common situation where one transmission line actually gets terminated by another transmission line, okay. In fact, many times one gets terminated by second, second gets terminated by three of different lengths and different characteristic impedances because you are trying to do some matching, concept that we will see later in a different module. What we are interested is how much voltage gets reflected from the first transmission line?



How much voltage gets transmitted into the second transmission line? Okay. To do so you just need to invoke KVL and KCL very simple. Let the-- incidence voltage on the first transmission line have an amplitude of  $V_{01+}$  and the reflected voltage because of the load at  $Z=0$  to  $V_{01-}$  so the line voltage on the first transmission line of characteristic impedance  $Z_0$  one will be  $V_{01+} + V_{01-}$  from this side.

And from the right side that is from the second transmission line you only have  $V_{02+}$ . What happen to  $V_{02-}$  well the transmission line is continued towards infinity therefore there is no possibility of having reflected voltage on the line, okay. So therefore right side voltage if  $V_{02+}$ . So  $V_{01+} + V_{01-}$  must be equal to  $V_{02+}$ . Now with KCL, you will see that the total line current here is  $i_{01+} + i_{01-}$ , okay incidence and reflected currents plus there is some current with the load itself which is  $I_L$  plus some current going onto the second transmission line.

Again there is only forward going current  $i_{02+}$  two plus, okay. You can equate this substitute for  $i_{01+}$  and  $i_{01-}$  in terms of  $V_{01+}$  and  $V_{01-}$ . Also note that the load current  $I_L$  is give by  $V_{02+}$  divided by  $Z_L$  which is correct because  $V_{02+}$  is the voltage on the second transmission line but that voltage  $Z=0$  is precisely the voltage across the load impedance in fact that is also equal to  $V_{01+}$  by  $V_{01-}$  but this one will give us easier answer.

So I am writing  $I_L$  as  $V_{02+}$  by  $Z_L$  at  $Z=0$ , so I hope that you guys are convinced about this equations, if you are convinced substitute  $i_{01+}$  and  $i_{02-}$  in terms of all these  $V_{01+}$  and  $V_{01-}$  and you can see that the ratio of transmitted voltage onto the second transmission line to the incidence voltage from the first transmission line which we will call as the transmission coefficient.

This was very simpler to an interface plane wave coming in and some transmitted voltage, right or a transmitted electric field that is given by  $Z_{\text{parallel}}$  by  $Z_{\text{parallel}} + Z_0$  and the reflection coefficient  $\gamma/L$  is equal given by which is the ratio of reflected line voltage-- reflected voltage on the line 1, two incidence voltage on line 1 given by  $Z_{\text{parallel}} - Z_0$  by  $Z_{\text{parallel}} + Z_0$ .

But what is the Z parallel? You can actually substitute all these and find that Z parallel is nothing but the equivalent impedance, the parallel combination of characteristic impedance  $Z_0$  with load  $Z_L$ , okay. So this parallel combination will be the effective impedance that you are going to see—first line actually going see and you can use that effective impedance to calculate transmission and reflection coefficient.

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### Power relations

- Average power  $(P_{av})_i$  delivered by source to input terminals of T-line is  $\text{Re}\{V_i I_i^*\}$  at  $z = -\ell$
- Average power  $(P_{av})_L$  dissipated by  $\text{Re}\{V_L I_L^*\}$  at  $z = 0$
- If T-line is lossless  $(P_{av})_i = (P_{av})_L$

There is last thing about power relationship. There is what I feel is undue emphasis placed on power relationships. Well at this point all need to remember is that average power in terms of phasors given by half real part of  $V_i I_i^*$  or  $V$  into  $I$  conjugate so the average power delivered by the source to the input terminals of the transmission line will be—I forget a half here but please do keep that half in mind, so this is half real part of  $V_i I_i^*$  okay.

We have already seen what is  $V_i$ ,  $V_i$  is nothing but these particular things right so  $V_i$  is  $Z_i I_i + V_g$  so given  $V_g$  find out what is  $Z_i$  the input impedance of length transmission line of length  $L$  and then right down this as half real part of  $V_i I_i^*$ , okay. The average power dissipated by the load is half real part of  $V_L I_L^*$  and this occurs at  $Z=0$  because that is where we have actually kept the load.

If the transmission line in between is lossless whatever the input power average input power that is given to the input terminals of the transmission line will exactly be the one that is delivered across the load, okay. So there is no loss out there.

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### Example: Cascaded T-lines

- For the circuit, find  $Z_{i2}$ ,  $Z_{i1}$ , and  $V_o^+$ .
  - Transform load impedance  $Z_L$  through length  $l_2$  to get  $Z_{i2}$
  - Now  $Z_{i2}$  is load to first T-line segment; transforming through  $l_1$  we get  $Z_{i1}$
  - From  $Z_g$  and  $Z_i$  find  $V_o^+$



In this expression if you want to find out how to find  $V_0^+$  okay. This is the same transmission cascaded transmission line experiment case that I am showing except that the load has been move to the second transmission line. Now, the analysis is very simple you can do KCL KVL and all but the idea is to actually transform the load okay. First consider  $Z_L$  which is the load connected to second transmission line.

But the second transmission line has a length  $L_2$  therefore, the impedance seems looking at this terminal right at the output terminal of the first transmission line will be  $Z_L$  transformed through to length  $L_2$  with characteristic impedance  $Z_{o2}$  call that has some  $Z_{i2}$  okay and that  $Z_{i2}$  further gets transformed to  $Z_{i1}$  via length  $L_1$  on the first transmission line. Now you get the effective impedance from seen from the two transmission lines, this is the infective input impedance.

This forms a voltage divider with the given generator impedance  $Z_g$  and you can find out what is  $V_0^+$  by following the same logic.

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## Smith Chart

- Graphical plot of normalized resistance and reactance functions in reflection coefficient plane
- Useful in solving several problems; finding impedance at a point on T-line, reflection coefficient away from load, matching network design, determining VSWR etc.
- Best suited for lossless lines and qualitative understanding



Figure : P. H. Smith; inventor of Smith chart

In this module we will discuss Smith chart one of the most widely used graphical tool and before that advent of personal computer this was probably the only way in which Microwave and problems RF antennas problems were solved. And the principle rules of Smith chart is to solve several transmission line problem. You can actually use Smith chart to find impedance at a point on a transmission line.

You can find out the reflection coefficient at any point on the transmission line away from the load. You can design matching networks; you can determine the standing wave ratios; you can determine where the maximum occurs where the minimum occurs all these things could be done by Smith chart and Smith chart was “The most” popular tool a graphical tool especially for doing all these problems before PC’s arrived-- now you can actually use readymade programs.

Or you can write simple program for yourself to manipulate the equation; to solve all the problem that a Smith chart. So in this case the question might be that why are we studying Smith chart right? The answer is that, for a first design-- especially when your lines are lossless you can actually use Smith chart to get reasonably accurate results and moreover this Smith chart gives you qualitative understanding okay.

So this Smith chart is something that gives you a qualitative understanding of the problem before you actually can plug in to get the quantitative numbers. Moreover, Smith chart is such a nice

graphical tool that most RF engineers and Microwave engineers or even antenna engineers visualize most of the problem via Smith chart, before learning Smith Char is something that will be very valuable for you when you decide to pursue RF or Microwave engineering.

Now what is exactly is a Smith chart? Smith chart is fairly simple to define what it is. It is actually a graphical plot of normalized resistance and normalized reactance functions in the reflection coefficient plane. There are several (( )) (23:08) is involved here, let us take it up one by one. What do we mean by normalized resistance and reactance? What we mean is that impedance  $Z$ .

If you divide that impedance with respect to the characteristic impedance of a transmission line then that forms the normalized impedance. If the characteristic line that you are considering-- if the impedance is real which is what you would mostly consider then if you have load resistance, then that load resistance divided by the characteristic resistance will be normalized resistance; load reactance divided by-- a load reactance normalized to the characteristic impedance will be the normalized reactance.

So any load in the problem that you are considering can be normalized with respect to the characteristic impedance of the transmission line, okay and that is what we mean by normalized resistance and reactance, in general normalized impedance. This normalized impedance is plotted in a reflection coefficient plane. Why is reflection coefficient plotted in a plane or why is reflection coefficient even considered to be plot able in a plane?

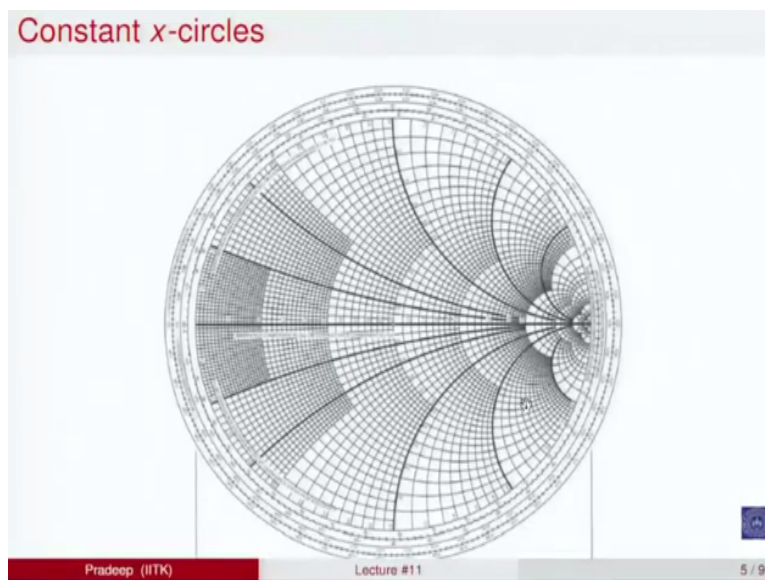
If you remember reflection coefficient is not always a real quantity it is defined by  $Z_L$  and  $Z_0$  and even by  $Z_L$  by  $Z_0$  and  $Z_L + Z_0$ . And when your load impedance happens to be complex then your reflection coefficient will also be complex. So a complex reflection coefficient can be plotted as a point in a complex plane with real axis giving with the real part of  $\gamma/L$  and the y axis is giving with the imaginary part of  $\gamma/L$ .

Therefore, every point on the reflection coefficient plane will correspond to a particular  $\gamma/L$  and hence correspond to a particular  $Z_L$ . And if you normalized that  $Z_L$  there is a one

to one correspondence between the two very nicely okay. And normalization is done, so that numerically everything is manageable otherwise you can work with unnormalized resistance and impedances as well.

So on the reflection coefficient plane every point corresponds to  $\Gamma/L$  and every such  $\Gamma/L$  corresponds to a given load impedance. And here is a picture of a person who invented Smith chart called Philip Smith and is written a very nice book on how to use Smith chart, if you can get hold off that book it is very, very valuable to read that.

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This is how the Smith chart would actually look. A Smith chart is basically collection of circles, some of this circles are called as constant resistance circle; some of this are called-- some of the other circles are called as constant reactance circle. Now reactance could be positive or negative-- positive reactance correspond to inductance; negative reactance corresponds to capacitance, so you can have constant positive reactance or constant negative reactance.

If you look closely at a Smith chart you will actually see that they are actually labeled here as constant resistance or conductance circle okay resistance and conductance are interchangeable in some sense in the sense that resistance is a real quantity conductance is also a real quantity. This chart can be split into two chart the upper hemisphere consists of positive reactance okay and lower hemisphere consist of negative reactance okay.

If the Smith chart is being used for impedance plotting so the circles are constant resistance circle; upper half is inductance plane; the lower half is the capacitance plane. If you are using this as an admittance chart, then the circles are constant conductance circle because on an admittance or in the parallel when talk about conductance and substances so this would be the capacitances of substances.

And this would be the inductive substances when the chart is being used as an admittance chart. These are some of the circle which I have marked these are constant  $r$ -circles you can actually see that the circle with the largest diameter actually corresponds to  $r$  equal to zero and then you keep increasing  $r$ . So this circle which is almost to the center is  $r$  equal to one circle okay the normalized impedance is equal to one here the normalized resistance is equal to one here okay.

These values of circles are for increasing values of  $r$  is as  $r$  equal to zero;  $r$  equal to say .3 .5 and then eventually coming to  $r$  equal to 1;  $r$  equal to 2; or  $r$  equal to 3 I believe this one and then keeps increasing and these circles which you cannot really see are for really large values of  $r$ , typically it will not really work in those regions, okay. So clearly this would correspond to the short circuit; this should correspond to the open circuit.

These are some of the constant reactance circles that you can see. The circle at the center is actually looking more like a straight line which has a radius of infinity corresponds to 0 reactance. All these circles in the upper hemisphere correspond to the inductance if the Smith chart is impedance chart and all these circles corresponds to – these are rather a semi-circle; these circles correspond to capacitive reactance when the Smith chart is consider as a impedance chart.

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## Properties of constant $x$ -circles

$$(\Gamma_r - 1)^2 + \left(\Gamma_i - \frac{1}{x}\right)^2 = \left(\frac{1}{x}\right)^2$$

1. The centers of all  $x$ -circles lie on the  $\Gamma_r = 1$  line; those for  $x > 0$  (inductive reactance) lie above the  $\Gamma_r$ -axis, and those for  $x < 0$  (capacitive reactance) lie below the  $\Gamma_r$ -axis.
2. The  $x = 0$  circle becomes the  $\Gamma_r$ -axis.
3. The  $x$ -circle becomes progressively smaller as  $|x|$  increases from 0 toward  $\infty$ , ending at the  $(\Gamma_r = 1, \Gamma_i = 0)$  point for open-circuit.
4. All  $x$ -circles pass through the  $(\Gamma_r = 1, \Gamma_i = 0)$  point.

Here is a short derivation of Smith chart, first of all we denote by this small ZL the normalization of the load impedance with the real characteristic impedance. The characteristic impedance is Z0 but when the line is lossless and the characteristic impedance is real you can write the R0 indicating that this is only resistance, okay. So ZL in general is RL plus J XL can be normalized and these normalized values are written as R + jX.

Now look at gamma, the reflection coefficient gamma is nothing but ZL - R0 by ZL + R0 this can also be written in terms of the real part of gamma and imaginary part of gamma because gamma is a complex number. Now substituting for ZL after normalization will give you ZL - 1 by ZL + 1 you can invert this relationship to write ZL in terms of gamma also, okay. And then write down that ZL as R + jX right this ZL is R + jX.

And then you replace this magnitude gamma e power j Theta gamma which is the phase angle by gamma itself and gamma can be written as gamma R + j gamma/L and then you can use complex number normalization to do this things and find out that this is given by this and R will be real part of this one given by  $\frac{1 - \Gamma_r^2 - \Gamma_i^2}{2\Gamma_i}$  and X which is the reactance part normalized is given by  $\frac{1 - \Gamma_r^2 + \Gamma_i^2}{2\Gamma_i}$ .



What you have to see from this boxed equation is that no matter what  $\Gamma_R$  and  $\Gamma_I$ , I consider which would be a point on the reflection coefficient plane, there will be a corresponding  $R$  and a corresponding  $X$ . Sometimes you might consider  $\Gamma_R$  and  $\Gamma_I$  which is unrealistic say  $\Gamma_R$  is equal to 100  $\Gamma_I$  is equal to 50. And the magnitude of  $\Gamma$  will be greater than 1 which is unphysical for a passive transmission line.

There will be a corresponding  $R$  and  $X$  but they would not be completely meaningless physically. So you have to understand that the magnitude of  $\Gamma$  will always have to be less than 1 and therefore you are not considering the entire  $\Gamma$  plane but only a restricted  $\Gamma$  plane. What are the properties of constant  $R$  circles, this  $\Gamma_R$ -?

So if you actually look at this equation and then rearrange the equation, see that the equation can be rearrange in the form of circle equations where the center for given by  $R$  by  $1+r$   $\Gamma_0$  this on the horizontal line so from the horizontal line the centers are given by  $R$  by  $1+r$  and 0 and the radius of this circles are given by  $1$  by  $1+r$ . The centers all lie on the  $\Gamma_R$  axis right on the horizontal axis  $\Gamma$  is  $\Gamma_R$ .

So all the centers for all  $r$ -circles will lie on  $\Gamma_R$  axis, okay. And the special case with the largest radius is when  $r$  is equal to 0 the radius is equal to 1 okay that center is given by  $r=0$  is given by 0  $\Gamma_0$  okay and this is center at the origin has a largest radius. What you observe is that  $r$  starts to increase the radius starts to increase because  $1$  by  $1+r$  starts to decrease. Moreover, the center starts moving towards the right side right.

Because what is happening  $r$  is increasing, eventually when  $r$  is equal to infinity the radius will be 0 and if  $r$  equal to infinity corresponds to open circuit. What would be the center? there center? Center will be 1, 0. Therefore, all of the center for the constant  $r$ -circle are located between origin and 1,0 on the real axis. Moreover, the  $s$  is very crucial. All the  $r$ -circles pass through the  $\Gamma_R$  equal to 1,  $\Gamma_I$  equal to 0 point which is the open circuit point.

So if you are not convinced you look at the constant  $r$ -circle every circle is passing through this right side point where  $\Gamma_R$  equal to 1 and  $\Gamma_I$  is equal to 0 corresponding to open

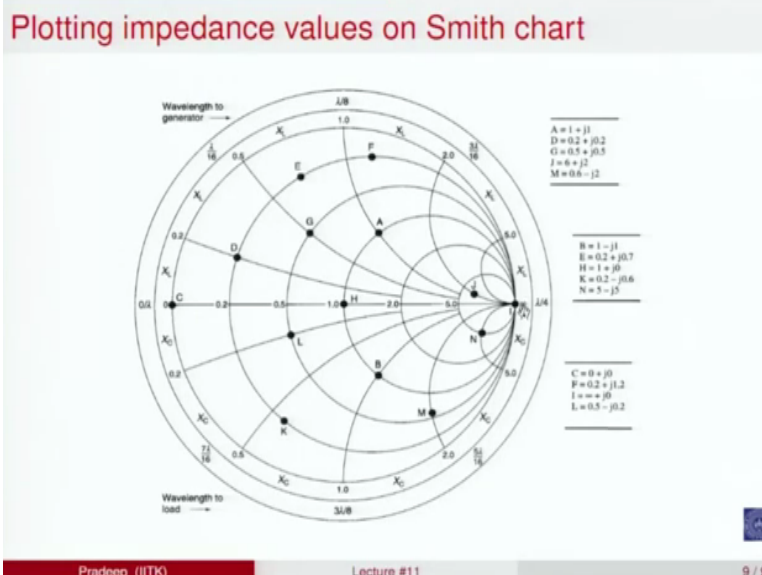
circuit. This outer circle is the one which is  $r$  equal to 0 has a largest radius, okay. Similar if you look at the constant  $x$  circle the center of this  $x$  circles are at one gamma one by  $x$  the radius is one by  $x$ . Now at  $x$  equal to 0 the center will obviously be at one gamma infinity, right.

So on the  $y$  axis the center is lying at 1 gamma infinity what is the radius of this one? The radius of-- this is one by  $x$  equal to 0 so the radius is infinity; so you have the center at 1 gamma infinity and the radius of infinity which is actually a straight line. the centers of all other  $x$  circle will lie on the gamma  $r$  equal the centers will be on the  $y$  axis. And for inductive reactance's the centers lie above the gamma  $r$  axis.

And for capacitive reactance's they would lie below the gamma  $r$  axis. Further,  $x=0$  is the circle of the gamma  $r$  axis because as I just said  $x=0$  corresponds to radius of infinity which means that it is a straight line and the center is a 1 gamma infinity, okay. The  $x$ -circles again become progressively small as magnitude of  $x$  increases from 0 towards infinity. At infinity point the radius is 0 and the center will be at 1 gamma zero.

And all the  $x$  circle pass through the right side point. So this right point-- the point out here is very interesting because every circle will constant reactance circle as well as constant resistance circle all them pass through this particular point, okay. So this is all about Smith chart.

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Now we will end this module by looking at some of the impedances that are plotted on the Smith chart. So I have shown enough points on the Smith chart what I would suggest is every point you try to locate on the Smith chart you get hold of a Smith chart in a store in a place where you can actually get that one and use that Smith chart, okay. This Smith chart does not show all the circles this is just for the clarity.

But the Smith chart that you will get will have a lot of circles but you take a pen or a pencil and then start marking every point here or you can print out this slide and then cover up all these points A to L, okay? A to M, N actually you can cover up all these points and then try to see whether you are actually getting these values of A correctly, okay. For example, what is the impedance at point A?

You have to see that this is at the meeting point or the intersection of the constant  $r$  and a constant  $x$  circle right. What is the constant  $r$  circle here? The constant  $r$  circle is 1; constant reactance circle is 1, so A is at  $1 + j1$  and there is precisely what you get. What about D? D is at point  $2 + j2$ . What about C? C is at  $r=0$ ;  $x=0$ , so C will be equal to  $0 + j0$ . What about B?

B will be at 1 right real  $r=1$  and for  $x$  this is 1 but this is at the lower hemisphere therefore you have to mentally add a -1 here, so this will be B will be  $1 - j1$  as we can see here, okay. What about M? M will actually be slightly higher than point because if you actually draw a circle out here it will be slightly greater than .5 although it is not shown, okay. So this would be say .6 and then on the  $x$  reactance this is at 2 so this is a  $.6 - j2$ , okay.

So this is the value of M, anything else that is interesting you can find out. For example, what is F? F is not shown to lie on any of these points, so you have to interpolate slightly, right. On the constant  $r$ -circle this .2 but for the impedance this is lying between 1 and 2 so mentally if you try and interpolate this one this might turn out to be around one 1.2 so this would be  $.2 + j1.2$ , okay. So this is how you would try to find point  $i$  of course being open circuit is given by infinity, right.

And for  $x$  this would be equal to 0. In the next, module we will actually start using Smith chart for various applications as we said and we will take up from that one. Thank you.