Electromagnetic Theory
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Lecture No - 63
Application of smith chart - II

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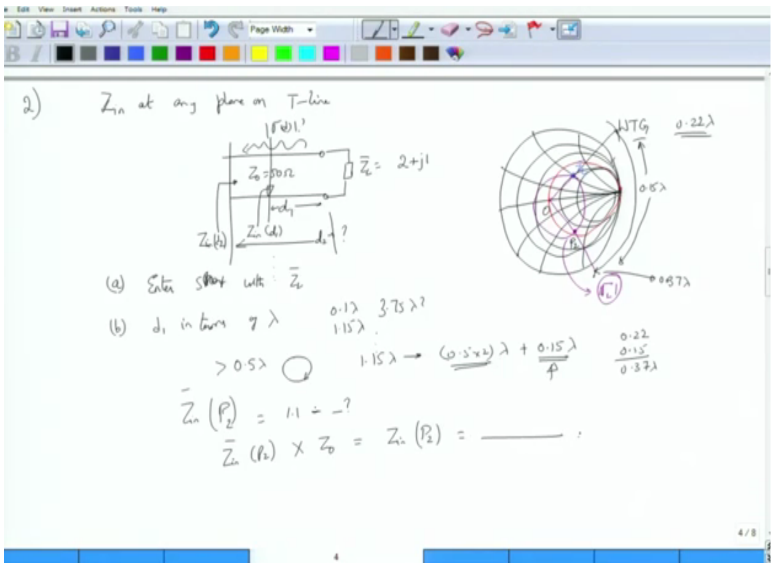

One of the major tasks of smith chart would be to find impedance okay, at any point on the transmission line. So we need to find out the impedance at any point or any plane on the transmission line. What do we mean by this? So I have a transmission line with Z zero say which is fifty ohms, connected to the load okay whose normalized load value is 2 plus j 1 okay, I have just changed the value over here, this is 2 plus j 1 normalized impedance value, so you do not have to normalize Z in this case.

Now let us say what is the input impedance scene at this plane which happens to be at the distance d 1 from the load? What is the input impedance scene at this plane which is at the distance of d 2 from the load and so on okay? So the input impedance scene at all these places can be obtained by this smith chart in a nice and easy manner okay. So as before we begin by drawing some representative circles on the smith chart okay.

So I am going to draw some representative circles, I will also draw the unit circle okay. So if they do not look like circles as I said please excuse this drawing. You should look at the smith chart that there is in your hand okay. Now this is my smith chart okay, so let us say this is one more circle which I have drawn just to complete my chart okay. So how do I find the input impedance on the transmission line?

Is there anything else that I need to know? Well you do not really need to know anything else right. So all you need to know is what distance you want to calculate the impedance. Now here is a crucial thing, as you move along the transmission line what happens to the magnitude of Gamma d? That would remain constant. Therefore, you actually moving on the constant Gamma circle or the constant SWR circle okay.

So this is what you are moving, so first step would be to enter okay enter your smith chart, S chart with Z L bar okay. So since Z L bar in this particular case happens to be 2 plus j 1, so in my mind this is my graph, this is 2 and then this is 1, so this would be the point that I am looking at. This is my Z L bar okay, now with that Z L bar as one point and with the center as another point, draw circle of this radius okay.

So this circle is the constant Gamma L circle okay, this is the constant Gamma L circle okay. So any movement that you are going to do would be on this particular circle okay. Next what you do? From the center you simply extend this point and read what is the value shown on the wavelength towards generator circle okay. So let us say this turns out to be some point two two lambda okay, this is just an arbitrary number that I am taking it does not really matter.

You have to verify that this number is correct okay, so you enter your smith chart with 2 plus j 1 and then find out that you actually at that particular point. So if you extend and then find out the reading on the WTG scale now all you have to do is, express d 1 in terms of lambda. That is say it is point one lambda; is it one point one five lambda; is it three point seven five lambda and so on okay and remember whenever your distance d 1 goes beyond point five lambda that means you have actually taken one complete revolution okay.

Therefore, one point one five lambda should actually be point five into two lambda plus whatever the component that is less than lambda. So it is kind of modulo point five lambda that you are doing and this is the distance that you have move. Because if you move this distance you actually back to the same point okay. So you extend, find out what is point two two lambda. Now on this WTG scale how much distance you have to move?

You have to move point one five lambda. So on this scale you move point one five lambda so let us say on the scale your, you draw at this point okay. So this distance is point one five lambda okay. So at what point you have ended? You have ended at point two two plus point one five which is point three seven lambda, so let us say this is my point, three seven lambda okay, reading on the WTG scale. Now what you have to do?

You have to now draw from this one; you have to draw a line until it reaches the center. Now once you have reached the center, you look on the constant Gamma circle where you are intersecting okay. This would give you the value of input impedance. What is the input impedance here? This is approximately so this is slightly greater than one, so let us say this is around one point one and then this would be somewhere around say on the x axis this would be something like this okay.

So I am just writing this as point P 2 and I would invite you write what is point P 2. So something around one point one minus something times to un-normalize the input impedance at P 2, so this is actually input at P 2, this used to be multiplied, this is normalized so this need to multiplied by Z zero to obtain the un-normalized input impedance okay. You find this out. I will leave this other exercise to do okay.

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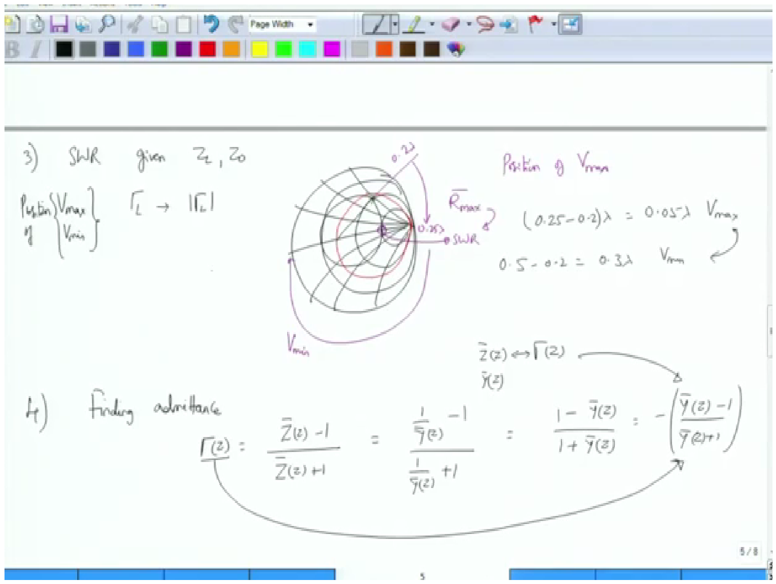

Now let us look at a third example of finding SWR, given Z L and Z zero. Now one way to do this one is to actually use equation which is what the way I would suggest. However, if you still want use smith chart you have to first find out, what is Gamma L right, draw the constant Gamma L circle and then see on the smith chart okay, so you enter your smith chart with the normalized Z L okay. So you have your smith chart in this way, there is a unit circle here okay.

So you enter your smith chart with a given value of Gamma L okay. Let us say this is the value of the Gamma L that I am entering and from the center you find out the radius and then draw a circle okay. So this circle corresponds to constant Gamma L circle and we have just told you in couple of minutes ago or maybe slightly more than that, wherever this constant gamma circle cuts the x axis or sorry cuts the horizontal axis that would be the value of standing wave ratio right.

Because this would actually correspond to R max which is normalized; which is actually equal to SWR. Incidentally if you extend this line and then find out how much you had to move. So let us say this is around point two lambda and this is point two five lambda right, so the distance that you had to move to get from point two lambda to point two five lambda such that you got to SWR actually tells you the position of V max.

Because on this side right, the voltage would be maximum that is why you get R max bar over here. And if you continue moving you would reach a certain point right, which would be on the left side of this axis okay, at this distance right you start from this one and you move all the way up to this distance that would correspond to the position of V min. So in fact what we have found out is, not just SWR can be found we can also find out position of V max and position of V min okay.

All from this particular exercise. So where, what is the position of the first maximum and as you come over here this could be the maximum that you would reach first. So to get to V max, the first V max would be located at point two five minus point two lambda right, which is actually point zero five lambda and if the lambda is given to be let say ten centimeters then this will be point zero five into ten right so and so centimeters for your location to the first maximum.

And what would be this value? This position of minimum will be point five because this is actually point five right minus point two which is point three lambda. So this is your position of V min okay and the distance and the difference between this two must obviously be equal to lambda by four and that is what is it right? So lambda by two corresponds to one complete rotation, lambda by four corresponds to one half rotation and when you actually move one half, you are inverting all the impedances.

Something that we are going to talk about shortly okay. So what is the next application for Smith chart that we are interested in? We have seem to found out almost all of this different quantities but we have not talked about finding the admittance okay. What is finding admittance mean? Right, so your smith chart you have even hard to tell you that this is actually impedance chart but (()) (09:42) smith chart can also be used as admittance chart. Now how do you do that one?

You recognize that the reflection coefficient Gamma right, at any point on the transmission line, let us say Gamma of Z is nothing but the normalized impedance scene at that point so call this Z of z, I know this is slightly confusing but please bear with me. Minus one divided by let put a bar over there, so the bar of Z plus one, this is your reflection of coefficient. Now what is the corresponding expression in term of admittance coefficients?

I know Z bar z is nothing but one by Y bar z minus one divided by one by Y bar z plus one, this can be written as one minus Y bar z divided by one plus Y bar z which can also be written as minus Y bar z minus one by Y bar z plus one right. So what it means is that reflection coefficient changes its sign by one eighty degrees if you go one eighty degree away right.

So you start with the impedance, you find out some Gamma Z, now if you go one eighty degrees on to other side which is diametrically opposite, you actually express Gamma of z in term of Y bar of z which is the admittance. Now there is a one to one correspondence between Z bar of z and Gamma of z and this equation seems to tell you that there is actually one to one correspondence between Gamma of z as well as Y bar of z.

Therefore, you can use smith chart both as impedance chart as well as admittance chart

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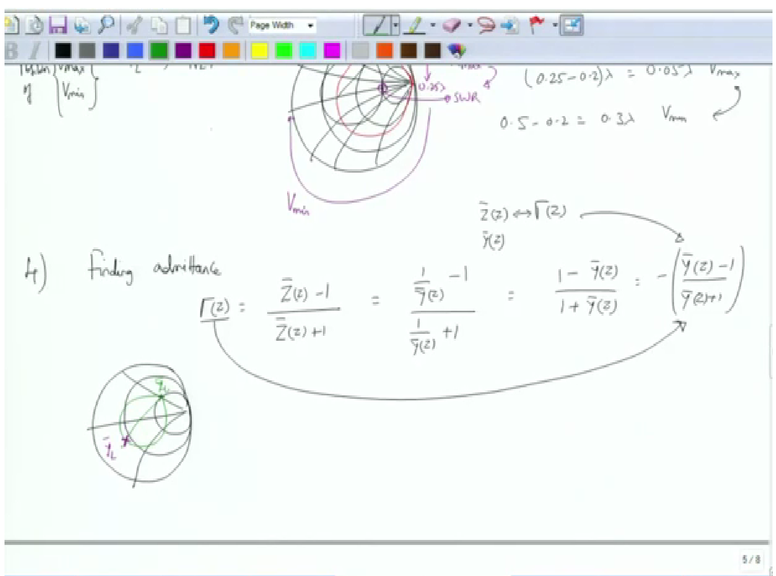

Okay. So to find the admittance, all you have to do is start on any point on the smith chart which is dependent which is given by the load or whatever the impedance okay and from that point right, so you first move on the constant Gamma circle okay until you find yourself on the diametrically opposite point okay. So this is the diametrically opposite point which corresponds to the normalized admittance.

For example, this is the normalized load impedance then this would correspond to right, so this is Z L bar then this diametrically opposite part would correspond to normalized admittance Y L bar okay. So this is how you find admittances at any point by going one eighty degrees or finding, you know diametrically opposite points okay. Now we have already talked about finding the location of the V maximum and V minimum.

You can try this V max V min positions okay with this particular Z L bar which is 1 minus j 1 okay. This is already normalized okay you do not really require the value of Z, for someone has sat down and normalized already this one to you. Now find out where you are going to get your first position of maximum and minimum okay. I will give you the answers. You try to find out whether these answers are correct.

So d max is point four one two lambda okay and if you if since the lambda is given as five centimeters in this particular problem so this would be point four one two lambda which would correspond to two point zero six centimeters from load okay. So if this is your load Z L and this is your transmission line, this one, so this is at two point zero six centimeter okay.

It turns out for this particular case because the capacity of reactants down here the location of this point will be on the lower hemisphere of the smith chart and as you go clockwise towards to the generator the first position that you would hit is actually V minimum. So the first position should actually be V minimum and this at around point one centimeter okay. So this is how the standing waves might look on the transmission line okay.

So the first thing that you would hit would be minima at point eight one centimeter okay and the next you would hit a maximum. The difference these two must essentially correspond to lambda by four okay.

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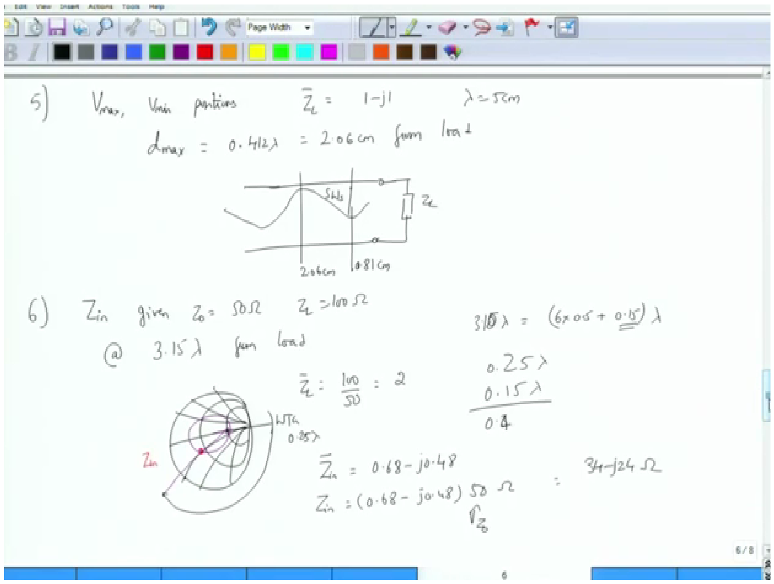

Let us look at another example, this time its kind of interesting to look at this example in which you are going to find out the input impedance okay. Given Z zero is equal to fifty ohms okay and Z L equal to hundred ohms okay. You might say that there is what is so great about this, that the thing is that I want to give you a taste of what you would be seeing if your length happens to be more than point five lambda.

So if you say at three point one five lambda from load, find out what is Z in, you would actually start with the smith chart. So enter the chart. This is your Smith chart, you enter the chart with the value of normalized load impedance. What is normalized load impedance? Hundred by two, normalized load impedance is hundred by fifty sorry which would essentially be two. So let us say this is on the axis too, then you draw with this as the radius, one circle which is your constant Gamma circle okay. So on this constant Gamma circle you now have to move.

What is the distance you have to move? Three point one five lambda. What is the value if you take this point and extend it on the WTG scale what would be the value that you would read? This would be point two five lambda but clearly three point one five lambda is nothing but six into point five plus point one five lambda right.

So which means you start from this point six times you come back the same point and from there you move point one five lambda right, because that is the know value that is lesser than point five. So you start with point two five lambda then you move point one five lambda for a total of point three or rather point four lambda. So point four lambda happen somewhere over here.

So on the WTG scale you move okay and once you have moved, you then enter the smith chart by drawing a line until you reach the center and see where it is actually intercepts this particular point and this would be the value of input impedance okay. I will give you the answer for this problem so that the input impedance is actually given by point six eight minus j point four eight as the normalized value, the un-normalized value will be point six eight minus j point four eight into fifty ohms right.

Because this is your Z zero and the answer would turn out to be roughly thirty-four minus j twenty-four ohms okay. So this is the input impedance as seen from at a distance of three point one five lambda okay. So if lambda if given as five centimeters you are looking at a distance of around three point one five into five which is approximately fifty centimeters from the load okay.

We now look at one of the other important examples of smith chart calculation and this example can be used

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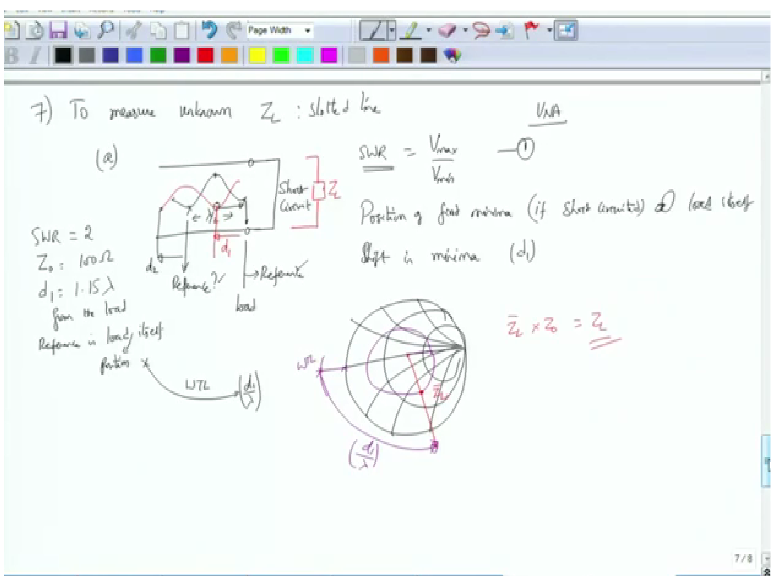

to measure unknown load impedances okay, to measure unknown value of the load impedance Z L. That method that we will follow okay, other methods but this is the classic method in which the unknown load was measured. What would happen if you first have a transmission line terminated in unknown load impedance okay, before you terminate put it into unknown load impedance, what you do is you terminate this one with a short circuit okay.

So you replace the load by a short circuit on the transmission line and then make measurements to find out what is the standing wave ratio. Now to find standing wave ratio you would need minima and maxima measurements right. So minima where does it happen? The minima happens right on the short circuit or on the load itself right. So minima happens, the first minima happens on the load itself okay when the load is short circuited.

Please note that when you replace the actual load, unknown load by a short circuit the first minima happens to be on the load itself then after every lambda by two this minima repeats okay. We do not want to know that second maxima so on so. Any maxima would serve as a sorry any minima would serve as a reference. To find a SWR, you found out what is V minima; you also find out what is the v maxima and then find out SWR ratio of V max to V min okay.

So you keep this calculation ready with you. You also find out position of first minima right if load side is short circuited okay. So if the load side is short circuited, position of the first minima would be at the load itself okay. If you are unable to identify at the load you can go slightly away from the load okay or from this particular point and then take this as a reference. It does not really matter which one you take as a reference okay.

So if its-- sometimes it is difficult to identify you know short circuit right on the load so therefore sometimes people go to the next reference load okay which happens to be the next position, position of next minima okay. So anything can be chosen as a reference, so you can choose one or the other, it does not really matter. For this example, I am choosing this reference to be on the short circuited point itself okay. Now what you do?

You replace the load, the short circuit with the actual unknown load that you are going to measure okay. What would happen if Z L is not equal to short circuit? Then there will be a shift in the minima okay. So now the minima would shift and produce its own standing wave pattern okay. What is the distance to which there has been a shift? Note down that point okay. Note down shift in minima.

You might ask, why should we note down shift in minima; why not note down shift in maxima? Technically the answer is that for it is easier to measure with much better accuracy, the minima points rather than the maxima points okay. Therefore, when you actually calculate SWR there is some amount of uncertainty in this calculation and because of the smith chart a slight change in SWR can totally change the values.

This method was not very accurate okay more accurate methods used, what is called as a vector network analysis or analyzer to find out the unknown load impedances okay. So this method of finding unknown load impedance which was called as slotted line measurement okay because this was standardized by using a slotted line. Is not very accurate but for an application of smith chart this might actually look okay.

So okay we will assume that SWR is known but in practice SWR slightly unknown and the slight unknown will actually change the value of the load impedance. Anyway with that that practical thing in mind, note down the shift in the minima. So in fact note down what is d 1. If suppose you have taken second one as reference note, no problem. You note down this reference right, so which would be d 2 let us call and this distance is what you all noting from the reference.

How much did the minima shift okay? Now once you have done that one, all you have to do is you start from the short circuit point okay and then move towards the load okay. Move towards the load until you reach a distance of either d 1 by lambda or d 2 by lambda depending on the which one you have chosen but at any case the reference has to be on the short circuit okay. What would it mean?

It actually simply means this, you start from the minima right and then move towards the load until you reach short circuit but because you do not know what this d 1 corresponds to, we are basically doing this on the opposite direction. You start from the position of the short circuit and

I know that if you move from this point towards the load, I would actually be able to find out where the minima I mean where the minima shift is located right.

So with that you simply move towards the load for a distance of d 1 by lambda right. So you would have actually measured this d 1 in term of some centimeters or something okay and lambda is given in some centimeter so you normalize it. Find out what is the given by lambda and move on WTL scale from the short circuit until you get to this point okay. So on the smith chart the steps are simple.

You first start with the smith chart okay and then you write down a constant SWR circle okay. That is something that you can do okay. So write down the constant SWR circle by choosing the center and SWR is this point okay, so on the constant SWR circle you need move. How much? You start from the short circuit okay on the WTL scale, you start from the short circuit and then you move until you make a moved a distance of d 1 by lambda and landed this point on the WTL scale. Next what you do?

You simply bring in this value to the center and this would be the value of unknown load impedance which is normalized. To un-normalize you need to multiply Z L bar by Z not, to give you the value of Z L okay. So please carry out this analysis okay for the values which I will give now. SWR is two, Z zero is hundred ohms okay and the distance d 1, which the minima has shifted is one point one five lambda from the load okay.

Assume that the first position, the reference is taken at the load itself. Reference is load position itself okay. So this is load position itself. So you find the answers and let us see what you are going to get okay. Know there is one last application that I would like to consider and this is perhaps very important application of a smith chart,

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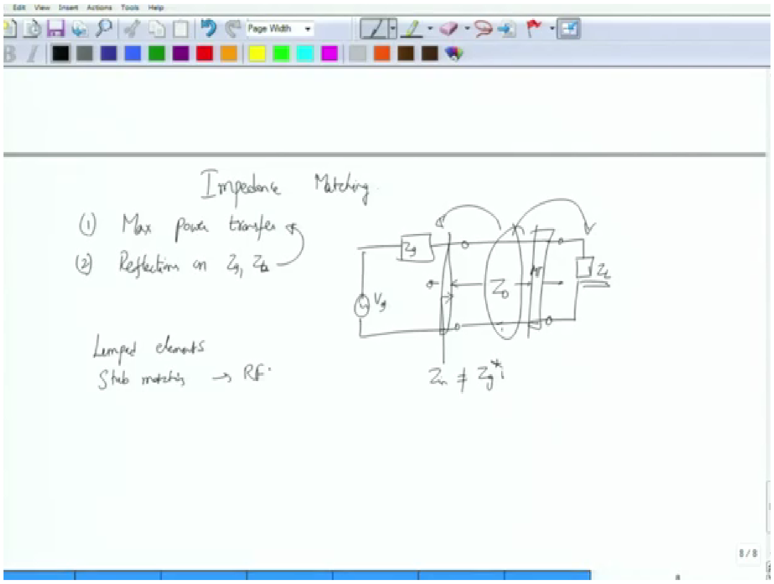

I mentioned the problem itself quite complicated. This is called as impedance matching okay. So the first question that you might ask, why should I match the impedance? So there are couple of reasons why you want should match the impedance. The first and foremost important reason is, maximum power transfer right.

So you will get maximum power transferred to the load when you actually match the load impedance with the transmission line impedance or actually you would have matched. So let us say this is your generator impedance and let us say this is your generator voltage okay, this is connected through a transmission line to some load okay. If this is all that is there, then the input impedance scene will not be equal to Z g or rather it should not be, it will not be equal to Z g complex conjugate right.

So only when the load impedance happens to be equal to this complex conjugate of the generator impedance then there will be maximum power transferred to the load right. So because this will not happen for a general this one, what you have to do is, you have to adjust the transmission line in such a way that you should get the Z g bar condition, Z g complex conjugate condition. Now in many cases we will not be able to adjust the transmission line.

Therefore, they will have to come up with a different way to match the impedance of Z L, via the transmission line of impedance Z zero (()) (25:13) Z zero such that this condition is satisfied okay. This is for maximum power transfer. The second thing is to basically minimize reflections on either Z g side or Z L side. This is called as load matching or generator matching okay. Again the primary reason would still be to transfer maximum power okay.

In a general case of impedance matching, there will be reflection but this reflection are actually cancelling each other, therefore you would be able to do the impedance matching okay. So only when you want to avoid reflection then you have to choose Z zero equal to Z L or Z zero equal to Z g or you put a matching network such that this Z zero looks as Z L and this Z zero on this side matching network would look as Z g in order to minimize the reflection on the line.

There are several techniques of impedance matching. There are lumped element techniques or there is stub matching which is quite popular in RF scenarios okay. We will be looking at stub matching in the next module okay. So in the next module we will be looking at stub matching. Thank you.