Electromagnetic Theory
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Lecture No - 64
Impedance matching

In this module we will discuss impedance matching. Let us first look at the reasons for impedance matching. Why would one be interested in matching impedances?

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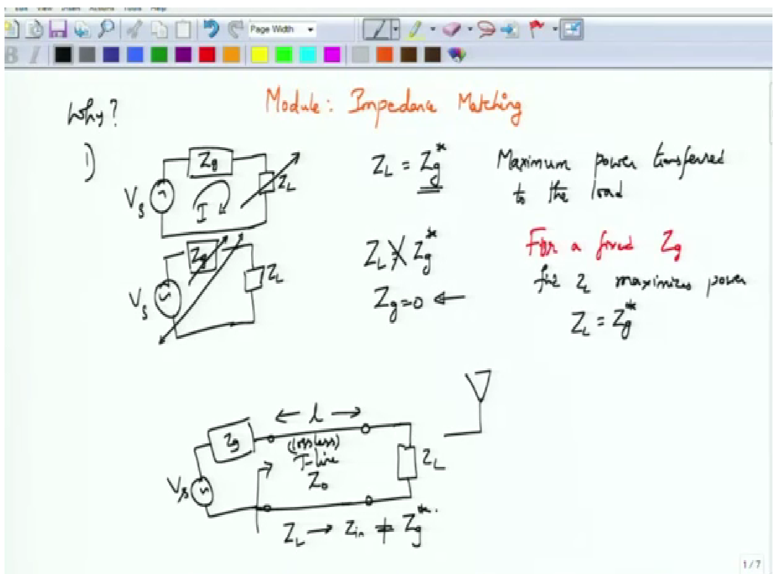

One of the reasons why we do impedance matching is to actually transfer maximum power. Now you know probably from your early circuit courses that if you consider a source okay, so let us assume a sinusoidal source or it could be any other source for that matter, which has a certain impedance okay because we are assuming sinusoidal excitation so for that particular frequency there is a certain force impedance or generator impedance that is called in the microwave RF literature.

So this is fixed okay so Z g is fixed and if we have an ability to choose the load impedance okay and this load impedance could be choose at our will then we can show by circuit analysis or we have probably shown in the circuit analysis courses, we might remember from that one that the maximum power will be transferred across the load when the load impedance is chosen such that it is equal to the conjugate of the generator impedance right.

So this is the case where we can actually expect or we actually have maximum power to be transferred to the load okay. Of course there is a slightly thing which people do not realize that, if at all we just have a source which has no internal impedance okay, so there is no internal impedance. Suppose we have situation where the load impedance is fixed and we are actually free to choose whatever the generator impedance we have for given value of the source voltage V S.

Then what is the generator impedance that you would choose in order to maximize power transfer across the load? Most people would say that, well maximum power transfer cannot be wrong because Z L is equal to Z g complex conjugate is supposed to give you the maximum power. But they would actually be wrong, maximum power transfer assumes that Z g is non-zero and it is fixed and Z L which is being varied.

If at all I have to choose a source whose internal impedance I can choose, you know something that you would not normally have but if hypothetically I am able to choose the source with internal impedance, I would actually set Z g equal to zero. So this is not the condition that would maximize the power transfer to the load but Z g is equal to zero would actually correspond to the maximum power transfer.

Because simply there is no drop across the Z g okay, so this aspect of maximum power transfer should be emphasized or I would like to emphasize that one. Basically what it says that for a fixed Z g okay find Z L that maximizes power okay so that maximizes power transfer, in this case would be Z L is equal to Z g complex conjugate okay. So you do not normally encounter a situation where the load is fixed and you can actually vary the Z g component.

I mean the source impedance but there are additional equivalent methods of achieving the second result. Anyway so going back to the maximum power transfer theorem or maximum power transfer idea that is one of the primary reasons why we would like to impedance match. So if we can get this condition Z L is equal to Z g complex conjugate for a fixed generator impedance, then I will be able to transfer maximum power across Z L.

And that is something that I would like. Suppose I have an antenna okay, the antenna presents a certain equivalent load impedance okay so let us call that as some complex load Z L and it is to be connected via a transmission line to the generator. Okay this is something that we would be doing in your labs if you were to perform some microwave experiments or antenna measurement experiment.

You actually have an antenna; you have a cable okay so this cable assume that it is lossless and its characterized by the characteristic impedance Z zero okay, as I said we only assume lossless transmission in this course okay. So for this lossless transmission line characterized by the characteristic impedance Z zero, we obviously know that if the transmission line has a length l, in general Z L gets transformed.

You know by the impedance formula and the input impedance Z in will not normally be equal to Z g complex conjugate correct.

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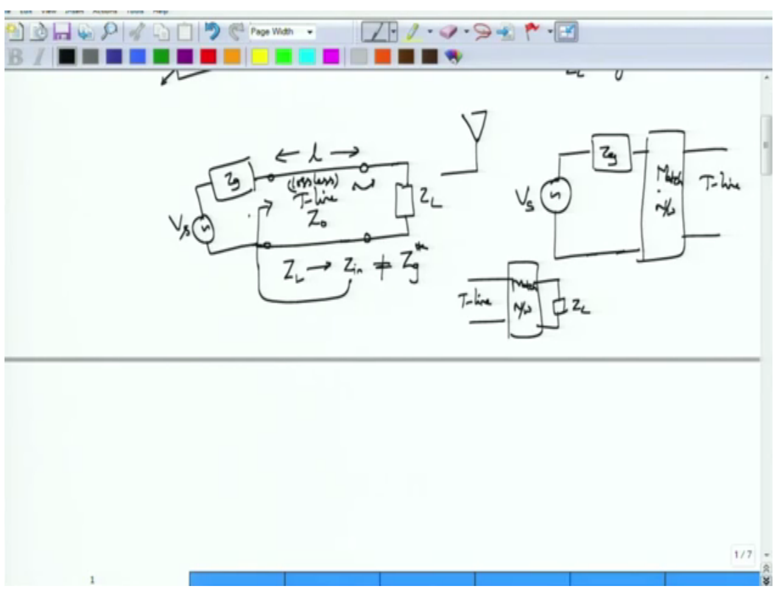

So this is not normally equal, of course in those rare occasion this would happen then it is good fortune and we will be able to satisfy maximum power transfer but the power will be transferred across only to the transmission line okay. So the equivalent impedance in this point will be the input impedance Z in and if this gets matched to Z g then maximum power is launched on the transmission line but the same maximum power may not be delivered to the load because this two are not matched.

Right so thus the general situation is that, if you have this kind of scenarios right you want to actually put matching networks everywhere. So you actually put a matching network at the source site or you do impedance matching okay. So this is the matched network that you have to put, so that the maximum power is launched on the transmission line and similarly on the load side you actually put a matching network so that you satisfy maximum power transfer theorem over here.

So you actually transfer maximum power to the load, so you typically have a source matching network as well as load matching network. All with the goal, same goal of maximizing power across the load so you want, do not want any power to be wasted because of the reflection that would arise when the loads are not matched. So you understand that one right?

So if there is a generator impedance Z g then if that is if not equal to Z zero, there will be reflections at the source and similarly when Z zero is not equal Z L there will be reflection at the load site and this reflection will create power losses. Some energy will be stored in the transmission line in the form of reflected voltages and currents. So we do not want any power to be wasted in the transmission line because of this reflection problem, so that brings us to the next requirement for impedance matching.

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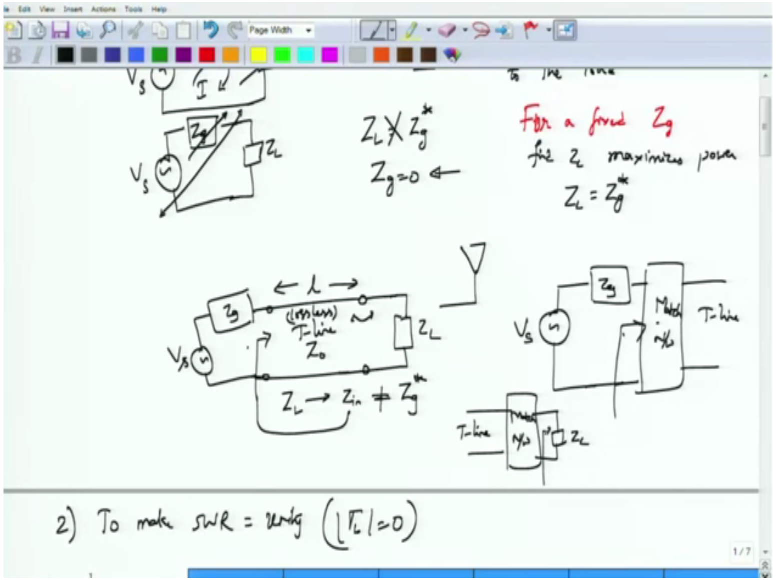

We want to impedance match to make SWR equal to unity. That is we want to keep reflections to be zero okay. Now what happens if the reflection is not zero? If the reflection coefficient is not zero then SWR will be greater than one and the input impedance that this source sees, you know at this particular point will actually depend on the transmission line Z zero; it will also depend on the length of the cable; it will depend on the load.

Suppose you have two antennas to perform measurements, when you are performing antenna pattern measurements you will actually have a reference antenna as well as you will have antenna under test which you are using. So if you have those two antennas, the impedances that would present to the generator will be totally different if the SWR on line is not equal to one. If SWR is greater than one, losses increase; the input impedance scene depends on Z L as well as l.

On the other hand, when SWR is unity then the input impedance scene will be completely independent of the length of the cable, it would be equal to Z zero. You know if SWR is almost equal to one, remember SWR will be always be greater than one but in the lower limit if it is very close to one then the input impedance scene will be very nearly equal to Z zero and independent of the load that you connect.

And this is something that you want. Just because you change an antenna, you do not want to recalculate everything or you do not want the efficiency to suffer and if you want to avoid that one you better put matching network, such that there is no reflection at the source side; there is no reflection at the load side okay. So to avoid these reflections

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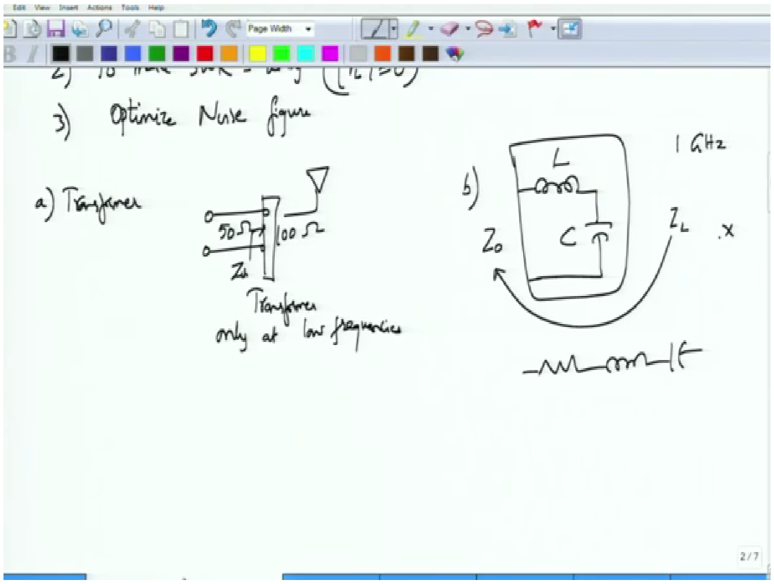

or to make Gamma l equal to zero or SWR equal to unity, another reason why we impedance match. If you go to active microwave devices there are other reasons why you would have you would try impedance match you know. If you are trying to optimize a noise performance okay so you would actually try to optimize a noise figure, in this case you would again perform impedance matches okay.

So we have seen couple of reasons why impedance matching is important but we have not really talked about how one can go ahead and perform impedance matching. So what does this matching network consist of? Suppose you assume that for some reason the antenna impedance happens to be about hundred ohms okay and I have a transmission line which maybe you know just a two wire line, which has a characteristic impedance of fifty ohms.

Now if it was a low frequency application, if this is not real antenna but some sort of amplifier then it would have been very easy for me to match them. What would I have done? I would actually put a transformer and chosen the appropriate ratios of the turns of the transformer so that the impedance scene will actually be equal to so. I can actually transform this hundred-ohm impedance it to be looking as a fifty-ohm impedance and when I do that one there will not be any reflections.

So this is something that you can do only at low frequencies right. Only at low frequency I can transform this hundred-ohm impedance into a fifty-ohm impedance scene looking at this particular point. So this is one method, so transformer matching or matching network using transformer which is very widely or used to be very widely prevalent in low frequency regime is one approach.

The other approach is to kind of mimic the transformer using a complicated networks of L's and C's okay. It turns out that a simple circuit such as this, would be able to know transform load impedances. Of course there are various ways in which you have to arrange this one depending on load and the characteristic impedance of transmission line whether which one is greater or which one is lesser.

But the basic idea is that network consisting of L's and C's is able to match or able to transform a load impedance on to Z zero okay. So it can actually transform if you were to put this lumped element matching network as we would call so. Why we call this lumped element? Because for frequencies up to say one Gigahertz or slightly higher than that, the values that you obtain for the inductor and capacitor can actually be realized even on the printed circuit board, can be realized using lumped elements.

You can actually buy these inductor circuit capacitors and then place them at appropriate point on the matching network. Now that is obviously not going to work if you are frequency goes beyond say one Gigahertz. One Gigahertz already stretching it too much so if you go beyond these frequencies then lumped element matching networks are no longer the preferred choices. Because you do not get an inductor to act like a inductor at high frequency, an inductor actually acts like an RLC circuit okay.

So this is how a typical inductor would look like probably the resistance should come in parallel but that does not really matter. Similarly, a capacitor would also look like an R C circuit. There is a L part in that capacitor, the inductor part in the leads of the capacitor but that can be usually neglected. But the point here is that, the lumped element matching network cannot be sustained at high frequency.

And if you are designing some microwave or radar application or antenna application at high frequencies, this is not the way to go for impedance matching. What can we do?

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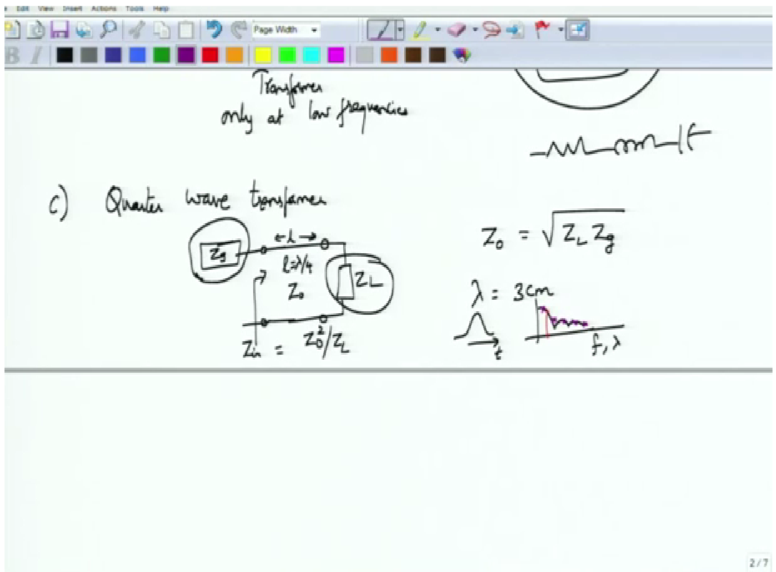

There is analogous transformer matching in transmission line, using transmission line and this is called as quarter wave transformers okay. Quarter wave transformer works on a very simple idea. Suppose I have a load impedance Z L connected to a transmission line of length l is equal to lambda by four having characteristic impedance Z zero okay, so in this case the input impedance scene, if this is the length l the input impedance scene at the input terminal of transmission line is actually equal to Z zero square by Z L.

So if I were to match, so for example this Z in supposed to be equal to Z g okay so if this Z g must be equal to Z in, then I should simply choose a characteristic impedance of Z zero of a transmission line of length lambda by four such that this is equal to Z L into Z g. So choose a transmission line of length lambda by four at that particular operating frequency to satisfy this condition.

Then any load which is typically real load to a typically real generator impedance okay. This is something that is usually used to match real impedances even at high frequencies. The problem here is that, the band width of this match is not very good because it actually works only at particular wavelength. For any other wavelength, you know the length will not be exactly equal to lambda by four.

For example, if you have performed all this designs with lambda of three centimeters okay and you know instead of sending a nice sinusoidal signal at three centimeters, you actually end up sending a pulse like this and we know that a pulse consists of multiple frequencies. So this is the frequency domain picture of this pulse and you see that all is different frequency would effectively be characterized also by different wavelengths.

And if you have designed this one only for one particular wavelength then all, so which let us say is this wavelength all the other wavelengths that are residing here are not matched. So what would happen? There would be reflections at other point and the pulse actually gets distorted. So the quarter wave transformer is typically used for sinusoidal steady state matching but that cannot be used to match over a broad band of wavelengths okay, such as something that you would find in a pulse.

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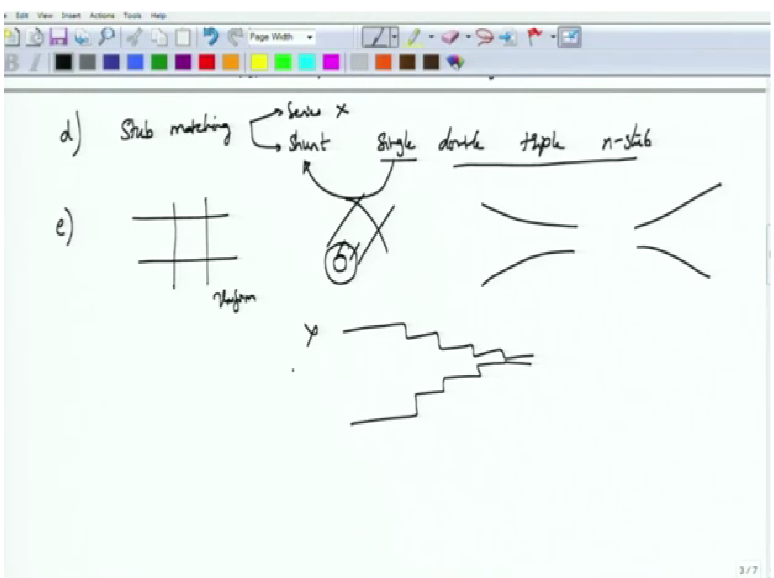

So what is the other option for us? There are additional options, one of them is known as stub matching okay and the number of elements that you use determines what kind of stub matching you are looking for. Again this stub could be series okay, normally not used in printed circuit boards because it is very difficult to cut apart of the printed circuit board and insert stub matching network there.

Unless you design it at beginning or you can use shunt stub matching in which the stubs, which are this small opened circuited or short circuited transmission lines are called as stubs and these stubs could be connected across in parallel or in shunt or they could be connected in series with the transmission line and the load impedance okay, in order to minimize reflections. Now this series stub is not widely used, so we not going to discuss that one.

In shunt stub that depending on the number of elements you can have a single stub, you know only one stub placed at appropriate location and chosen its length; you can have double stub or you can have triple stub or in general you can actually have a n-stub matching network okay. The formulas and procedures for this double, triple stub and n stub are quite complicated and not particular suited at this level.

So we will not look at double stub matching, we will only look at single stub matching and we will look at stub matching in the shunt method that is to say we connect the single stub in parallel to the main transmission line okay. There are variations of this stub matching which actually employ the exploit the non-uniform nature of transmission line. So far we have look at transmission line and we have drawn (()) (16:09) a generic circuit which consist of a uniform, in a straight line and whose cross section is not changing.

Alright although in picture I might not able to show that one, the cross section taken at this plane of transmission line; cross section taken at this place on the transmission line would all look the same okay. So you take coaxial cable and this would be the cross section of the coaxial cable for any length that you can choose okay. So at any point you cut and look at cross section, the cross section would remain the same.

Such transmission lines are known as uniform transmission lines okay. But it is also possible to taper this transmission lines okay or exponentially increase or you connect multiple sections of the transmission line okay. So you connect the multiple connections of this transmission lines, probably the figure did not come very nice but this different kinds of non-uniform transmission lines are quite widely used in microwave circuits in order to perform impedance matching.

Because of their wide band width characteristic and something that is required when you are transmitting pulses on a transmission line. So unfortunately we will not be able to deal with even this type of transmission line matching networks. So let us concentrate only on one type of transmission line matching network which is the single stub matching network.

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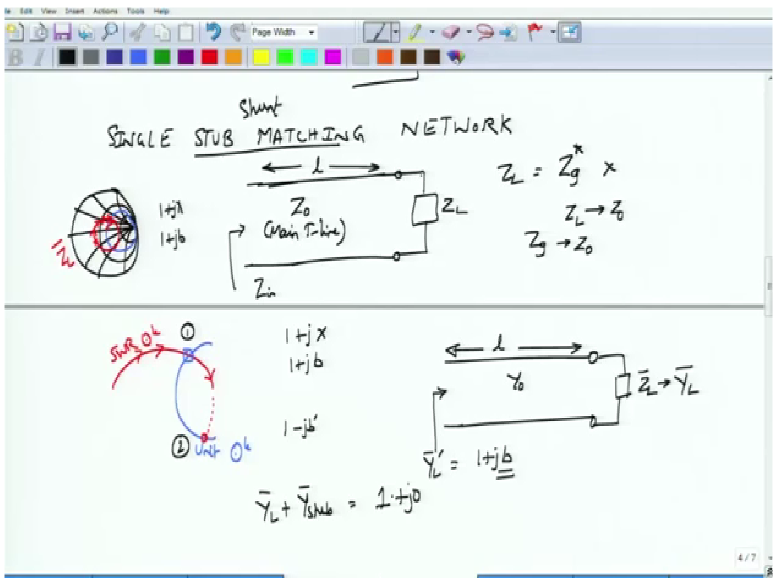

So our goal is to consider single stub matching network. It is not really a network, it just a single stub out there but sometime this matching networks were used, you know in the earlier literature so we colloquially call this a single stub matching network okay. So the basic idea is this, so go slowly over here understand the basic idea the numerical examples will very easily follow. So let us begin by having a certain load Z L.

Remember we are not maximizing the power transfer, we are not choosing to impedance match in such a way that Z g complex conjugate is actually equal to Z L, we are not doing this. What we are trying to do is to minimize SWR on the line which is slightly easier to perform. So we try to make Z L look like Z not, so there is no reflections on that particular transmission line and in case Z g also happens to be different from Z L.

Then we try to make Z g also look like Z zero so as to avoid reflection at the generator side okay. So this is the load okay, this leads are very short assume to be of ideally zero length so do not bother about this length but to this load terminal you connect a uniform lossless transmission line having the characteristic impedance of Z zero okay. So you connect a transmission line having an impedance of Z zero of any length that you are considering.

Now if you go back to the impedance transformation formula then you know that if the transmission line has a length l then the input impedance scene at this point is given by one complicated formula but on a smith chart I can actually find out the input impedance given Z L, Z zero, length and the operating wave length of this particular network right. So in general of course Z zero is not equal to Z L, that for Gamma l will be non-zero.

But it is possible to rotate this Z L or transform this Z L to some Z in. Now if we were able to transform this Z L into form which is useful for us then we can talk about matching okay. So what we do there? Before that let us just put some convention, the main transmission line is the one which is connected to the load and upon which we are trying to minimize the reflection, so let us call this as main transmission line, main t line okay.

Now what we do here is that, as you go through Z L, let us plot the corresponding smith chart over here. Do not take this smith chart too literally. So just to give you an idea okay. So I have this constant r and this x circles correct, on the smith chart I have all this constant circles. Now I also know that there is unit circle on the smith chart right. On the unit circle what are the impedances? The impedances on the unit circle are of the form one plus j x, correct?

That is the impedance on that form. If I were to use this smith chart as admittance chart, then the admittance on the unit circle will be one plus j b okay. So with this unit circle in place, what we would like to do is to take any load which is described by a certain SWR circle right upon which this would propagating since this color is not very nice let me redraw this particular thing okay. So let me write this smith chart in this way and write down all this constant x circles okay.

Let me write down the unit circle that is the unit circle which I have drawn okay and now this is the SWR circle okay. So this is the SWR circle upon at some particular point there is a load here okay, so this is the load point Z L which we have drawn and of course I cannot actually plot Z L but I will be plotting the normalized value of the load impedance. How do I obtain normalized values? By dividing Z L by Z zero okay.

Now you look at this red curve okay which represent the constant SWR circle characterized by the Z L and Z zero okay. Any movement towards the generator, so as I move towards the generator, I look at the Z L bar and then I keep moving along the constant SWR circle. Now what happens at this point? At this point something interesting has happened. I am moving on the SWR circle; this is my constant SWR circle.

What actually happened was, I hit upon a unit circle right; so I hit upon the unit circle on the smith chart. So at this intersection point okay the SWR on the circle as you keep moving when it meets the unit circle it indicates that the impedance at that distance from the transmission line is of the form one plus j x, correct? So this is what actually implies as you move along the constant SWR circles, so you meet the unit circle and the impedance at the point will be one plus j x okay.

Now this is what we have obtained so far. So what we had was a transmission line, you moved a certain distance on the transmission line, so as to make this Z L or equivalently the admittance Y L okay as one plus j b. Of course if you keep continuing right, so you continue this movement on the SWR circle you will actually reach a second intersection point. At that intersection point the impedance happens to be one minus j b or j b prime okay, it would be one minus j b prime.

You can choose any of this as your possible solutions, I will explain later. Usually you will try to find the first intersection into the unit circle okay, usually you try to find the first intersection to the unit circle call this as one; call this as two. So you are usually interested in where the SWR meets the unit circle for the first time okay and read of the values of the admittance at that point. Now why am I talking about the admittance?

Because as I said, this is the single stub matching but the stub will be connected in shunt, so it is better to deal with admittances rather than the impedances. So Z L becomes Y L, of course it needs to be normalized so that for Z L bar actually becomes Y L bar. We know how to do this one on the smith chart already. And then you have actually moved a certain distance on the transmission line okay, so sorry we used l here.

So you have moved a certain length l on the transmission line okay. So as to make the admittance out here okay some Y L prime, let us call this one looking into the transmission line terminals of the characteristic impedance Y zero to be equal to one plus j b okay, where b is the normalized non you know imaginary part of the admittance. But it is at least one step closer to matching. Why? Because you only now have to take out this j b. How do I take out this j b?

Well mathematically simply add something to this, let us call that as the stub admittance such that the result is one plus j zero right.

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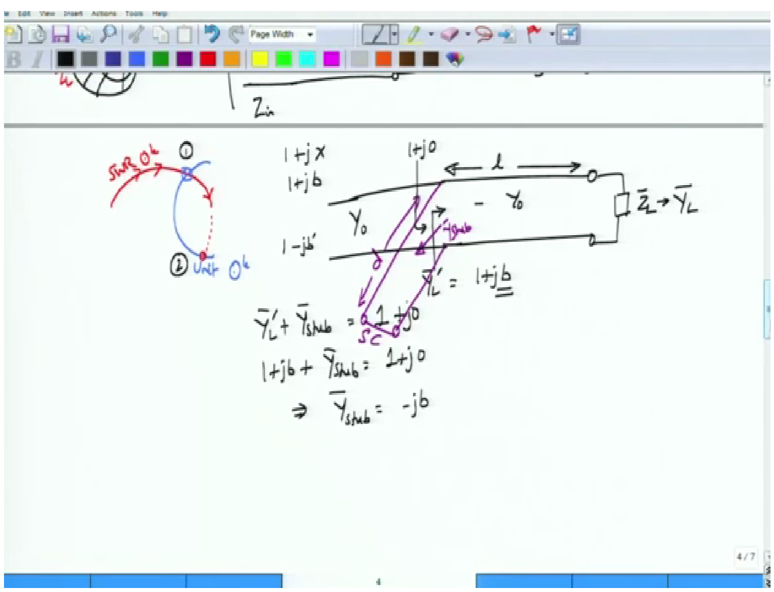

Now I already know what is this Y L prime which is the impedance or the admittance scene looking at a distance l which is the point where the SWR circle has actually met the unit circle, so Y L prime is one plus j b plus the stub impedance is equal to one plus j zero. This obviously implies that the stub should be equal to minus j b. So another word this stub must be chosen such that its admittance is equal to minus j b.

So when you choose that one and then put up the stub, you are ready to do the stub matching. Again these impedances can be obtained, remember open circuited or short circuited transmission line would always give you capacity or inductive reactants right it would always give you imaginary parts of the reactants. So you choose the length of this appropriately so call this as a d, so you choose the length of this one appropriately.

If d is chosen according to whatever the value of b required, then it is possible to either use a short circuited stub or an open circuited stub. In order to provide an admittance scene looking into this terminal at the same plane as Y stub, so that the total admittance scene into the left of the stub okay would be would actually be equal to one plus j zero and the transmission line is actually matched and there will not be any reflections okay.

So this is basically the idea. So let us try and put this idea into practice okay. So there is a small way in which I would like to show this one. I will not draw the complete smith chart but if you have the smith chart with you actually you can follow the idea over here okay.

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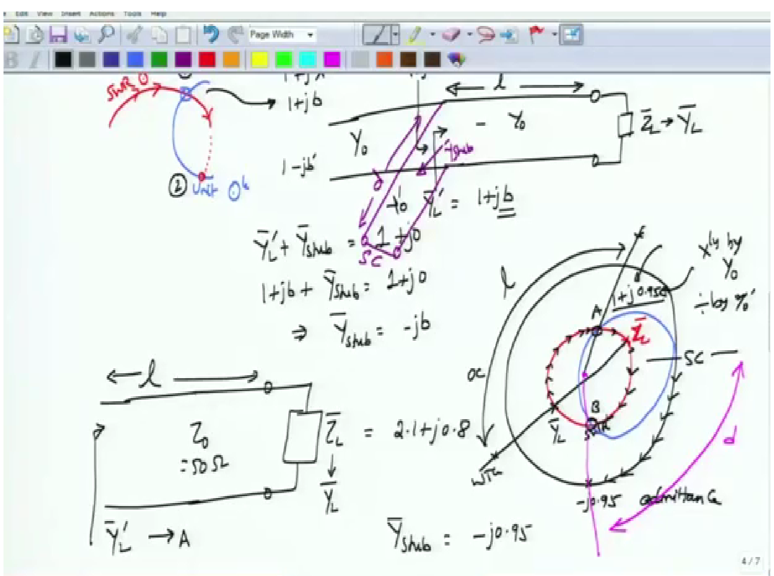

So let say I have the load impedance okay, there is actually two steps in this smith chart. The first step would be to determine the length to which you have to move along the main transmission line in order that you meet the unit circle. Once you have met the unit circle and read off the corresponding value of the admittance there, you need to consider whether it is a short circuited or opened circuited stub.

And then move in such a way that you are actually be able to obtain minus j b okay so that the total admittance actually adds up to one. So let say I have a Z L which is equal to two point one plus j point eight okay. So on the smith chart you can locate this one and from Z L you can actually obtain Y L okay. Let also assume that Z zero is there okay. So if you assume that z zero is a let us assume that this is about fifty ohms okay.

So assume a characteristic impedance of fifty ohm. Sorry this Z L I have already given you in the normalized form, so do not worry what is Z zero is but remember this normalization has been done with respect to main transmission line okay Z zero. Now with Z L bar you can actually obtain Y L bar mathematically you can do that but on the smith chart it becomes easier. So let us go ahead and construct our smith chart. This particular circle is my constant SWR circle okay.

So this is the SWR circle with the load having a value of two point one two plus j point eight and that should acquire somewhere over here right, so on the two circle so this is where somewhere two circle happen and then this is point eight so let say this is somewhere over here. So this is my load Z L bar and how do I obtain the admittance Y L bar? Y on the SWR circle I have to move one eighty degree which means I have to find the opposite point okay to obtain Y L bar.

So I obtain Y L bar on moving along the SWR circle okay. You can find out what is the SWR for this one or you assume SWR is given to you okay. So with the SWR circle, so let assume because for that one you still need some Z zero, so assume that on the corresponding SWR you actually moved one eighty degrees which means finding the diametrically opposite point and that is where you are Y L bar is. So this is your starting point right.

So this is your starting point. Now we have to move a length l or normalized to the wave length l by lambda, you have to move. To do that one you also require where the unit circle is located right. So let us say the unit circle is this particular circle okay, so this is my unit circle and locate the unit circle okay. So now you start at this Y L bar and you keep moving along the constant SWR circle until you hit the intersection to unit circle.

Again you have two intersections, one at A, if you continue to move towards like this you would move and intersect this SWR I mean unit circle at point B as well okay. We do not want to consider B. Lets only consider point A. So at A, you can read the value so let say this is about one plus j point nine five okay, so again this is not an exact value this is something that you have to read it from your smith chart.

And I am making some simple numbers over here just to show you how to perform this stub matching. Now what is this point A? Point A is the intersection of SWR and the unit circle. Where would they intersect? It is actually the corresponding length over which you have moved along the transmission line right along the transmission you have move a distance l such that the admittance scene at this point is Y L prime right so this is Y L prime point.

Y L prime point is point A on the smith chart and how much is the distance you have moved? You can easily find out. Extend this to the outer periphery. Note down what is the WTG scale. On the WTG scale, note down the starting coordinates and then at point A, so from the center of this one you go to point A and then extend that one okay, on the outer WTG scale you note down what is the difference and this length is actually the length that you have moved okay.

So this is the length along which you have moved in order to arrive at Y L prime. So at Y L prime we have seen that it is equal to one plus j point nine five, the admittance therefore the stub normalized admittance that you require in order to obtain impedance match is minus j point nine five. And on this stub network where should I move towards? From the short circuit point, I should be moving towards the generator right.

So that at certain length d, the stub admittance normalized will be equal to minus j point nine five. Now where is the short circuit located? Remember if the smith chart is being used as admittance chart, short circuit is located here okay and open circuit is located over here. And after locating Y L bar, from this point onwards we will treat the smith chart as an admittance chart okay.

So since we are treating chart as an admittance chart the short circuit is located at this point. From this on the short circuit, on the outer, the larger the circle we keep moving until you hit this point which presents minus j point nine five admittance okay. So this is the admittance that you would obtain when you move along the outer circle, you know on the circle r equal to zero or you know equivalently g equal to zero.

The question is why am I moving on the outer circle? Because what is the SWR for the shorted line or an open circuit line? The reflection coefficient magnitude is one therefore SWR on this line is equal to infinity right, so that would correspond to gamma l is equal to one right and that is actually the outer most circle. So on the outer most circle you keep moving until you hit minus j point nine five which is exactly required to cancel out this plus j point nine five okay.

How much distance you had to move? Again from the center of the chart, you note down from the, sorry from the short circuited point, you note down what is this length okay. So this length will give you the stub length which is required in order to obtain an impedance, sorry, admittance which can be used to cancel the admittance obtained at point A okay. So these are the two steps. Finding l and finding d.

Just to give you summary, start with Z L, locate Y L bar, once you have located Y L bar you are now considering the smith chart as an admittance chart. Move on the Y L bar on the SWR circle until you meet the unit circle okay. So that unit circle movement meeting happens at the point A okay. At this read out the value of the impedance, sorry, read out the value of admittance and you find out what is the corresponding point that you need to cancel.

So here you have one plus j point nine five to cancel, so to cancel j plus point nine five you need minus j point nine five, that you can obtain by starting at the short circuited point because we are assuming that the stub to be short circuited and then moving on the outer periphery or on the outer circle, the larger circle the gamma l equal to one l circle until you reach the constant minus j point nine five arc okay.

So from there, from the center you draw line extend find out the distance. This is called single stub matching okay. There is only one complication that might arise. For example, if someone specifies that I do not have the characteristic impedance of the stub equal to Y zero, let say I have something which is Y zero prime. How do I proceed to this problem? Nothing. You first do the first step as it is, locate Z L, go to Y L bar, move on the SWR circle until you reach point A.

Now what you have obtained is an admittance which is normalized with respect to Y zero, so you can actually first un-normalize okay. Multiply by Y zero of the main transmission line so therefore this becomes un-normalized impedance okay. Once you have done that one again normalize with respect to Y zero prime okay. Not the real part only the imaginary part, normalize with respect to Y zero prime okay.

So by dividing by Y zero prime you have normalized with respect to Y zero prime, you take that as the imaginary part that you need to cancel. If it is plus you add a minus; if it is a minus you make it plus and then move along the new imaginary part value okay, so obtain the matching. So this brings us to the end of stub matching and impedance matching. In the next module we are going to discuss time domain or transient analysis of transmission lines.