Electromagnetic Theory
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Lecture - 66
Transients on Transmission line – II

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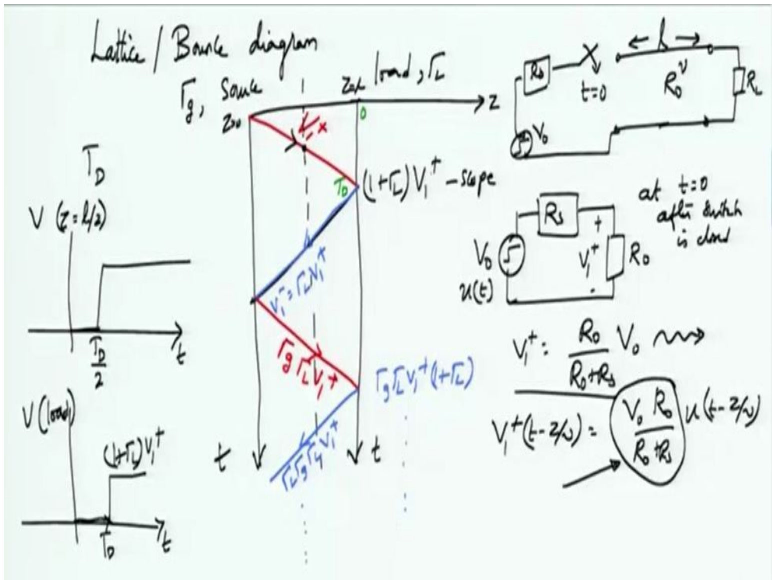

In this module we will continue discussion of analysis of a step voltage on a transmission line using lattice or bounce diagram. So I hope this particular slide should be familiar to you if you have seen the last module. We had a transmission line excited by a step voltage by a source of amplitude V zero connected through the source had an internal impedance of R S or internal resistance R S connected to the load via transmission line of length l and propagation velocity v.

So therefore, the total delay in this transmission line was T D okay. So what we were discussing is that at T is equal to 0 when the switch is closed and this step voltage appears on the transmission line. The voltage source that will be here say R L. You know it is like this. You imagine that there is at a very large distance there is some object which is glowing okay. For example, a star, a star is glowing.

And then when you see the light from the star it is actually not the light that you are seeing which has just left the star's surface. What you are actually seeing is that light which actually left the star a millions and millions of years ago because that time delay is the factor over there okay. So the light which just leaves the star surface today or you know the surface from the earth today if you have to shine a high power laser beam does not immediately see its load which may be located millions of miles away.

What it sees is that the first time when you launch in the slide it will see the atmosphere. Whatever the equivalence impedance that the atmosphere presence is the impedance that we will see okay that the light will see. So in the same way R L becomes visible after the voltage is actually crossed over and spend time to reach R L and even then it would not be visible it will actually have to come back okay.

So until two-time T D the source does not really see the effect of load. This is like a radar thing okay. You take a pulse; you throw a pulse at a distance until the two times the propagation delay has finished you will not be able to receive that pulse back okay. And only when you receive the pulse back you have some idea of what is the target there. So in the same manner this is essentially the idea.

The pulse will not or the step voltage will not see this load R L but instead it sees that the transmission line R 0 as its immediate impedance and that creates the voltage divider which is the voltage that is launched and the voltage across the R z as the voltage that is launched on the transmission line okay. So with an amplitude of V 1+ given by R 0 by R 0 + R S will be launched on the transmission line.

We represent that by writing this as V 1+ okay. So we represent that one by writing here as V 1+ as initial voltage that is launched on the transmission line okay. So at the receiver when you are in the load side when this time delay of T D is elapsed okay right. There is a time delay of T D so on the time axis this is zero but this is T D okay. So at time T D the initial voltage V 1+ from the source appears.

But immediately as soon as it reaches the load there will be a reflection created and that reflective voltage will be equal to gamma L times V 1+ where gamma L is Z L or R L -R 0 by R

L + R 0 okay. So that voltage is reflected back and then begins to travel from the load to the source side. What would be the total voltage on the load side? It will be the sum of incident and reflected voltage. Incident voltage is V 1+, reflected voltage is V 1-.

The sum is actually equal to 1 + gamma L multiplied by V 1- that would be the voltage that is seen on the source side. So if you have to put for example put an oscilloscope here okay. So you put an oscilloscope here you would say that there is nothing happening until time T D right. At that point you will see a voltage which is 1 + gamma L multiplied by V 1+ okay. So this is the voltage at load side okay. This is as a function of time that we plot okay.

Similarly, as I was saying you can put an oscilloscope in the middle of the transmission line so V at Z is equal to l by 2 v that is V at Z is equal to l by 2. So at this half way distance right until some half time T D by 2 because it will take if it takes 1 T D to reach from the source to the load side then it will take T D by 2 to reach half way through across the transmission line. So until that point there is no voltage thereafter the voltage will jump.

This can be obtained by as you drawing a straight line and then following it up where it intersects that is the point where you actually start seeing the voltage. So this is how the voltage waveforms look half way and at the load point okay. This voltage which is actually beginning to go backward right so this V 1- which is going backward, would actually reach the source. And at that point if gamma g happens to be is equal to zero then this would be observed.

There will not be anything that is going back now once if gamma g is equal to 0 this voltage would be observed and this would be the end of all the voltages. The load voltage continues to remind at 1 + gamma l V 1+ and then the source voltage would actually receive whatever the voltage that have been sent plus this voltage gamma l V 1+ and then it would actually stop. There is nothing out there that is happening.

However, if gamma g is not equal to zero as in case when R G is not equal to R 0. This would act like an incident voltage on the transmission line and create a next voltage which is reflected from the source which would then travel again in the same direction as the initial V 1+. It would travel from source to the load okay. What would be the amplitude of this voltage? Well this voltage amplitude is gamma g times gamma L V 1+.

Because gamma g times V 1- is the reflected voltage but V 1- is gamma L multiplied by V 1+ therefore this would be gamma g gamma L V 1+. This is the voltage that is going backwards now at this point there will again be a reflection right. This reflected voltage will have amplitude of gamma L gamma g gamma L V 1+ and this cycle continues okay. What is the voltage here? What is the next bit of the voltage here?

This is gamma g gamma L V 1+ gamma L and so on okay. So the total voltage the steady state voltage would actually be the sum of all these individual voltages okay. So we can actually do that sum.

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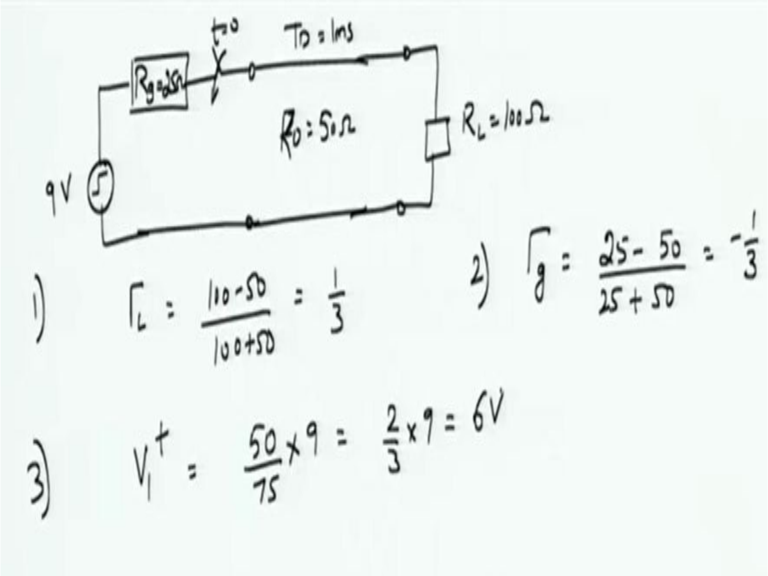

So probably a better way to understand this one or to get more intuition on this particular technique is to as you take an example and solve it. So let us look at this example. I have this source impedance which is 25 ohms okay and I have a source which is essentially a step voltage of amplitude 10 volt which I am going to connect at T is equal to 0 so this is the switch at T is equal to 0 and connected to a transmission line such that the delay is 1 millisecond okay.

So here we don’t have to bother what is the length and the propagation? We just assume that the transmission line has a delay of one millisecond which we have calculated by other means. Z 0 or R 0 of the transmission line is 50 ohms and we connect a load which is say R L of 100 ohms. Now we want to find out the voltage at the load side, voltage at the source side, voltage in the middle of the transmission line. How do we do that? First step okay calculate gamma L.

Gamma L is 100 - 50 by 100 + 50 which is 1 by 3. Calculate as a second step what is gamma g? So you are in bad luck because gamma L is not zero there will be reflections. What is gamma g? Is that zero? Unfortunately, no. Gamma g is 25 - 50 by 25 + 50 which is -1 by 3. So what you see is that gamma l and gamma g are equal in magnitude but there are not equal in sign. So there is also the fact that gamma g is also not zero.

There will be reflections, multiple reflections on the transmission line okay. The third step is to actually calculate what is V 1+? This is the initial voltage amplitude that is launched on the transmission line to calculate this one you need to form a voltage divider between R G and R 0 right and whatever the initial voltage source I mean whatever the voltage source that you have. And this gives you 50 by 75 multiplied by 10 which is 20by 3 volts.

So let us actually change this one from 10 volts to 9 volts just to get some initial real nice numbers. So what we get here is 50 by 75 multiplied by 9 which say 2 by 3 multiplied by 9 or this is 6 volts okay. So this is the initial voltage that we are launching on the transmission line.

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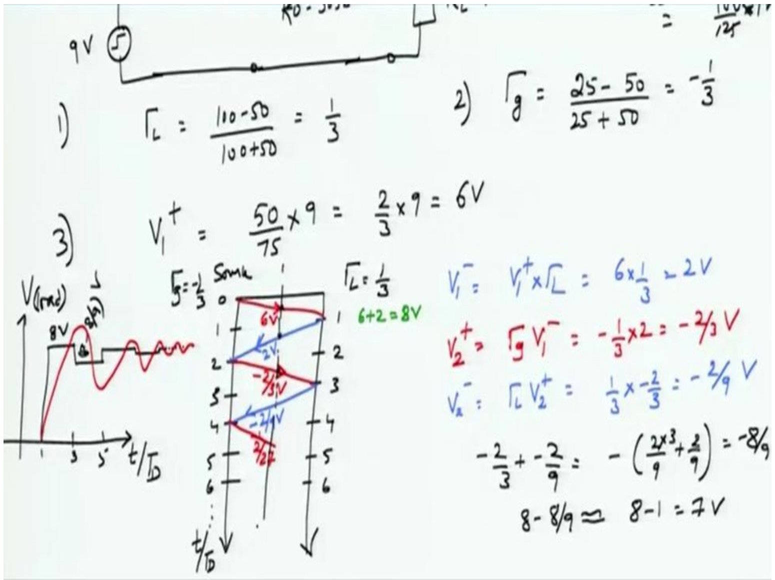

Now you will draw the bounce diagram okay. So this is my bounce diagram which I am drawing okay. And I can normalize this one with respect to T D therefore this would be 0 time or 1-time delay, 2-time delay, 3, 4, 5, 6 and so on okay. Here also the same thing 1, 2, 3, 4, 5 and 6. So this is the source side okay write down gamma g. Gamma g is equal to -1 by 3. Write down on the load side gamma L is equal to 1 by 3.

Now you will being moving the voltages so initially the voltage that will be launched will be six volts and it would propagate like this so this is 6 volts okay and then right after this 6 volt reach there will be reflection. So the reflected voltage V 1- is equal to V 1+ multiplied by gamma L which is 6 multiplied by 1 by 3 which is 2 volt right. A 2 volt is actually launched back. So you can write this as 2 volt okay and this is 6 volt.

What will be the total voltage at this point? The total voltage will be 6 + 2 is equal to 8 volts. So the voltage actually jumps up to 8 volts at t is equal to 1 millisecond. Now these 2 volts has come back. What would be the reflected voltage? Well the reflected voltage will be two times so this will be V 2+ will be gamma g multiplied by V 1- which is -1 by 3 multiplied by 2 which is -2 by 3 volts okay.

So a -2 by 3 volts is launched towards this side so -2 by 3 volts is launched. What would be the voltage reflected from load side again? This would be V 2- will be gamma L times V 2+ right.

Gamma L is 1by 3 and V 2+ is we have seen -2 by 3. Therefore, this is equal to -2 by 9 volts okay. So this would be -2 by 9 volts. So further reflected voltage will be minus into minus will become plus so 2 by 9 multiplied by 1 by 3 so that it would 2 by 27.

So you can see that the amplitude of the reflected voltages keeps on decreasing and for 2 by 27 or something we can probably stop at this particular point. Now let us try to sketch the waveforms okay. So let us try to sketch the waveform here. First at the load side okay so at the load side in terms of time normalized with respect to the time delay of the circuit I am trying to plot the load voltage okay. So how does it look?

Initially at up to time t is equal to 1 time delay which is one millisecond there won't be any voltage. So after one, we actually see a total voltage of 6 + 2 which is 8 volt. So this is 8 volts and this would continue to remain until the second voltage waveform reaches here. That is until this -2 by 3 volts which is 2 by 3+ reaches this load side at 3 milliseconds right. So at 3 milliseconds what would be the total voltage?

Whatever the initial voltage that is there plus the voltage that is now here. So this would be -2 by 3 + -2 by 9 okay. So this is – (2 multiplied by 3) you can do and this would be 9 + 2 by 9 right. So this would be 6 + 2. This is around -8 by 9 so approximately -1 volt so the voltage actually drops now 8. So the total voltage is 8 – 8 by 9 which is approximately 8 - 1 volt this is about 0.9 or something. So this is roughly 7 volts okay.

So I am not really interested in writing down that one so we can actually do all these calculations. Now 8 – 8 by 9 volts is this voltage and this change happens at 3 milliseconds okay. Now again at 5 milliseconds something must happen right. So let us right this one this is 8 – 8 by 9 volts okay. So at 5 milliseconds now get this 2 by 27 and then there would be one more voltage that is gone back so there is actually a positive voltage up there.

So the voltage actually just slightly increases okay. It would actually increase by a very small amount okay. And this is again decrease further and it would again increase further slightly and then eventually reach a steady state okay. What would be the steady state voltage? Under the steady state condition, the transmission line can be removed and the voltage simply becomes 100 by 25 + 100 multiplied by 9 volts which is 100 by 125 multiplied by 9 volts.

This would be the steady state voltage at the load side okay. So you can convince yourself by plotting this particular thing. Also if you look at the way this voltage is there you can imagine that there is some sort of an under-damped circuit right. So if you look at the under-damped amplitude oscillations it will look like this. It will go up come down, go up come down and this is the under-damped circuit that one might think okay. There is a slightly different circuit okay.

We will see that one later which would look like an over-damped circuit okay. This was how we could draw the voltage waveform at the load side but what if I want to find the voltage at midway through here right. All I have to do is keep moving along this line until I hit the corresponding line okay carrying the voltage and keep adding that one okay. At this point the voltage is 6 volts then when I hit here it would be 6 + 2 volts that is 8 volts.

Then I come here that would 8 -2 by 3 volts then it would be 8 -2 by 3 -2 by 9 and so on.

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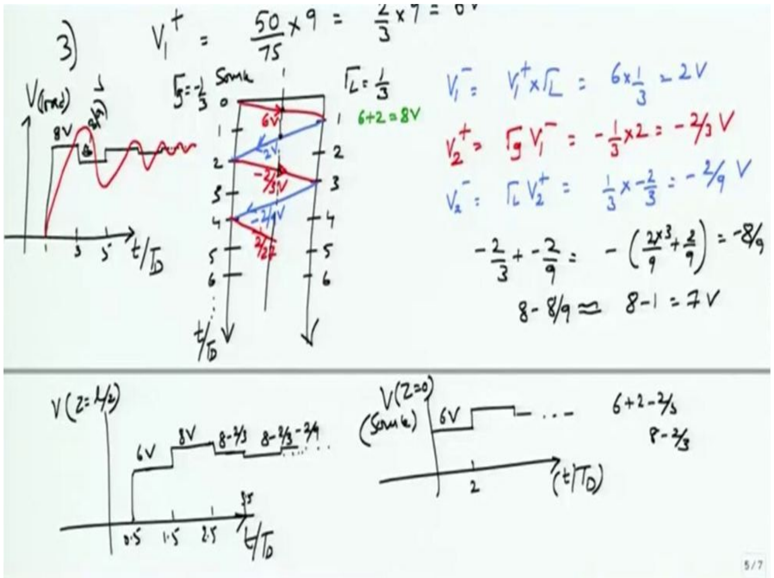

So if you want to plot the voltage waveform as a function of time at Z is equal to l by 2 that is half way through the transmission line. Initially until T D by 2 that is until 0.5 you don't see anything. At 0.5 you see 6 volts okay. Then it will again change at 1.5 millisecond or 1.5 time durations at which point it would again rise slightly because this is 2 volt jump so this becomes 8 volt. Then again at 2.5 this will drop somewhat because there is -2by 3.

So it is 8 -2 by 3 and then at 3.5 again this would drop a little bit. Because now this will be 8 – 2 by 3 – 2 by 9 so on, so here it would again rise and then eventually go something like this. Now at the source side what would happen initially there is a 6 volt that is launched so at the source side if you were to draw again normalizing t with respect to T D. So this is V at Z is equal to 0 which is the source side voltage that we have to write.

Immediately as the switch is closed a 6 volts is launched thereafter at 2 millisecond that is after 2 time delay the voltage actually changes over from 6 to 6 + 2 – 2 by 3 which is 8 – 2 by 3. So now it would actually changeover to 8 – 2 by 3 so you know whatever that voltage is right. That would be the voltage here and then it would again change over right. So I do not whether it is - or + does not matter but this is how you would actually calculate okay.

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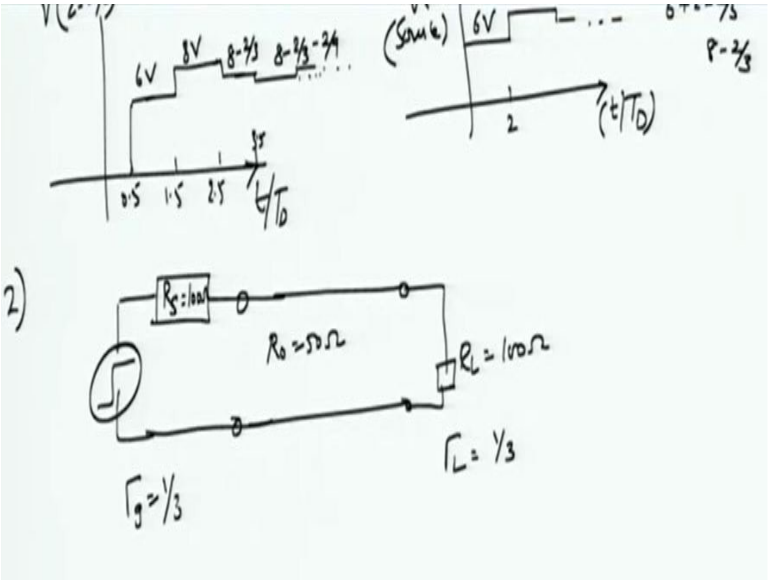

So this is one of the circuits which I wanted to show you how to do latest analysis or bounce diagram. For completeness sake let me also consider an case where I have a step voltage waveform okay connected through an internal impedance of say R S is equal to 100 ohms and then I have a transmission line which is R 0 of 50 ohms and then I also have R L which is 100 ohms. So in this case clearly gamma L happens to be 1 by 3 gamma g also happens to be 1 by 3. Here the voltages seem to be having only positive reflection coefficient.

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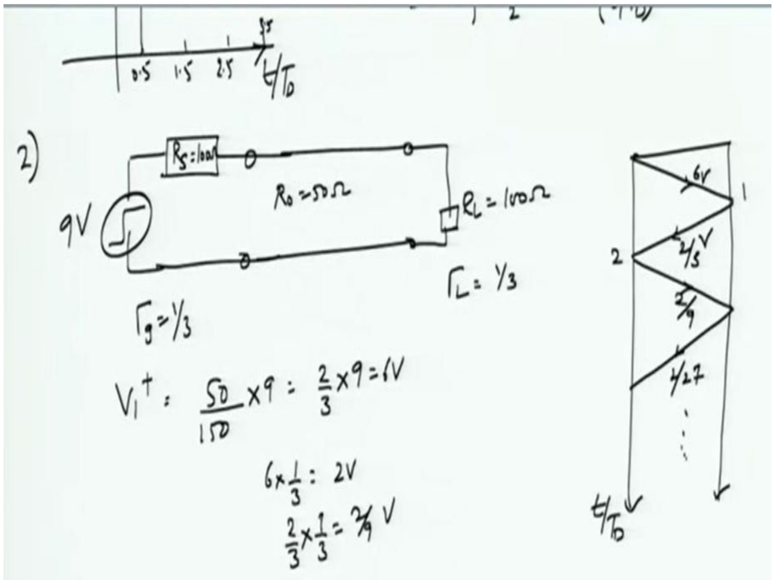

And that shows up when you draw the bounce diagram. So you draw the bounce diagram again by normalizing t with respect to T D okay. Again assume that this is 9 volts okay so with 9 volts the initial voltage that is launched now of course be different. The initial voltage launched V 1+ will be 50 by 50 multiplied by 9 volts not really so this is 2 by 3 multiplied by 9 so this is 6 volts launched again.

So 6 volt goes through okay at t is equal to 1 or t by T D is equal to 1 it reaches here. At this point, 6 multiplied by 1 by 3 which is 2 volt launched back so this is 2 by 3 volts. At this point again so this is the time t is equal to 2. At this point or at this time again 2 by 3 multiplied by 1 by 3 which is 2 by 9 volts is launched. So this is 2 by 9 and then this again is launched as 2 by 27 and so on okay. Now if you want to write down what is the waveform for load?

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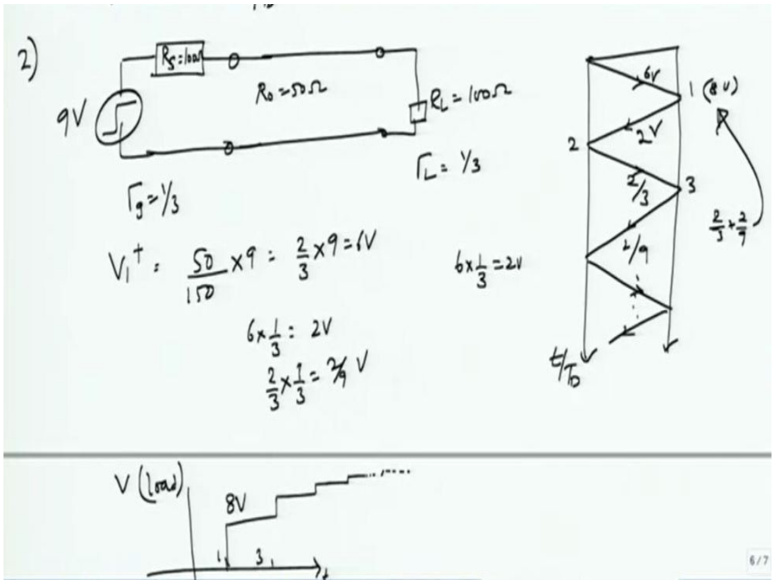

You will see that the load voltage actually goes something like this. At t is equal to 1 time delay the voltage at this point jumps up to 6 volts then at 3 the voltage again jumps so this would be 6 + 2 by 3 okay. So this is 6 volts launched and then 6 by 3. I have made small mistake over here this would actually be 6 multiplied by 1 by 3 which is 2 volts launched here. And then 2 by 3 is launched back again at this point and then from 2 by 3 you launch again 2 by 9 over here.

So this is 2 by 9. Initially, the voltage will be 8 volt because this is 6 + 2 so it is 8 volt that is there at t is equal to 1 then at t is equal to 3 you have 8 volts already so this is already 8 volts + 2 by 3 + 2 by 9 that is the voltage that you need to add to this 8 volts. It would actually jump up okay. Then again once more when you get this reflection thing it would again jump up slowly okay. So this way it would keep on doing until it basically reaches the steady state.

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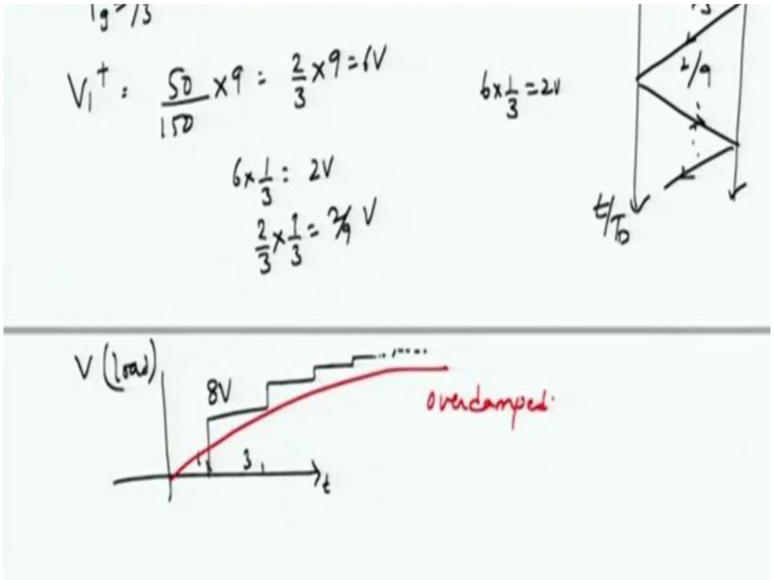

You can see the response difference in that this response keeps on growing until it reaches a steady state as some sort of an over-damped circuit okay. So this is an over-damped circuit okay. Don’t take these words too seriously these are just kind of analogy with control system because the waveforms look like that. This is an under-damped circuit because the waveform was actually oscillatory okay.

To get oscillatory waveforms you either need gamma g to be negative or gamma L to be negative. If both gamma g and gamma L are positive, then you get an over-damped kind of a response. So we will close our discussion of transients on transmission lines with this example. We will talk about capacitive termination of a transmission line in the next module.