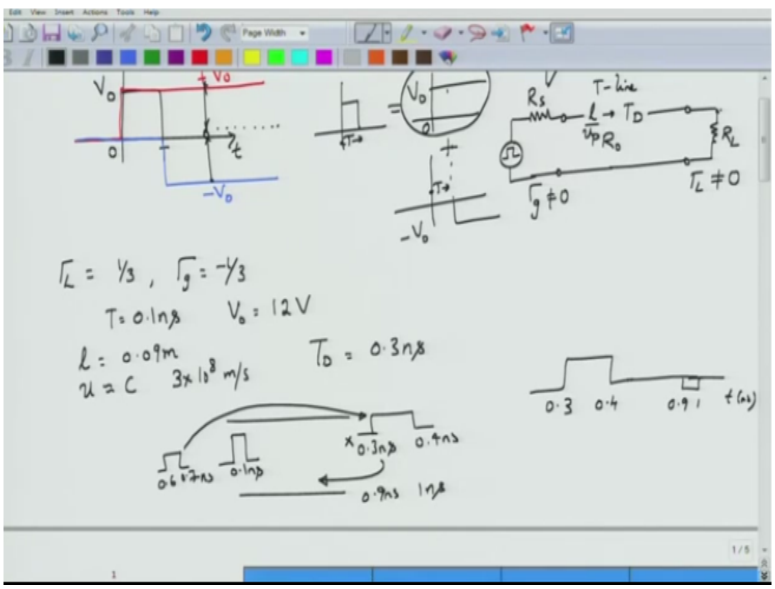
Electromagnetic Theory
Prof. Pradeep Kumar K
Department Electrical Engineering
Indian Institute of Technology – Kanpur

Lecture No - 67
Pulse on Transmission line

In this module we will conclude our study of transmission lines by looking at its response to pulse input and then also consider the capacitive termination which is something that you would find in most transmission line problems. Where the load happens to be not just a pure resistor but combination of resistor and capacitors for this course we will consider only a pure reactive termination such as a capacitor.

So let us begin by first considering the pulse response

(Refer Slide Time: 0:44)



of the transmission line. So, what we actually mean here is that we will launch pulse on transmission lines and then we will try to find out what is the voltage at the load whether the pulse would be reproduced faithfully or whether the pulse actually gets distorted because of the reflection problem, not matching at the load and not matching at the source side. So, the problem that we will consider will involve launching a certain plus.

Let us say, the pulse has a amplitude of V zero and for our convenience seek we can consider this pulse to be starting at zero and being none zero until a duration of T seconds that means the pulse essentially is there for a duration of T seconds and we want to see what happens when such a pulse source is connected to a transmission line. As I said in many digital circuits you have one digital device driving another digital device.

This entire thing in between is what is called as an interconnect and in high speed digital designs this IC interconnect actually has to modeled as a transmission line. This transmission line has a certain length and therefore there is a certain propagation delay T D associated with this transmission line and all though we show and we will be using only resistive loads under resistive sources.

Slightly, more realistic representation of this one would actually be a complex load which will have both the resistor and a capacitor and similarly you will have mostly a capacitor plus some resistor either in series or in parallel being driven. So, the load is not just pure resistor and source resistance is also not just pure resistor. So, there are these reactive elements and how the pulse would actually propagate in this realistic scenario is something that you will have to do it.

When you study this in further detail for this particular module and for this course we will assume that for the pulses to be propagative in the transmission line both the source as well as the load terminations are resistive. So, we do not consider the complication that would arise because of the capacitive reactance also being found at the load or the source side. So, far as the source will be represented by a resistor as some resistance R s and then there is a pulse source which we are connecting.

Then there is a transmission line which basically is assumed to be lossless and has a characteristic impedance of R o or Z o and has a propagation delay. One side propagation delay of T D which is given by l/u and u is the face velocity or u or u p of the face velocity of the waves that we are considering. So, this will be terminated by a certain load resistor R l which again is resumed to be real.

Now, how do we actually start this problem? I mean, we have already seen how the transmission line would carry a step voltage from the source side to the load side, right. So, there would be multiple reflection is involved when gamma l reflection coefficient at the load side is not equal to zero. Or when gamma g is also not equal to zero so you actually have multiple reflections which means that the voltage that you get at the load side would not be a pure step away from but rather it would be away from which would tend to be some value.

It could be oscillatory or it would be damped or it could not be exactly a step away from. In another words, it is actually getting distorted. The wave is getting distorted as it propagates through this particular system. I mean as not propagate but because of the multiple reflections. So, that was for the step response. Now, how are we going to analysis the response or how do we understand the pulse propagation on the transmission line?

Well, you remember from your signals and system courses probably that's what we call them. This pulse source can actually be broken up into two step wave form. How? You have one step which I am representing here by red line which has an amplitude of v o and is switched on at T equal to zero. Then, we also have another step wave form which does not appear until T equal to capital T but then at that point the pulse actually has an amplitude of minus v o. So, this pulse has an amplitude of plus v o.

This negative step has a amplitude of minus V o and it would be turned on at this time T what would happen between zero to T the capital T there is only one step wave form therefore voltage actually is equal to the original pulse voltage. Whereas after T, that is after capital T there is pulse as supposed to be going to zero. It does indeed go to zero if you consider this as a sum of two opposite amplitudes, right.

They are equal amplitude but they are opposite in sign, right. Then you add them up so at every point T greater then capital T what you get is essentially zero. So, if you add this two up what you get here is a zero here and then it would continue all the way. So, the lesson of this one was that a pulse can actually be thought of as sum of two step wave form of equal amplitude and switching on after a certain delay. That delay is the pulse duration T. So, this is a pulse duration T.

Now, this must make our life very easy because we already know how to fall, for one step away from the transmission line and because this problem is essentially linear, the transmission line is linear this can be analysis within super position in the sense that you establish the response of the transmission line circuit because of one step away from which is turned down at T equal to zero and then, we examine the effect of the same circuit for step that is actually starting at T equal to T.

And has amplitude of minus v o and both of these problems can be solved by appropriately modifying lattice diagram. So, let us actually try out on a numerical example over here. So assume that gamma l happens to be 1/3 and gamma g happens to minus 1/3 for some combination of loads that are really important for us or if the load values and the characteristics impedance of the transmission line as well as the generator impedance R s is given.

You can actually calculate these values. So you could calculate and then you have, what you will be using is this equation. So, I have actually given you that equation. So I mean given you the values. Gamma L is 1/3 gamma g is minus 1/3. So, now you can actually imagine even before something would happen that there would be –if the original step wave form is applied then they would be an oscillations happening.

Because in one way you are going then when you come back to the generator side the voltage has to be negative of whatever that has come through. So, there would be certain oscillations and this would correspond to the underdamped situation that we show in the last module. So, let's also assume that the pulse last for about say 0.1 nano seconds in one case and then we will extend that to 0.8 nano seconds in the second case.

So, far as the duration here at this time is 0.1 nano second. The amplitude of the pulse you know is 12 volts these are chosen so that number further calculations are slightly easy. Now, the only thing that you have to understand is what is the time delay of the transmission line? So, for the time delay of the transmission line if it is specified that the length of the transmission line is 0.09 meters.

And u is specified as approximately the value of c free space propagation of light which is 3 into 10 the power eight meter per second then you can calculate the one-way propagation D L T D as 0.3 nano seconds. Now, this is interesting the pulse itself is only 0.1 nano seconds. The transmission line has a delay of 0.3 nano seconds. What would you think would happen spend few minutes on this problem thinking over what would happen?

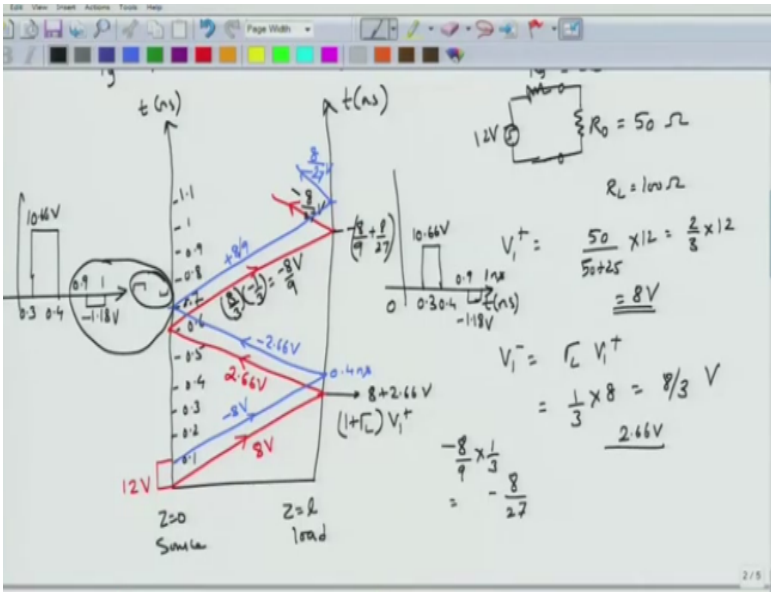
The essential idea is that you are launching a pulse whose duration happens to much smaller than the propagation delay. So, this pulse would propagate and arrive at the load side at 0.3 nano seconds after the pulse. So, the entire pulse is available from 0.3. So, since the duration of this one is 0.1 nano seconds the pulse is available from 0.3 to 0.4 nano second at the load side at which point it further get reflected and would arrive back at the source at 0.6 nano seconds and last until 0.7 nano seconds.

So, between that there is nothing coming in here because the pulse is already completely come to the load side. So, the load would get exerted only from 0.3 to 0.4 nano second and then it will wait for the next bit of reflected plus to come through. The next bit of reflected plus will come at 0.6 + 0.3 which is 0.9 nano seconds and last until 1 nano seconds because 1 nano second is the duration. It last until 0.9.

So at this point you might imagine because there is a negative thing out there and there is this one. The voltage that would could come back would have a value that would be lesser compared to the original one. So, you would actually have something that would look like having a negative value here because gamma g is negative in this case. If gamma g were to be positive, then this pulse amplitude would be positive.

So, this pulse would start at 0.9 and last until 1 nano second. So, if you calibrate all your time access in terms of nano seconds then this is how you would expect even before being any of calculations this is how you would expect the voltage to appear at the load side. Let us actually verify that this conclusion is correct by drawing the lattice diagram. So to draw the lattice diagram remember what you need to do

(Refer Slide Time: 11:08)



This z equal to zero and this is z equal to l this is the load side and this is the source or the generator or the driving side. The time axis is still along this one. All calibrated in terms of nano seconds. Now, let me first write down this point 0.2 let me write down the scale here. Otherwise, I will not be able to draw this one fit all the time or at least the time that is significant for us this one. So you have 0.8, 0.9 and 1.

So this is probably sufficient for us if you want you can also go to 1.1 but this is probably sufficient for us because we do expect by our previous understanding that you will have pulses one at 0.3 and one at 0.9 and lasting until 1 second. Thereafter, there will something else at 1.6 but that is way long time that we need not look at that part. So, let's also write down what is gamma l, gamma l is 1/3. Let's also write down what is gamma g? Gamma g is minus 1/3.

Now, let us launch a pulse but you know that between times from 0 to 0.1 we actually have a pulse of amplitude 12 volt. But that 12 volt will not be the one that is actually launched on the transmission line. So, you imagine that there is a step way from here which will then continue to be there and then at this point this will be the negative step. So, you simply propagate this one. So, there would be two steps that you need to propagate one with the positive value.

The other one with the negative value I will try to write down both of them with two different colors so that we can actually see that one. What is the initial pulse sample that is launched? Well, the load does not yet appear at the transmission line at the source side when you close and apply the switch I mean apply the pulse. So, initially it is only this R 0 of the transmission line that is relevant to us. Then, there is certain R g over here and then a 12-volt step is applied.

So, what is R 0 let us assume that or let us look at this one R 0 is given to be 50 ohms from which you can actually calculate what is R g or you do not have to calculate that one that would be 25 ohms. So, here we do not have a match over here and you can actually see that gamma g will be minus 1/3. Similarly, R l is 100 ohms. You could have obtained this value from knowing what is gamma l. All right so now let us launch 1 pulse.

What is the initial pulse amplitude that we need to launch? So, V 1 plus will be divided between the R 0 and R j and voltage appearing across R 0 is 50/50 + 25 into 12. This is nothing but 2/3 of 12 and this is 8 volts. So, initially an 8-volt step wave form is launched on the transmission line and it begins to propagate and it will reach the load end at 0.3 nano seconds. So, here you have 8-volt pulse that is being launched on the transmission line and arriving at 0.3.

As soon as this step arrives there would be a reflection of this step. So, the step (()) (14:19) value gets reflected and what would be the reflected value? Reflected value is V 1 minus which is again gamma L which is the reflection coefficient here times whatever the initial value V 1 plus that we obtained gamma L happens to be 1/3 V 1 plus c g 8 therefore this is 8/3 volts that gets reflected. So this 8/3 volts is approximately 2.66 volts.

You can use the calculator the check whether this value is correct or not. So once you have done that one now you begin to propagate this reflected value. So, what is the value that is getting reflected and when will it arrive? It will arrive at the source sided 0.6 nano seconds. So, this is how the reflected wave would travel. Now at this point immediately the waves get reflected and it would arrive at 0.9 nano seconds. What is the value that is reflected here? So, this is 2.66 volts or 8/3 volts and the value that goes over here will be 8/3 into minus 1/3 which is minus 8/9 volts.

So, it is actually negative and it is very, very small. It is a very small number. So, it is minus 8/9 volts. So, I hope I got all the numbers correct over here and again once you get value here it gets again reflected but we do not have to calculate that one. The amount that gets reflected I mean even have to propagate this one but the amount that gets reflected is minus 8/9 into 1/3 which is minus 8/27. So, this is minus 8/27 volts. This is how the first step would behave.

Now, this is not the end of the story we need the second step and the second step begins at 0.1 nano seconds but it has an amplitude of minus 12 volt. So, all you have to do is basically change this value from plus to minus as we propagate the second step. So, pick a different color and then when will this arrive the step, negative steps starting at 0.1 nano seconds would arrive at 0.4 nano seconds.

So, it would arrive at 0.4 nano seconds and what is the value minus 8 volts gets reflected. So, what would be the value minus 8 into 1/3 which is minus 2.66 where would this arise? So, this was at 0.4 nano seconds. So, 0.4 + 0.3 is 0.7 so you should arrive at this point and at this point the reflected voltage is minus 2.66 volts again the voltage gets reflected and it would arrive at 1 nano second and then get further reflected.

So, what would be the value here it is minus 8/3 multiplied by minus 1/3 that would be plus 8/9. So, this would be plus 8/9 and then this fellow would be 8/27 volts. It is actually comforting for us because this step would continue to have a value of minus 8/7 for all the time but this step will have a value of 8/27. Therefore, after 1 nano second the sum of these two will be equal to zero. Now, if you were to draw the voltage away from at the load side this is what you are going to observe.

So, let us call this as 0.3 nano second and it would be 0.4 here. This is zero so let me calibrate T in terms of nano seconds. What would be the pulse value? This is 8 + 2.66. So, this is the value over here 8 + 2.66 volts which is 10.66 volts. Because you remember the voltage that is actually seen at the load is 1+ v gamma l times V 1 plus. So, there is some of incidence as well as reflected voltage and that happens to be 8 + 2.66 which starts at 0.3.

And actually it would have continued like this. But unfortunately this minus 8, minus 2.66 which happens to be minus 10.66 volts begins at 0.4 and therefore that would be zero. So, in another words this step response actually would have continued like this until the next reflection did happen. But the step response because of this negative input would start and have an equal amplitude there by canceling each other and living you with the pulse which again last only 0.1 nano second and begins and ends at 0.3 and 0.4 nano seconds with an amplitude of 10.66 volts.

Now, when does the next pulse arrive? The next pulse would arrive at 0.9 nano seconds at this point what would be the value of this one that would be minus 8/9 plus 8/27 actually this is in brackets. So, we actually have 8/9 which is around less than 1. So, if it was 9 then it would have been 1, it is about 0.9 something plus there is some other value that is here. So, if you add them together what you get is about minus 1.18 volts.

So you will get minus 1.18 volts and that would be negative here. So, it would start at so this 0.4 and then this would start at 0.6 and this is at 0.9 nano second and then last at until 0.1 nano second. So, this would start at 1 nano second and this would have a value of minus 1.18 volt. So let me redraw that one over here just clear up all the confusion. This pulse amplitude is 10.66 starts from 0.3 ends at 0.4 then the next pulse which is because of the reflection.

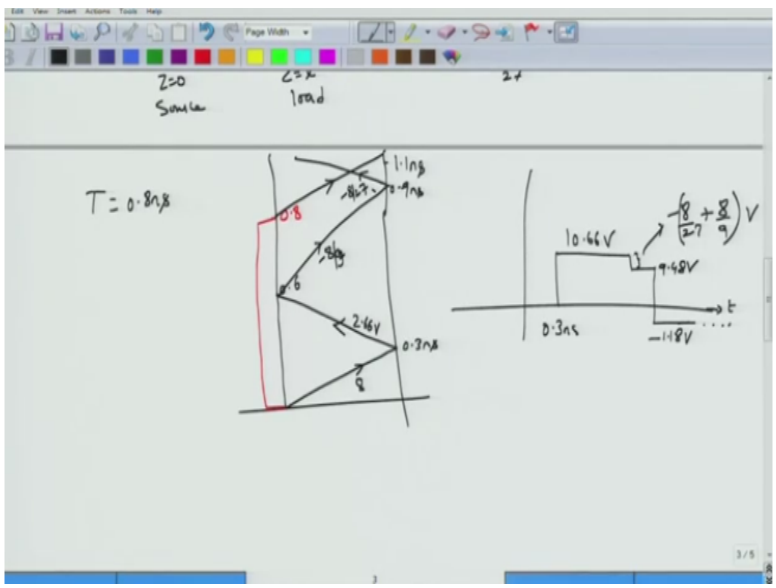
And due to the mismatch between source transmission line and the other one you are going to get another pulse at 0.9 nano second and lasting until 1 nano second and this value of minus 1.18 volts. Next, time you can actually calculate when you will get the next pulse and that pulse would come at a later time then it would have a small positive value and again you will get a negative value and eventually this transient would try to go to zero.

I mean they would go to zero exactly at T equal to infinity. But if you wait for about 10 or 15 propagation delay times then all the secondary pulses would go to zero. Now, one of the things which I want to emphasize here is that this kind of a situation is actually very bad. It is like one of the problems is that they are the secondary pulses. There after pulses are certainly not required.

Because if you want to launch not just one pulse but multiple pulses then you have to wait unit all these transients have died down otherwise they would actually start to interfere. So, because of these pulses that would interfere your actual pulse output would not be the same as what your input it. So, if the CMOS driver that you have connected is driving another CMOS driver assuming that the load impedance and the source impedance can all be modeled as real resistive impedances.

This kind of a situation with the interconnect delay that is quite large would limit the operating speed of the inverter and hence the operating speed of the overall circuit. Something that you do not want to happen. On the other hand, which I will not do that one in this module is that what happens when your transmission line delay is actually less than the pulse duration. So, you can actually see that for yourself I will not do this thing.

(Refer Slide Time: 21:35)



But if you assume T is equal to 0.8 nano second while the one step has gone and come back the next the negative step is still has to begin like you see here this is your diagram. So, if we go up to 0.3 nano second with the first step. The pulse itself now has to last all the way up to 0.8 nano seconds. So, this is 0.3 so the first step goes 0.3 then it will begin to come back and it will line here at 0.6 nano seconds.

So, at 0.6 nano seconds the second step has not even begun to go. It still all these points are going right. So, this is the first step point then the next point would have gone it would arrive here the next one would go. Next one would arrive. Then negative step of value whatever that initial value, whatever the value minus 8 volt would not even begin to propagate until this 0.8 nano seconds. So, at point nano seconds it would begin to propagate but what would be the time it would take to arrive at here 0.8 + 0.3 is 1.1 nano seconds.

So, this is the time at which the initial pulse which is lasting longer then the propagation delay would take to arrive. Here is where the cancellation starts to happen but before that these case would also come back so 0.6 + 0.3 would be 0.9 and again there is something else here. So, the actual pulse output would look something like this. So, initially you have a value of 8 plus whatever that 2.66.

So that would be 10.66 and this 10.66 which beings at 0.3 nano seconds would last until. So, this is 0.3 nano seconds let me draw remove all the other lines that are not required. So, you have 0.3 nano seconds this is 0.6 nano seconds and then 0.9 nano seconds. So, at 0.9 nano seconds you will have this is 10.66 because this is 8. This is 2.66, this is exactly the same things as we have calculated earlier and this is minus 8 by you know what is this value which we found.

This is about minus 8/9 and then this fellow is minus 8/27. So, at 0.9 nano seconds you actually have to add up the value is coming to you minus 8/9 and minus 8/27 and that would be the value to which this fellow would have to decrease.

So, this decrease that would happen is actually about minus 8/27 + 8/9. So, this is the voltage to which this should decrease until you hit the next negative pulse until you hit this 1.1 nano second where the negative step would begin to appear. So, once the negative step beings to appear you will actually see that this voltage. This is 9.48 volts and this voltage is minus 1.18 volt and this would be the way in which the pulse would propagate.

In some sense, this is much milder then what you have but unfortunately the pulse duration has increased sufficiently. The original pulse actually lasted the pulse duration is all right. But what would not be all right is the small distortion that would occur. Of course, the real problem with this one is that you are trying to actually limit the speed because this is 0.8 nano seconds. So, its frequency gets limited you cannot drive many more pulses because the pulse duration itself is quite large compared to the propagation delay.

So, this is all that I would like to talk about the pulse response. Next we will look at what happens when there is a capacitive termination at the load and we will begin from that one. Thank you.