Electromagnetic Theory
Prof. Pradeep Kumar K
Department Electrical Engineering
Indian Institute of Technology – Kanpur

Lecture No - 06
Capacitive termination in Transmission line

In this module, we will discuss one last aspect of transmission lines and we will conclude with that.

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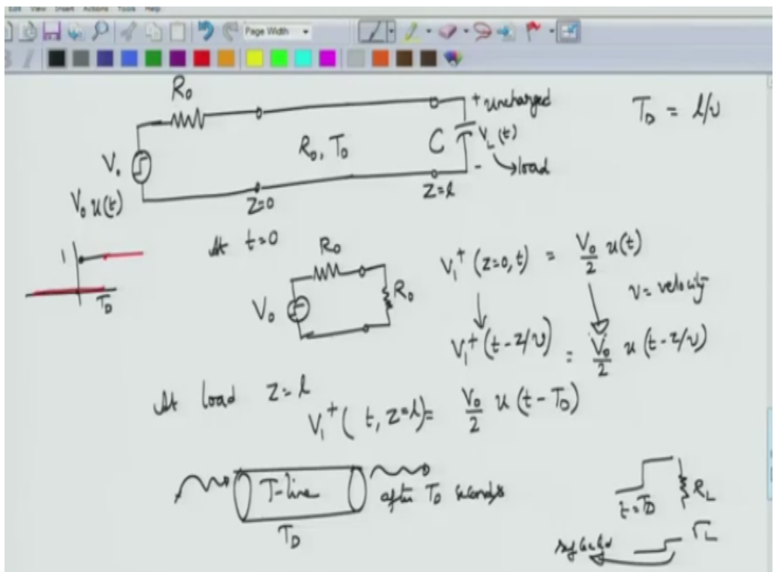

This is a situation where the driver connecting the load and via a termination line is actually driving a capacity load something that you would see when a CMOS is pulling up or pulling down and it is driving one more CMOS which will of course pull down or pull up in response to the initial CMOS invertor. And on a printed circuit board or on a computer motherboard you would actually see lot of these circuits.

Because they actually form a lot of high speed digital circuit modules. So, you will see lot of this situation where one driver is driving not a resistor but a capacitive load because their CMOS invertor can actually be modeled much better as a capacitor then as a resistor in the pull up and pull down times. So, when you have such a capacitive termination then the bounce diagram thing that we discussed in the last module does not really work to understand how the incident.

And reflected voltages would behave on the transmission line. So, one needs to go back to the differential equations at least in the simple context and then try to obtain insight into the behavior of reflected voltages. So, we will actually consider that one. Once you understand capacitive termination it is actually fairly simple matter for you to understand other types of termination as well inductive termination or possibly a combination of resistor and capacitors.

All you have to do is to write appropriate differential equations. So, what is the situation that we have we will assume one thing. Because it would simplify our analysis we will assume that somehow we have been able to match the generator with the transmission line. So, I have the transmission line being again lossless and having a characteristic impedance of R 0.

But on the source side or on the driver side that we actually managed to match it to the source impedance then I again have a switching waveform which would be represented by step connected to this one with an amplitude of V 0 connected at T equal to zero. So, you can actually represent this one by actually writing this as V 0 u of t, where u of t is a wave form which would be zero for time t equal to zero.

Then it would eventually begin to V equal to 1 for T greater than or equal to zero. So, it is discontinuity represent the switching action and that is how we model this switching action by this mathematical function u of t. Now you have a transmission line which will have a certain propagation T d or equivalently a velocity u, we will not even talk about velocity. Let us just simply talk about the propagation delay over here.

Because the concept of face velocity is not really, really valid in the nice way for this scenario. So at the load side I have a capacitor which let us assume that initially is uncharged. So I have an uncharged capacitor with the value of C and the voltage across the capacitor that I am looking at let me write this as V L of t. This L stands for load. So this is the situation that I have. Now one thing does not really change, what does not change is that?

At t equal to zero when you switch on this step input to the transmission line there is the generator side still does not enough time for it to see the capacitor the capacitor becomes visible only after two propagation delay times. So only after some deflection happens and it comes back then it would see the effect of capacitor out there. How the reflected wave form comes back is something that we are going to discuss.

So at t equal to zero the equivalent circuit would still be something like this. So it would have R 0 the transmission line impedance is R 0 and the step away from the value of the step away from basically gets split between these two. Since we have matched the generator and the transmission line the initial voltage that is launched at z equal to 0 which happens to be the generator side for all time t will be equal to V 0/2 u of t.

So this is the way from that would be transmitted or propagation until the load side. So this is my z equal to 0 source side. This is z equal to load side. How would this wave travel? We already know that a wave that is launched on a transmission line would travel like V 1 + t minus z/v. v stands for velocity. So this expression is still valid because this is something that we show in the transient analysis that any wave form which has this particular form corresponds to a propagating wave along the positive direction.

And step wave is also a wave which is propagating along the transmission line. So clearly writing from this expression we know that this nothing but v 0/2 amplitude and u of t minus z/v. Now at the load side what happens? Now at load side Z equal to l. So the initial wave which you have launched would now start to arrive at the load side that would be l/v but we already know that the propagation delay T d is given by l/v.

Therefore, you can write this as t minus t d the voltage that is appearing at the load side, as z equal to l is actually equal to v 0/2 u of t minus l/v so let us actually write down this one as v of t, z so z equal to l and then this is u of t minus T d. What is this expression u of t minus T d? What would that correspond to? It is exactly the same unit step except that it is now delayed by T D units. So this would be the unit step which is delayed and then begins to go up at T D.

So this is the wave form that is appearing at the load side and it actually makes sense because you have a transmission line. So you have a transmission line whose propagation delay was T D so you launched something over here. This thing would come at the load side after T D seconds. So after T D seconds this initial whatever the value that would start to appear at the output of the transmission line.

So this is actually the wave that is going into the uncharged capacitor. The way to analysis this one after this becomes very difficult because now the capacitor has not simply returned a replica of the input. So if it was a resistor load what would have happened? You have a step input which is appearing at T equal to T D and part of this one depending on what the value of gamma l would have been reflected.

So if gamma l is positive, so let us say this is the part that is actually getting reflected and would propagate backwards. But now this is not the case, this is not a resistor. It would not simply reflect the replica of the signal that is incident. But it would change the wave shape because the capacitor voltage cannot change instantaneously. So the capacitor does not show the same kind of behavior as that of a resistor.

The reason for all this has to do really with the frequency behavior of these capacitors. The step input can be analyzed using Fourier transform as consisting of an infinite numbers of sinusoids and then the capacitors reactants depends on the frequency. So depending on different frequency that are incident which happens to be a step away from their responses of the capacitor would be different and when you apply Fourier synthesis you would see that the way form is not actually not exactly a replica of the input.

So, this is what would happen and therefore we do not have this kind of a simple behavior when you a capacitive or inductive or a combination of these type of loads. So instead what you should do? You should actually use Thevenin equivalent circuit. What is Thevenin Equivalent?

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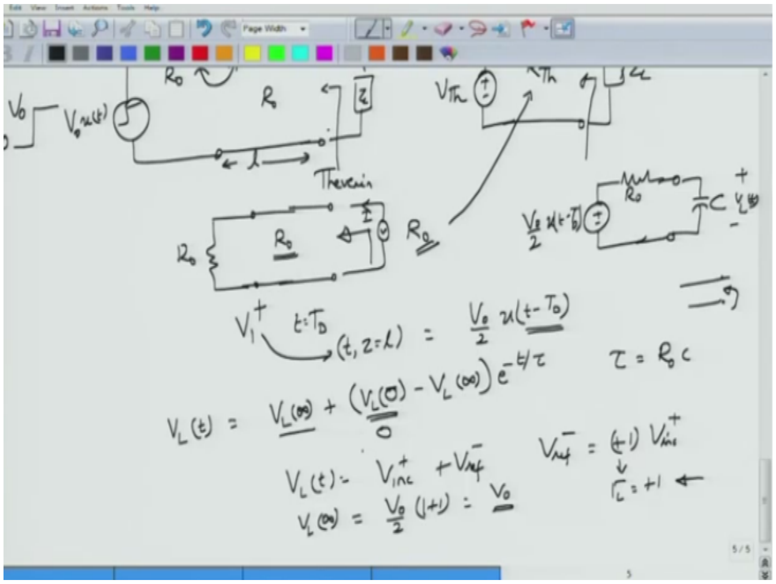

Well, I have this source side R 0 with this V 0 u of t at the switching way form and then I have a transmission line which also has a characteristic impedance R 0 and then I have this load. So, in this particular case the load happens to be a capacitor it could be any capacitor. What I want to do is, I want to find the Thevenin Equivalent at this point. What would the Thevenin Equivalent look like?

Well a Thevenin Equivalent will consist of the Thevenin Equivalent voltage and a Thevenin resistance. So, this was what it would look and then you have the load connected to this Thevenin Equivalent circuit. How do I find out R Th? Well, all I have to do is terminate this V Th in another words make V Th equal to zero and whatever the impedance that I would be seeing will be the impedance that would be the Thevenin impedance right.

So, I would actually be able to obtain the Thevenin impedance - So, go back to the original circuit if I turn off this V O if I make this V O source go away what would be the equivalent circuit? There would be a matched impedance R 0 and then there is a transmission line which also has R 0. What is the resistance looking into this one, no matter what the length of the transmission line is the resistance seen or the impedance or the resistance seen looking into this output terminals of the transmission line which is terminated by R 0 is actually equal to R 0.

If you do not believe me you can actually show this one by driving a certain voltage and then observing what would be the current and then taking the ratio of the voltage to current which would turn out to be equal to R 0 or you can use your intuition from transmission line. A transmission line which is terminated in its characteristic impedance is actually equivalent of an infinite length transmission line.

Whose impedance would always be equal to R 0. The point here is now that R Th is actually equal to R 0. What would be the Thevenin voltage? If you want to find the open circuit voltage here, now you don't have to terminate the voltage source. If you find the open circuit voltage, the open circuit voltage would be the initial voltage that is launched. It would be V 1 plus value that is coming towards the source, I mean the load side. But when would this V 1 plus appear?

The V 1 plus voltage would appear after a time T equal to T D and we already know what is this voltage, V 1 plus at t z equal to l on the load side. This we already know, this is actually V 0/2 u of t minus T D. So, this is a step voltage which appears at T equal to T D where T D corresponds to one propagation delay. So, in a very simple way what you think of is that when the switch is applied and the voltage changes to say zero volts to some volt v, some volt V 0 that voltage would appear at the load side after one propagation delay.

So, that is what this t minus T D is representing. Therefore, the Thevenin Equivalent of this circuit is fairly simple this is V 0/ 2 amplitude. This V 0/2 amplitude comes because the voltage initially launched will be divided between the transmission and internal impedance R 0. If internal impedance R 0 were not to be there, then V 0 would be launched but when that would lead to further complications because now the source site does not get matched.

So, here you have an amplitude of V 0/2 launched that would arrive at T equal to T D onwards and it could act like a step away from. This is the Thevenin voltage, u of t minus T D and thereafter there is Thevenin resistance R 0. This would be now connected to a capacitor or this would be trying to drive a capacitor whose voltage I am representing as V l of T. Now how they analysis this circuit? This is simple R c circuit.

We know that for a simple R c circuit the voltage across the capacitor is given by the voltage at infinity, right plus the voltage at T equal to zero minus the voltage at infinity times e to the power minus tau and tau is the time constant of the circuit. In this case, what is the time constant of the circuit? This is simply R 0 into C because this is Thevenin resistance. This is R 0 into c is the time constant. What is the value of the capacitor voltage at T equal to zero.

This is zero because the capacitor is initially uncharged. What would be the value of this voltage at T equal to infinity that is what would be the value in the steady state situation? Remember in the steady state, the capacitor becomes open, correct. But when the capacitor becomes open and this is actually a transmission line if you remember the voltage that you see at the load side is actually the sum of incident and so left side this as incidence and reflected voltage.

But reflected voltage for an open circuited situation, in the steady state open circuit situation would be equal to plus one times V incident. Why? Because the reflection coefficient for the open circuit is equal to plus 1. Now you might actually object over here, you might say that well you did not use reflection coefficient value here to calculate the reflected voltage but you are using reflection coefficient over here to calculate.

What is the voltage V L of infinity the steady state voltage? The answer is this when you open circuit the impedance can be thought of as a resistive impedance right for which I can define gamma L and when you have an open circuited load connected through a transmission line the open circuited load or even a short circuited load would actually send back a replica of the input signal as its reflection.

Therefore, because of this reason I can actually use reflection coefficient only for this calculation between the initial switching time and the final steady state while the exact reflected voltage will be defendant on how the capacitor is getting charged. So, gamma L is equal to + 1 simply implies that the load voltage at infinity would be equal to V 0/2 into 1 + 1 which is V 0 itself. This is correct.

Because eventually the transmission line should not play any role if it is loss less it would simply delay everything that is happening. So, if you remove that one a steady state the capacitor is simply connected through this R 0 to the step voltage V 0. The entire V 0 would then appear across the capacitor. So, we have now all the information that is required we know that V L of zero is zero and the capacitor is initially uncharged.

As the capacitor is initially uncharged I also know what is real of V L of infinity, V L of infinity is V 0. Now, I can find out what is the capacitor voltage?

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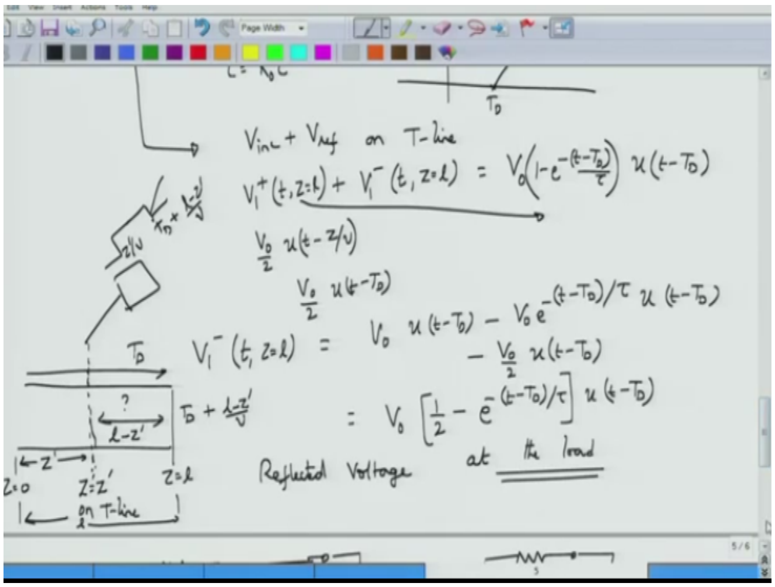

So, capacitor voltage V L of t is given by V L of infinity V 0, V L of zero is zero and tau is R 0 C. So, V L of t the capacitor voltage will be equal to V 0 into 1 minus e power t/ tau. So, this is how the load voltage would look like. So, this would of course start at T equal to T D and therefore you need to account for that one as well. So, this is the load voltage across the capacitor and this is how it would behave.

It would start at T equal to T D and then it would began to rise towards V 0 and it would so with the time constant of tau where tau is equal to R 0 into C. So, this is the capacitor voltage. Now this is not completely what I'm interested. I also need to make a small change here in terms of t, It should be t minus T D. This is just a mathematical requirement that I have to show this t. Otherwise, this equation would not give you the correct values. So, t mine T D will give you the correct value. You can actually check this one.

Before T equal to T D the unit step of function will make the voltage across the capacitor equal to zero that is what you have over here. At T equal to T D, this exponential factor will be equal to one. So, 1 minus 1 is zero and the voltage would be zero because the capacitor voltage is also zero here. So, this is captured nicely by this equation and then eventually it begins to charge. To what value it would charge? It would charge to a value of V 0.

This is all that is there for the capacitor voltage. But there is something that we have not talked about. Will there be reflections? Yes, there will be reflections. How will we calculate the reflection? Well, I know that the load voltage is actually because of the KVL is nothing but incident plus deflected voltage on the transmission line. Because you have a transmission line so this is the transmission line voltage let us say V line.

And this is the load voltage let us say V L and these two must be equal to each other at Z equal to l. So, what is the incident voltage? It is V 1 + t minus z/v and at z equal to L you can substitute this one and the reflected voltage, let us call this as V 1 minus and then just leave it that point. So, we will just call this as V 1 minus and say t at z equal to l and similarly for V 1 plus, this one we can write at V 1 plus of T at Z equal to L.

So, this must be equal to V 0 1 minus e power minus t minus T D divided by tau into u of t minus T D. So, this is the incident voltage and this is the reflected voltage and the sum of this two must be equal to the line voltage or the total voltage. We already know what is V 1 plus of t at z equal to l. We already know what is V1 plus of t at z? Which is nothing but V 0/2 u of t minus z/v. So, at z equal to l this would be V 0/ 2 into u of T minus T D.

So, what would happen to V 1 minus? V 1 minus at t equal to z equal to l on the load side will be equal to V 0 u of t minus T D minus V 0 e power minus t minus T D divided by tau into u of t minus T D minus V 0/2. This is coming from the V 1 plus voltage which has been taken to the right hand side. So, this would be u of t minus T D and what you see is that this V 0 and V 0 might two will together give you V 0/2 and there is V 0 of something.

So, I can actually write this as V 0 into 1/2 minus e power minus t minus T D by tau and u of t minus T D. Of course, this is the reflected voltage right at the load. This is the reflected voltage at the load but I also want us know how the reflected voltage would propagate backwards. How would it propagate towards the force side? You can actually easily obtain that one. So, let us say this is the load z equal to l.

And then considering some other point along the transmission line which is z equal to z prime. This is on the transmission line. At what time will the reflected voltage appear here? Remember the incident voltage took T D time to go all the way from the source side to the load side from there will be an additional time lag. How much would be the additional time lag? What is this distanced?

This distance is nothing but l minus z prime because z equal to zero is this one. So, z equal to zero and z equal to l this is the total length l here and this is z equal to z prime, right. This is z prime. So, this fellow must be equal to l minus z prime. So, the reflected wave begins at T D and takes some additional time of l minus z prime/v in order to appear at this plane Z equal to z prime.

So, if you hook up an oscilloscope you see a voltage initially at z prime/v. So if you cup an oscilloscope over here initially you would see something at z prime by v and there after a change in the voltage at T D plus so at T D plus l minus z prime/ v is where you see the change voltage. So, to just account for that one I can simply all these time units appropriately and what I obtain is this expression

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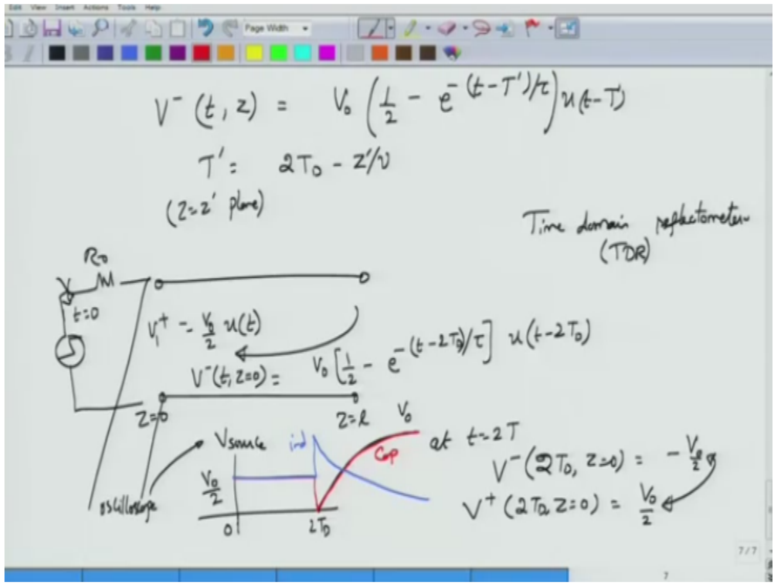

Which is v minus at t for any value of z will be equal to V 0 into 1/2 minus e to the power minus t minus t prime. It is the time at which the plane, the oscilloscope at z equal to z prime sees a change in the voltage divided by tau. So, nothing has changed over her except that the time is not exactly the same. So, T prime is actually equal to two times T D minus z prime/v. You can see that this is correct.

Because initially I assumed that z prime is equal to l that is at the load side you are in. So, for z prime is equal to l this would correspond to having the time T prime is equal to T D itself and this is correct because the reflected voltage will begin at T equal to T D. So, this is okay. So, consider now z equal to zero which happens to be the source side. So, for source side what you get as T prime that is the point at which the voltage begins to change that would be 2T D.

And this is also correct because it takes one T D to go here and again one more T D to come back therefore the change or the total propagation distance would be 2 times T D. So, this is what the change would happen and this change would begin to show up as the reflected voltage arrives starting from T equal to 2 D. So, let us also write down the v of t minus T prime as well. Just to show you that T prime is also over here. The unit step away form.

So, T prime is the time at which voltage changes at z equal to z prime plane. So, the time at which this would change would be equal to 2 T D minus z prime/v. Now, look at what happens as the reflected voltage comes back. What is the incident voltage? The incident voltage is still v 1 plus and it is a nice way from which is going as step voltage at starts at T equal to zero and goes with an amplitude of V 0 /2.

So let us write down this one down here to say that this is z equal to zero. This is the load side, z equal to l. So, if you are looking at the voltage by connecting an oscilloscope to this one while you still have a certain R 0 and a pulse source or a step source connected you can actually hook up these two needs to an oscilloscope and keep looking at what you see here. Initially, as you turn on the voltage source at T equal to zero you would see that the voltage begins to change.

And then it becomes V 0/2 because this is actually equal to the voltage that is launched on the transmission line. So, this is the voltage at z equal to zero launched on the transmission line. So, the incident voltage launched V 1 plus. Now what happens, as the reflected voltage arrives backwards, right. So, the reflected voltage arrives at so V 1 minus, in this case you do not have to write down one because this is only one reflection that is going to happen.

Because the source is matched everything that is reflected would be absorbed. So, V minus t at z equal to zero from this above expression is given by V 0 1/2 minus e power minus t minus 2 T D. Because this is what the value is that prime is equal to zero divided by tau U of t minus 2 T D. So, until 2 times the propagation delay nothing changes on the source wave form or the wave form that you have connected to the oscilloscope over here.

However, at T equal to 2 D something happens. So, what would happen? At T equal to 2 D this exponential value will be equal to1. So, at t equal to 2 T the reflected voltage V minus at 2 T at the source side Z equal to zero will actually be equal to minus V 0/2. At the same time, your V plus at 2 D Z equal to zero is actually equal to V 0/2 because it is the continuous step that is happening.

So, what would be the total voltage at T equal to 2 D as the reflected voltage comes back to us the source. This voltage actually dropped down the zero. Thereafter, this step is still continuing but there is also this negative voltage I mean this also the charging voltage because of the reflected wave form. You remember how the reflected wave form was going on. It would be this particular wave form. So now it would begin to rise and to what value would it go over?

Well, as T minus T prime becomes very large compared to infinity this exponential goes to zero and this reflected voltage will be equal to V 0/2. There is also an incident V 0/2 voltage still continuing from the initial value and these voltage beings and goes towards—exponentially V 0. So, let me highlight this particular graph by sorry one second. So, I highlight this graph by showing you that the voltage for a purely capacitive terminated would be equal to zero at T equal to two T D and eventually rises towards V 0.

In fact, if you were to connect this to an oscilloscope and observer this wave form you can clearly tell that the load actually has a capacitive reactance and this principle is used in what is called as time domain reflectometer. In the time domain reflectometer what you do is you launch a step or a pulse more often, reflectometer or TDR. In TDR, what you do is launch a pulse or a step voltage which could travel.

And because of the faults that are located along the transmission line which could be capacitive or inductive or anything there will be reflection from that. You can estimate the time by looking at what propagation delay has elapsed. So, from the time you can go back and calculate what is the length at which the fault occurs? And by looking at the shape of the wave form you will be able to see whether this is a resistive fault or a capacitive fault or an inductive fault.

Just to complete this one the inductive fault would look like this it would actually jump up to a value of V 0 and then eventually starts to drop towards zero. So this would be the inductive and this is the capacitive and these are widely used to calculate or to locate faults much like a radar.