Electromagnetic Theory
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Lecture - 74
Waveguide: Wavelength, Impedance and Power Calculation

In this module, we will begin our discussion on some of the aspects of waveguides and kind of finish this topic of waveguides.

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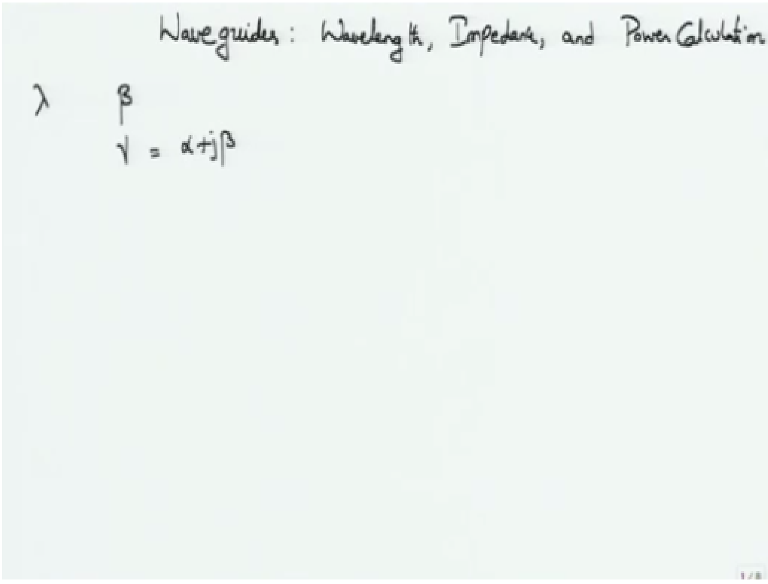

We will discuss wavelength, impedance, power calculation and if you know the module permits, we will also talk about attenuation inside a waveguide. Well, we have been talking about waveguide and examining its frequency for different modes, we obtained expressions as also for that one. Sometimes, it is quite interesting to actually look at the wavelength aspects of the waveguide.

You know in some high frequency such as say 20, 30 gigahertz are above or the 60 gigahertz waveguides, instead of quoting the frequency, it is sometimes convenient to quote in terms of wave length and because of that let us look at how wavelength and frequencies are related inside this waveguide okay. We already know that wavelength has to be defined as in terms of the phase of the distance between or the distance between two points which have the same phase right. That is the basic definition of a wavelength.

We also know that this wavelength is related to a parameter beta and we need to find out what is beta. We have already seen the relation for beta and gamma. Gamma is equal to alpha plus j beta, which would essentially be present for any mode if the mode is lossy or if the frequency is less than the cut off frequency, then alpha would be the dominant factor, beta would not be there.

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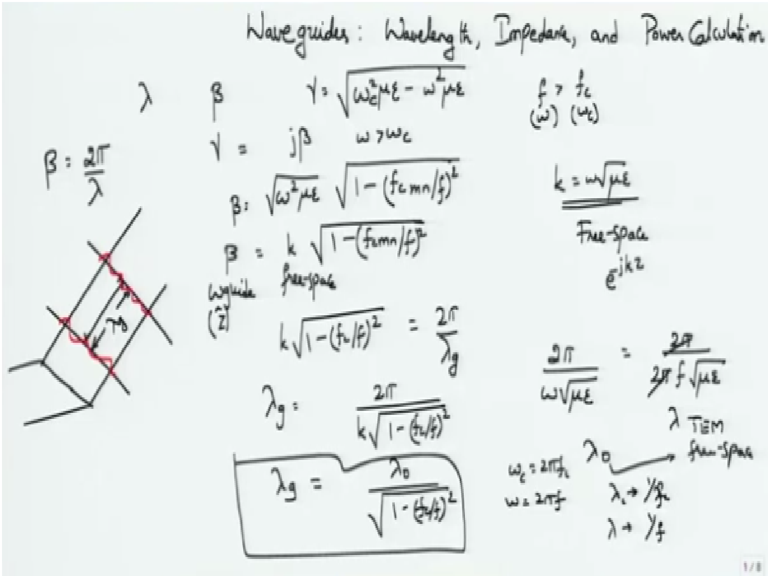

Whereas for the case when the wave is actually propagating, then gamma will be equal to pure propagation constant, right. For that to happen, we need to recall the relation between gamma and omega and that relation is omega square mu epsilon minus omega c square mu epsilon under root. Since, we know that this is gamma and the other around, so this should be omega c square mu epsilon omega square mu epsilon.

So only when the frequency or equivalently omega becomes greater than the cut off frequency fc or equivalently omega c does the second term dominate the first term and convert gamma into a pure imaginary quantity and that imaginary component you know would be the propagation constant beta right. That would describe how the wave is propagating along the z direction and terms of e to the power minus j beta z.

So the way to relate gamma and beta would be when omega is greater than omega c, you can take this omega square mu epsilon outside and then adjust these equations as to obtain the propagation constant beta as omega square mu epsilon under root square root of one minus fc since there would be a cut off frequency for different modes. So let us just write this out as fc mn divided by f whole square.

This is your beta and omega square mu epsilon can be conveniently written as k and therefore square root of that one will become k. So what is k, k is omega square root mu epsilon and this would have been the propagation constant had we considered propagation in free space right. So in free space, you do not have any other aspect to the propagation constant, you just have this k factor and we saw that the fields would go as e power minus jkz right.

So this is how they would have propagated where k would be equal to omega into square root of mu epsilon right. This is a free space propagation. However, in this case you see the propagation constant beta inside the waveguide is related to the free space parameter, but the free space parameter is getting multiplied by some factor right. This factor takes into account that propagation will not happen when f is less than fc, the cut of frequency okay.

So this is the propagation constant for the waveguide, which would be the propagation constant for the free space multiplied by this one. Of course, if you actually take the waveguide and start increasing the dimensions, you know you start expanding a to infinity and b to infinity, you would essentially end up with free space propagation correct, because a going to infinity and be going to infinity will cause the cut off frequency fc to go to zero.

So all modes would be possible and that would correspond to free space propagation, but in practice of course a is never infinity or b is never infinity and therefore this situation does not really arise for us. What is interesting is that, because we have bounded the waveguide, there would a certain cut off frequency and the frequency f has to increase this cut off, I mean has to go beyond this cut off frequency.

We know beta and lambda are related in terms of transmission lines or propagation constant of a transmission line or for the free space we have to calculate this beta and said beta is related is two pi by lambda. Beta is given by two pi by lambda. In this case, can I simply call this as two pi by lambda, unfortunately no.

This beta corresponds to the propagation constant along z direction, right this is the component along z direction, along the propagation direction and along the propagation direction if you find out two planes such that the phase between these two planes is essentially the same. So the phase of this wave here is the same as the phase of the wave component at this point okay.

This distance should be called as lambda g, what is lambda g, it is the distance between two points along the waveguide, so you have to imagine now, right. So this is your waveguide component that I have drawn here. So along the waveguide as you propagate the distance between two components with same phase, equiphase distance would be the guide wavelength, you know wavelength along the direction of propagation or along the guide.

So this beta that we have obtained in terms of k and whatever this one minus fc by f, I am just dropping this mn for notation simplicity, but you should not drop that one. So this fellow should actually be equal to two pi by lambda g okay, which implies that lambda g can be written as two pi divided by k square root of one minus fc by f whole square. But I also know that two pi by k can be rated because two pi by k is nothing but two pi by omega mu epsilon, omega is nothing but two pi f right.

So it would be two pi f times square root of mu epsilon two pi goes away and one by square root of mu epsilon divided by f is nothing but wavelength in free space itself right. So wavelength lambda is the wavelength of the TEM waves, transverse electromagnetic waves or the lambda in free space. Sometimes this lambda in free space is given by a notation lambda zero, zero indicating that this is the free space wavelength.

So in terms of that lambda zero, I can write this expression for guide wavelength lambda g as lambda zero divided by one minus fc by f whole square. Also omega c will be two pi fc, omega will be two pi f right and the ratio of fc by f can also be related to the ratio of lambda because lambda c would be one by fc and lambda itself would be one by f. So you can rewrite this expression in the denominator as well, but you do not want to really do that one.

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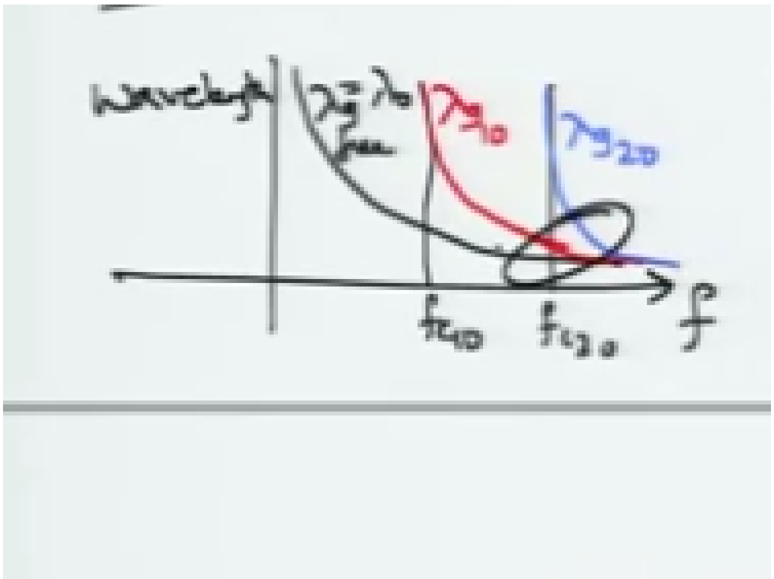

What is interesting is that, the guide wavelength that you have obtained is actually greater than the free space wavelength lambda zero right. So if you want to plot this guide wavelength as a function of frequency right. So if you plot this one as the function of frequency so the wave length in free space this is how it would go right. So f is equal to zero, the wavelength would be around infinity and thereafter it is a one by f kind of a relationship.

The ratio of these two at any given point would give you the free space velocity as well. However, for the waveguide nothing would propagate until you reach the fundamental mode and thereafter you will start getting the higher order modes. So before this wavelength is itself not properly defined and thereafter the wavelength will be defined in a same manner. It would actually start at a particular value at f is equal to fc.

It would be infinity and thereafter it would start to converge like this. As thus frequency increases higher order modes would begin to propagate and you start getting different kinds of lambda g. So this is lambda g of one zero, this is lambda g corresponding to two zero, the next higher order mode and this one would be lambda g corresponding to free space, in which case this would also be equal to lambda zero right.

So for the free space guide wavelength is equal to the free lambda g and because of one by f they would all start to drop and of course at f much larger or f tending to infinity, the wave length all would converge towards one another. This is the guide wavelength okay.

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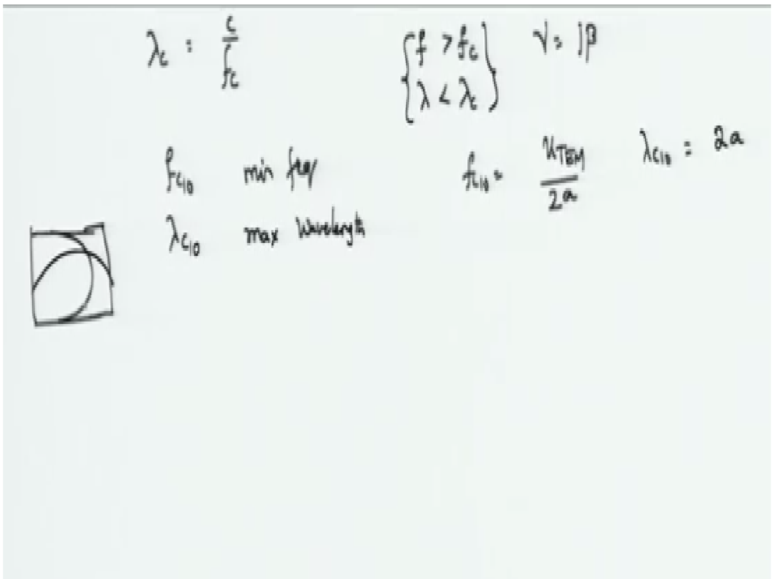

The cut off wavelength can also be obtained as lambda c given by c by fc where c is the free space propagation assuming that the waveguide is filled with air okay or even if the waveguide is filled with something else you can always take this as the definition and what it implies is that for the waveguide to have propagation f must be greater than fc. So that gamma is equal to j beta because lambda and f are inversely related.

The same condition means that lambda must be less than lambda c for the same condition to occur okay. So fc one zero would correspond to the minimum frequency right before which the propagation does not begin corresponding lambda c one zero corresponds to maximum wavelength okay. If the wavelength happens to be less than this, then only the wave would be propagating if the wavelength happens to be greater than this then that particular mode would not propagate.

Since fc one zero is given by UTEM divided by two a, as you can see in the last module that we discussed this one. Lambda c cut off wavelength for the t one zero mode would be given by two a itself. So this is actually the two a condition. At this two a, you will be able to obtain the fundamental mode, which will have one half cycle variation along x or along y walls okay.

So along these walls, you will have one half variation and that half variation comes because you have chosen lambda c as two a right. So this is the maximum lambda that you can have, anything else that would not correspond to a propagating mode.

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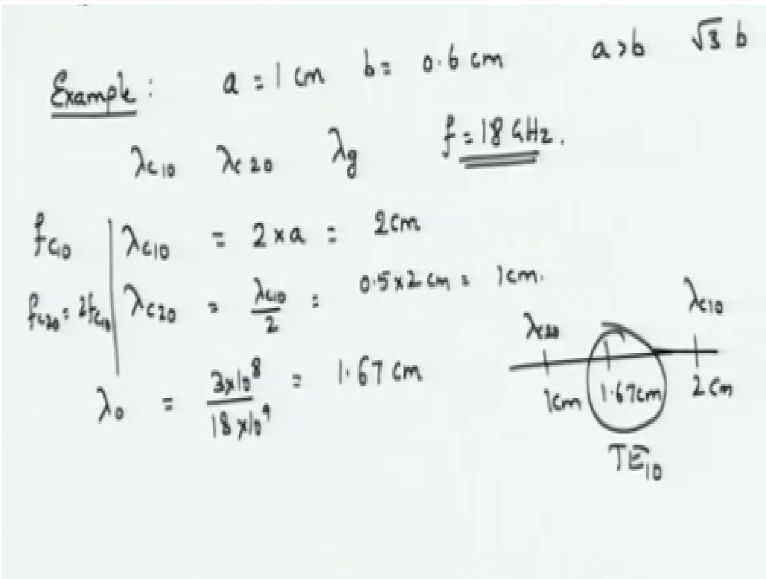

Let us look at a simple example, this example is to illustrate calculation of this quantities, this is not much of an interest in this example other than that, but these calculations are important for you because you need to get some proficiency in calculation of cut off frequencies and cut off wavelength okay. Take a is equal to one centimetre, b is equal to point six centimeter waveguide.

Clearly a is greater than b and we know that the fundamental mode will be t e one zero because a is greater than b and is also greater than square root of three b. We also know that the next higher order mode will be t e two zero. Can you calculate what are the cut off wavelength for one zero, cut off wavelength for two zero and also calculate the guide wavelength lambda g and compare that one to the free space wavelength okay.

Assume that the waveguide itself is operated at a frequency of 18 gigahertz okay. So frequency of the generator that we connect to the waveguide is 18 gigahertz. First question, will this 18 gigahertz correspond to propagating mode or will it be attenuated. To obtain that one, we need to know what is fc one zero okay or equivalently one can try to find out what is lambda c one zero.

Lambda c one zero is equal to two times a, a is one centimeter, therefore this is two centimeter correct. Similarly, lambda c two zero would correspond to because fc two zero is nothing, but two times fc one zero, lambda c two zero would correspond to lambda c one zero by two, which would be point five into two centimeter, which is one centimeter okay. Let us now look at lambda zero, which is the free space wavelength.

If the free space wavelength corresponding to 18 gigahertz happens to be less than two centimeter, but greater than one centimeter, then this would be operating in the t e one zero mode, other than if it is less than one centimeter both modes (()) (12:51) look at that one. Lambda zero is given by c divided by 18 gigahertz, 18 gigahertz is 18 into ten to the power nine. C is nothing but three into ten to the power eight.

And if you do this calculation you will get this as one point six seven centimeter. So clearly one point six seven centimeter happens to be between one and two centimeters. This is lambda c one zero, this is lambda c two zero. So this particular waveguide is operating in the t e one zero mode as we can see and its lambda zero is given by one point six seven zero.

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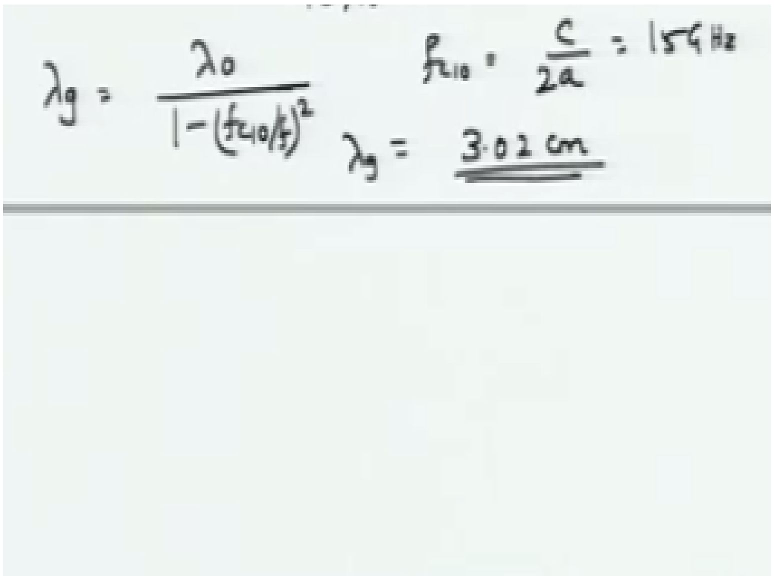

So this is actually a single mode propagation only t e one zero mode is propagating. What is the corresponding waveguide lambda g waveguide length. Lambda g is nothing but lambda zero divided by one minus fc one zero by f whole square right or equivalently one can find the relationship in terms of lambda also, but fc one zero is easy to calculate, this is c by two a which is nothing but fifteen gigahertz in this case okay.

So you can obtain what is lambda g by substituting into these expressions and you would get this as three point zero two centimeter. So you see here the free space wavelength is one point six seven centimeter, the guide wavelength is three point zero two centimeter okay. The waveguide wavelength is significantly longer than the free space wavelength because the waveguide is kind of operating very close to fifteen gigahertz right.

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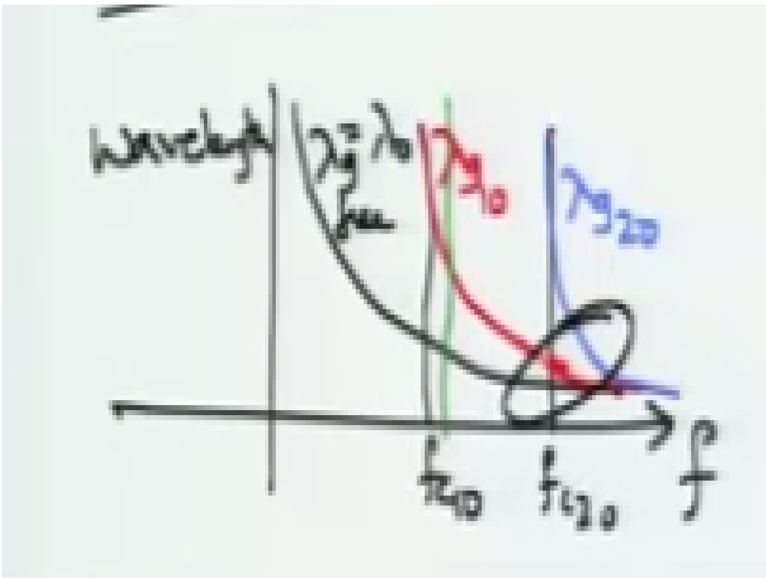

So it is actually operating quite close to the cut off frequency and therefore the waveguide lambda g would be quite large. You can see that one from this picture right. So because your operating very close to the cut off frequency, so you take a look at this one, you are operating very close to the cut off frequency lambda g would be quite large compared to lambda zero. However, if you operate at a higher frequency, then lambda g would correspondingly come close to lambda zero okay.

Now at this point, it is customise to discuss phase velocity and group velocity, but I would like to have a unified discussion of phase velocity and group velocity as a separate module. So I will not introduce that waveguide phase velocity and group velocity in this module. We will calculate that one and we will discuss the significance of this calculation after we have looked at dielectric waveguide okay, a separate module we will be talking about.

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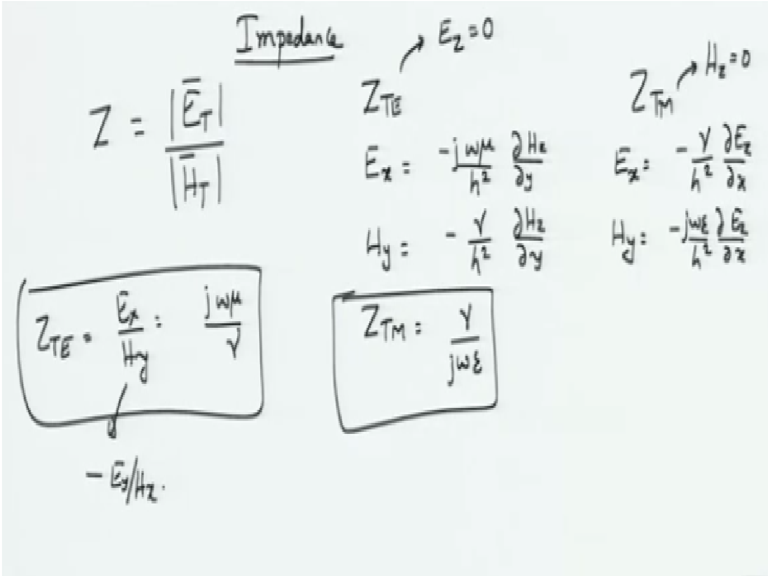

So we will now discuss impedance of the waveguide. For the impedance definition, you remember that for the free space we had defined it as the ratio of transverse electric field component to the transverse magnetic field component and the same expression will be used even for this waveguide as well, you have to define impedance as ratio of magnitude of the transverse electric field component to the magnitude of transverse magnetic component.

Let us try to find impedance for the TE case. For the TE case I know that EZ must be equal to zero and if you go back to the expressions for Ex Ey Hx and Hy that we obtatined in terms of EZ and HZ, you have to go to those equations, which ways no couple of modules are near and then set EZ is equal to zero. So when you set EZ equal to zero, you see that H EX will be equal to minus j omega mu divided by H square del HZ by del y.

You will also find out the corresponding Hy component to be equal to minus gamma by h square del HZ by del y. Now you take the ratio of Ex to Hy and what you find here is that you will see that this minus sign will cancel with each other, del HZ by del y will go away and what you are left with is that j omega mu divided by gamma okay. So this is your Z TE okay and similarly you can find out what would be Z TM.

What would be Z TM, for the TM case HZ must be set equal to zero and you will have corresponding components for EX let us say given by minus gamma by H square del EZ by del X and then you will have Hy, which would be minus j omega epsilon divided by H square del EZ by del x.

Again taking the ratio of these two, so this is Z TE which we want to write down, similarly for ZTM will be equal to the ratio here minus sign again will cancel, H square will cancel, what you get is gamma by j omega epsilon okay. These equations allow you to write down the corresponding transfers components in terms of the impedances. Of course, the ratio of EX to HY must also be the ratio of minus Ey to Hx in both cases because you go back to those equations and you will see that this is exactly the case.

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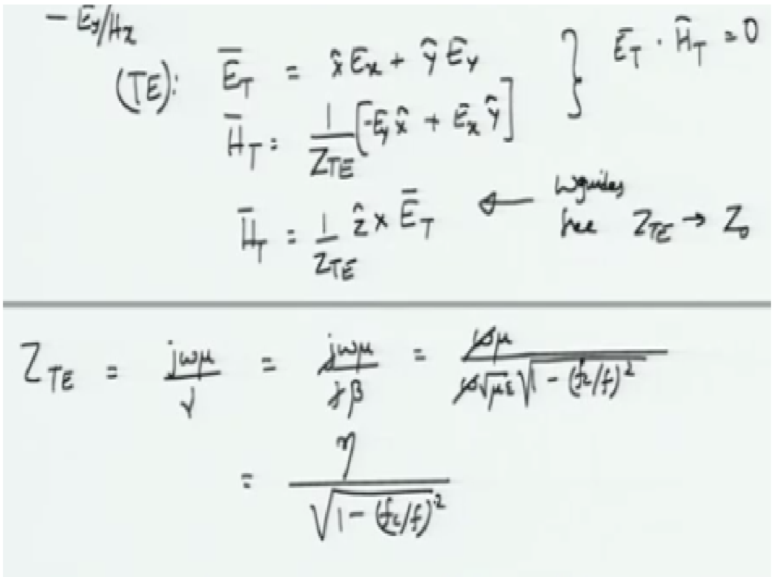

So the transfers electric field component for TE, I leave the TM as an exercise to you, ET is given by x hat Ex y hat Ey okay and the corresponding H components can be written in terms of the waveguide impedances as Ex is related to Hy right and Ey is related to minus Hx. So you can write this as one by Z T E along x would be minus Ey plus Ex because Ex and Hy, so Ex y hat okay.

So this is the advantage of obtaining the impedance. Now once you obtain the impedance you can actually calculate all the other quantities as well. You can also see that the dot product of these two is actually equal to zero right because we see that E dot H for the transfers components is equal to zero indicating that these components are perpendicular to each other or mutually orthogonal to each other.

We can also use them to relate because of this minus and plus kind of a reminding you of a cross product, you can find out what is HT in terms of the cross product corresponding to the propagation direction Z and the transverse component, this divided by one by ZTE. Just as we have done for the case of a free space propagation, you can express the H component in terms of the curl or the cross product of Z and E transfers components.

You can check that this equation checks out for both waveguides as well as for free space. In free space you need to replace this ZTE by Z zero, which was the intrinsic impedance of the medium right. So if there was no factor of FC in a particular free space case okay. As I said similarly you can find out ZM, but going back to what ZTE is, ZTE we obtained was j omega mu by gamma right.

So when the waveguide is actually propagating, I know that gamma can be written as j beta right. So gamma would be equal to j beta, therefore I can write this as j omega mu by j beta. J will cancel with each other and the impedance turns out to be real okay, but what is beta, beta is nothing but omega square root mu epsilon, which is what the free space component would be times one minus FC by F whole square.

So corresponding to a particular cut off frequency calculation M and N, this would be one minus FC by F whole square, omega cancels mu and square root of mu will cancel one of them and pull the square root of mu onto the numerator and what you get is the free space wavelength eta has we had written down divided by square root of one minus FC by F whole square.

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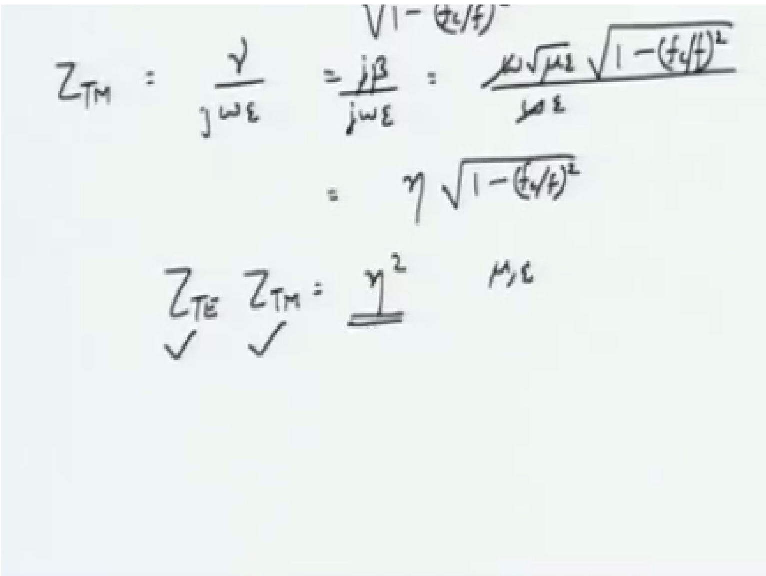

What about ZTM well, ZTM can also be calculated, ZTM is nothing but gamma divided by j omega epsilon. So substitute for gamma as j beta again j will cancel and the impedance turns out to be real and beta is nothing but omega square root mu epsilon, the free space part times one minus FC by F whole square right. Omega epsilon is there, omega cancels from the square root of epsilon cancels in the numerator that square root gets transferred to the denominator.

And what you get is eta into square root of one minus FC by F whole square. In case, you can if someone has calculated ZTE and ZTM for you and if you want to calculate what is the intrinsic impedance of the mode, you can find out. ZTE times ZTM will be equal to eta square okay. So once you have this equation you can actually find out what is eta. Now you might say that well. If I want to calculate what is eta I would calculate what is ZTE.

If I want to calculate, I mean if I want to calculate ZTE I need to know eta, if I want to calculate ZTM also I need to know eta. So why is this equation important, the answer is that for a waveguide, it is a kind of easier to calculate the ratio of the transfers E and H fields by making appropriate measurements and from there find out what is eta okay. This is especially true when you cannot really access the material that is sitting in between the waveguide okay.

Because of that reason it is sometimes easier to calculate ZTE and ZTM from that make an estimate of eta, from that eta you make an estimate of mu and epsilon. This kind of an inverse way finding out the material constants is quite common at very high frequencies and especially at printed circuit boards okay. So something that we were discussed at this point.

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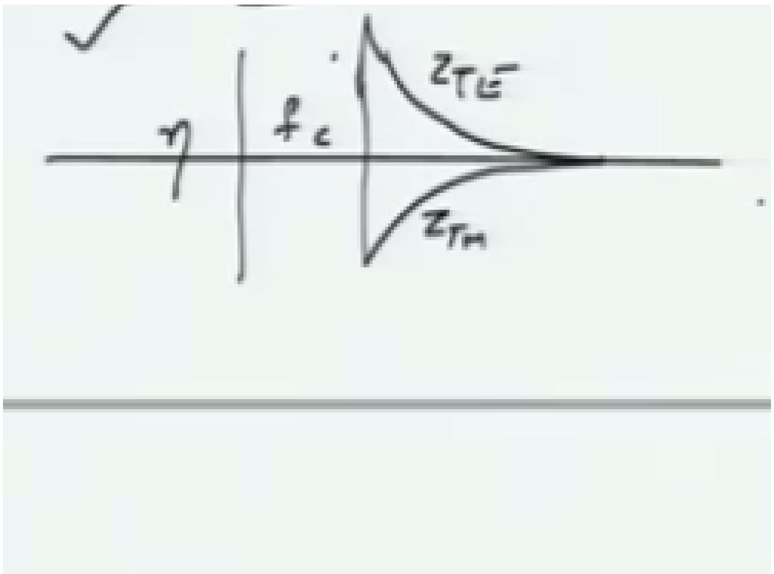

If you want a sketch graphically how this ZTE and ZTM goes, well they would all converge to eta for F much larger FC, you can see here that for F much larger than FC, you can see here that for F much larger than FC this term will be equal to zero right and then you will ZTM will converge to eta.

However, when F is less than FC there will not be any component here and then the ZTM would actually be equal to kind of minus infinity. Similarly, ZTE will be equal to plus infinity and propagation would begin at appropriate cut off frequencies okay. So you would actually see that this is how the corresponding ZTE and ZTM impedance values will vary.

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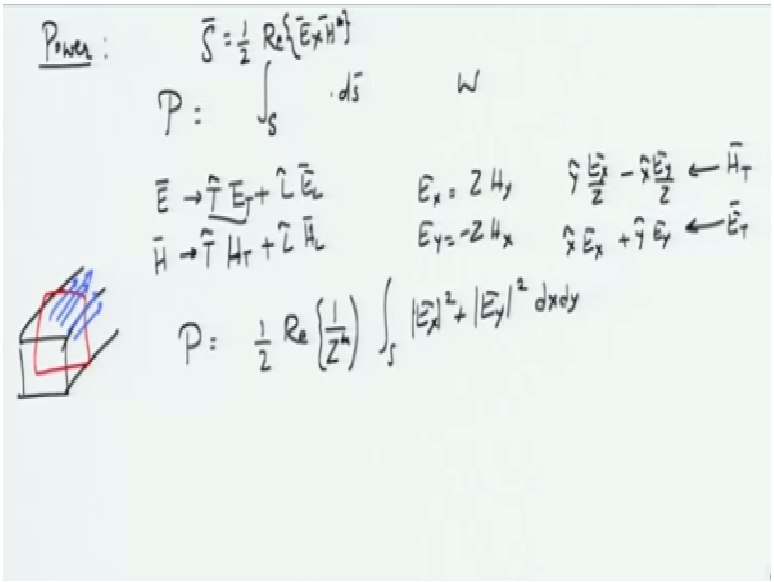

Let us discuss one final aspect. We will discuss what is called as power calculation or we want to know what is the transmitted power when we exact a particular waveguide mode. As before to obtain the power, we go back to pointing theorem or the pointing vector. The pointing vector would have a certain average, you know certain power density S. From the power density, we will calculate what is the actual transmitted power.

So the transmitted power P is given by integral over an appropriate surface that you have to choose, we will choose that one very shortly and then what is the pointing vector and integrate this pointing vector over the surface that you have chosen right. So this will give you the power in watts. Pointing vector itself will give you power density.

What is power density, for this case where you have expressed electric and magnetic fields as a phasor, it would be half real part of E cross H complex conjugate right. So this calculation you need to perform and then substitute for E and H. Immediately, you can see that if E can be broken up into its transfers okay and its longitudinal component similarly breakup H as transverse okay, H component and the longitudinal H component okay.

What you can see is that, since the power needs to be transmitted or taken from the mode from one place to another place on the waveguide, what is interesting is not the longitudinal components, but it is only the transverse components right. Since these are the only things, which are interesting and these relationships are already known in term of the impedances, you can safely substitute for the impedance and then relate E and H and obtain this expression.

What we mean here is that Ex is given by ZHy and Ey is given by minus ZHy right because Ex by Hyz and Ey by Hx is equal to minus Z. You can actually write down what is this E cross H complex conjugate in terms of this Z and Z complex conjugate, H will be Ex by Z along y along minus and then you will have along the x component to be Ey by Z along the x component, this would be your transverse magnetic field. For the transverse electric field, it would be x hat Ex plus y hat Ey right.

And this would be the transverse electric field component. Now you try to find E cross H, but then remember that when you find E cross H, H has to be complex conjugated. So when you do all that, you will see a simple relation as half real part of one by z complex conjugate, which would be the complex conjugate of the impedance z itself and integral over Ex square plus Ey magnitude square dxdy.

For the waveguide that we had in this way along the z direction, I have chosen the surface to be something along x and y plane. It makes sense right, because with this chosen surface the power will be carried along z direction. So you are interested in power being carried along z direction and therefore it would be wise to choose the surface, the open surface to be that along x and y encompassing the walls, just below the walls, but encompassing almost the walls for the calculation over here.

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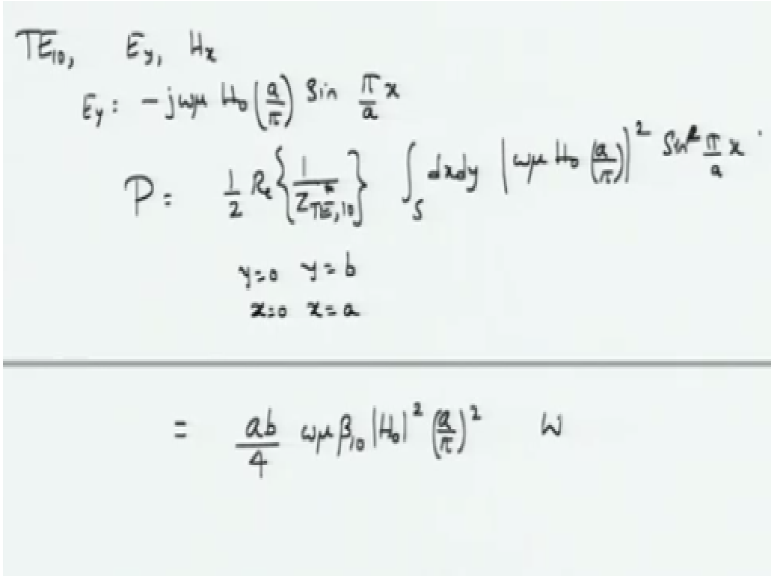

For the TE one zero case which is very important, we know that we have only Ey and Hx components and Ex component is zero and Ey component is given by minus j omega mu H zero a by pi sin of pi by a into x right. So this is what Ey is given to be and this is enough for us to calculate the power carried.

So P will be equal to half real part of one by ZTE corresponding to the cut off frequency of one zero right times integral over the surface dxdy and then put this Ey square that would be omega mu H zero a by pi, all these quantities mod square okay times sin square pi by a into x, you already have seen dxdy. Now along y you assume y equal to zero and y is equal to b and for x, you assume x equal to zero to x equal to a.

Although, you just have to consider the surface to be slightly less than that, but you know it does not really matter if you consider this to be x equal to a rather than slightly less than a okay so because of that reason we will consider the wall as x equal to a. Integration over y will bring out b into picture and then integrate the sin square after rewriting that in terms of one minus cos two something.

You carry out this integration, you also know that ZTE turns out to be real right. So ZTE turns out to be just omega mu by beta where beta corresponds to one zero. Do all these simple calculation, what you get here is ab by four okay, omega mu beta one zero, beta one zero corresponds to the propagation constant for the dominant mode de one zero times H zero magnitude square a by pi whole square watts. In fact, because beta one zero can be related in terms of FC and F you can rewrite this one in terms of the frequency as well.

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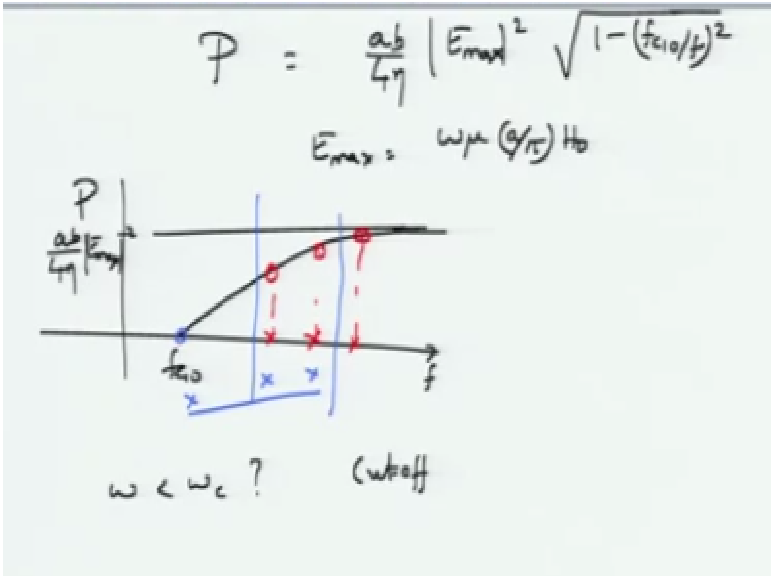

So you can write this as ab by four eta E max whole square okay times square root of one minus FC one zero by F whole square. What is E max here, E max corresponds to his amplitude okay. E max is omega mu a by pi times H zero. This is simply to set a certain constant to simplify this one, but if you know how much power you are actually putting into the wave guide then you can calculate what is E max, you know what would be the operating frequency for your waveguide mode.

If so you know F, you know a and b the dimensions of the waveguide, you know the intrinsic impedance, assume that it is filled air or some other material. You can find out what is E max and from E max you can find out H zero. Remember this was the only constant that we had not been able to pin down from boundary conditions.

Boundary conditions allowed us to write down whether a, b, c or d would exist and whatever that would remain would we put everything into under H zero or E zero depending on which modes we were analysing and to pin down those H zero or E zero values you have to use the power condition. So you know how much power you are putting in, then you will be able to find out the amplitudes H zero okay. Such calculations are quite simple, but something that you might have to try it out once or twice you just get familiar with this one okay.

So if you plot this power that is transmitted as a function of frequency clearly if when F is less than FC one zero, there will not be any power to be propagating at F is equal to FC one zero barely any power gets propagated. So this would be FC one zero, this is the F axis and as

F becomes very large, this factor becomes equal to one and then you would reach to ab by 4 eta E max square right.

So this would be the asymptote value to which you would reach, so as you start increasing the power okay. This would not be completely correct, because this only assumes that TE one zero mode is the one which would be propagating throughout the frequency, but we do know that after sometime FC two zero one start propagating then you have TE zero one then you have TEM one one TM one one. So all these things would actually drop the power.

In other words, they would also start carrying some amount of power and if your operating frequency happens to be say here, then you actually have power distributed in three different modes so some power carried by the fundamental, next power carried by the next higher order mode and the other higher order mode. So all these would share power depending on how much power you have put in and what are the corresponding factors for FC one zero and F that you have to calculate.

So if you want all the power to be concentrated within you have to operate it only in the fundamental mode, but at that point your waveguide is not actually carrying maximum energy. What would happen when omega is less than omega c, well all the modes would be cut off right. So the quantities would only be attenuation and will you launch some power with an operating frequency F less than Fc nothing would be carried by it.

And everything would be just attenuated okay. So we stop at this module and we will consider the attenuation calculations in the next module.