Electromagnetic Theory
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Lecture - 79
Hertzian Dipole Antenna

For now, we will consider a very theoretical antenna, called as a dipole antenna, or a Hertzian dipole.

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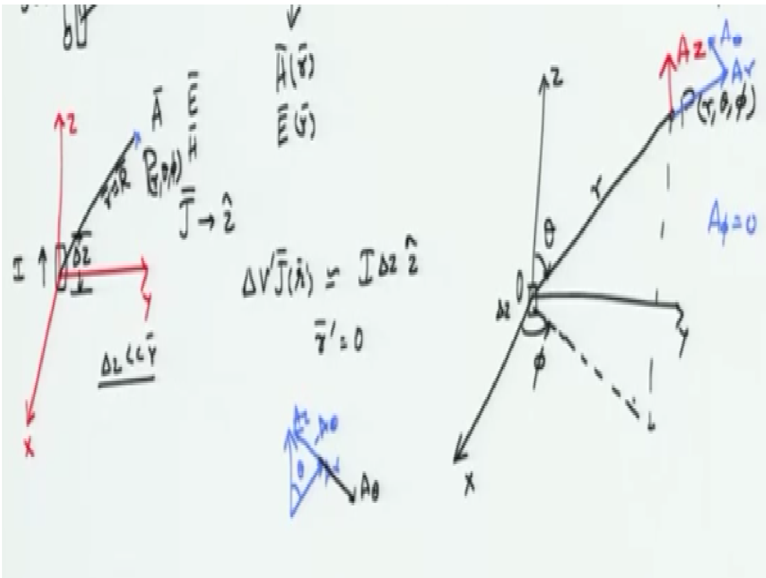

The basic idea here is that you have very short wire, which is carrying a uniform current I, okay. It is carrying a uniform current I. This is wire is located at the origin of a certain coordinate system, we call this xyz. And the wire itself is oriented along the z direction. The current at all points on this short antenna are uniform, okay. So the current is all uniform, so therefore, the J vector would also be along the z direction.

The current density is along z direction, we will assume that the length of the antenna itself is some delta z, okay. It is a very short wire, and this delta z is much, much less than r, okay. So we assume that this delta z is essentially the length of the antenna, then the volume, there is nothing about the volume, right. So delta v-prime into J of r-prime can be nicely written as I delta z z-prime, because that is the current that is being carried.

So the volume density of the current or the total current, volume current, basically becomes I into delta z, okay. So, with this we have, and we will also place this one at the origin. So we can easily set r-prime is equal to zero, which means that the antenna length is chosen to be very, very small so we have chosen the distance r and this length of the antenna itself is infinitely symbol. So, that is actually very, very small, okay.

Now, where do we want the fields, let us say we want the fields at this particular point. How do I obtain the fields? Well, first step would be to find out the A value here, from there you find out what are the different components E and H that you would like to find. Remember, even though A can be found the power carried by the wave is determined by E and H via Poynting's theorem. Therefore, you need to know what is E and H, from the calculation of A, okay.

We have established that we need to calculate A at this point, let us also call this point as P, and let us say it is located at a general r theta and phi. This would correspond to r, this would also correspond to the capital R, because this r-prime is considered to be very short, okay. So let us expand this coordinates over here, okay. This is x, this is y, this is z, but of course we are not in the Cartesian coordinate systems anymore.

This is my antenna, which has a very short length delta z, that is too small to even consider. And from here at a very far away point, or at any general point, I am looking for the fields. The point itself is marked at r theta and phi, where theta is this angle and the phi is the angle with which will be made by the projection of OP on to the xy plane. So, this is phi, this is theta, and that length of O to P where O is the origin is the radial distance r.

So, at this point, I would like to find out A, let us find out A by asking one simple question, what would be the direction of A. To answer that, go back to the relation between A and J. I know that J is now directed along z-prime, or J is directed along z. Therefore, the only component of A, that I will be obtaining will also be directed along z direction. So, I have A along z direction, so this is my Az.

But, I do not want Az, because I cannot use that in the spherical coordinate system, what I want is Ar, A theta or A phi. Obviously, if A is along z, the phi component will be equal to zero. So, A phi is equal to zero but what about Ar and A theta. Well, this is your Ar, and what about A theta, A theta will be along the direction that would be here, right. So this is minus A theta, so this is your Az, and you can split this Az into with this angle being theta.

You can split this one into Az is this, Ar is this and this one would be minus A theta, okay. So, you can split all of this, and write down Ar as Az cos theta, and A theta as minus Az sin theta. If you still have doubt as why this is minus A theta, please remember that theta is measured from the z axis. So, this should have A positive theta, okay. So, I find out Az, from Az I find out what is Ar and find out A theta, what is Az.

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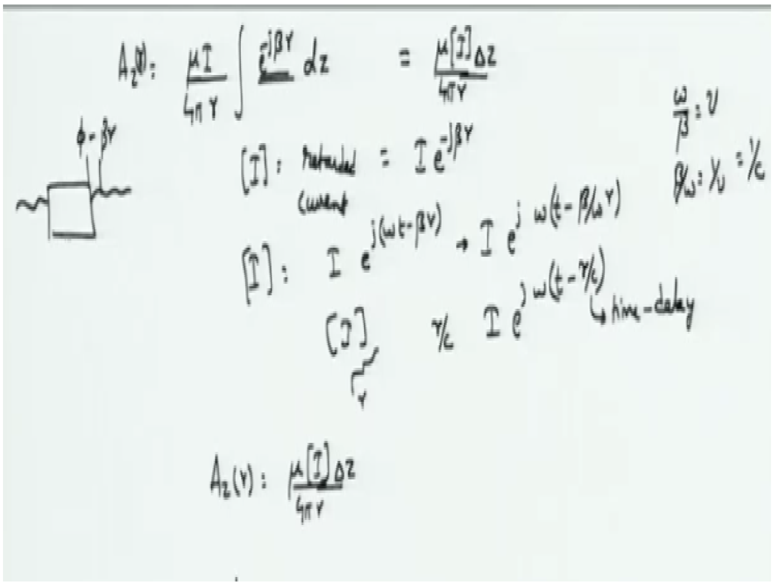

Substitute for J of r-prime dv-prime as I delta z z-prime, okay. So, I have mu by four Pi, so I have mu by four Pi, and this r would be the radial distance r that we are considering, because it is so small that I can simply push this one out. So, I have mu I by four Pi r, and there is e to the power minus j beta r and there is integration along dz, right. So, this is what I have, of course, what I have this e power minus j beta r is nothing the retarded value of I.

This is actually capturing the retardation or the time delay by having a phase change from zero to beta r, we know that, right. So, you have a certain element, and then you send in a sine wave or a cosine wave, and there would be a phase delay. That phase delay was nothing but beta into r, in fact beta is called as a propagation constant, and is measured in radians per meter. Therefore, this is radian per meter times r.

The integration of course, does not depend on z, therefore, I can simply pull this one out. We can also assume that, since we have also assumed that the length of antenna is delta z. So, integration of dz will simply pull out the delta z, or put that value of delta z. So, Az at any point r that we have obtained is independent of theta and phi. This is given by mu I divided by four Pi r delta z, where I have introduced the notation I as the retarded value for the current, retarded current.

This is given by whatever the value of the current at the source, times this phase factor e power minus j beta r. In terms of time, of course, this would be, so if you were to express this in terms time, this would be I e power j omega t minus beta r. You can sometimes rewrite this one as e to the power j omega t minus beta by omega into r. But I already know what is the relationship between omega and beta, omega by beta is nothing but the velocity v.

Therefore, beta by omega is one by v. So this would be e power j omega, and in case of free space, we know that it is essentially light, electromagnetic wave is essentially light, and it would be travelling with a velocity of c. So beta by omega will be c, so, you have r by c. This r by c would be the time delay, that we have been talking about, or the retardation that we have been talking about.

The notation here, simply captures that, whatever is happening at r was actually the value of the current that existed at r by c seconds earlier. This is the time delay or the time retardation, that we spoke about. So, you have Az of r, which is the expression for the component of vector potential Az in Cartesian coordinates. That would be given by mu I by four Pi r delta z.

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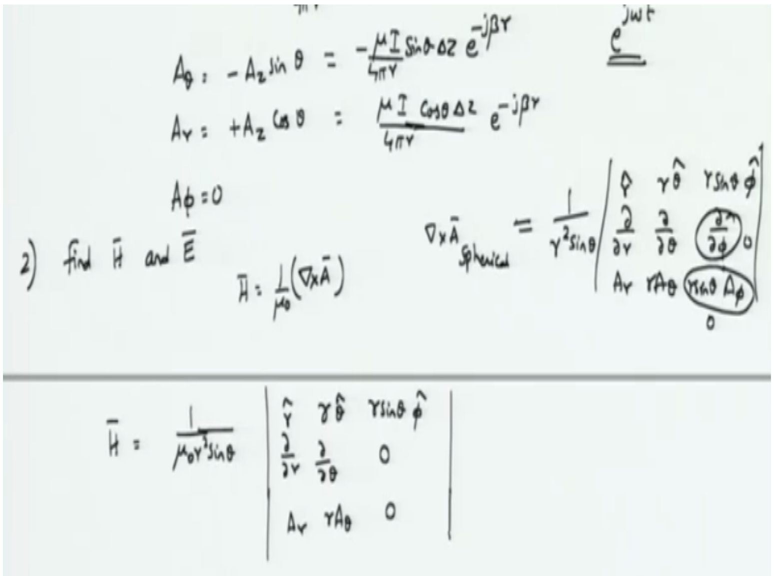

I also know that A theta can be written as minus Az sin theta, and Ar can be written as Az cos theta. So, what would be Ar, Ar will be mu I, so the current is I, and then divided by four Pi r, I am going to drop that notation on the harmonic part, or the time retardation part and consider just the value of I, because every component would be retarded, so it would be not okay, or it would be confusing for me to keep taking that retardation for every component.

You just understand that, whatever computations that we are doing, r for retarded values of fields, okay. So, you have mu I by four Pi r, there is a cos theta, delta z and of course there is this e to the power minus j beta r, right. So, I just told you that, we can drop this e minus j beta r, but it turns out that I cannot drop it, in the subsequent calculations I need e power minus j beta r, because there is a dependence on r.

What I meant was, we can drop this e power j omega t, right. So, we can drop this e power j omega t from all of our equations, because every equation would be harmonically waving at a frequency omega. What would be A theta, A theta is minus mu I by four Pi r sin theta delta z e to the power minus j beta r, okay. So, I know components A theta and Ar of course, I also know that A phi is equal to zero. Now what, this is the second step, find H and E.

How do I find H, H is nothing but curl of A one by mu zero, right, one by mu zero times curl of A. So, if I want to find this one, I need to know what is curl of A. So the expression for curl in spherical coordinates is one by r square sin theta determinant of r hat, r theta hat, and then r sin theta phi hat, you have the corresponding differentials, del by del r, del by del theta, and del by del phi and components Ar, A theta, and A phi, the determinant of these, okay.

If I have got some components wrong, you can appropriately substitute that one. Here, we know that A phi is zero, so I can put this equal to zero. There is no dependence on phi, therefore, this is also equal to zero, is there dependence on r certainly, is there a dependence on theta certainly. So, we just keep these components non zero. So, if I go back to this and say H, H must be equal to one y mu zero r square sin theta outside of this.

Then there is r hat, r theta hat, and r sin theta phi hat, del by del r, del by del theta, del by del phi is zero so I am going to consider this as zero itself, A phi is also zero. But Ar is this quantity, right, so Ar is this quantity, Ar and A phi is the quantity, A theta is r, A theta and this is r A theta. The corresponding expressions for the curl of A would consist of r A theta, and r sin theta. So let me just rewrite this one, this would be r sin theta A phi.

Luckily, we do not have to still consider this, because that is equal to zero. But, instead of A theta here, it becomes r A theta, okay. So this is the expression, a word of caution here, the formulas get too tedious, these are not difficult to evaluate. The only problem is that there are too many r theta, there are too many differentiations. The math gets really messy, but keep remembering the big picture, the big picture is that if you do all this math.

What you have done is to find A, from A you have found out H and E, and you will know how these fields depend on r theta and phi. The dependence of the electric field or the magnetic field on r theta phi is called as the antenna pattern. That is what you are trying to get to, you want to know what is the pattern in which the electromagnetic fields are radiated from this particular antenna, okay.

Whenever you get some trouble with the differentials and this one, I will just show you one or two steps and then I will straight out go to the answers, I mean the solutions. But I would request you to take some time off and understand that you have got all the steps correctly, okay.

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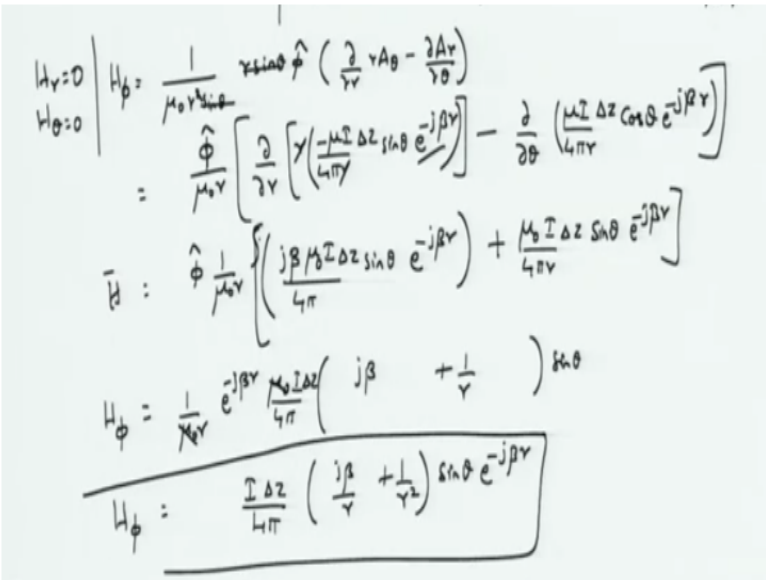

Alright, what components we will have, you have one by mu zero r square sin theta, there is r hat component, because del by del theta of zero minus zero times r A theta. There is nothing over there, so there is no component of H r, so H r is zero. What about theta component, again you can cover this one from the determinant rules, we know that del by del r of zero minus Ar into zero will have no component.

So, you are lucky that H does not have r, H does not have theta, so the only component that we are going to get will be H phi, and H phi is given by one by mu zero r square sin theta times r sin theta phi hat del by del r rA theta minus del Ar by del theta, okay. So, let us substitute for this one, so r sin theta cancels here, so there is only one by r sitting here, you have one by mu zero r and along phi direction.

There is no surprise this is a H phi component, r times A theta. So go back to what is r, A theta, A theta is minus mu I by four Pi r. So, you have r into minus mu I by four Pi r, there is also sin theta, there is delta z sin theta e to the power minus j beta r minus there is a r here in the numerator and r here in the denominator that will cancel with each other. Then, you have del by del theta of Ar, Ar is nothing but Az cos theta.

So you have mu I by four Pi r delta z cos theta e power minus j beta r, correct. As I said, these are going to be a little messy, so look at what you get, you have phi hat one by mu zero r r dependence is gone, the r dependence from the first expression comes only by minus j beta r, so dl by del r of e power minus j beta r is nothing but minus j beta comes out, minus and minus will cancel.

So you get j beta mu I or mu zero I, because free space, divided by four Pi delta z sin theta, there is still e power minus j beta r, minus del by del theta. So the dependence on theta is in terms of cos theta, so del by del theta becomes minus sin theta. Therefore, this becomes plus mu zero I by four Pi r delta z sin theta e power minus j beta r. Now let us try to untangle these expressions take some common factors.

We seem to have this mu zero I delta z by four Pi as a common factor, so I can push this one out, okay. So I actually have two elements, so I have e power minus j beta r, this is H. So H phi, I am dropping this phi hat, because this is H phi. So e power minus j beta r is a common factor that can be taken out and then you also have mu zero I by four Pi as a common factor that can also be pushed out.

So, inside here integral what you have is j beta, there is also one by mu zero r from the outside. So, you have j beta sin theta, because sin theta is over here delta z can also be pushed out and if you push out mu zero I by four Pi delta z there is only one by r here sin theta is there, which again can be pushed out. So let us push out sin theta also, so we will have somewhere here as sin theta, and what else is left out, that is all we are left out, okay mu zero cancels on this side.

So you have I delta z divided by four Pi j beta, just push this r inside by r plus one by r square sin theta e power minus j beta r. So this is the expression for H phi, you do not have to of course go to A, in order to obtain the electric field, you can either obtain electric field from H phi, and H phi is a function of r and theta.

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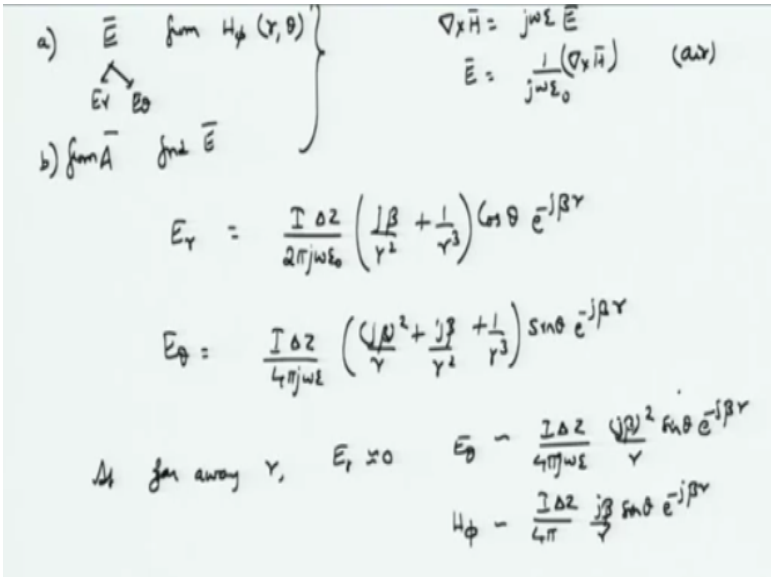

So therefore you will see that electric field will also have two components, you will have Er and E theta, okay. The other method is to go with A, from A find electric field E. I will leave this to as an exercise for you, I will not solve that, if you are trying to go with the first approach, for the first approach you can use curl of H expression, right. So you have curl of H is equal to j omega epsilon E.

Therefore, E will be equal to one by j omega epsilon curl of H, epsilon of course is epsilon zero, right, because we are considering air medium. So find out curl of H, H has theses components simplify you will get Er and E theta. I will give you the solutions for that, Er is equal to I delta z divided by two Pi j omega epsilon zero j beta by r square plus one by r cube cos theta e power minus j beta r.

And then you have E theta, which is I delta z divided by four Pi j omega epsilon divided by j beta square by r plus j beta by r square plus one by r cube and sin theta e power minus j beta r. So all these quantities, Er, E theta, and H phi are suitably delayed, there are picking up this phase factors of e power minus j beta r, which is perfectly alright. This simply indicates that all of this are waving at a same phase retardation.

But the expression for Er, E theta and H phi are complicated more over the expression for E theta consists of three terms inside the bracket, expression for Er consists of two terms inside the bracket. So, clearly when we have this different terms, if you start at the near the antenna and then you keep moving away from the antenna in the radial direction, you will see that at some time one by r cube term will dominate, then one by r square will dominate.

And eventually one by r term will dominate, because for r that is very large compared to the antenna origin, one by r square will drop to zero, one by r cube will drop to zero at very, very far away distances, at far away values of r, Er approaches zero E theta will approach I delta z by four Pi omega epsilon j beta whole square by r there is some sin theta e power minus j beta r. And H phi will be approximately equal to, again one by r square term will drop out.

So I have I delta z by four Pi, I delta z by four Pi and j beta by r there is j beta by r there is also sin theta e power minus j beta r, j beta square will be minus beta square by r, that is the one by j.

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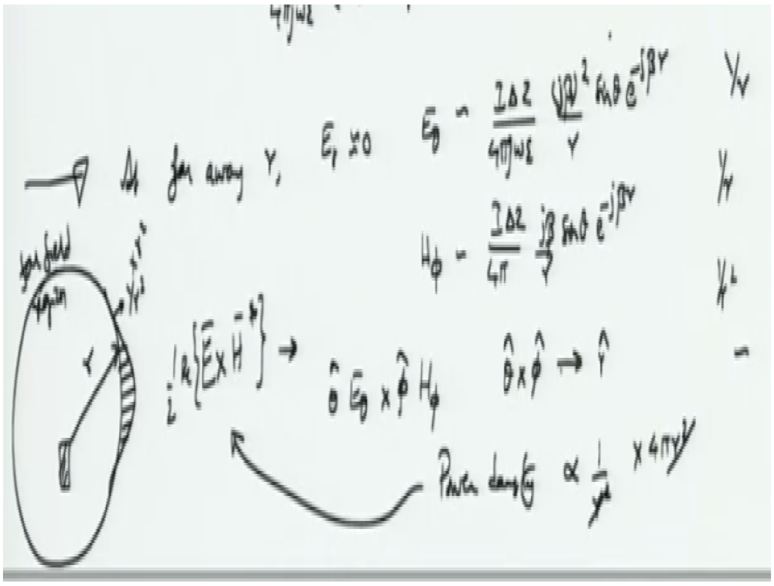

But now I know that in the spherical coordinate system E cross H complex conjugate, half of real would be the average power that is carried by the electromagnetic wave, right. And since E is along theta, H is along phi, these are the non-zero components that I get at far away distances, then E theta cross theta hat cross phi hat H phi will carry power in the direction of theta hat cross phi hat, which happens to be the radial direction.

So, this is somewhat comforting, you take an antenna, you excite this antenna with energy, and the energy is now propagating radially outward, okay. So, if you have a meter to detect power you place at a far away distance, then there would be power that is being radiated. The interesting part is that, because E theta goes as one by r, H phi goes as one by r, the power density, this is the power density, right, from the Poynting's theorem.

This would be going as one by r square. Therefore, when you multiply this one by the area four Pi r square you know of a spherical surface that you can imagine. So, this is my short dipole, imagine a spherical surface of radius r, okay. then the power density here is one by r square, the area itself scales up as times r square. Therefore, the power itself will be constant. So, there is actually constant average power being radiated around the antenna, okay.

And this antenna also to be an isotropic antenna in the sense, power density is constant at all points on the surface. If an antenna would be directive, then the power density itself would be different. Maybe the antenna is just having power density over here, and all other regions the power density would be zero. So, this kind of power density vector would tell you that the antenna is directional, otherwise the antenna is isotropic, okay.

So this region where we are very far away from the antenna, and the power density is essentially one by r square, and electric field components consist of only theta and phi, and these components when you consider in time domain they would turn out to be real, because there is minus r square by r and this j must have also gone some somewhere, I might have probably made a mistake here, okay.

So that should have gone away, when you consider at a very far away distance, sorry it will not go away, there is a minus j out there, it would anyway go away. So the point here is E theta would be going as one by r, H phi is going as one by r, the power density goes as one by r square, and the power itself goes as a constant. This region which is very far away from the antenna is called as the far field region.

Far field region is also called as radiative region, and this is the region where the electric and magnetic fields basically radiates power away. Alright, we will see what other regions will be there around the antenna. Suppose you are not very far away; you are pretty close to the antenna. For Er one by r cube will be much larger than one by r square. Therefore, this one by r cube will be dominant. Again for E theta one by r cube term will be dominant.

So when you are very close to the antenna it is the one by r cube terms that would dominate and because H does not have one by r cube, you can imagine H phi going to be zero. So the field consist only of the electric field, and this is the region of electro static, okay.

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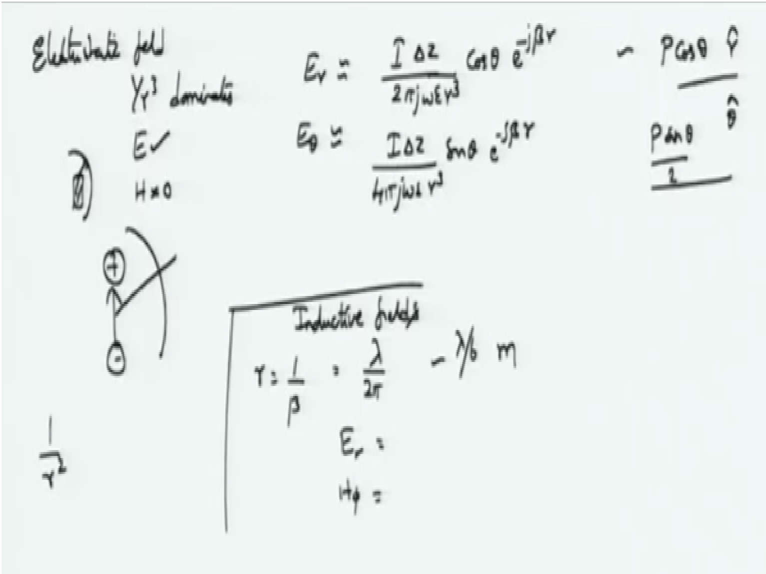

So electro static field region is that region around the antenna, so this is the antenna, around this antenna you are very close, so that one by r cube term dominates and electric, the fields around will consist only of the electric field, the magnetic field is approximately zero, okay. So with this you can find out what is Er, approximately E theta, approximately in this region Er will be I delta z divided by two Pi j omega epsilon r cube.

See there is an r cube out there, cos theta e power minus j beta r is still there, no matter how close you are, you are still away from the antenna. Therefore, there is a small retardation factor, but r is very close, so beta r will be very small by itself, E theta will be I delta z divided by four Pi j omega epsilon r cube sin theta e power minus j beta r. If you forget all these one by j omega and everything and write down this as some P cos theta, right.

And you write this as P sin theta, where P can be defined as, P is the constant that is surrounding. And this is along r and this is along theta, the strength of these two are one half the other, so this is actually about two. This s precisely the fields that we obtain for a dipole, right. So a short wire, that is carrying current actually is equivalent of having a dipole and then you are looking at the fields around that.

Obviously the fields around dipole would show this type of behaviour, right. So this is the dipole equivalent of the antenna in the electrostatic case. Now, if you go slightly away from the antenna such that, one by r square term must dominate, know, if you want one by r square term to dominate, what you need to see is that go back to this one, for example, you go to H phi when does one by r square term dominate here.

One by r square term will dominate provided, so you multiply this term here j beta by r in the denominator and r in the numerator, so that you may have a proper comparison, what you get is beta r divided by r square must be there as one by r square. So if this becomes comparable, right, then this beta r by r square or beta by r will dominate, so or this would be greater than this one, greater or comparable to this one, then beta by r term will be dominating.

But I know that if this happens, this condition is that beta r must be approximately one, because r square cancels so beta r must be approximately one. But I know beta in terms of lambda, right. So what is beta in terms of lambda, beta r, r will be equal to one by beta, but beta itself is lambda by two Pi. So two Pi can be approximated as 6, so at r of lambda by 6 meter, if lambda is expressed in terms of meters, the fields that you get.

So this is the second region, right. The field that you get are what are called as inductive fields, okay. The region around this region, around the antenna is called as inductive fields, and you can show that because it is one by r square that are dominating, H phi term would be there and amongst Er, there would be term for j beta by r square, and you will also get E theta terms, okay.

But the fields you are interested in are Er and H phi, you can obtain the corresponding expressions for these. The other region we have already discussed, this is called as the radiative region or the far field region, okay, and the pattern to determine the pattern of the antenna we would come back to that one in the next module. So in this module we will stop, in the next module, we will consider the characteristics of antenna. Thank you.