Electromagnetic Theory
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Lecture No - 83
Long Wire Antenna

In this module, we will wrap up discussion on Antennas by considering a more practical case of a long wire antenna. Here we do not explicitly make the assumption that the length of the antenna is very small compared to the wavelength, which anyway one cannot really ensure that would happen. Moreover, for such a short dipole antenna that we discussed in the last two modules, the radiation pattern turned out to be isotropic.

And isotropic radiation is not very useful unless you want your energy to be spread evenly throughout the space with no specific direction pattern. But when know that antennas are actually used mostly for line-of-sight communication or at least some sort of a directivity is required, which means that we need to consider non isotropic radiation and one of the ways in which we can produce a directed beam pattern would be to go for a long wire antenna.

The long wire would simply mean that the length of the wires is appreciable to the wavelength of the source.

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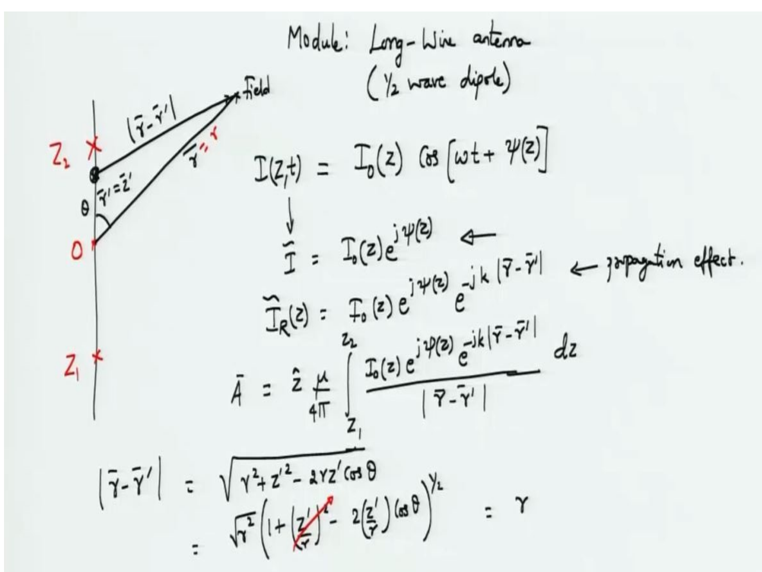

So, consider that we have a long wire antenna which let's say is placed between two points Z 1 and Z 2. Or we can consider this to be any other two points that does not really matter. Now what we want is that this kind of a wire which is spread between these two points once this is carrying a wire we need to find out what is the electric and magnetic fields which are very far away from the wire itself.

That is we are actually not interested in the electrostatic or the inductive fields. We are interested in the region, where we can have some radiation. So what we are about to find our only the radiation zone, electric and magnetic fields. Now, before we can proceed of course I need to also know the current. Let us assume that the current at any point on the wire is given as a function both of Z and t.

Why would it be a function of Z, because now the wire is no longer centered at Z equal to 0 in the sense that the wire length is actually quite long and we know from our transmission line theory that any length of piece of a wire, which is comparable to the wavelength will have waves on them. So there is this phase that you need to consider that comes simply because of the propagation effect.

So the current that we are considering on the antenna must be a function both of Z and t and what we are really hence considering is a current wave. And we will assume one of the simplest current distributions. Let us assume that in time, it would be a Cos Omega t + some Psi of Z. And in the spatial variation that is the dependence on Z comes about by having a current I 0 being a function of Z, or the current function being I 0 of Z.

Of course we never really work with time dependent quantities in this way or at least that is not we have done in this course. So we immediately switch the phasor notation and I know that the current phasor I is given by I 0 of Z e to the power j Psi of Z. So this is very reasonable but we also know that this not a phasor that we are going to work with. Because when you consider radiation pattern for an antenna you are actually interested in finding the retarded currents.

So the retarded current would come because of the propagation effect and that would be given by I 0 of Z e to the power j Psi of Z. This is the original current but to account for the propagation I have e to the power - j k, k being 2 Pi / Lambda times r - r prime. //correct//. This is the length or the distance between the source point or prime and the field point or where we are calculating.

So what is this r and r prime? Just to refresh your memory. So if this is the center of the wire, or the center of the Z axis so call this as 0 then r would be the field point, from the origin r would be the field point. And at any other point on which we are considering the current on the wire itself, on the antenna itself that would be the r prime point. This is the field source point and this is the field point and r - r prime would correspond to the distance between source and field.

So this is what we have, the retarded current simply corresponds to the propagation effects as we described in the last module. Now from the current, we know that we need to find out the retarded potential from the potential we will be able to use the relationship for magnetic vector potential A. In that assumed Lorenz gauge we will then find out the electric field and the magnetic fields, using the relationship of E A B and A.

Since the current is in the Z direction, our current density will also be in the Z direction, we will again assume that the thickness of the wire can be eliminated. So what we are really considering is a very very very thin antenna. And for that antenna, the retarded current phasor is known. This current is directed along the Z axis or it is a function of is Z. And therefore the vector potential A will also be directed along the Z axis.

So the vector potential A phasor of course I am not showing phasor at every point, but this is also the retarded vector potential. So, the vector potential is given in the direction of Z and given by this expression Mu / 4 Pi integral of whatever the current that you had. So, I 0 of Z e to the power j Psi of z e power - j k r - r prime / r integration is over dz. This is exactly like the short dipole antenna except that we have explicitly taken into account that we want the radiation pattern in the radiation zone.

So we will make that assumption very soon and this is not r but this is actually the distance between source and field point. So this is actually r - r prime. Now pay attention over here because this is where we introduce one of the most useful approximations that we are going to use in a antenna analysis, if you are to ever take up this subject. Of course the integration limits on this one is Z1 and Z2. But coming back to the approximation that I am talking about it is the approximation to the distance r - r prime.

If you look at this r - r prime and treat r and r prime as two vectors, then law of Cosines tells me that this magnitude is actually given by square root of r Square + z prime Square - 2 r z prime Cos Theta. What are these different terms that are involved? Well, the source points, you know the current is actually directed along Z axis, therefore r prime is actually z prime vector which is directed along Z axis. Therefore, instead of r Prime I can write this as z prime, z hat.

And Theta will be the angle measured from the Z axis. This is just like the spherical coordinates Theta that we are measuring. This angle Theta gives you the angle between r and r prime. So, this is your actual magnitude but now we are going to assume that this r, you know the magnitude of r by itself that is the distance from the origin to the field point. So, this is my field point, the distance from the origin to the field point itself is much much larger compared to the distance r prime.

That is to say, you no matter where you go on the antenna //ok/// and this antenna length is going from Z 1 to Z 2. No matter where on the antenna you are there. The distance r - r prime would look exactly equal to or approximately equal to the distance from the origin. So, the position of you on the antenna this one would not matter and therefore I can approximate this, by first using binomial theorem and then showing you where the approximation comes from so I can pick r Square out.

So, you have 1 + Z prime / r whole Square - 2 z prime / r Cos Theta. I hope that I have obtained this one correctly. There is a half and because there is a square root of r Square, this r Square and the Square and the root cancel each other and you get r times this factor. Now in this factor, we will neglect this term z prime Square / r Square. So we are going to neglect this term because this is second order Z. We have already said that r by itself is quite large and what is r? r is nothing but the magnitude of the distance from the origin to the field point.

So if I neglect this z prime divided by r in effect saying that it does not matter where on the antenna you are, that antenna length itself is white small compared to the distance r that is involved. But remember the antenna length is not small with respect to the wavelength. So this is where some people do get confused.

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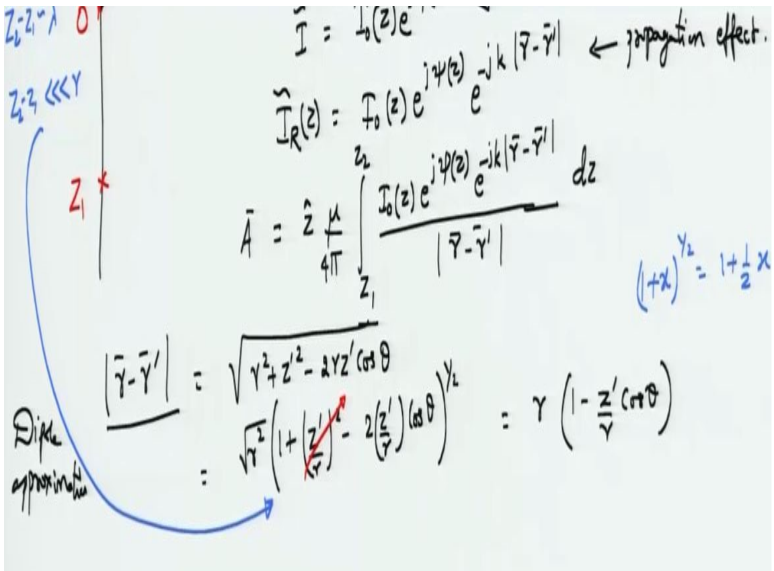

What we are saying here is that the antenna length, which is Z 2 - Z 1 is appreciable to the value of Lambda but Z 2 - Z 1 is still very very very small compared to the distance of the field point from the origin. So this is what we actually putting in this approximation. So if I neglect this and I know that binomial theorem tells me that 1 + x to the power half must be equal to 1 + 1 / 2 x.

So, I can use this binomial Theorem to write down this one as r into 1 - you get 1 - z prime / r Cos Theta. So, this is what you get for r - r prime and now, I have to substitute this r - r prime approximation in two places. One there is a numerator place and the denominator place. Now watch what happens now? This approximation is sometimes called as a dipole approximation that we have just did.

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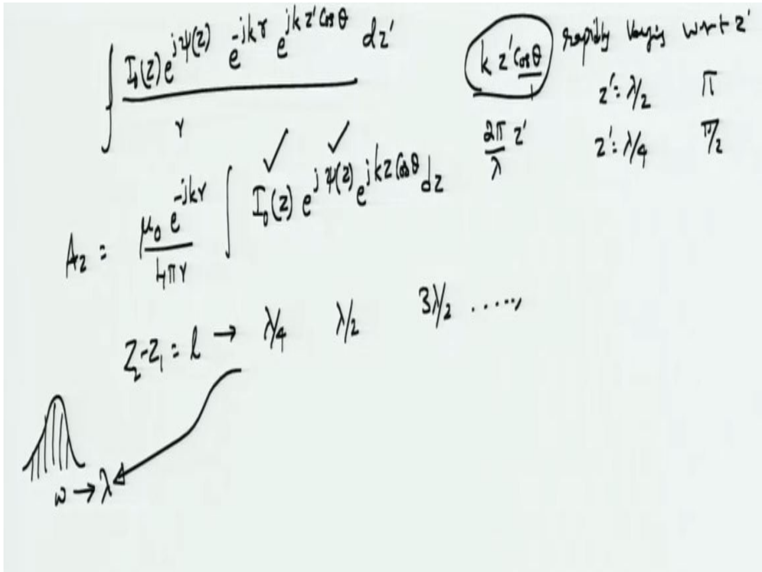

Now what we do is in the integrant I 0 z can be written whatever that hasn't changed e to the power j Psi of z also hasn't changed. But in the numerator I have e power - j k. Now substitute for the approximation of r - r prime. You get e power - j k r and then you also get e power j k z prime because r into r will cancel times Cos Theta the integration is along Z axis divided by, so in the denominator what I have is r - r prime. So if I expand this r - r prime and if I put the approximation I get r - z prime Cos Theta.

Now, z prime itself is quite small, the magnitude of Cos theta would not change by, it would not increase to greater than 1. Therefore, this term z prime Cos Theta would always be less than z prime itself which further is less than r. Therefore, in the denominator, I simply erase this z Prime Cos Theta. So I simply erase that z prime Cos Theta. Now you may ask can I do that same thing in the numerator.

Unfortunately, you will not be able to do that in the numerator. So what is the difference? Well in the denominator, I didn't have anything else. I just had r - z prime Cos Theta and I compared z prime Cos Theta to r and canceled that out. But in the numerator I have k z prime Cos Theta. Even if you assume that Theta is equivalent to maximum magnitude of Cos Theta would be equal to 1, I still have this k into z prime. But k is not small because k is 2 Pi / Lambda into z prime.

And this factor e to the power j k z prime Cos Theta is actually the propagation factor on the antenna wire. And if you go to z prime, which is equal to Lambda / 2, the phase that you get k z prime Cos Theta would be, so k z prime Cos Theta assuming Cos Theta is equal to 1 will be equal to Pi. Or if you go z prime is equal to Lambda / 4, the phase would be equal to Pi / 2.

So in other words the numerator quantity, this phase factor k z prime Cos Theta is rapidly varying quantity with respect to z and therefore or z or z prime doesn't matter. So I should not consider this k z prime Cos Theta is equal to 0. I cannot make this approximation; I cannot take z prime Cos Theta is equal to 0. So, we just leave everything as it is and I know that e power -j k r which corresponds the propagation delay which has been converted into the phase part comes from this e power-j k r is not a function of Z.

So I can move this e power-j k r outside, I can also move this r outside and what I am now left with is the magnetic vector potential which is still in the Z direction. Which is still in the Z direction and is given by Mu 0, I am assuming that we are in the free space. So, Mu 0 E power - j k r, the direction I have already indicated therefore no need to write that divided by 4 Pi r and the integration would still have I 0 z e to the power j Psi of z e to the power j k z Cos Theta dz.

I am dropping z prime which would I have represented by field point and then I am just writing that in the place of z prime as z. Now how do we proceed? Well again we are in a soup. First of all, look at this expression very carefully and understand why we are in soup. There is no problem with this term Mu 0 e power -j k r / 4 Pi r. There is absolutely no problem with this term.

However, if you want to further evaluate this I need to know what this I 0 of z would be and this Psi of z would be. Assuming that k is known because I know the wavelength, which I am exciting the antenna and everything else is known because Theta is also related to the distance is related to the direction in which I am standing in turn to find the electric field and the magnetic field. The only problem seems to be that I do not know what is I 0 of z and e power of j Psi of z.

And this is in fact, the central problem in antenna analysis. This is one of the most important problems in antenna analysis because you simply do not know the current distribution of a majority of antennas. In fact, even the simplest case of a half wave dipole with non-zero thickness conductor has never been fully solved. This all this happens because you do not really know the current distribution I 0 of z into e power j Psi of z.

Moreover, you do not even know the current distribution for different values of z. If the length of the antenna which is say z - z one is equal to l, actually is Lambda / 4. You get a different current distribution if it becomes Lambda / 2, the current distribution changes 3 Lambda / 2 the current distribution changes and it just keeps on happening. The second part which I would like to emphasize is all these l Lambda / 4, Lambda / 2 is all fine.

These all corresponds to a fixed wavelength but if you were to try and send in a Gaussian pulse through, you know as an excitation part to this antenna, you will immediately see that this pulse will have different frequencies. And these different frequencies will have different wavelengths and this length and the current distribution because it is a function of wavelength will now become a very complicated function of the actual signal that is being sent on the antenna itself.

So to re-emphasize the dipole approximation works very nicely to reduce the complexity of the expression for the magnetic vector potential. However, the central problem of antennas, which is that of estimating the current density or the current distribution is because it is very difficult in order to proceed further all you have to do is to start guessing. So this problem is complicated and what we do is, we simply resort to guessing. What sort of a guess should be make?

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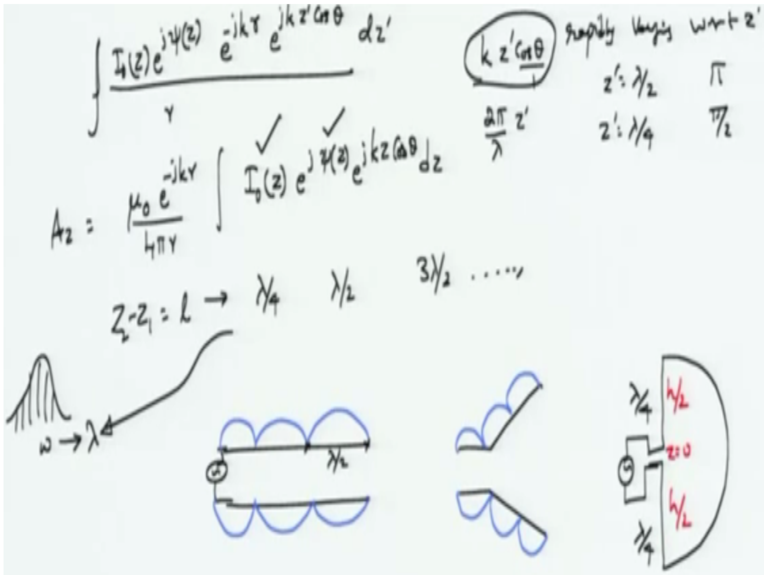

Well imagine that before this became an antenna it leaved as a transmission line. So these two pair of wires are a transmission line and I actually had a source which was connecting and exciting this transmission line. See at this end far end there is no connection, there is no load so which means that this is going to be an open circuit. If you were to draw the current pattern of this particular one you would see that the current has to go to 0 and then current would essentially be uniform over here.

Similarly, the current would be uniform and it would be something like this. Now imagine that I am going to flare up this transmission line. So I am actually flaring this transmission line. It is quite reasonable although not completely ok to assume that this current distribution would still look like this. So the current distribution is still looking in this way and finally if I flare the antenna out, I get one current distribution and I assume that this current distribution is Lambda / 4 here and Lambda / 4 here.

Because the distance between two minima is going to be Lambda / 2. And I am actually tearing this transmission line a part by flaring it up and by making the lengths completely perpendicular. So, the current distribution must go to 0 at the ends, because this forms the open circuits, you are actually kind of assuming that this open circuit condition is still valid. And you are also going to assume that the feed network are the voltage source that is feeding is actually feeding it through with a very small feed point.

So the field point is very small that is just enough to connect the voltage source. So, this is one current distribution that one can assume. It has been shown that the current distribution assumption is not completely ok. We had assumed a sinusoidal one, but depending on what length you are there would be different kind of current distributions. For example, there is a triangular current distribution and if your antenna length increases.

For example, instead of this Lambda / 4, Lambda / 4 total giving you a Lambda / 2 Length, if you had a Lambda antenna then you have Lambda / 2 here and Lambda / 2 for which the current distribution would be twice of this one. So, hopefully this is now I have captured it correctly but anyway I am not, maybe I am not capturing it completely ok. But the point here is that the type of current distribution depends on the antenna length.

And it is one of the most important problems to actually specify this current distribution. Since I have kept my z equal to 0 center over here. Now I am going to assume that this is h / 2 and h / 2, as the two lengths which have you know of the two antenna and the current distribution is now becoming sinusoidal.

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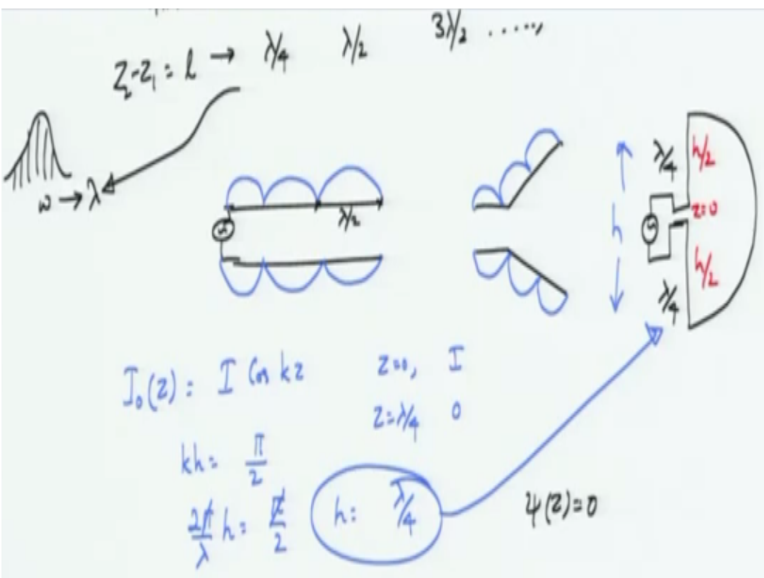

So, what is the distribution? The current distribution I 0 of z is given by some I, which is a uniform current, which I am assuming Cos k z. At z equal to 0, the current distribution is I and at z equal to Lambda / 4, + or - Lambda / 4, the current distribution must go to 0. For in order to ensure that we need to have k into, if you consider the total length as 'h' because there was 'h by two' and 'h by two', so 'k into h' must be equal to the integral multiples.

So, when does the cos function go to zero? It goes to zero at 'm pi / two'. So for 'm' equal to the fundamental, you are looking at 'pi / two'. Cos of 'pi / 2' is zero. But I know that 'k' itself is given by '2 pi / lambda'. Therefore, multiplying this by 'h', will give you 'pi / two. 'pi' cancels on both sides and 'h' is basically given by 'lambda / four'. Well, there is no surprise.

We were actually looking for a 'lambda / 4' antenna itself because that is the current distribution, which we said be for a 'lambda / two' antenna, from the transmission line thing. All that this last statement as told is that, we are consistent with our approximations. Now, once I know this, I can now substitute for the current distribution 'I 0 (Z)'. Let us also assume for simplicity that chi of Z is equal to zero throughout the antenna length.

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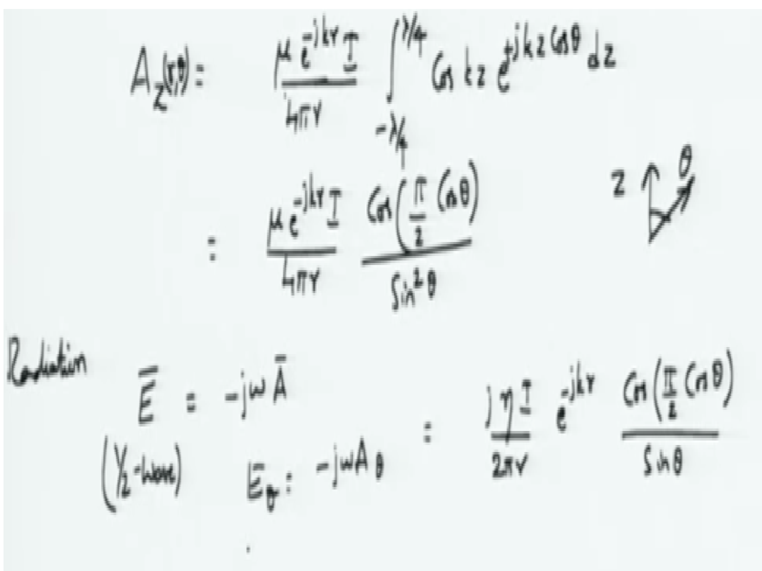

So if you do that and substitute for 'A of z' and carry out the further analysis, so you see that 'A z' will be equal to 'mu e power minus jkr divided by four pi r' the current value 'I' is constant, therefore I can pull this out of the integral and you have 'minus lambda by four to lambda by four', which is the length of the antenna that I am considering, 'cos k z' this is the current distribution, ‘e power minus j’ that phase factor has gone, so you have ‘e power jkz cos theta d z’. So, clearly this ‘A z’ is a function both of ‘r’ as well as ‘theta’.

You can evaluate this integral. I am not going to do this evaluation, but this is tedious and straight forward integration, divided by ‘4 pi r’. So what is the vector potential that I am going to get? This would be ‘cos (pi / two, cos theta) / sin square theta. Now let us not evaluate the electric fields and magnetic fields. You know you can evaluate them. I am not going to evaluate this completely, but I will just give you what the expressions for the electric fields are.

So the electric field ‘E’ for the half wave dipole or a lambda by two dipole as sometimes it is called, is given, ‘E’ will also have two components, ‘E theta’ and ‘E r’, and therefore ‘E’ itself is given by in the radiation zone, so ‘E’ must have only theta component and ‘H’ must have the phi component. The electric field ‘E’ is related to ‘magnetic vector potential by minus j omega A’. Therefore ‘E theta’ will be equal to ‘minus j omega A theta’.

Find out the ‘A theta’ component for this vector potential. This is ‘A’ along ‘z’. But this is the ‘z’ axis and if this is your theta axis, then there is a relationship between ‘z’ and theta. You can find from those relationship that the electric field ‘E theta’ is given by ‘j eta I divided / 2 pi r, e to the power minus jkr cos (pi / two cos theta) divided by sin theta’. One of the sins will go away because theta will be ‘z sin theta’ that angle along with that one.

And it would be in the direction of theta itself. So, this is the electric field pattern. From this you can find out, if you are interested, you can find out what could be the ‘H five’ component.

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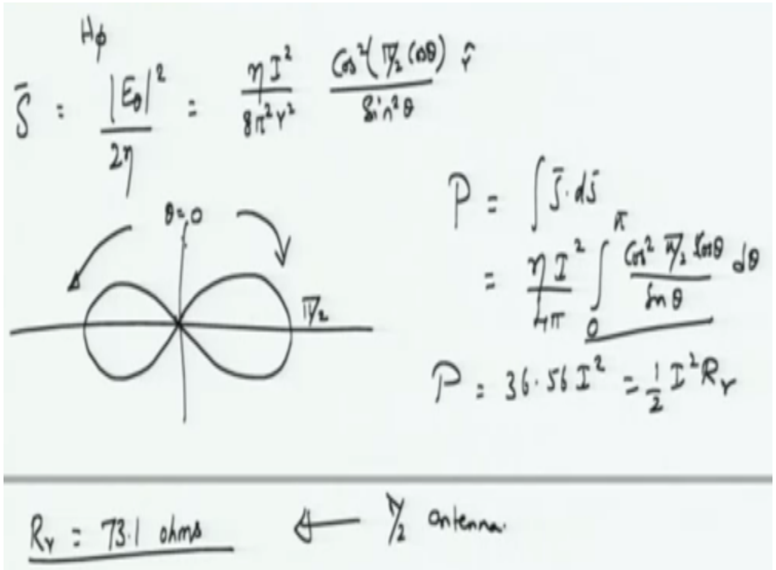

But you do not really need to do that one because the power density or the average power density, in order to obtain the average power density, you just need the magnitude of ‘E theta’ itself. So it could be ‘E theta square / two eta’. And what direction would the pointing vector be? It would be along the radial direction. So, this is given by ‘eta I square / 8 pi square r square, cos square of (pi / 2 cos theta) / sin square of theta’. And this would be along the radial direction.

Now what is the radiation pattern? Well, you see that this is actually ‘cos square (pi / 2 cos theta). At theta equal to zero, so at theta equal to zero, cos theta is one, but sin theta will not be equal to zero. So if you actually look at the radiation pattern, you would find that the pattern would look in this following way. So you have, this is one, but you have ‘cos square of pi / two’ which is also going to zero, so you can use L'Hospital's rule to find out what would be the value of ‘S’ at theta equal to zero.

And in terms of that, the power density at theta equal to zero is also equal to zero. But more importantly, you want to find out where this could be maximum. So, where this power direction will be maximum? Maximum would occur, when the numerator term would be equal to zero. And that would happen when theta is equal to ‘pi / 2’, ‘plus pi / 2’ or ‘minus pi / 2’.

So the radiation pattern, if you have to plot it as a polar axis, mark different values of theta, would actually start having zero at the center and then it would actually have a maximum at theta equal to ‘pi / 2’. So this is theta equal to pi / 2, this is theta equal to zero and this is pi / 2. You will get one more pattern up here. The elementary dipole antenna that we actually looked at, that also had a similar kind of a beam pattern.

So we looked at that one. And this is the beam pattern for the half wave dipole. One last thing about the half wave dipole. What would be the power radiated? The power would be integral of ‘S.ds’. And you can evaluate this integral, it turns out that the evaluation of the integral terms is not quite so simple. What you need to do is, use some numerical methods. So, you can do that and what you are left with is, this integral to evaluate. So, if you evaluate this integral, you will show that this would be equal to about 36.56 ‘I square’.

Assuming ‘I’ to be constant, theta as given this value, integrate this numerically to show that the power radiated would be 36.56 ‘I square’. But remember we also talked about the power and the radiation resistance of an antenna. The radiation resistance of an antenna was something that was an equivalent way of looking at an antenna as a circuit element. So, the power dissipated would be equal to ‘half I square R A’ where ‘R A’ is the radiation resistance of the antenna. ‘R A’ or R r’.

You know both will work, so we will use ‘R r’ because that is what we have used. So this is a way of thinking an antenna as circuit element. And this equivalent circuit element is that of a resistor. So cancel ‘I square’ on both sides to obtain ‘R’, the radiation resistance of the antenna as about 73.1ohms. So, this is something that might be asked in your interview. So you remember this value. This is for a ‘lambda / 2’ antenna. The radiation resistance is 73.1ohms.

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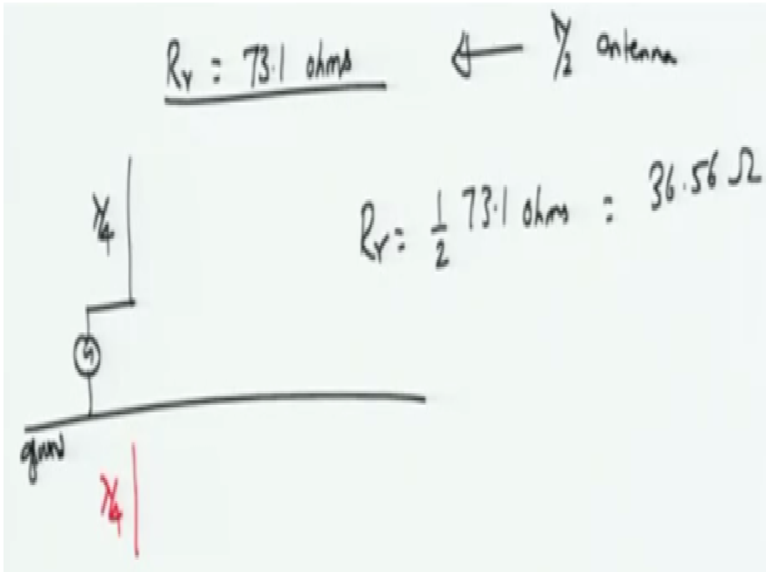

There is one curious antenna, called as the monopole antenna. In the monopole antenna, what you have is a ground plane, to which you are connecting your signal. And then the antenna itself has a length of ‘lambda / 4’. What would be the directional pattern for this antenna? In terms of that because of this antenna, there would essentially be an image antenna of the same length ‘lambda / 4’, having the current distribution in this way.

Now if you look at the characteristics, the characteristics would not change and whatever you have obtained from a ‘lambda / 2’ dipole antenna, you essentially obtained it from a ‘lambda / 4’ antenna. The only catch here is that that radiation resistance of this antenna, is actually half of the radiation resistance of the other ‘lambda / 2’ antenna. So this is again, this is actually equal to 36.56ohms. And do not really want to use an antenna with a lower radiation resistance because then its power handling capacity becomes very limited.

So, this is for the half dipole antenna. There are other exotic topics in antenna analysis, which we are not going to see. There is only one final thing, which I would like to leave you with, is what is called as freeze formula. This freeze formula derivation is not really important for us.

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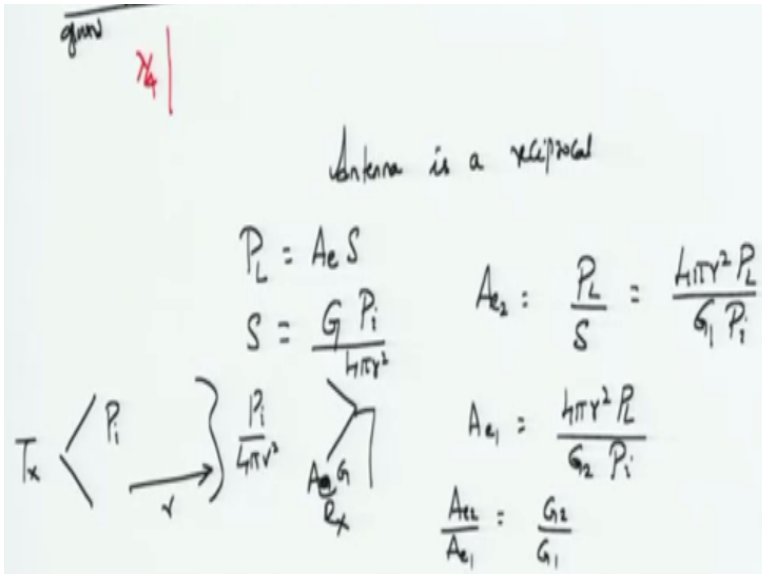

All that you have to remember is an antenna is a reciprocal device. A reciprocal device means that the antenna pattern behaves exactly the same, while if it is operating as a transmitting antenna or if it is operating as a receiving antenna. So, antenna is a reciprocal device. So, the same beam pattern gets excited, when you transmit some signals using an antenna or the same energy would be received by the antenna in the same beam pattern.

Moreover, if you assume that the antennas are placed far apart or the antenna in such a way that the incoming radiation is essentially plane wave, then the power density that the antenna sees would be multiplied by some effective area. So it is like a catchment area of the antenna, which will capture the incoming energy, assuming that the beam patterns are matched, it will capture the energy coming in from the, in the form of a plane wave.

And it would capture in what is called as an aperture area. So if, so this aperture area or the effective aperture area of an antenna is described by looking at its power that it is absorbing and assuming that the antenna is exerted by a plane wave or it generates a plain wave of power density ‘S’, then it is capturing ability is captured by effective area or the aperture area ‘A e’. If the antenna has a certain gain, then the signal with the power density that would be received, will also be scaled up similarly.

So, the power density 'S' will be equal to 'G times whatever the power density that you have'. So, if you have a transmitting antenna, so you have a transmitting antenna, which is putting out a total power of 'P I', the power density at a distance 'r' will be equal to 'P i / 4 pi r square. Now have a receiving antenna with an effective aperture 'A e' and having a gain 'G', we will assume that the antennas are placed in such a way that the gain 'G' is maximized.

Otherwise you need to consider some angles theta also. So, with that the power density that is heating on the receiving antenna is given by 'G 4 pi P i / 4 pi r square'. So, now if you look at the second antenna, so if the first antenna is the transmitter and the second antenna is the receiver, the effective aperture of the second antenna will be given by 'P L / S' that is the power density that it is receiving and what is the power that is actually delivered to the load.

And this would be given by '4 pi r square P L substitute for S' and you get the gain of the first antenna because this is the first antenna, which is shaping the gain, so 'G 1 into P i'. Now what you just have to do is, just to understand that the same antenna can act as a receiver or a transmitter. So if this acts as a receiver, then you substitute, in place of 'G 1' you substitute 'G 2', in place of 'A e 2' you substitute 'A e 1'.

So the effective aperture for the transmitter antenna or the antenna one, which is now acting as the receiver is given by '4 pi r square' its power that is dissipated in the load divided by the second antenna gain G two times the same power that I am putting out. So this is essentially the reciprocity theorem that is telling you. And if you now look at the ratio of the two aperture antennas 'A e 2' to 'A e 1', that ratio will be equal to the ratio of their respective gains G 2 / G 1. So with this knowledge of effective aperture, we are now ready to complete Friis transmission formula.

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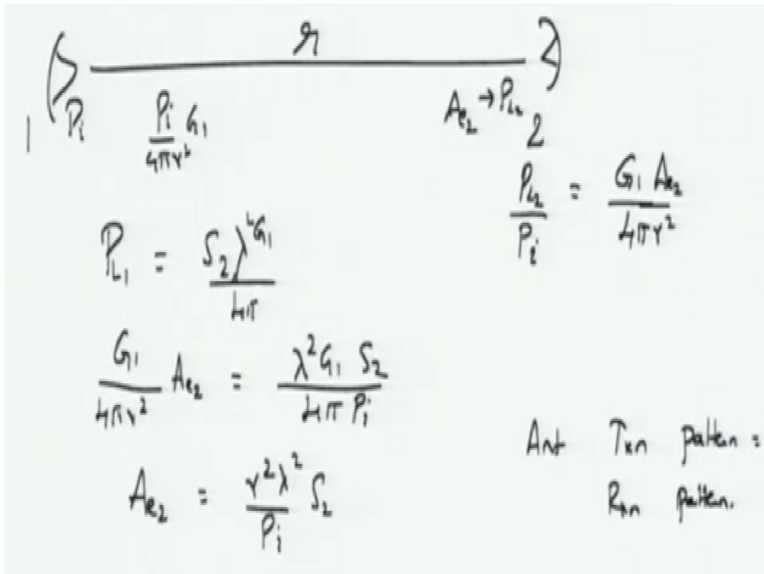

We have a transmitting antenna, we have a receiving antenna. There is a large distance between them, so that everything can be considered as a plane wave component. So, we know that the power that is received by the second antenna will be given by ‘power received by the second antenna P L 2 to whatever the power that is incident from the first antenna’ is given by ‘G 1, which is the gain of the first antenna, times the effective aperture of the second antenna’. So this is what it would actually capture from the second antenna.

It could be ‘G 1 / 4 pi r square, A 2’, which actually comes from this earlier expression for ‘A e 2’ itself. So go back to the earlier expression for ‘A e 2’ and then rearrange it in such a way that the power dissipated in the second antenna is given by ‘G 1’ gain of the first one divided by 4 pi r square into A e 2’.

Now, this power because the antenna is essentially acting as a reciprocal device, we can show that power absorbed by the first antenna would be equal to ‘the power density that is being produced by the second antenna times that G 1 divided by lambda square by four pi’. So this ‘lambda square / 4 pi’ actually comes from a slightly different expression. We are not going to derive that expression over here. But the basic idea is that, the first antenna transmit some power, which is ‘j p I’, there is a power density of ‘P i / 4 pi r square’.

It is transmitting with a gain ‘G 1’. This is getting intercepted by the second antenna with an effective aperture of ‘A e 2’ and gets converted, the power captured will be delivered to the load ‘P L 2’. So, with that and with ‘P L 1’ is given by the power that is delivered to this one is given by ‘G 1 A e 2 / 4 pi r square into P i’. Now with aperture two, what you get here is ‘G 1 / 4 pi r square, aperture A e 2’ is equal to ‘lambda square G 1 S 2 / 4 pi P i’.

So what this has established is that the antenna pattern is essentially reciprocal. So, what it shows is that when you have this, let us just complete this expression. So, since I know this, I know this, I know this result, I can rewrite ‘A e 2’. And I say ‘A e 2’ as ‘r square lambda square / P i times S 2. So, what this relation is simply telling me is that the antenna’s effective aperture depends on the angle theta, in the same way as it has been transmitted by the transmitted radiation.

Thus essentially telling you that antenna transmission pattern is exactly equal to its receiving pattern. And therefore, antenna is essentially a reciprocal device. Now, coming back to this free transmission formula, which we would want to be establish, we have seen that if you have an antenna one with the input power ‘P i’, it would provide a power density of ‘P i / 4 pi r square’. Now because the antenna has a certain gain ‘G 1e’, so this power density gets multiplied by gain ‘G 1’. And this power density is transmitted to the second antenna.

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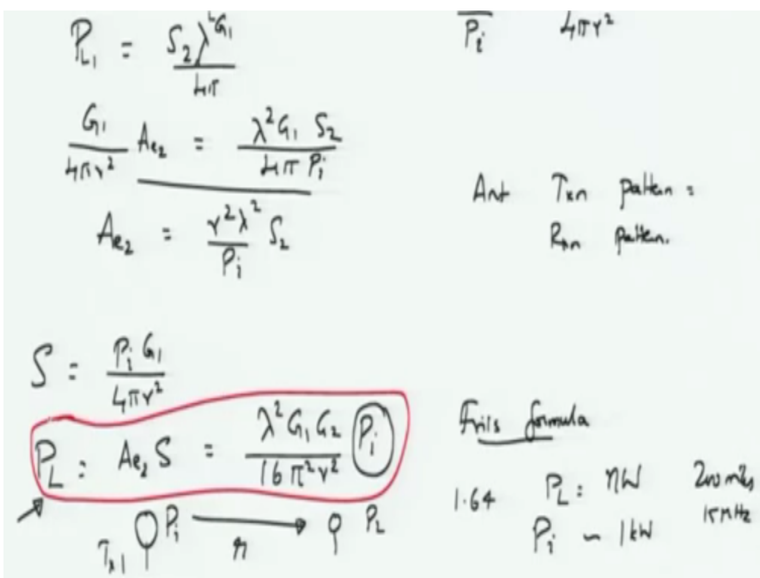

So at the second antenna, the power density that the second antenna is receiving is given by ‘P i G 1 / 4 pi r square’. This power is actually converted into the load power by the second antenna, not the entire power is converted, only the effective area times this ‘S’ is converted. So, because the effective area captures whatever the antenna’s ability to convert the incident power density into the load power.

So power dissipated in the second antenna load or the load connected to the second antenna, depends on the effective aperture of the second antenna times the power density that is incident on it. So substituting for the values of this one here, for the aperture ‘A e 2’ from this expression over here, what you see is, ‘lambda square G 1 G 2 / 16 pi square r square, P i’. This formula is known as Friis formula or Friis transmission formula

And it actually tells you that if you start with a power ‘P i’ of two antennas, whose gains are ‘G 1’ and ‘G 2’, the kind of power that was eventually available at the load or the output terminals of the antenna goes inversely proportional to ‘r square’, so it is ‘1 / r square, directly proportional to the product of the two antenna gains and also proportional to ‘lambda square’. This is actually used in estimating the range of power that is getting loss.

In fact, this kind of formula or the modified types of these formulas are used to estimate the path loss. From one transmitter antenna to the receiver antenna, if the power obtained is ‘P L’ and the incident power is ‘P i’, the kind of reduction from ‘P i’ to ‘P L’ and the range over which this reduction happens, both can be calculated by Friis formula. For example, the gain of a half way dipole would turn out to be 1.64.

From the Friis formula, if you have to put in the numbers, you would have power delivered to the load in the order of Nano watt, while your input power ‘P i’ happens to be around one kilo watt. So this of course is assumed that the antennas are separated around two hundred miles and working at around fifteen megahertz. The numbers are not really important. But what you have to understand is that the power delivered to the load is inversely proportional to ‘r square’.