Principles of Communication Systems - Part II

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Lecture - 02
Spectrum of Transmitted Digital Communication Signal, Wide Sense Stationarity

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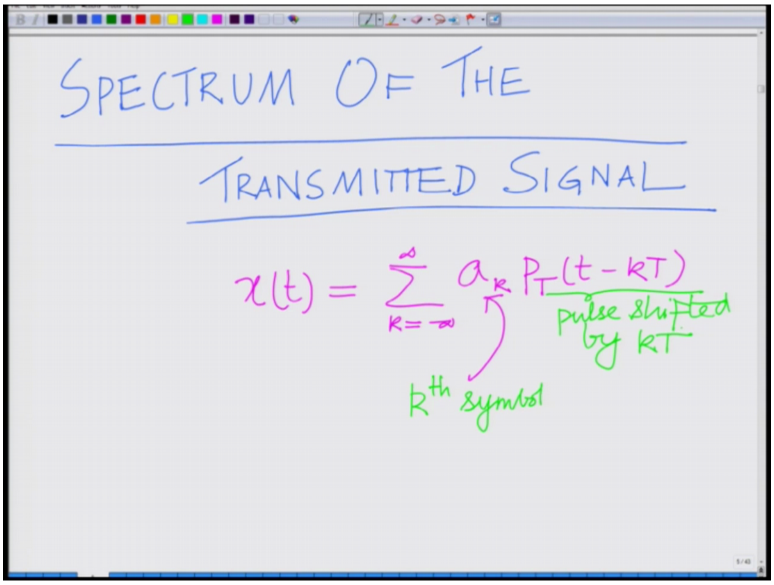

Hello, welcome to another module in this massive open online course. In this module, let us start looking at the spectrum on the transmitted signal in the digital communication signal. So, what we want to look at is for a digital communication system, we want to look at the spectrum of a or spectrum of the transmitted signal. Now, we have seen that the transmitted signal x t equals summation well k equals minus infinity to infinity a k P T minus k T, a k is the well k th symbol k th digital modulation symbol. This is pulse shifted by k T, where P T is basically it is a pulse shaping filter correct is a response of the pulse shaping filter.

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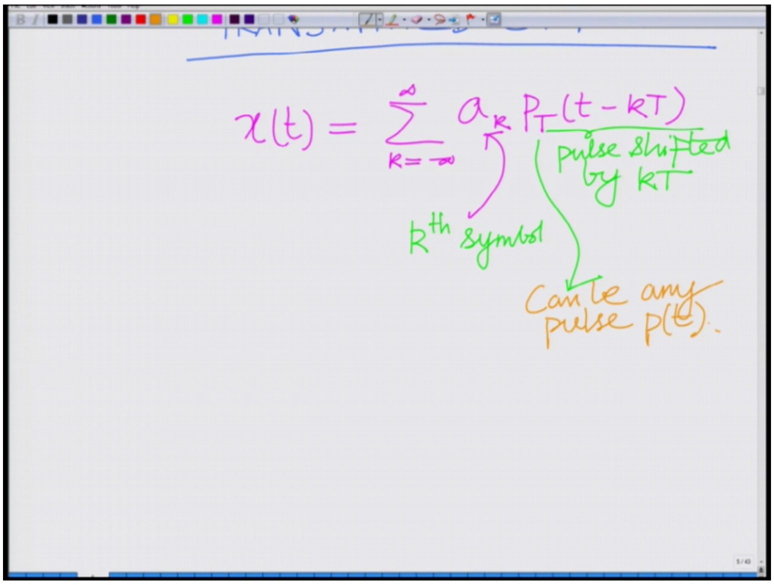

And in general, P T can be any pulses need not be necessarily P T this can be any pulse all right. Although I am considering this specifically example of P sub capital T t, which is the rectangular pulse this can be any pulse.

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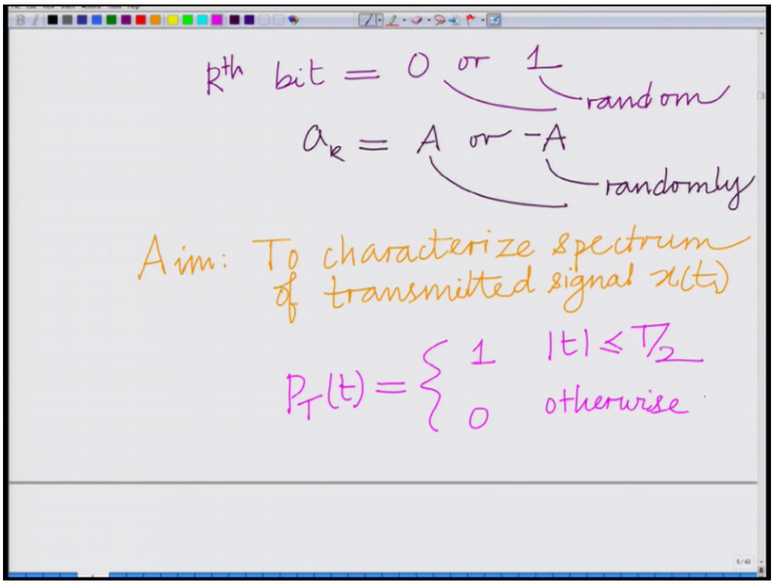

Now, the k th bit we have seen the k th symbol, now k th bit equals 0 or 1 information bit can be 0 or 1, and this is randomly all right. So, it can randomly take either 0 or 1. So, which implies a k equals remember 0 is map to a you know what B P S K binary phase shift k system or 1 is map to minus A correct, 0 is map to a, one is one. So, it takes a k takes A or minus A randomly. So, depending on if the bit is 0 or 1 randomly the corresponding symbol is A or minus A randomly. Now, we have to we want to characterize the spectrum of the transmitted signal x t. So, we want so let us put it, this; your aim is to characterize transmitted signal x t. Now, P T remember P T of t is the pulse 1 although we can choose any pulse where choosing this in this example are 0, 1 for mod t less than equal to capital T by 2 or 0 otherwise remember P t or t we have looked at several times.

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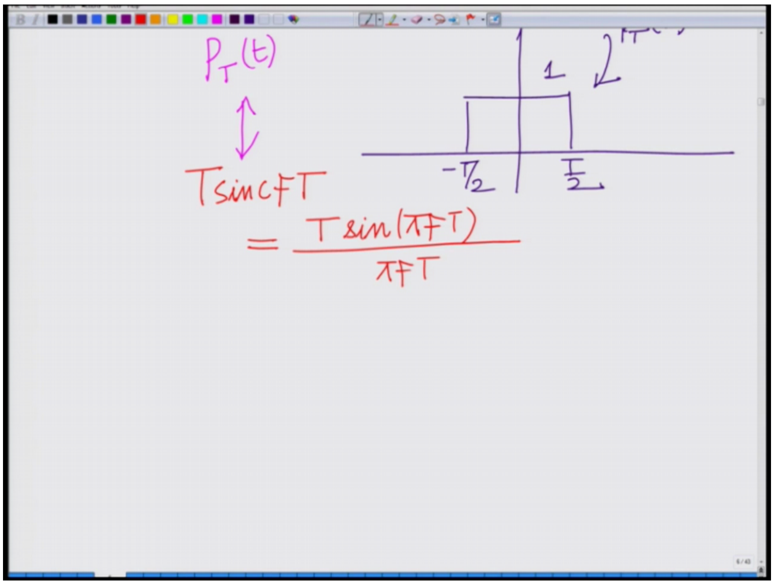

P T over t pulse of height 1 of minus T by 2 to T by 2, so this is your one of the pulses P sub T of t that can be chosen. And remember this P T of t P sub T of t has the Fourier transform we know from the theory of Fourier transform that is the Fourier transform of this is T sinc F T. We will see this spectrum of this as a role to play in the spectrum, naturally the spectrum of this pulse shape has a role to play in the spectrum of the transmitted signal x t. So, this is sin c T sin c F T which is T sin pi F T divided by pi F T this we know from the properties of the transform and also from the previous course where we have explicitly derived the spectrum of this pulse.

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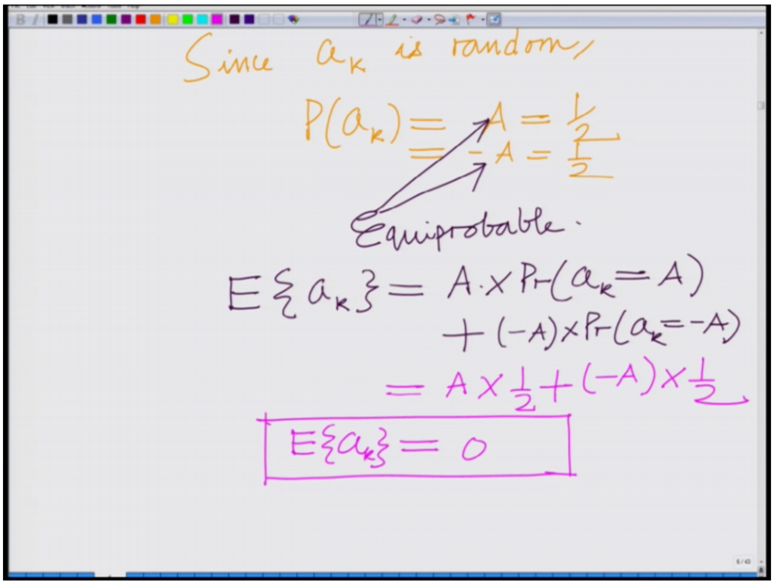

Now, we will invoke our knowledge of the properties of probability. So, let since a k since a k is random, we will set probability a k equals A equals half, and probability a k equals minus A equals half. So, we are assuming this a k and A are equiprobable, equiprobable is probability half. There are 2 symbols of they are equiprobable probability is half. So, these are equiprobable. So, therefore, now when we use our knowledge of probability if you calculate the expected value of a k that will be well half well that will be A times probability of a k equals A plus minus A times probability a k equals minus A, which is equal to probability a k equals is A x half; half plus minus A into half is 0. So, expected value of a k this is equal to the average value of a k equal to 0, this has an interpreter. So, we are saying a k takes values plus A or minus A with equal probability, probability half each. So, if you calculate the expected value of a k what you can observe is that the expected value of a k is 0.

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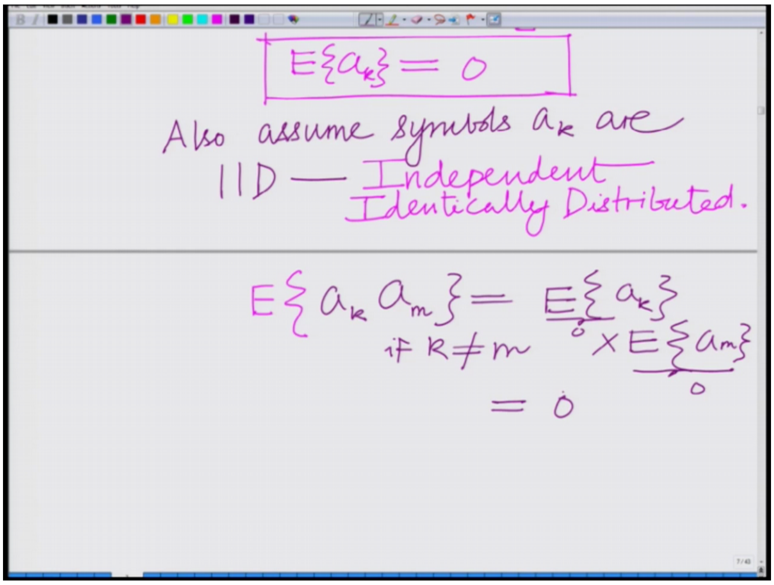

Now, we will also assume that the symbols are independently distributed. Also assume symbols a k are IID this means independent identically distributed. Identical distribution means each has takes minus A or plus A with probability half. Independent means basically these random variables are independent, and it follows from independence that they are uncorrelated. Uncorrelated is a property when the random variables are independent then it follows that they are uncorrelated that is if you would considered 2 symbols a k and a m expected value of a k into a m if k not equal to m this is equal to expected value of a k into expected value of a m. And observe that both these quantities are 0, therefore this quantity is 0. And this are basically this is uncorrelated. So, if the expected value of a k a m is 0 is termed as uncorrelated. So, a k and a m are uncorrelated we are assuming that they are independent of course, from independence it is follows that they are uncorrelated which means expected value of a k into a m is 0 if k is not equal to m.

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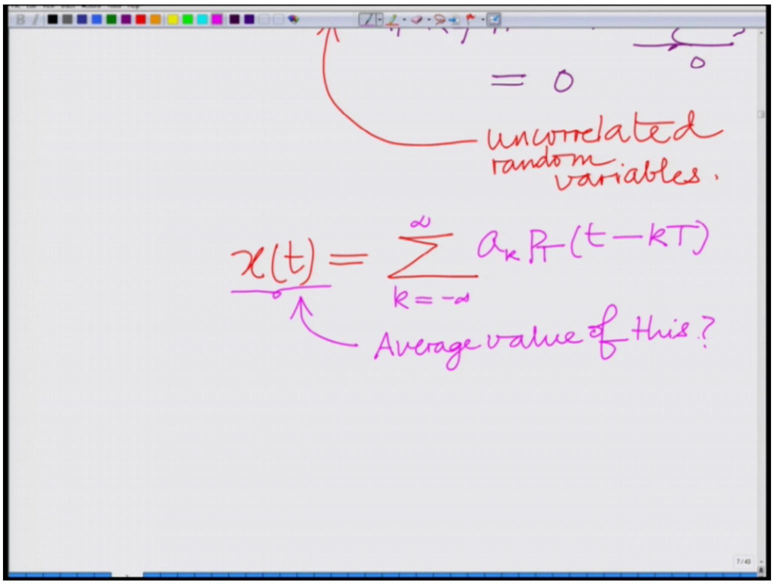

Now, if you look at now let us go back to our transmitted signal x t equals summation well k equal to minus infinity to infinity a k P T minus k T. Now, if I take the expected value we want to ask the question what is the average value of this, what is the average value of this, we want to ask that question.

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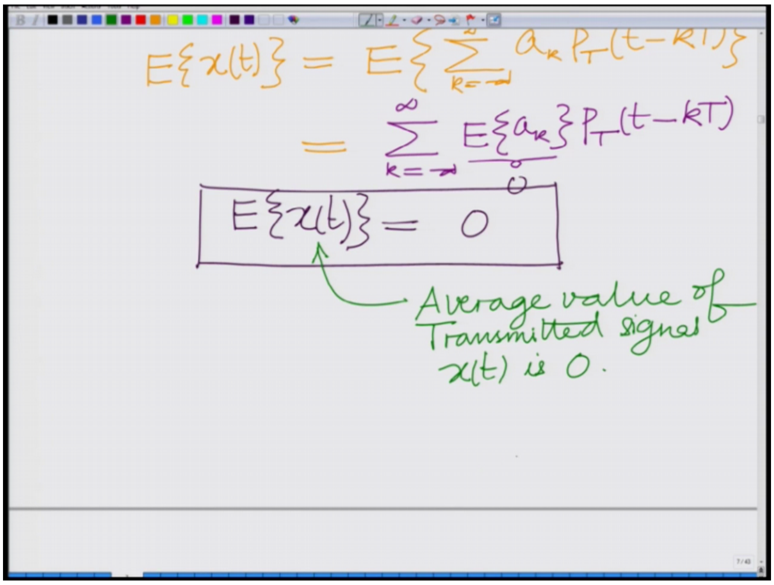

And you can see that expected value of a k if you look at this expected value of x t the transmitted signal is well expected value of summation k equal to minus infinity to infinity a k P T minus k T, which is expected value of well taking the expectation operator is linear. So, expected value of a sum is sum of the expected values, so that is expected value of well a k because P T is a deterministic pulse it is not random. So, the expectation applies only to a k and we know expected value of a k is 0. So, this is sum of 0 which is 0. So, we have this interesting property that if you look at the expected value or the average value of x t is 0 that is average value of transmitted signal is 0, which results from the property that the average value of the transmitted symbols is 0. The average value of each a k is 0 which implies that the average value of transmitted signal is 0, average value of the transmitted signal x t is 0.

Now, let us find the spectra. We want to find the spectrum. So, let us employ a conventional approach. Whenever we want to find the spectrum of a signal, we use the Fourier transform all right. When you want to find the spectrum of a continuous signal we use the Fourier transform, but does it work well when we want to find the spectrum of a random signal. Let us try to first explore that.

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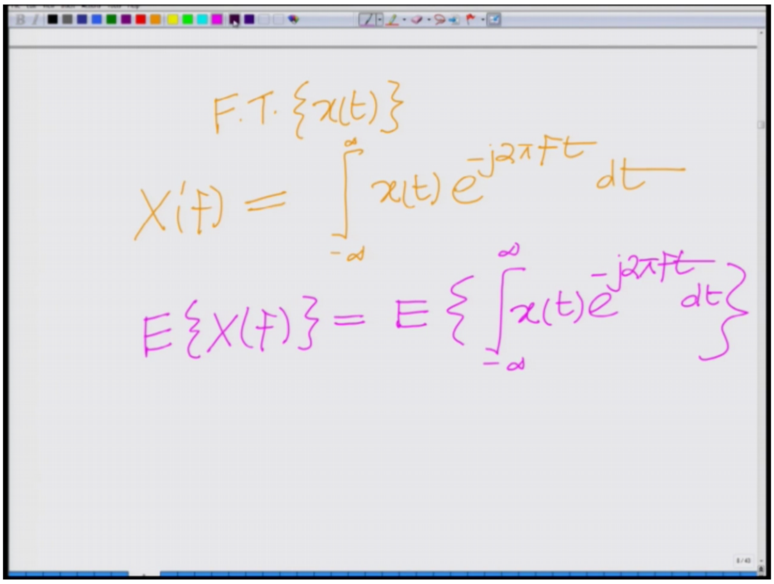

So, if you look at the Fourier transform of x t which is a random signal, you will observe that this is equal to or if I denote this by x f is equal to minus infinity to infinity x t correct e to the power of minus j 2 pi F T d t. And if I therefore, if I take the expected value of X F this is again expected value of minus infinity to infinity x t e power minus j 2 pi F t d t.

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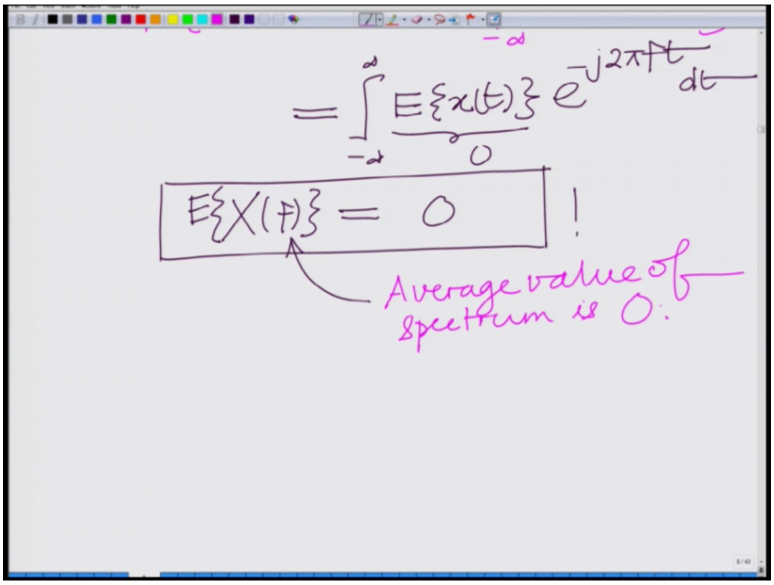

Taking the expectation operator inside since the expectation operator is linear, I can take it inside the integral this is integral minus infinity to infinity expected value of x t e to the power of minus j 2 pi F t d t expected value of x t is 0, this is 0. So, which means that expected value of the spectrum we have very interesting property expected value of the average value of the spectrum is 0. So, very surprising if we look at this we can see that the average value of the spectrum is 0 or average value of the Fourier transform the transmitter signal is 0.

So, the average value of the spectrum is 0, which is very interesting property does that mean that basically there is no spectrum or there is it does not have the transmitted signal does not occupy any spectrum. If that were true then we do not need any spectrum to transmit a digital modulator or a digitally modulated signal x t, which is really obviously, it is not true. And it is in fact, observed because we need spectrum to transmit any signal. So, the fault of the fallacy lies in the way in which we are calculating the spectrum of this random signal.

So, what we have to ensure, so what we have to do is to develop a technique for appropriately calculating the spectrum of this random signal all right. So, our technique is erroneous and we have to develop or use a technique to calculate the spectral or the power in the various spectral bands of a random signal and that is given by the power spectral density of the signal.

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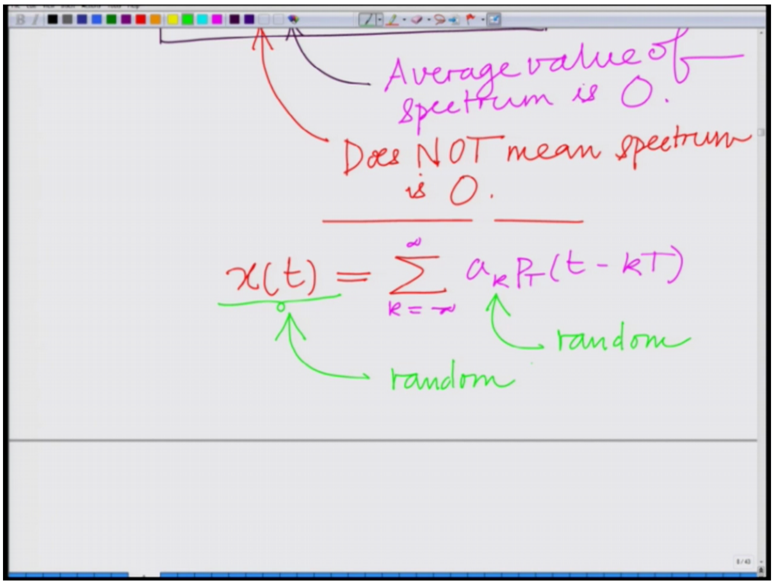

So, what we want to calculate rather than a signal, so we want to I want to emphasize here again this does not mean spectrum is 0. So, the right way and error lies in and therefore, the right way now x t is a random signal. And therefore, if you look at this x t equals summation k equal to minus infinity a k P t minus k T each a k is random, this implies x t is also a random quantity, correct.

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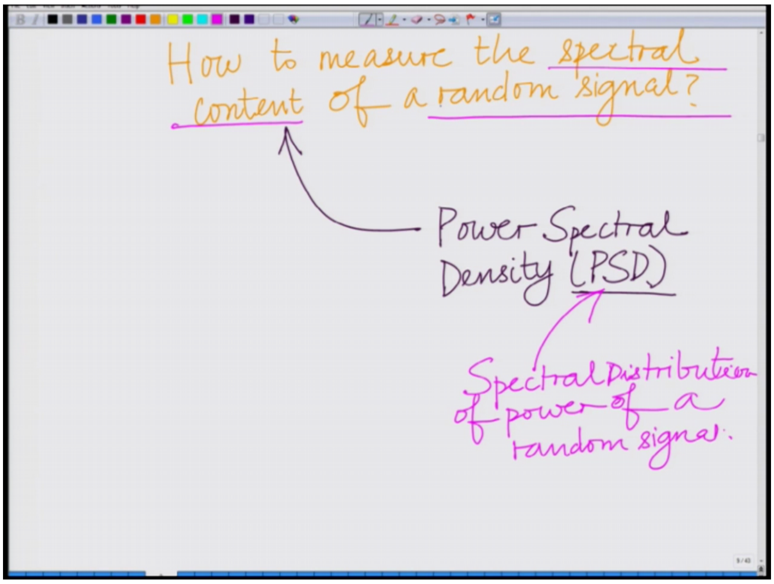

Now, how to measure this, we want to ask the question how to measure the spectral content or the spectrum how to measure the spectral content or the spectrum of a random the spectral content of a random signal, and the answer is through the power spectral density. So, you want to do this through the power spectral density. To do this we want to use the concept of power spectral density or what is termed as PSD. So, the PSD or the power spectral density gives us the spectral distribution of this gives us the spectral distribution of power of a random signal.

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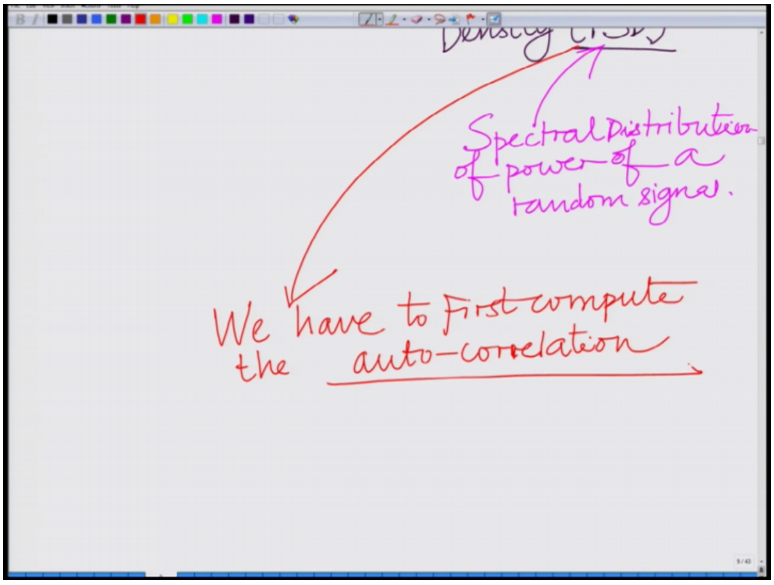

However, first we have to compute for the power spectral density we have to first compute the autocorrelation. To compute the power spectral density, you must be familiar with this concept of autocorrelation for the power spectral density is the Fourier transform the autocorrelation.

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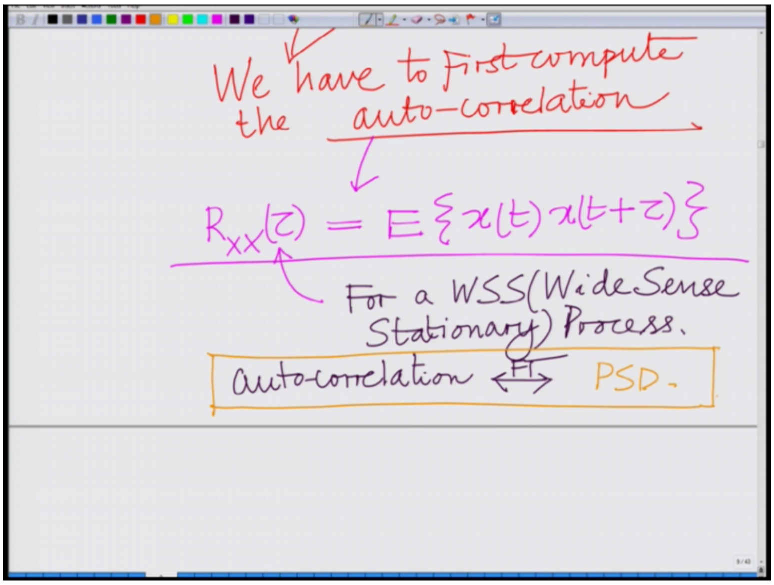

And this is defined as autocorrelation is expected value of x t into x t plus tau. For wide sense stationary process I will describe this again later this is r x sources of tau for a WSS process, for a WSS that is for a wide sense stationary. So, first we want to compute the autocorrelation. Autocorrelation, if you take the Fourier transform pair that gives us the PSD - the power spectral density. So, what we want to do the approach we want to take rather than simply taking the Fourier transform taking the average value of the Fourier transform because that does not seem to yield anything sensible. What you want to do is to model this first of all it is a random signal time varying random quantity which is the random process correct. Therefore, we want to compute the power spectral density or the spectral distribution of the power. To do that first we have to start with the autocorrelation function alright and we will do this in the subsequent module.

Thank you very much.