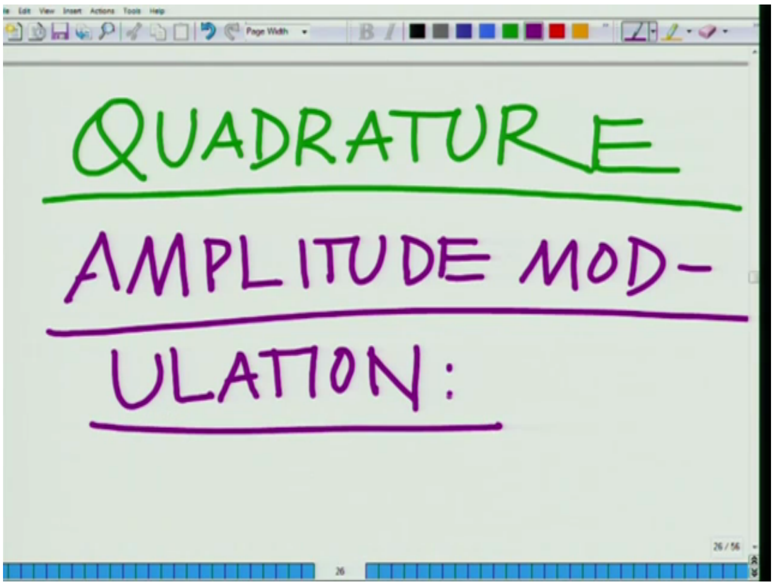
Principles of Communication Systems – Part II
Prof. Aditya K. Jagannatham
Department of Electrical Engineering
Indian Institute of Technology, Kanpur

Lecture – 22
M-ary QAM (Quadrature Amplitude Modulation) – Part I, Introduction, Transmitted Waveform, Average Symbol Energy

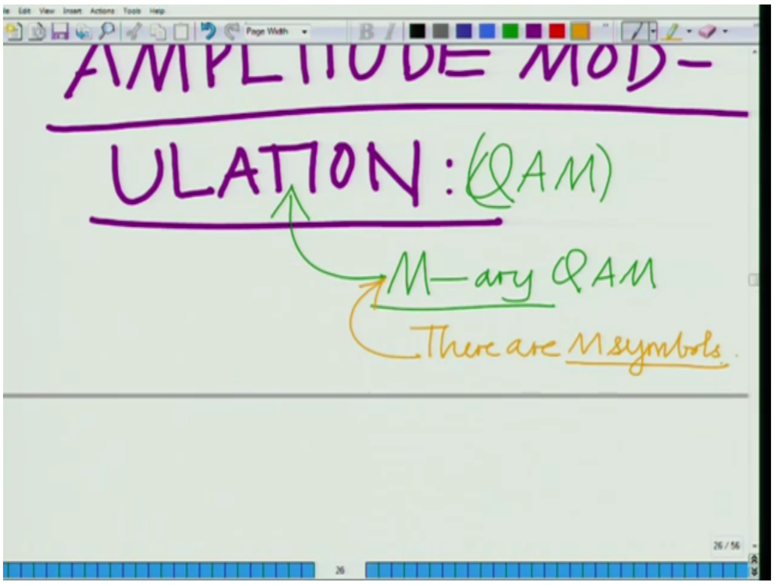
Hello. Welcome to another module in this massive open online course. So, let us consider another digital modulation scheme. In fact, which is a very general and powerful on digital modulation scheme which is one of the most commonly used digital modulation scheme which is used in all the most all the modern wireless communication standards such as 3 G 4 G and also a potentially future and future in the future in 5 g wireless communication standards, alright. This is known as QAM or quadrature amplitude modulation and to some extent is a generalization of all the schemes that we have seen before that is your BPSK amplitude shift keying a PAM also. So, QAM is a general version of all these things.

(Refer Slide Time: 00:58)



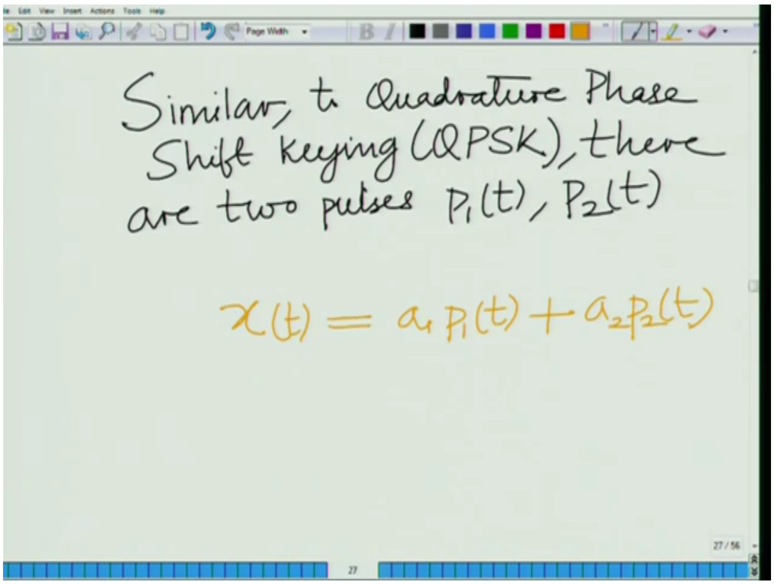
So, let us start looking at quadrature. So, let us start looking at quadrature amplitude modulation or this is basically this is also known as QAM.

(Refer Slide Time: 01:38)



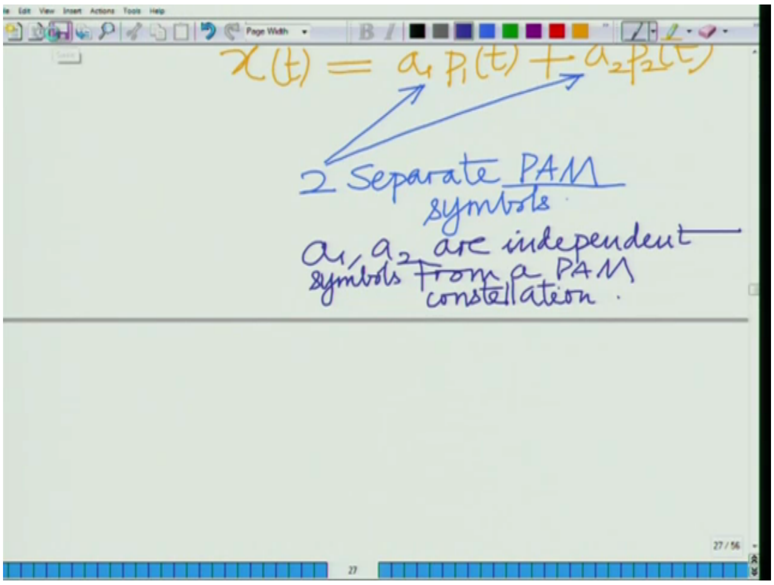
And to be a bit more precise this is known as M-ary QAM, M-ary quadrature amplitude modulation basically this implies that it has there are M symbols as you seen M-ary PAM in M-ary QAM there are M symbols.

(Refer Slide Time: 02:08)



Now, similar to quadrature phase shift keying as I said this is generalization of QPSK similar to quadrature phase shift keying, similar to quadrature phase shift keying, there are 2 pulses P 1 and P 2, P 2 t which are orthonormal. And we have X t equals a 1 times P 1 t plus a 2 times P 2 t.

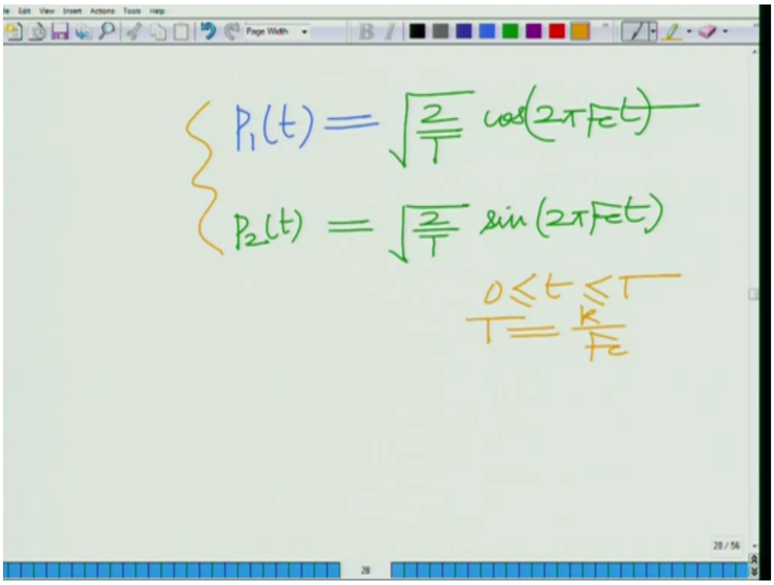
(Refer Slide Time: 03:12)



So, and now a 1 and now in QPSK each a 1, a 2 are 2 BPSK symbols alright. a 1 a 2 element of plus or belongs to either plus or minus P here a 1 and a 2 can be 2 separate PAM symbols that is the whole idea there can be 2 separate, there can be 2 separate PAM symbols.

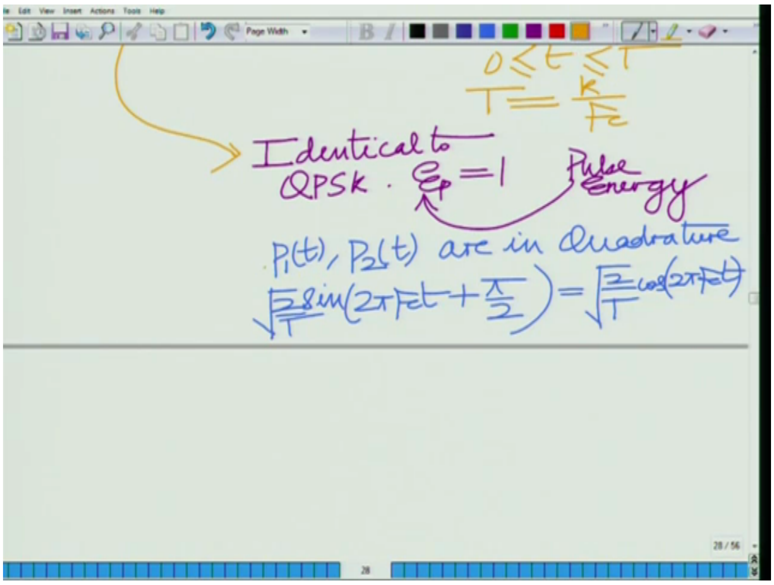
So, a 1 and a 2 are drawn from a PAM constellation independently independent symbol drawn from a PAM constellation, PAM constellation these are independent symbols from a PAM constellation. So, a 1; so, we have pulse P 1 t on which you are transmitting symbol a 1 pulse P 2 t, in which you are transmitting symbol a 2. Both a 1 and a 2 are individual PAM symbol. So, now, in quadrature phase shift keying QPSK a 1 and a 2 are individual BPSK, binary phase shift keying that is a 1 is plus or minus A; a 2 is plus or minus A here we are generalizing this concept. So, a 1 can belong to a PAM pulse amplitude modulation, a 2 can belong to PAM and a 1 and a 2 are independent symbols. So, we are transmitting 2 PAM symbols per every waveform X t, 2 PAM symbols for every waveform X t.

(Refer Slide Time: 04:48)



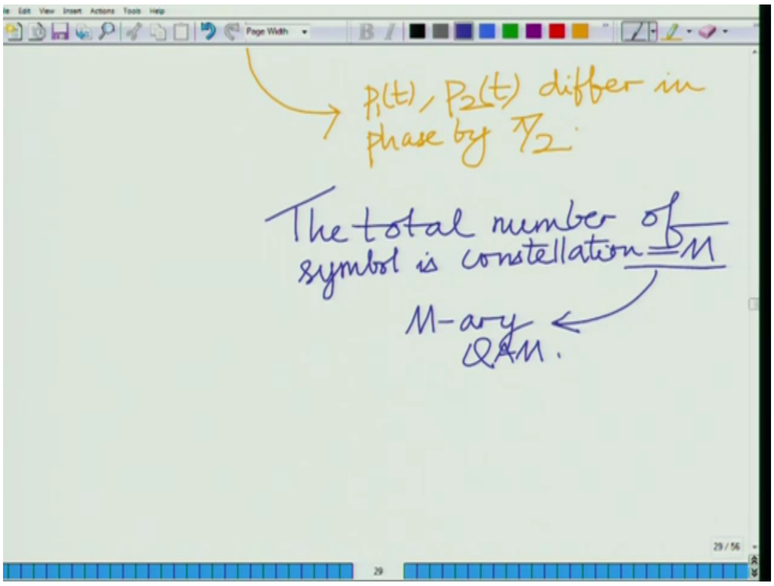
Now, as we have seen in QPSK our P 1 t, this is similar to QPSK that is unit energy pulses, to square root of 2 over t cosine 2 pi F c t, plus P 2 t is square root of 2 over t sin 2 pi F c t, 0 less than or equal to t less than or equal to t, and the t is contain that duration of the symbol contains an integer number of cycles that is the same P 1 P 2 t are unit energy that is unit energy orthogonal pulses that is a set of orthonormal pulses.

(Refer Slide Time: 05:38)



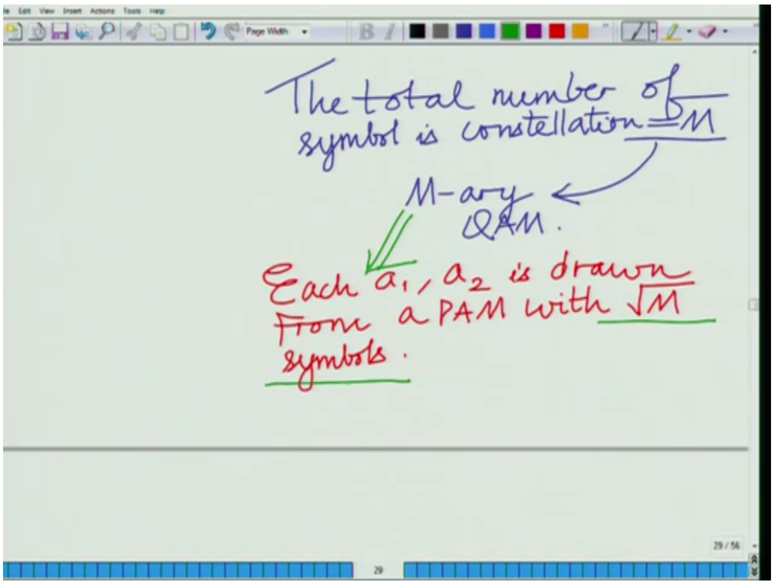
This is similar to in fact, identical. In fact, not even similar this is simply identical, that is these are in quadrature 2 QPSK E p equals 1. Remember this is your pulse energy the pulse energy normalized to 1, and the basis P 1 P 2 t are in quadrature P 1 t, P 2 t are in quadrature. Because remember sin 2 pi F c t, plus pi by 2 root 2 over t sin 2 pi F c t plus pi by 2 equals root 2 over t cosine 2 pi F c t.

(Refer Slide Time: 06:51)



So, differ by, what is it mean to say they are in quadrature we have seen this before, pulses P 1 t P 2 t differ in phase by pi 2 pi by 2, differ in P 1 to P 1, t P 2 t differ in phase by pi by 2. So, the pulses are in quadrature. And the phase difference between these pulses is pi by 2 implies they are in quadrature. Now let the total number of symbols in the QAM be m. So, we are saying this is an M-ary QAM total number of symbols. So, the total number of symbols in fact, the total number of symbols in this constellation is equal to M, that is why it is known as we have said that that is the reason it is known as M-ary, QAM M-ary QAM means total number of symbols in the QAM constellation is M which means each now this implies that each.

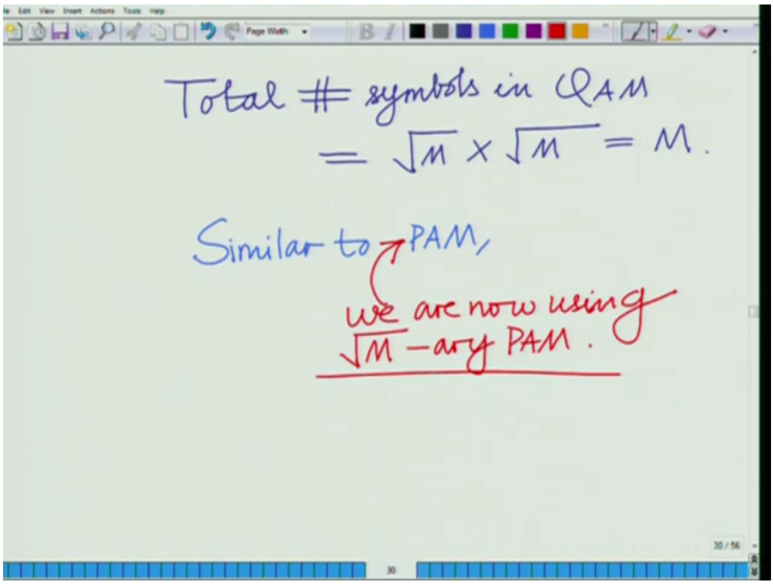
(Refer Slide Time: 08:25)



Now, this is something that is you have to understand each A 1, a 2 is drawn from a PAM with square root of M, this is a key observation. So, e this is each a 1 and a 2 is drawn from a PAM that is PAM constellation PAM symbol set with square root of M symbols. The reason being that a 1 belongs to a PAM constellation with square root of M symbols that is a 1 can be chosen in square root of M way M ways a 2 belongs to a PAM constellation is square root of M symbols a 2 can be chosen in square root of M ways.

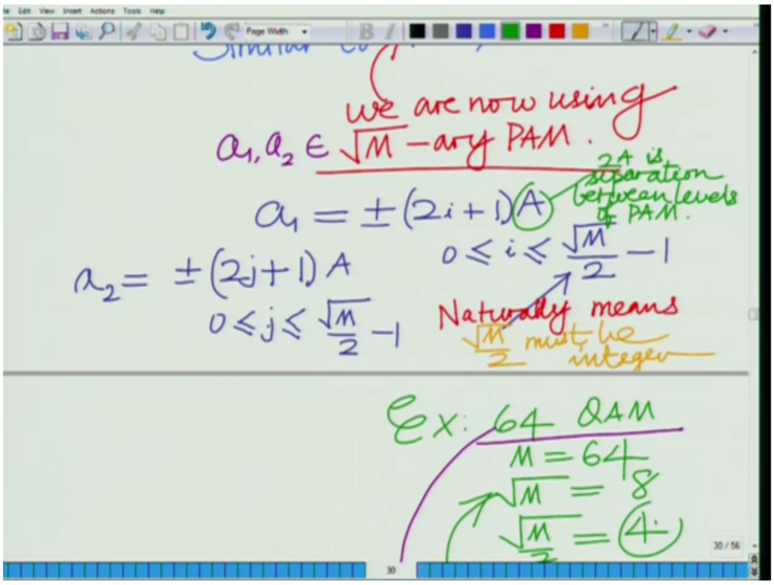
So, the total QAM symbol can be chosen in square root of M into square root of M that is M ways. So, the total number of symbols in the QAM then becomes square root of M into square root of M this is similar to QPSK for instance a 1 is in QPSK, a 1 is chosen from plus or minus A that is a set containing 2 symbols BPSK a 2 is chosen from a BPSK set containing 2. So, the total number of symbols constellation points in QPSK is 2 into 2 that is 4, similarly in M-ary QAM we are choosing a 1 and a 2 from PAM from PAM constellation with square root of M symbols.

(Refer Slide Time: 10:06)



So, therefore, what happens is total number of symbols in QAM, this is equal to square root of M into square root of M which is basically simply equal to M. Now similar to what we have described over PAM, except now we are using square root of M-ary PAM not M-ary PAM, note we are now using. So, everything remains the same except M-ary PAM becomes square root. So, M-ary QAM can be thought of as comprise as being form. So, M-ary QAM can be thought of as being formed from 2 independent square root of M-ary PAM constellations alright. So, what PAM is going to be now a square root of M-ary PAM. That is each a 1 a 2 belongs to a square root of M-ary PAM that is the interesting aspect.

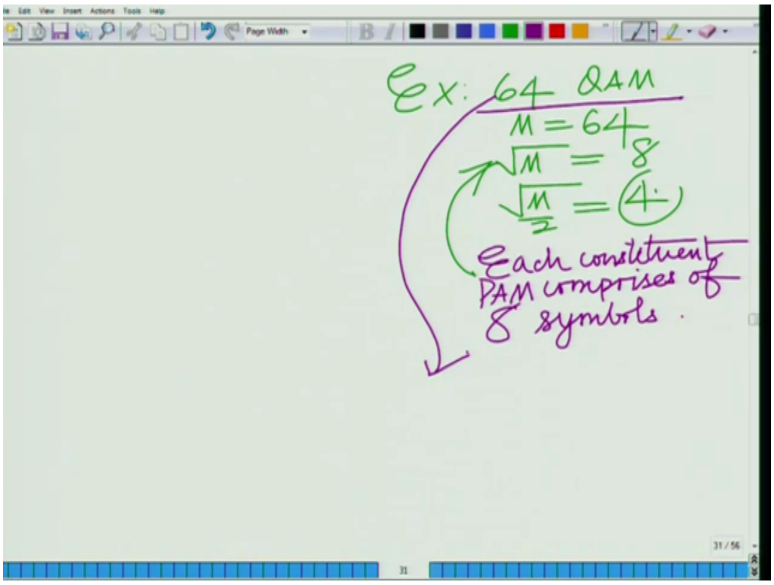
(Refer Slide Time: 11:29)



So, a 1 comma a 2 belong to a square root of M-ary PAM, which means that a 1 equals plus or minus 2 i plus 1 0 less than or equal to i, remember in M-ary PAM they have 0 less than or i, i less than or equal to M by 2 minus 1 this will be equal to square root. So, naturally I have to replace M by square root of m. So, this will be square root of M by 2 minus 1.

So, naturally since we are considering 0 less than or equal to i less than or equal to square root of M over 2 this means that this naturally means that remember naturally means you cannot chose any value of M square root of M over 2 must be an integer. Naturally means that square root of M over 2, naturally means is square root of M over 2 must be an integer.

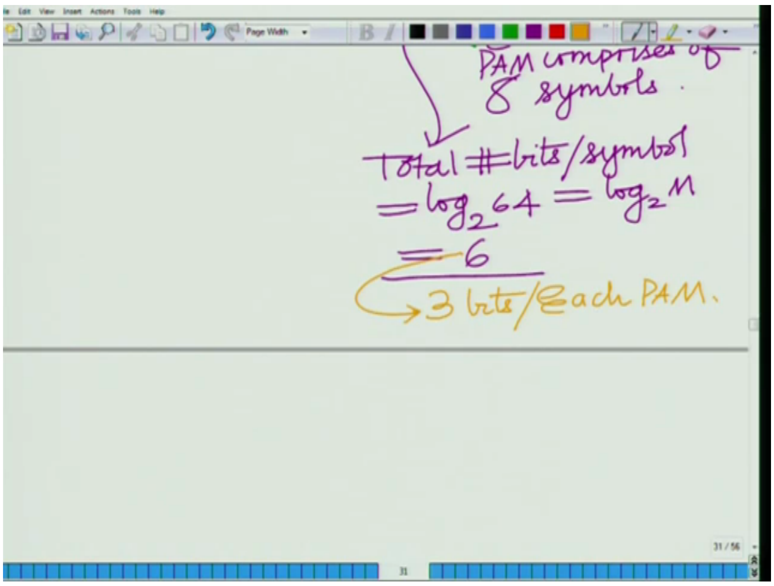
(Refer Slide Time: 12:38)



For example we have 64 QAM, 64 QAM that is M equal to 64 square root of M equal to 8, square root of M by 2 equal to 4 which is an integer.

So, each PAM consists of 8 symbols each constituent pam. So, each constituent PAM comprises of 8 symbols in 64 QAM. And naturally total number of course, we have not seen of course, it goes without same total number of bits per symbol.

(Refer Slide Time: 13:39)

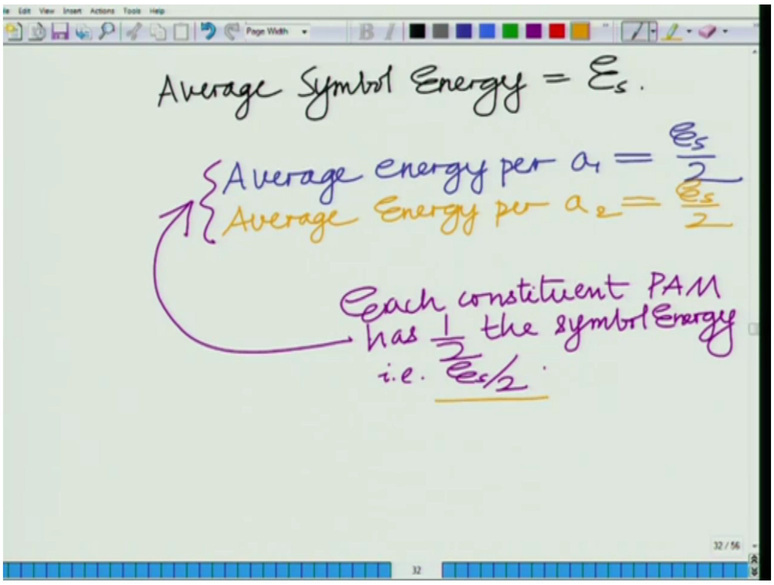


In this a QAM 64 QAM total number of bits per symbol equals to log to the base 2 M that is log to the base 2 64, this is basically log 2 to the base log M, to the base 2, which is basically 6. And you can see this corresponds to 3 bits per each PAM which is 8 symbols log a to the base 2 is 3. So, 3 bits per each constituent PAM, so, the infest component PAM, that is along P 1 t carries 3 bits, PAM along the quadrature sin 2 pi F c t pulse square root of 2 over t sin 2 pi F c t carries 3 bits. Total number of bits is 6 bits per symbol in the 64 bit QAM. So, that is the point that we are trying to make.

Now, similarly let us complete this discussion over here, a 2 is also plus or minus, now 2 j plus 1, a where a is remember a is again the same 0 less than or equal to square root of M by 2. Now a is similar to before where 2 a is separation is the separation or distance between levels separation between levels of the PAM successive levels of the PAM is 2 a. So, a 1 is plus or minus 2 i plus 1 a; a 2 is plus or minus 2 j plus 1 a and both i and j lie between 0 and square root of M over 2 minus 1. So, that is the point.

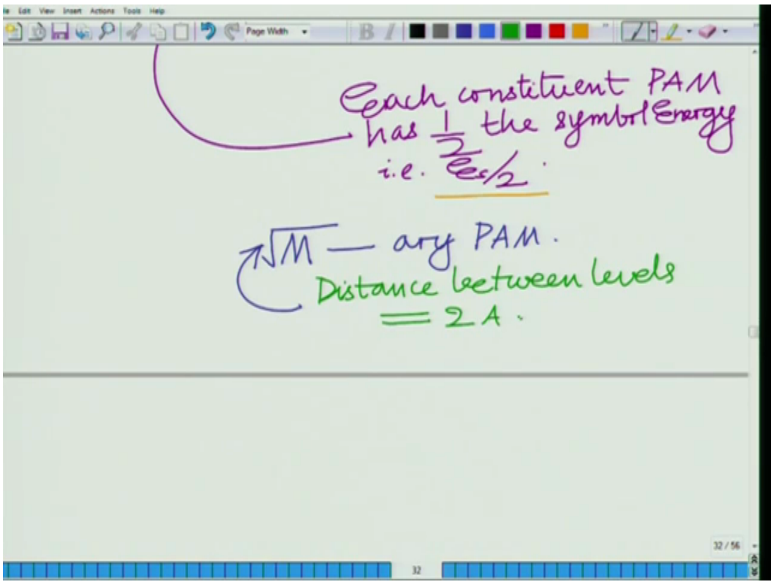
Now, further if average symbol now let us find again the value of A remember, we have to keep the average symbol energy constant. So, let us say average symbol energy is Es for the M-ary QAM alright now what is the corresponding value of a.

(Refer Slide Time: 15:59)



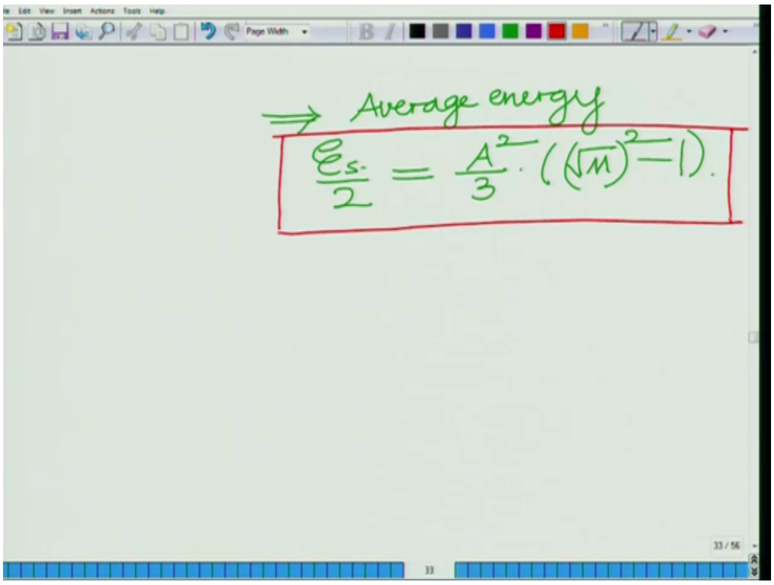
Now, let us say, let us say average symbol energy equal to Es which implies average energy now there are 2 QAM. So, each shares half of the energy. So, average energy per a 1 equals Es by 2, average energy per a 2 equals Es by 2. So, we say each QAM, each constituent PAM, each constituent PAM has half, the each constituent PAM has half the symbol energy that is Es by.

(Refer Slide Time: 17:25)



Now, again going back to our PAM, now remember we have M-ary PAM or square root of M-ary PAM which means an difference between levels is 2 A, distance between levels is 2 a which means average energy, equals well a square by 3 times square root of M square minus 1.

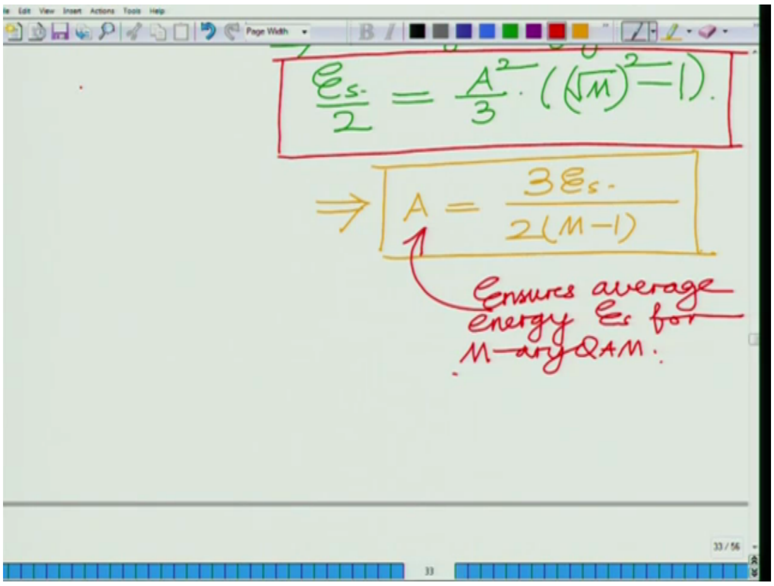
(Refer Slide Time: 17:51)



Now remember when you had M-ary PAM the energy was a square over 3 times M square minus 1; however, here we have a square root of M-ary PAM. Therefore, the average energy per PAM is A square over each constituent PAM is A square over 3 times square root of M square minus 1 square root of M whole square minus square root of M square minus 1, which is basically M minus 1 and this should be equal to the an average energy per PAM which is Es over 2.

So, we have Es over 2. So, this is a key step average energy per constituent PAM is Es over 2 and each PAM is only a square root of M PAM. So, therefore, we have this equation now if you look at if you do not remember this derivation, you can look at the derivation we have done for the M-ary PAM, there we had Es equal to square root a Es is equal to a square over 3 times M square minus 1.

(Refer Slide Time: 19:15)



Here we have Es over 2 equals a square by 3 into square root of M square minus 1 which basically implies Es is equal to well this basically implies or the other way round. Rather a s or rather a is equal to 3 Es by 2 M minus 1 and this is an important.

So, the amplitude a or the distance 2 a is such that a for each square root of M-ary PAM a for each square root of M-ary PAM constituent square root of M-ary PAM is chosen as 3 Es by 2 3 Es by 2 M minus 1. So, this ensures remember what is the constraint here ensures average energy Es for average energy Es for M-ary for the M-ary QAM. So, alright so, what we have done here is basically we have introduced a different modulation which is generalized version of PAM. In fact, for that matter generalization of several scheme VPSK, QPSK is generalization of BPSK PAM is in a generalization of BPSK and QAM basically is a generalization of all these such modulation scheme.

So, it contains a combination of 2 PAMs there are 2 constituent PAM constellation pulse amplitude modulation constitution. Each of containing square root of M symbols therefore, that QAM is M-ary, alright. So, M-ary QAM is form from 2 constituents square root M-ary PAMs, each transmitted on one is transmitted on the cosine 2 pi F c t component and the other is transmitted on the sin 2 pi F c t pulse both of the pulses are in quadrature. So, it is quadrature amplitude modulation all right, previously we have pulse amplitude modulation.

Now, we are using 2 pulses which are in quadrature. So, it is quadrature amplitude modulation each constituent PAM is square root of M symbols. And what we have derived is basically to ensure an each constituent PAM has a average symbol energy Es by 2 that ensures energy Es for the a M-ary QAM. And then we have derived a relation for the for the constant A all right correspond to each constituent A square root M-ary PAM such that the average symbol energy is Es for the overall QAM constellation.

So, we will stop here and in the next module you will look at what is a appropriate receiver the processing at the receiver, and the resulting bit error rate and symbol error rate and also the this overall symbol error rate for the QAM constellation.

Thank you very much.