Principles of Communication Systems – Part II
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Lecture – 25
M-ary PSK (Phase Shift Keying) – Part II, Optimal Decision Rule, Nearest Neighbor Criterion, Approximate Probability of Error

Hello, welcome to another module in this massive on open online course. So, we are looking at M-ary PSK modulation, and we have said that in M-ary PSK constellation points are arranged inside as on a circle right circle of radius a equals square root of S.

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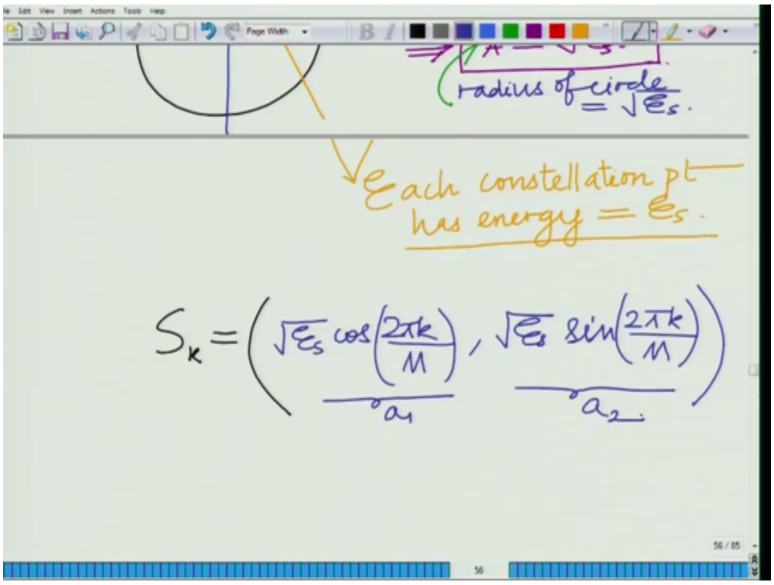

So, each symbol has an energy exactly equal to E s is. So, the average energy is; obviously, E s all right. The kth symbol is given as S k equals where we have seen what the kth symbol is the kth symbol is given as therefore, square root of E s cosine 2 pi k over kth symbol is given as square root of E s cosine 2 pi k over M and so this is your a 1, this is your a 2.

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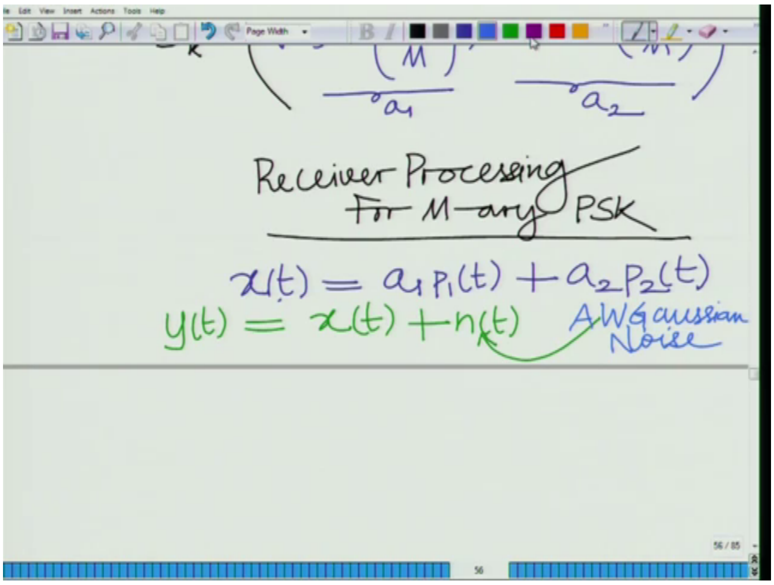

Now, what we want to see is the receiver processing. You want to see the receiver processing for M-ary PSK, we have x t equals a 1 p 1 t plus a 2 p 2 t; obviously, the received signal again we have seen this several time before you are considering in an a w j n channel. So, this is x t plus n t where n t is additive white Gaussian additive white Gaussian noise.

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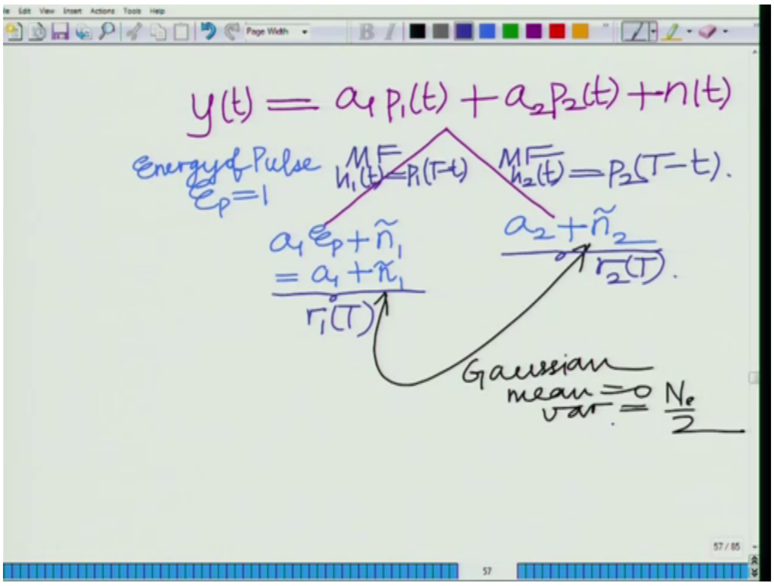

And well therefore, I can write y t equals a 1, p 1 t plus p 2 p 2 t plus n t. And now I have to match filter with similar to QPSK QAM etcetera, match filter with both h one t equals p 1 t minus t followed by sampling at t, and match filter also with h 2 t equals p 2 t minus t. And we have seen before this match filtering operation because p 1 and p 2 the pulses are orthogonal this one will give us a 1 plus e p plus n 1 tilde plus e p equals 1 energy of pulse.

So, therefore, this is a 1 I keep writing e p again just to account for the general scenario and this thing is e 2 plus n 2 tilde again note e p equals 1. This is energy of the pulse n 1 tilde n 2 tilde these are Gaussian noise samples mean equal to 0 variance equal to that is n naught divided by 2. And now you see there is a slight change in the detection scheme for M-ary PSK in QPSK for instance and QAM what we could do is we could look at r 1 t and r 2 t separately.

That is for instance if we call this as r 1 t r 2 t. We could look at r 1 t and r 2 t separately. That is remember M-ary form can be thought of as comprising of 2 constituents square root M-ary PAM constellation, correct. So, I can look at r 1 t as corresponding to one constellation M-ary square root M-ary PAM constellation along the x direction x axis or the horizontal axis and I can look at the other r 2 t as corresponding to the statistic for the square root M-ary PAM along the y axis all right. So, these can this are d coupled, alright. So, I can decide, I can chose or I can make decisions about a 1 and a 2 independently; however, that is not possible in M-ary PSK that is what makes the decision scheme for M-ary PSK slightly complex in comparison to for instance to let say a square constellation, I just QPSK or QAM. So, in M-ary PSK what happens is if you again. Remember let me draw by picture this is my circle the points are arranged around a circle.

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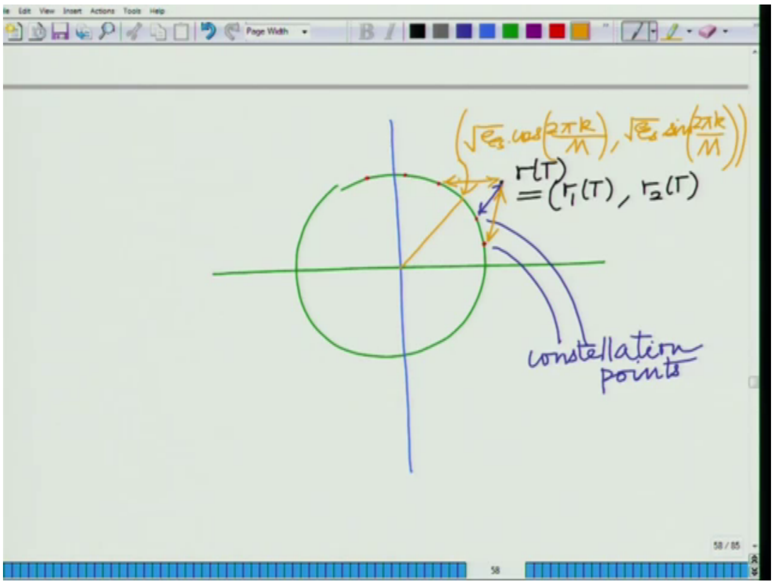

For instance, you have this point which is square root of E s cosine 2 pi k by M this is the kth point this is the square root of E s cosine 2 pi k over M and you have several such points. So, these are the points of the constellation. And your decision statistic r T for instance let say lies somewhere over here that is r t which is equal to now a combination of r 1 T and r 2 T. And now there is no other way except remember we can always rely our minimum distance criterion. So, you have to measure the distances with respect to the various constellation.

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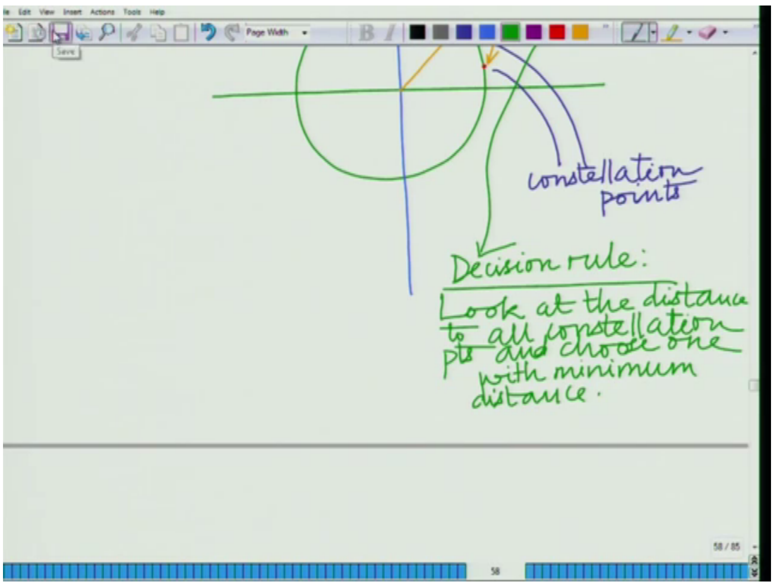

So, there is no other way other than measuring the distances with respect to the various constellation points and chose the one. So, what we have to do is look at the distance. So, the decision rule is to look at the distance to each or look at the distance to all the constellation points, and chose one with minimum distance with minimum and chose the one with minimum distance that is the interesting aspect.

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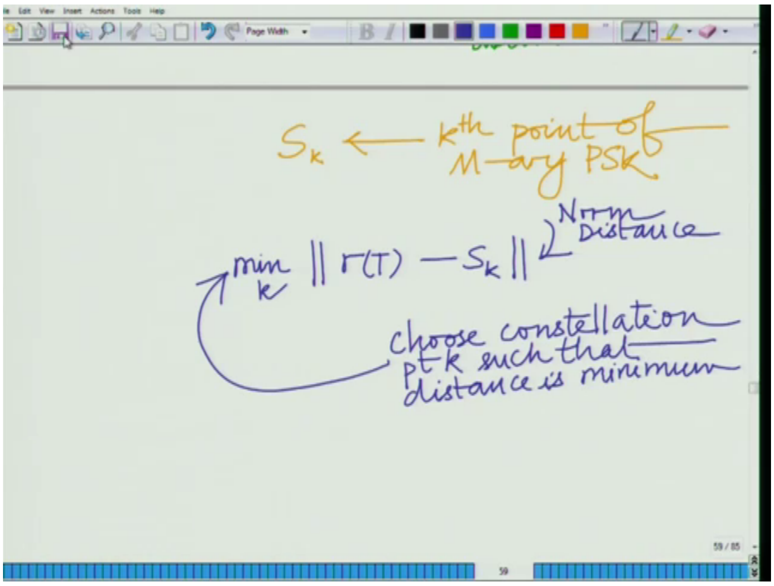

So, basically I have to chose, so r T. So, I have to chose point k, S k let say this is again kth point of the M-ary PSK constellation. Then chose k such that r T minus S k the distance remember this is the norm, 2 norm which can be use to represent the distance. So, we have to chose k such that minimize over k. So, chose k such that distance is minimum choose constellation point k, choose the constellation point k such that the distance is minimum.

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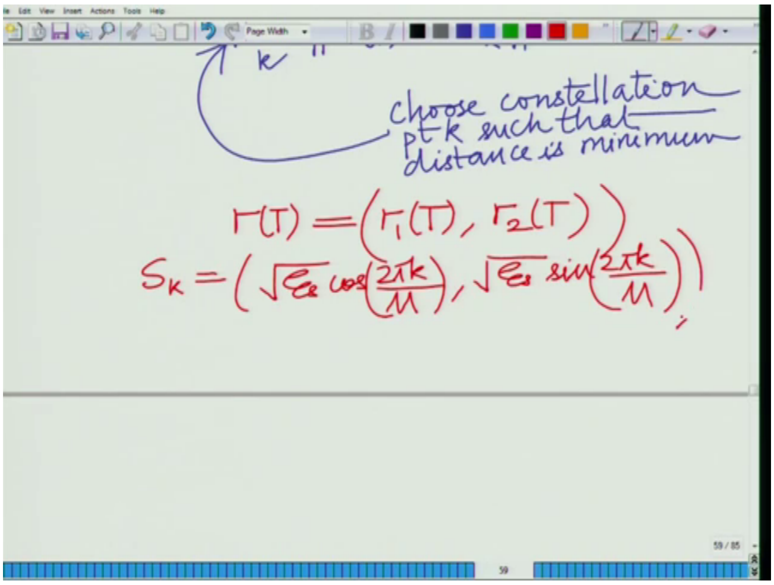

And I can express remember once again we have remember, r T equals r 1 T for r T 2 is a 2 dimensional point, S k equals square root E s cosine 2 pi k pi M, square root E s sin 2 pi k pi m. So, the minimum distance.

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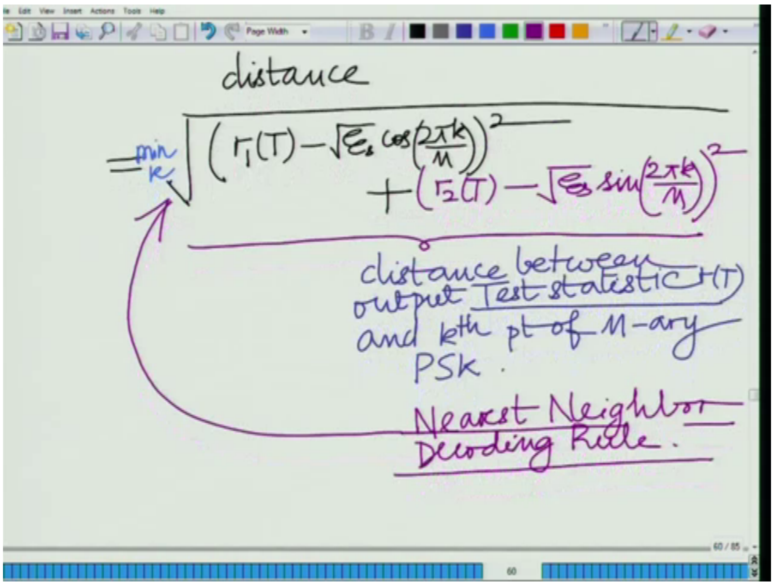

So, the distance will correspond to will be well we know how to measure the distance between to 2 dimensional points. So, this will be r 1 t minus square root E s cosine 2 pi k over M whole square plus, r 2 T minus square root E s sin 2 pi k over M whole square whole under root. This is the distance between the output, what we call test statistic which is obtained after match filter and match filtering and sampling output test statistic r T, and kth point of M-ary to PSK. And then what we have to do is we have to do to nothing, but basically choose the k which has the minimum.

And this is known as your again the minimum distance criterion or the nearest remember we have already seen this in the context of in, formally I had explain this in the formal in the context of will pam QAM etcetera this is the nothing, but the nearest neighbor decoding rule. This is nothing, but the nearest decode. So, that is the decision rule for M-ary PSK. So, that is the decision rule for M-ary PSK.

So that is the decision rule for M-ary PSK and the that pretty much, now what we have to do is basically we have to look at the bit error rate for M-ary PSK, we have to look at the bit error rate for M-ary PSK.

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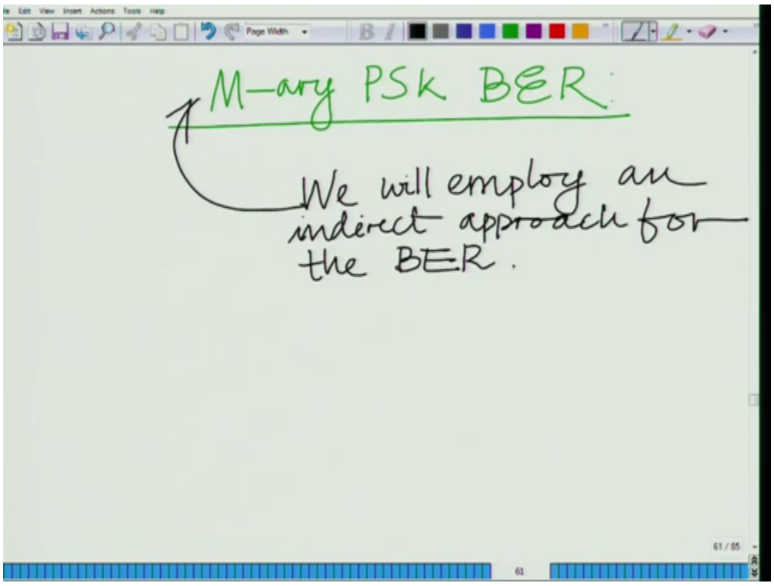

So, for M-ary PSK we have to look at the bit error rate for M-ary PSK, now for this find the bit error rate previously we had look at for instance pam QAM etcetera, we had in a elegant way in look at bit error rate; however, this because of the nature of the decision rule it is not very simple to find the bit error rate, but we have a way to get around this all right. So, we will employ a different approach which is basically based on finding the number of nearest neighbor is so in fact, an interesting approach and. In fact, it can be use to tremendously simplify the bit error rate computation for several other digital communication applications right where it is not very easy to compute the bit error rate all right. So, far we have seen scenario such has pam QAM etcetera where it is relatively simple to compute the bit error rate all though elaborate, it is relatively simple, alright.

So, here we have starting to see scenarios where it is not. So, simple to compute the bit error rate, I will illustrate a general approach it can be used in such scenarios to compute the bit error rate. And this is known as basically it is known as the nearest neighbor decoding rule or it is based on the number of nearest neighbors of each constellation point. So, we will employ an indirect approach to calculate the bit error rate. Let me just highlight that, we will employ an indirect approach for the bit error rate for instance let us start with QPSK again. Let us go back to QPSK for a movement to illustrate this consider QPSK. In QPSK if you remember if I draw the constellation for QPSK I have 4 points.

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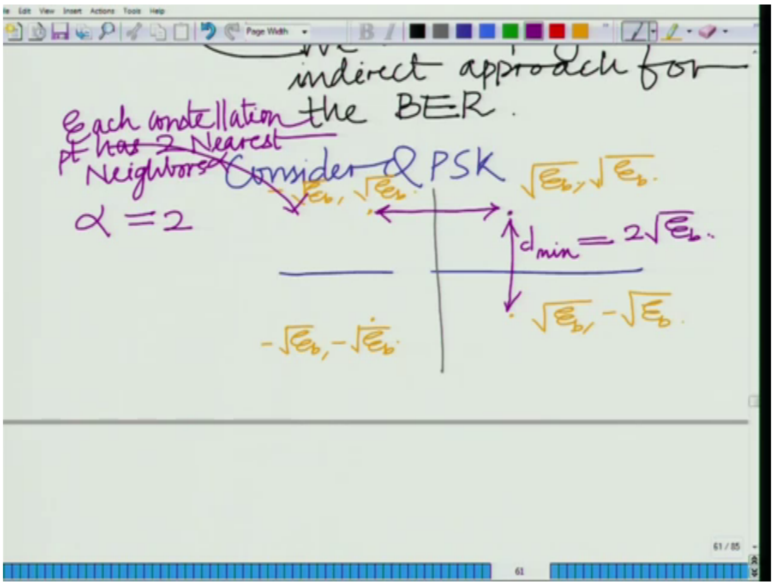

Square root of E b, square root of E b, square root of E b, minus square root of E b, minus square root of E b, minus square root of E b and finally, minus square root of E b square root of E b.

Now, if you look at any particular constellation point for instance, this each constellation point has 2 nearest neighbor so called nearest neighbors. So, each constellation point observe that in QPSK, each constellation point has to nearest neighbors. Let us denote the number of nearest neighbors by alpha. So, alpha equals to 2 nearest neighbors alpha equals 2. Now further the distance to the nearest neighbors is well if you look at the distance between these 2 points it is twice square root of E b. And we can formulate the bit error rate approximate expression approx symbol error rate not the bit error rate rather approx. In fact, symbol error rate is of an abbreviated as SER just as bit error rate as abbreviated BER.

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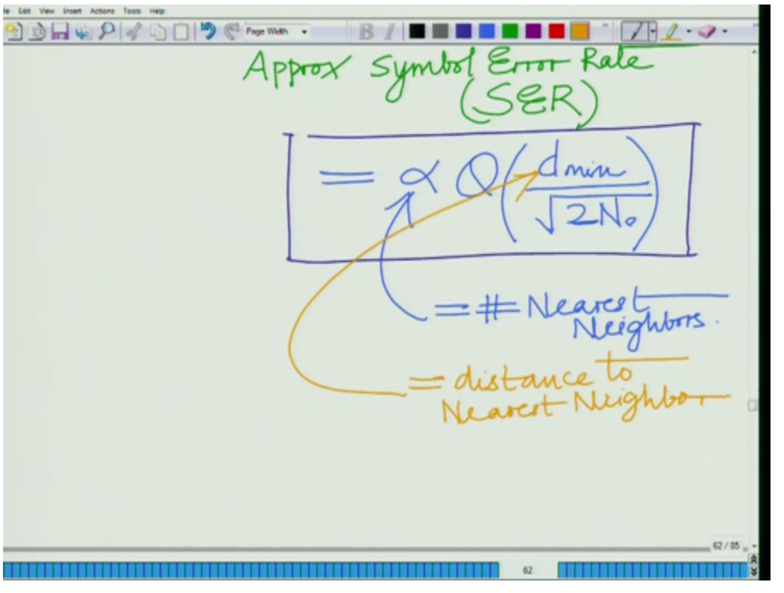

This is equal to and this is an interesting expression alpha which is number of nearest neighbors times Q in to d min distance to the nearest neighbor divided by square root of twice n 0.

So, this is an interesting expression for the bit error rate and it is very helpful in such scenarios where it is difficult to compute the exact bit error rate. So, alpha equals number of nearest neighbors as we have already said number of nearest neighbors, d min that is equal to distance to nearest neighbors nearest neighbor and this is the very useful expression.

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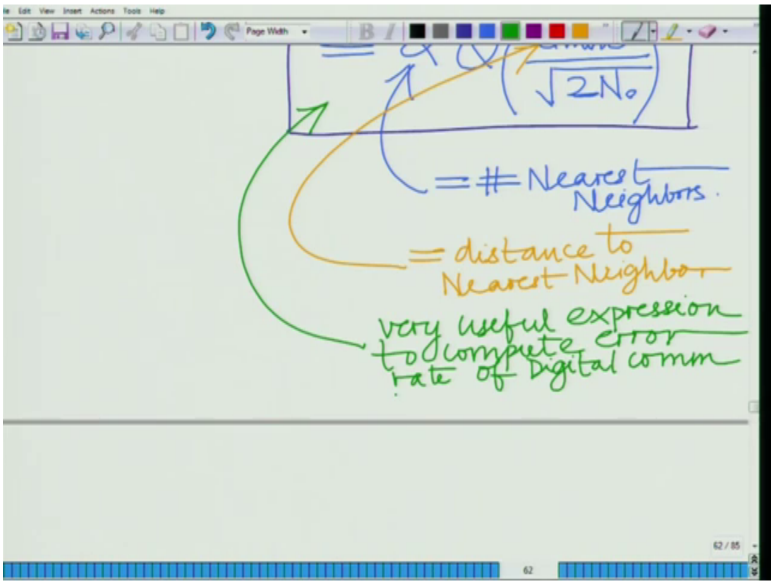

Let me note that also it is a useful expression to compute the error rate of a digital communication system, or a digital modulation scheme; error rate of digital communication when exact expression is difficult to obtain and exact expression is difficult to obtain. Now let us go out to a M-ary PSK.

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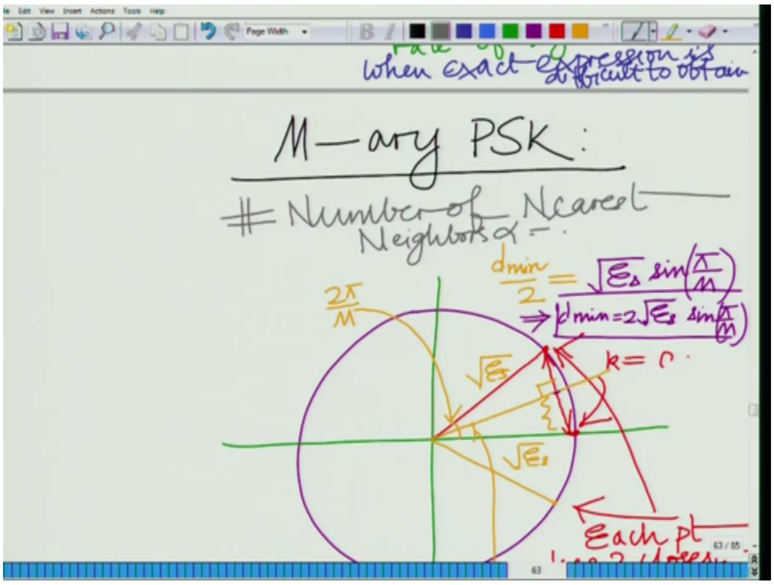

So, now let us go to our M-ary PSK constellation, in here we want to find again the symbol error rate using the nearest neighbor criteria using this approximation based on the number of nearest neighbors. And again if you go to a M-ary PSK, I draw the circle let me just draw it a little bit better and the little bit.

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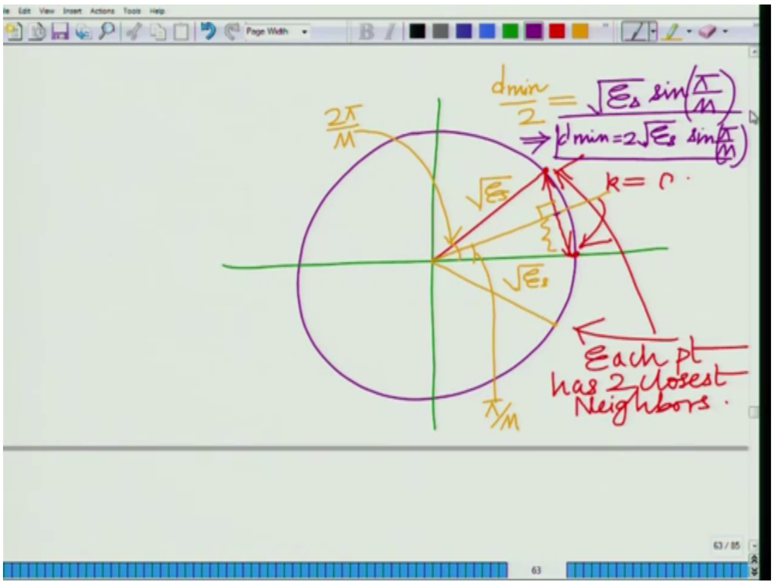

So, has to illustrate, it now if you look at this now if you look at this for instance let us look at any constellation point or let us look at any constellation point you can look at any constellation point for instance corresponding to k equal to 0 remember phase is 0. Now each constellation point has 2 neighbors. So, each constellation point has 2 closest neighbor. Each constellation point has 2 closest neighbors, and now remember this is equal to square root of E s, this is equal to square root of E s. If I look at the distance if I look at the distance correct.

Let's now look at that distance d min if I draw a line from the center to the line joining these 2 constellation points, it is perpendicular and this angle remember the total angle the total difference the total phase difference between 2 points is 2 pi by m. So, this angle is 2 pi by M which means this angle is pi by m. So, this angle is pi by M. Therefore, this half of the distance, d min by 2 and you can check this d min by 2, or let me just write this over here, a d min by 2 you can verify this is equal to square root of E s in to sin pi by M that is an important result. So, this is square root of E s in to sin pi by M and this basically is your d min by 2. And therefore, implies d min equals twice square root of E s in to sin pi by M that is the forth point.

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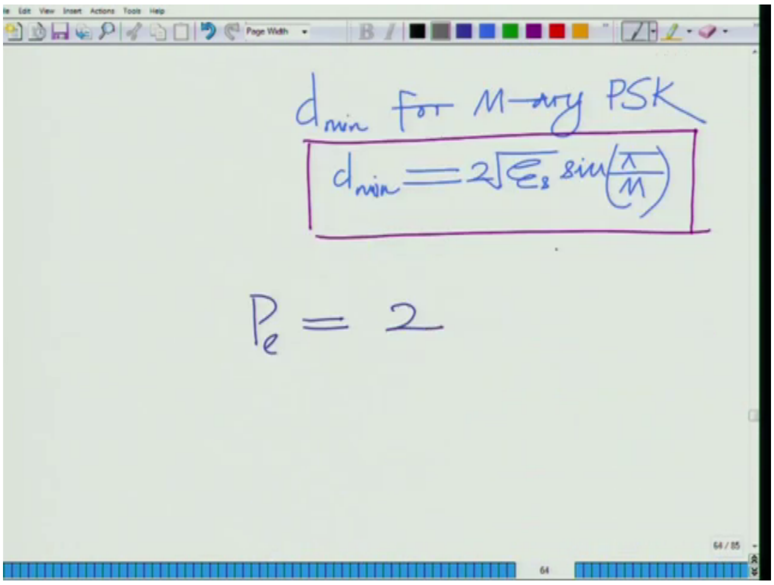

So, d min for M-ary PSK and you can check this equals square root E s or 2 square root E s sin pi by M. I hope that brief derivation it is rather simple it is based on your high school geometry principles of high school geometry, it is based on principles of high school geometry that is we look at cord right of the circle, if you draw the line from the center to that cord line from the center is perpendicular to the cord that is one and we use the fact then the radius of the circle is square root of E s, and the phase difference between the points is 2 pi over M. Therefore, if you look at this cord line joining the center to the cord it bisects an angle, right.

So, the angle is now pi by M you can use this simple principles derive it, but might my just take a little bit of looking at it carefully. And therefore, now it becomes really simple I do not have to go through an elaborate derivation. In fact, thanks to the result that we have we have probability of error is 2, well number of nearest neighbors alpha equal to 2 that is the first thing, let us note that also I think that is also equally important. So, number of nearest neighbors alpha equals 2 let us note that.

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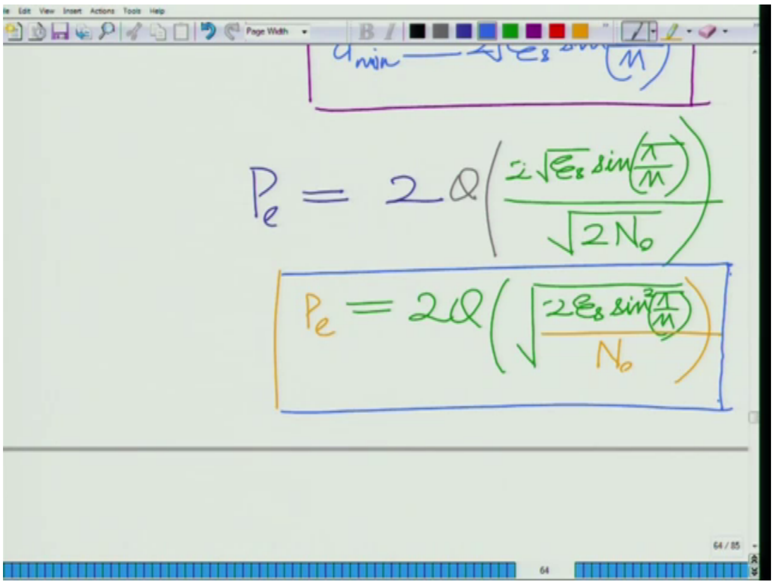

So, we have probability of error equals twice Q, well d min is twice square root E s sin pi over M divided by 2 n naught, which is equal to twice Q square root of twice E s sin square pi over M, divided by n naught. That is the expression for the approximate probability. Remember this is not remember this is not an exact expression this is an approximation and In fact, this approximation is very close.

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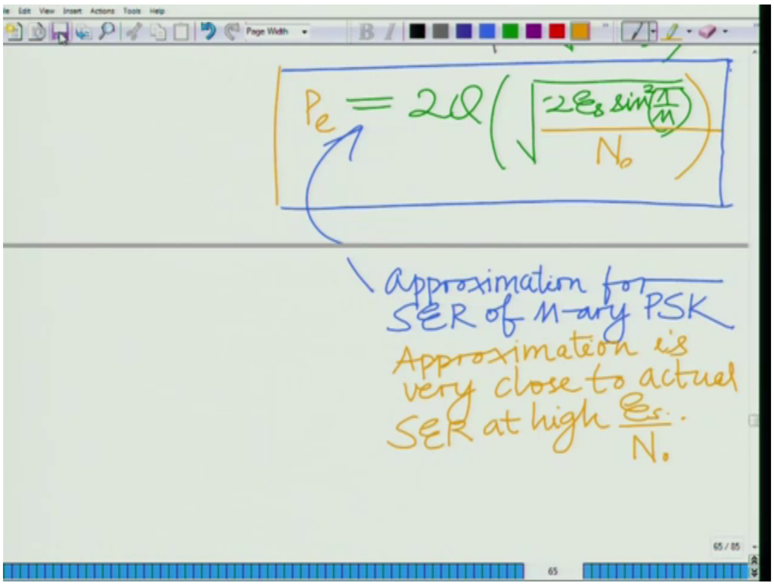

So in fact, this is an approximation, let me just again emphasize that this is an approximation, but this approximation is very close as E s by n naught increase at highest. So, it is an approximation rather very useful. It is an approximation for symbol error rate of M-ary PSK. And the approximation is very tight at high E s over n naught or we can say instead of tight, let we use a simpler word the approximation is very close to actual SER at high E s over n naught. So, that is pretty much what we have for M-ary PSK all right. So, we have seen M-ary PSK, we have seen the receiver design for M-ary PSK in the previous module we have seen the constellation the wave form for M-ary PSK all right.

How to compute the average symbol energy and now we have seen in this module we have seen the receiver design the match filtering operation correct and the optimal decision rule which is based on it is nothing, but based on the nearest neighbor decision rule and we have also show seen an interesting frame work all though we have intere ill interesting frame work to compute the approx to compute the approximate probability of symbol error, for any general constellation and or although I have illustrate in the context of QPSK and M-ary PSK. This is the very general the nearest neighbor criteria or the nearest neighbor cre criterion based approximation that we have seen to compute the similar rate is very general and can be used, you know wide variety of scenario, using that we have computed the symbol error rate of this M-ary PSK constellation all right. So, we will stop here and continue with other aspects in subsequent modules.

Thank you very much.