Principles of Communication Systems - Part II
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Lecture – 04
Spectrum of Transmitted Digital Communication Signal, Relation to Energy Spectral Density, Introduction to AWGN Channel

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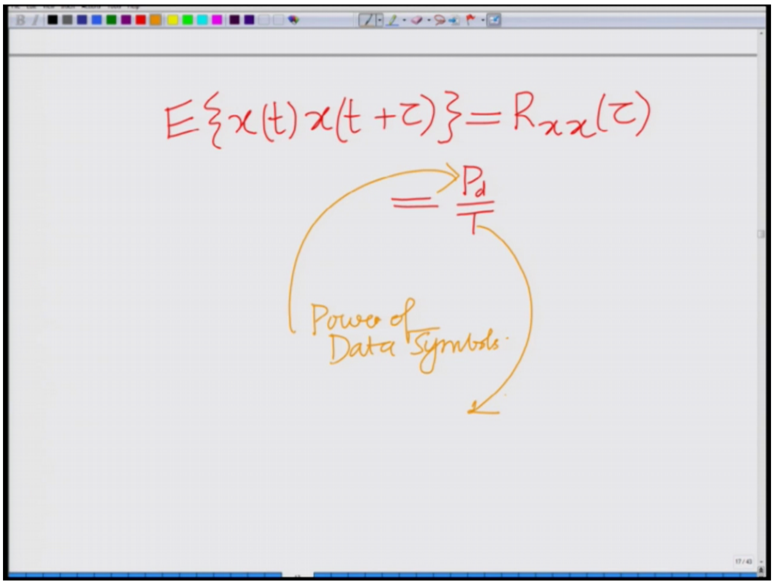

Hello, welcome to another module in this massive open online course on communication massive open online course alright and we are looking at the power spectral density of the transmitted digital communication signal. And we have derived autocorrelation, the autocorrelation of the digital communication signal as follows; expected value of x t into x t plus tau. We have shown this is basically your R x x of tau. We have shown that this is equal to that R x x of tau is equal to P d over T where P d is the power of the data symbols, T is the symbol duration times R P T P T of tau where this is your autocorrelation of pulse P T t.

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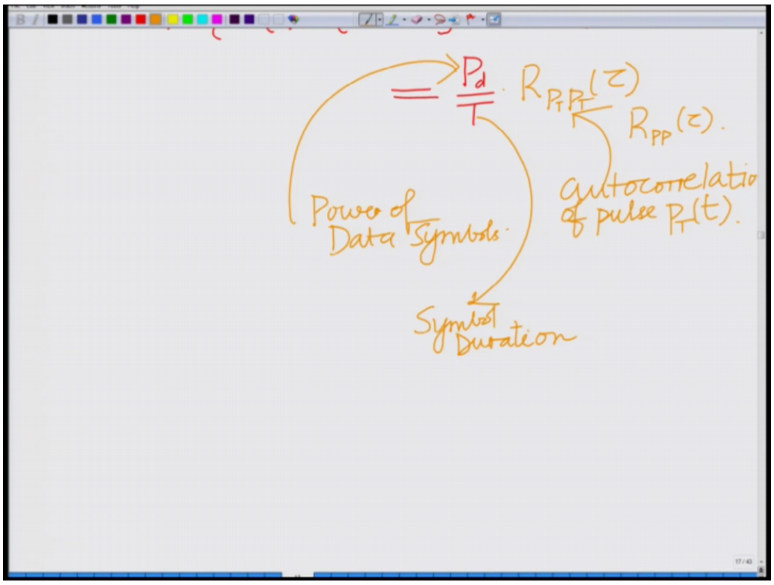

In fact, for any pulse P, this will simply be R P P of tau we can use any pulse tau we cannot we do not need to be necessarily we can use any pulse P T. We do not need to be necessarily limited to the rectangular pulse P T P sub capital T that was simply an example.

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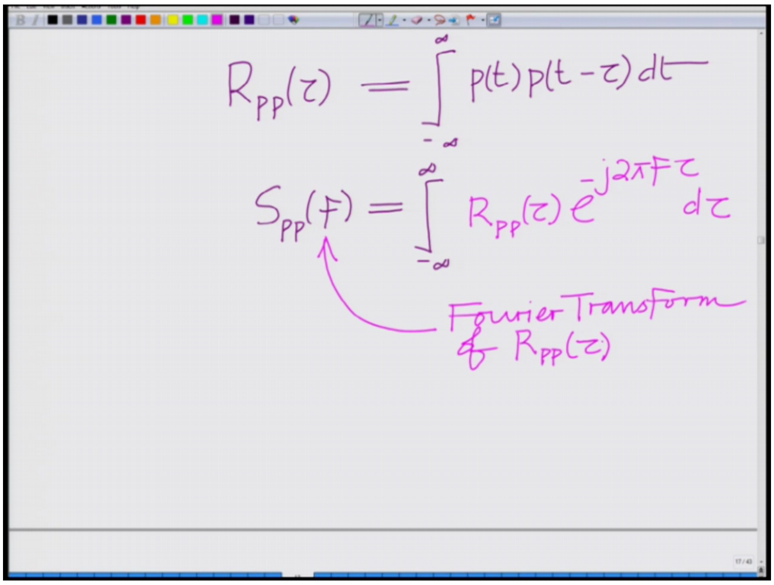

For instance, and now we also know that this R P P of tau is of course, this has Fourier transform this is basically your R P P of tau which is autocorrelation of pulse P t is defined as minus infinity to infinity integral P t P t minus tau d t, this is R p t p t tau. And if I take the Fourier transform of R p t p t tau, I obtain S P P of F correct S P P of F which is the Fourier transform that is Fourier transform of R P P tau, which is R P P tau integral minus infinity to infinity R P P tau e to the power of minus j 2 pi F tau d tau. So, S P P F, this is the Fourier transform of R P P tau and this can be shown to be interestingly.

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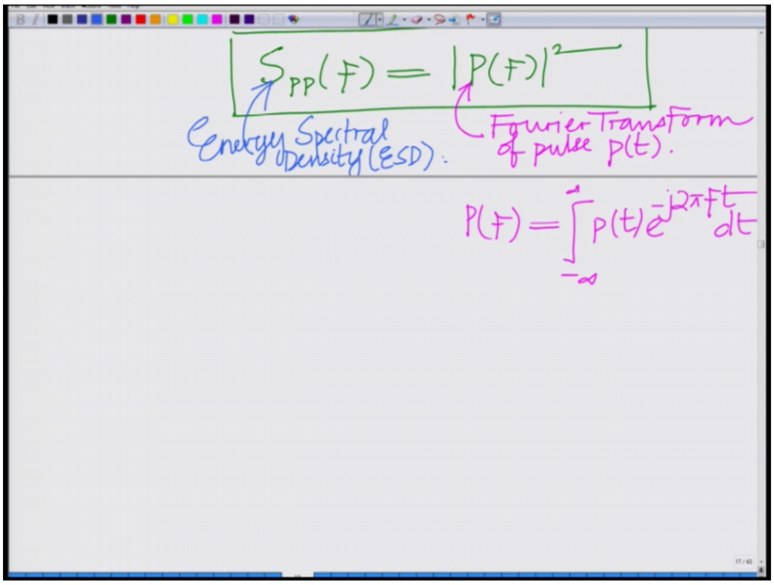

We can show that S P P F is equal to magnitude of P F square, where P F is the Fourier transform of pulse P t that is what we mean is. P F equals integral minus infinity to infinity P t e to the power of minus j 2 pi F t dt and therefore, this is magnitude of P F square where P F is the Fourier transform of the pulse P t. And therefore, this quantity S P P of F this quantity is termed as the energy spectral density or simply the ESD energy spectral density of pulse P t which is a Fourier transform the autocorrelation function of the pulse P t.

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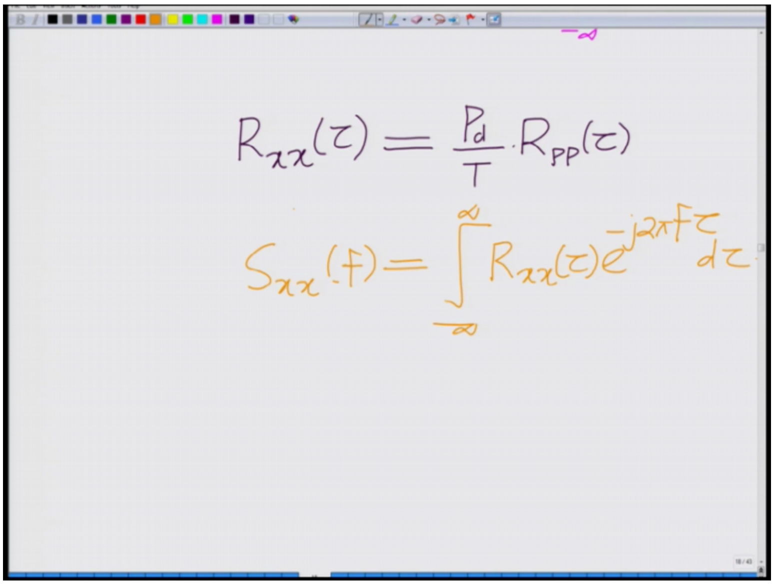

And therefore, what we have is we have the autocorrelation of the digital communication signal equals P d over T times the autocorrelation of the pulse R P P. If you take the Fourier transform on the left to get the we know that the power spectral density S x x of F equals minus infinity to infinity R xx tau e to the power of minus j 2 pi for a white sense stationary process correct.

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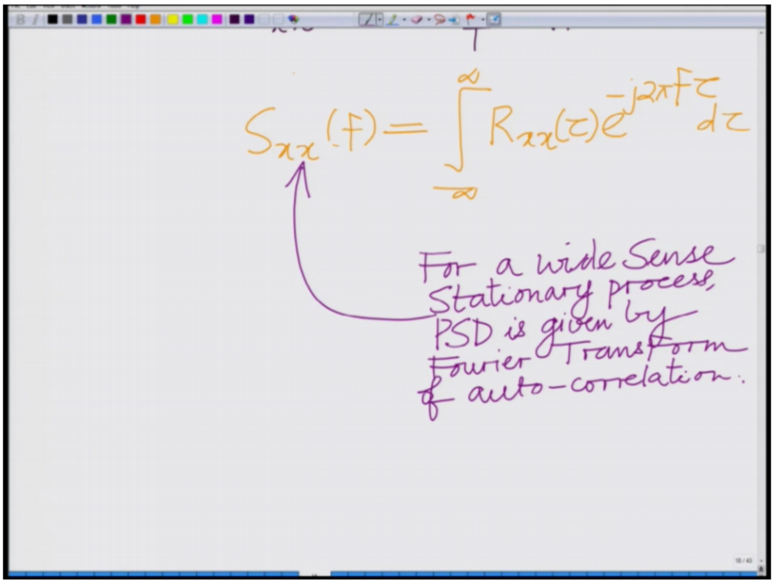

Power spectral density is given by the Fourier transform of the autocorrelation function for a and this is a important property for a wide sense stationary process the power spectral density PSD is given by Fourier transform Fourier transform of the autocorrelation function.

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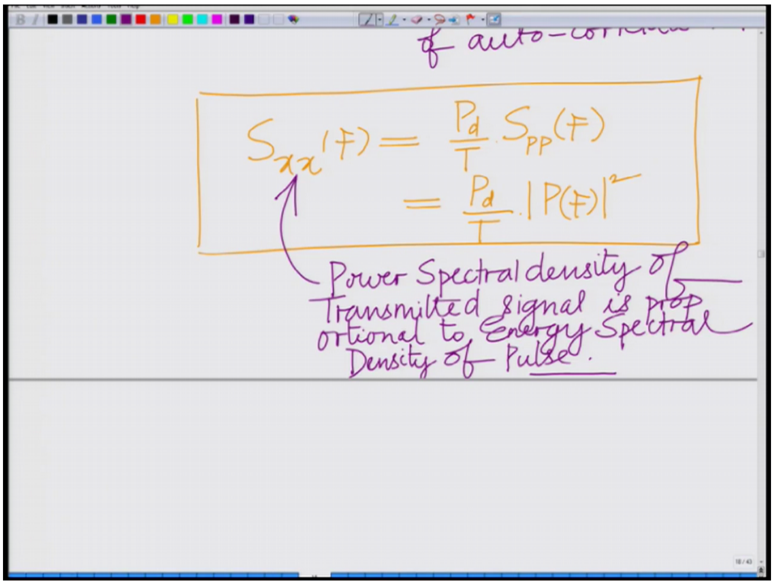

And therefore, what we have is we have well S x x of F which is the Fourier transform of R x x tau is equal to P d over T times the Fourier transform of R p p tau which is autocorrelation of the pulse, so that is your s. You can call that as your Fourier transform of the autocorrelation S p p of F, but S P P of F is nothing, but the energy spectral density all right this is the Fourier transform the autocorrelation of the pulse which is the energy spectral density. And therefore, we have this is equal to P d over T times magnitude of P F square.

So, basically what we have is a very interesting result that the power spectral density of transmitter signal is proportional to energy spectral density of pulse this is a very important result that is the power spectral density of the transmitted digital communication signal is basically proportional to the energy spectral density of the pulse P t. So, this is an important property which helps us characterize what is the power what is the spectral distribution of power of the transmitted digital communication signal, and what this result tells us is that it is nothing but a scaled version of the energy spectral density of the pulse.

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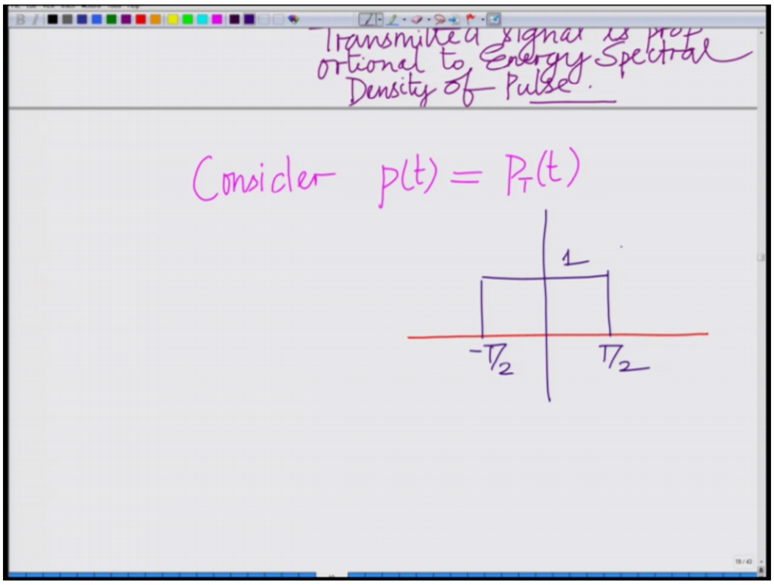

And now for instance let us take a simple example. Let us consider again for the purpose of this example consider the pulse P t equals the rectangular pulse. We are going back to our rectangular pulse correct, which is our rectangular pulse 0 to capital T or rectangular pulse 0 to capital T or let us go back to our rectangular pulse. We have defined it minus t by 2 to T by 2 for a height of one that is what we have defined the pulse to be if you remember.

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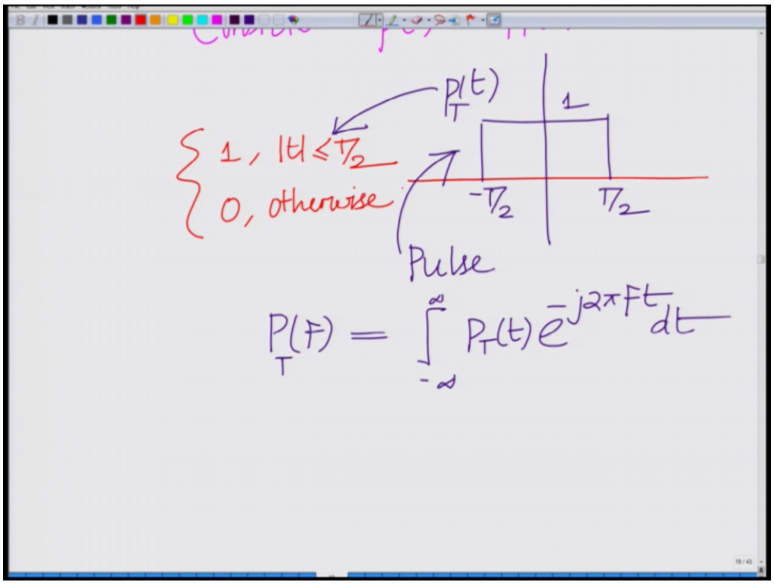

Now, therefore, now this is our pulse P t. This is the pulse or rather this is our pulse P subscript capital T of t. And therefore, now if you look at the Fourier transform of this pulse that is P subscript T of F is minus infinity to infinity P T of t e to the power of minus j 2 pi F T d t. And we have seen this pulse is nothing but this is equal to 1, if mod t less than equal to T by 2 and 0, otherwise. And this we have derived the Fourier transform this already.

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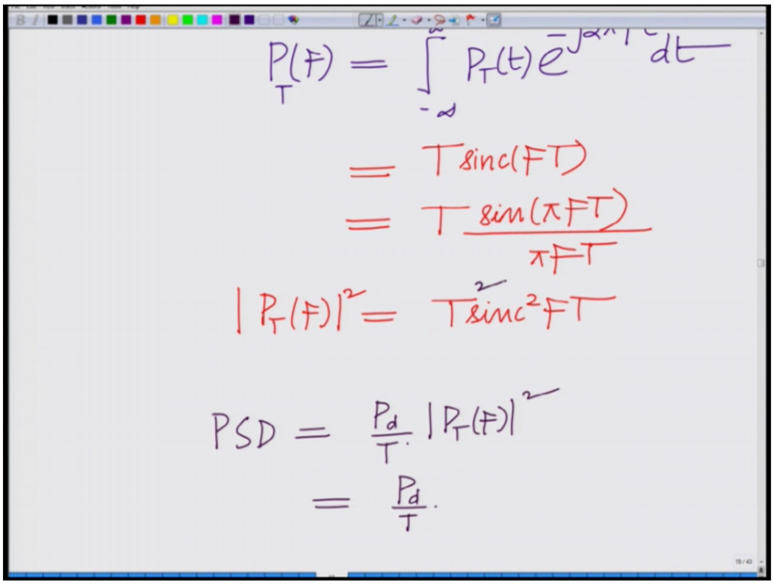

This Fourier transform is T sinc F of T which is equal to T sin pi F T divided by pi F T. And therefore, the magnitude P T F square equals well this is equal to T sinc F sinc square F T, which implies the power spectral density.

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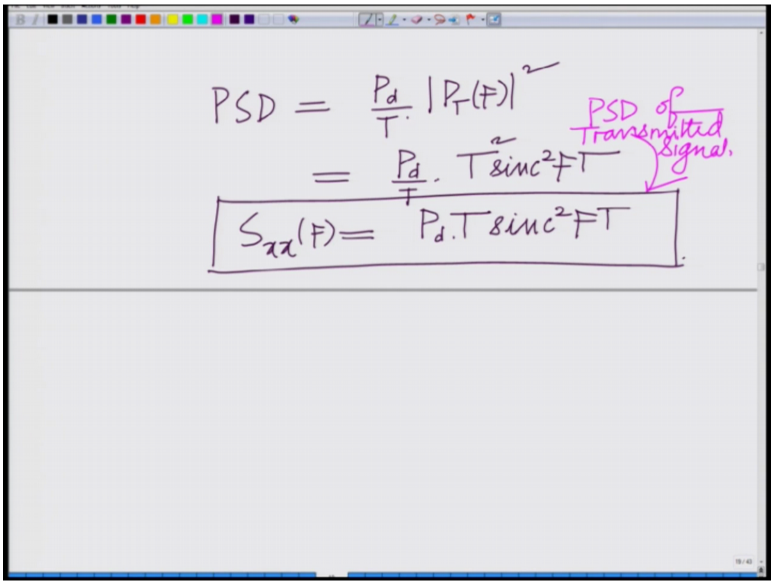

The PSD equals well P d over T times magnitude P T F square which is equal to well pd over T I am sorry this is t sinc F T this is T square sinc square F T. So, this is times sinc square F T, so this is P d well P d into T sinc square F T. So, this is the power spectral density of the transmitted signal. So, this is the sorry this is the S x x F, this is your power spectral density, this is the power spectral density of the transmitted signal. And thereby we have derived the expression for the power spectral density of this transmitted signal, so that completes our analysis with respect to the power spectral density of the transmitted signal.

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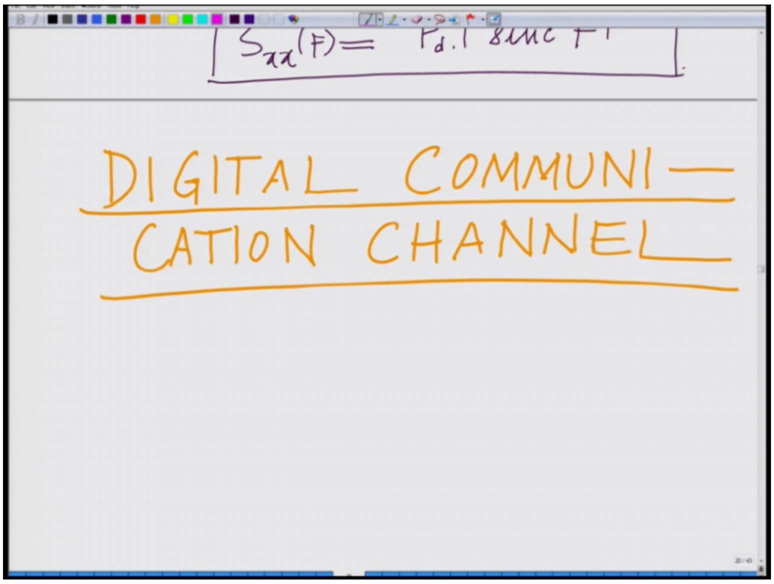

And now let us look at the properties of the digital communication channel. So, let us now proceed to look at the properties of a digital communication channel. So, let us look at the properties or let us simply call this as the digital communication channel, how do we model a digital communication. Now, simply put the channel is the medium through which the transmitted signal propagates from the signal propagates or signal traverses from the transmitter to receiver in a communication system. So, the channel is nothing but you can say it is the medium through which or right the medium through which the signal propagates from the transmitter to the receiver.

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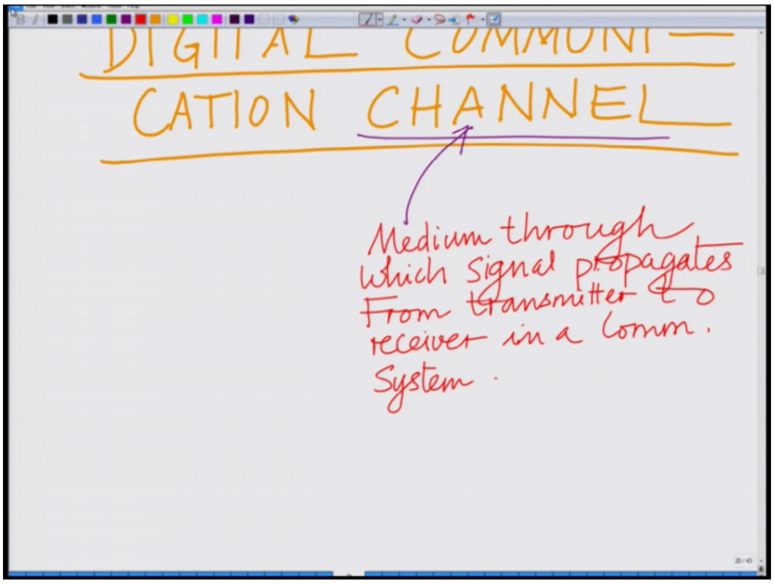

So, channel which is not very clearly defined in many context. It is simply a you can think of it as a medium because the transmitter and receiver are separated medium through which signal propagates or signal travels from transmitter to receiver in a communication system not necessarily a digital communication system only, in any communication system, for instance in any communication system correct not necessarily. So, there is a channel all right. So, the need in the communication system by definition there is a transmitter, receiver and these are separated. The transmitter transmit the signal which has to reach the receiver the medium through which the signal propagates correct from the transmitter to receiver that is termed as a channel and there can be various kinds of channels.

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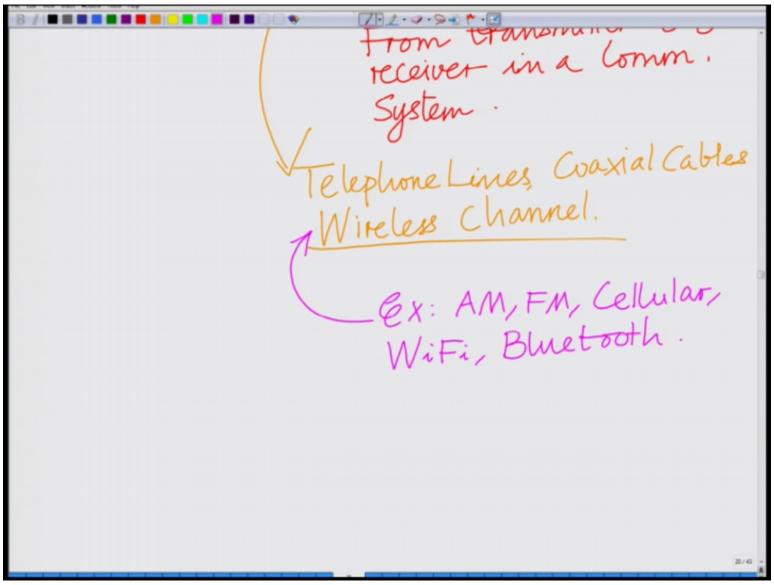

For instance, we can have some examples common examples are a for instance your telephone lines coaxial cables which connect basically your set of boxes all right. Coaxial cables which transmit which are used for the transmission of TV signals correct or your cable basically. And also the wireless channel all right, when there is no physical when there is no particular guided propagation medium when the read electromagnetic waves are transmitted over the radio channel over the air right for instance such as your AM, FM. In fact, this is turning out to be the most dominant mode of communication there is for instance your all your broadcast services such as AM, FM your cellular at all this thing, cellular, Bluetooth, Wi-Fi set all of these are based on the wireless channel. When there is no particular physical a guided propagation medium all right, it simply transmitted over the air all right the radio channel EM waves can propagate of course, correct.

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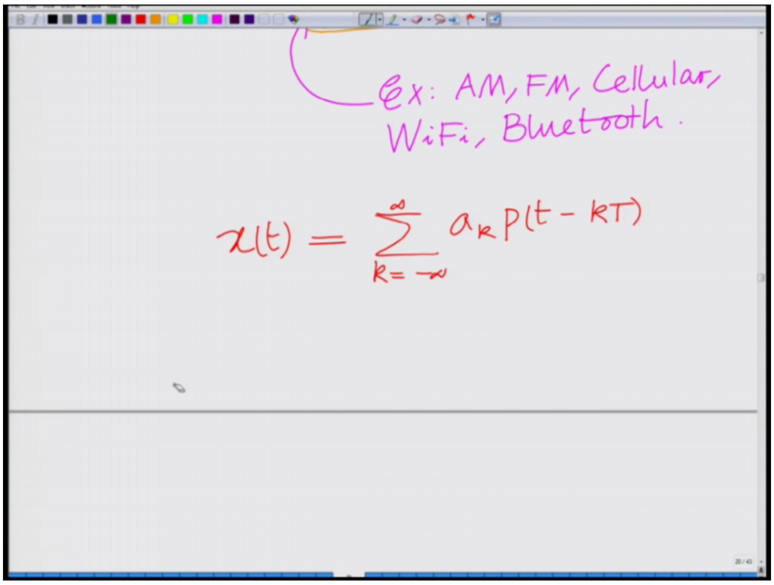

And now therefore, we have our signal correct we have our transmitted signal which is x t equals summation k equals minus infinity to infinity a k P t minus k T where capital T is the symbol time.

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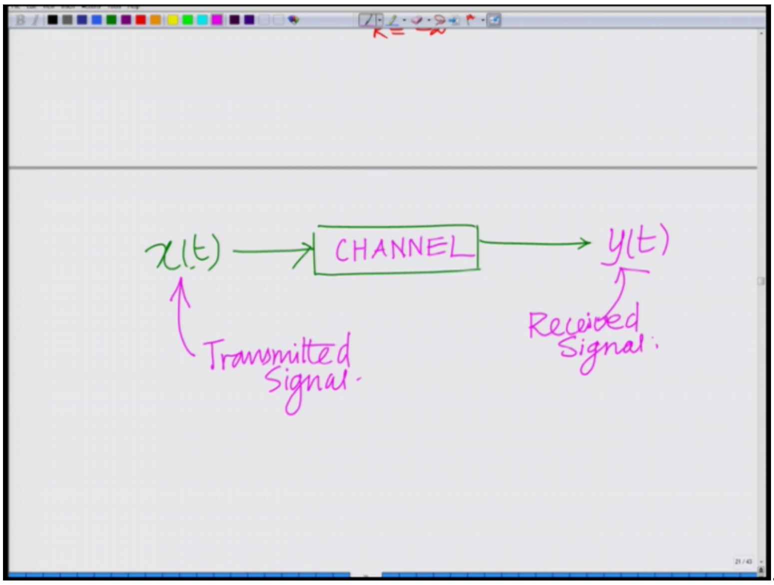

Now, this signal x t; we have the signal x t, which traverses a channel, which passes through a channel. So, this signal x t passes through a channel passes through a channel to the receiver and you have y t. So, this is the transmitted signal, this is your received signal transmitted signal, received signal and it passes through a channel. Now, one of the simplest models for a digital communication system or for any communication system for that matter is what is known as an additive white Gaussian noise channel.

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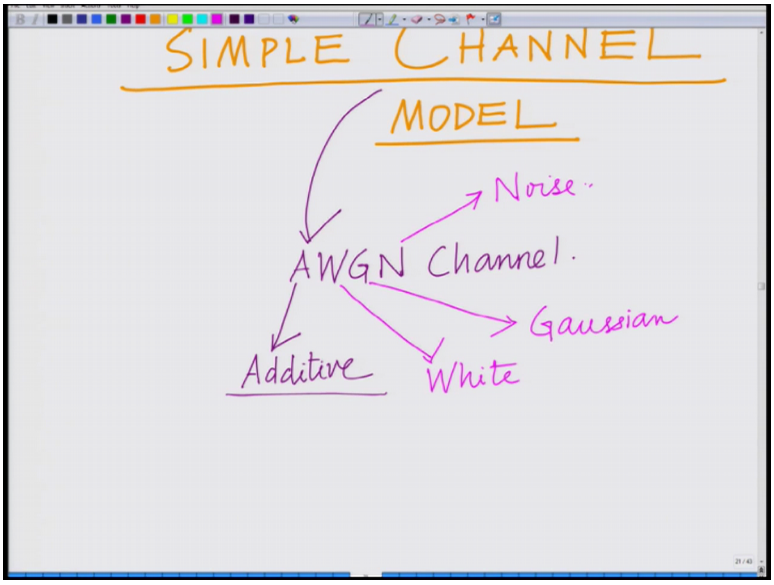

One of the simplest; a simple communication system model, now a simple channel model is what is known as an additive white Gaussian noise channel termed as an additive white Gaussian noise - AWGN. This is a very important, simple, yet very important where let me first explain what these different terms mean A stands for well additive, W stands for white, G stands for Gaussian and N stands for noise. So, this is basically an additive white Gaussian noise channel.

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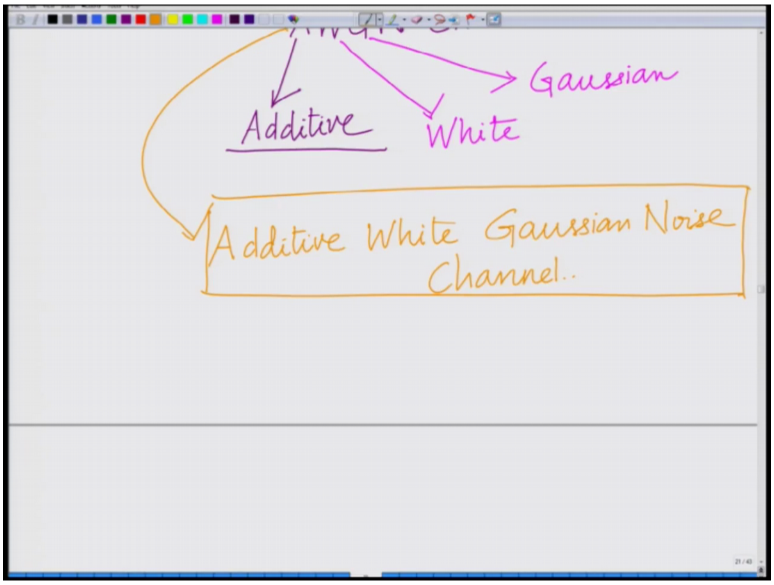

So, AWGN basically represents additive white Gaussian noise channel, which is one of the most prominent channel models or of the most simplistic channel models employed to model the behavior to understand the behavior model and understand the behavior and performance of communications. Not just digital communication systems, but also for that matter any communication systems also analog communication system and so on analog communication systems and so on.

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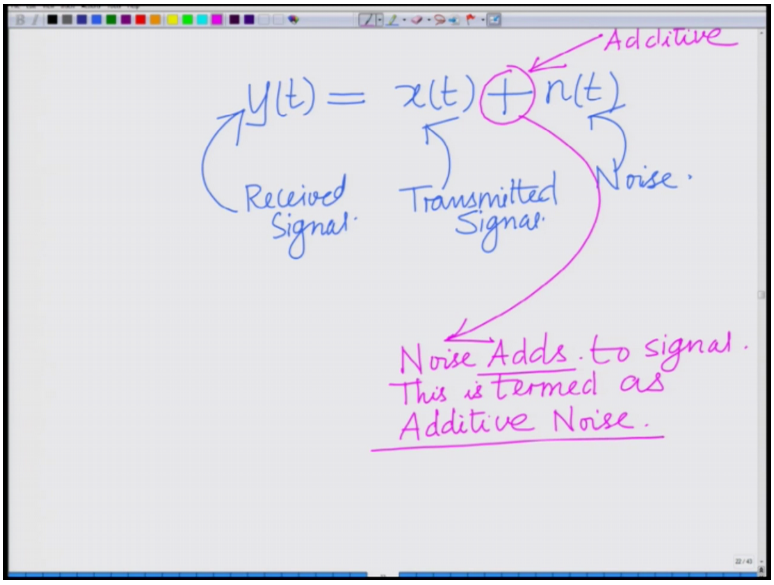

So, in an AWGN channel we have the received signal y t, it is a very simple model as I have described it. As I have said already y t equals x t received signal equals transmitted signal plus the noise. So, this is your received signal, we have already seen that. This is the transmitted signal, transmitted signal and noise. In particular, look at this adds to the signal, this noise is additive in nature the noise this very symbol where the noise adds to the signal noise. So, we have noise adds to signal this is term as this is a very important assumption that the noise is additive in nature. So, this is termed as additive noise. So, this is an additive noise channel that is one of the important. So, in the name AWGN where the phrase additive, the term additive means that the noise adds to the transmitted signal x t. And of course, there are other aspect that is the white and the Gaussian which need to be defined in order to complete the definition of the noise as well as the definition of this channel. So, we will look at these aspects in the next module.

Thank you very much.