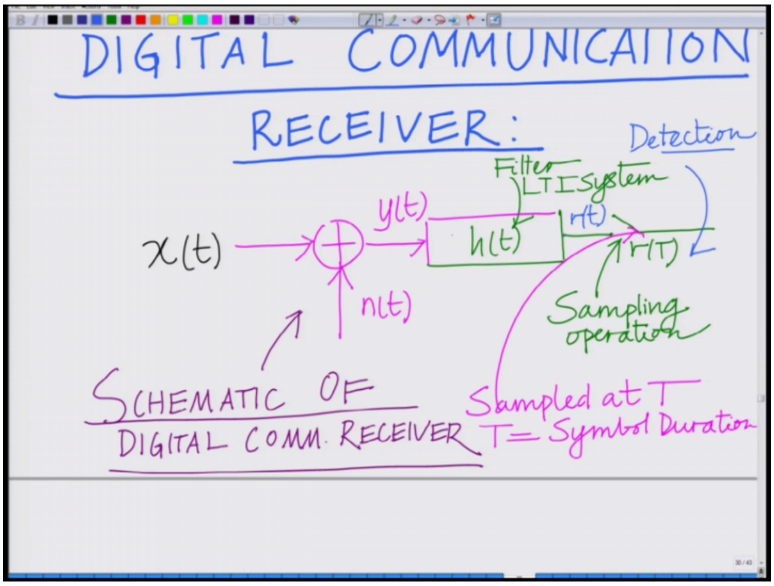
Principles of Communication Systems - Part II
Prof. Aditya K. Jagannatham
Department of Electrical Engineering
Indian Institute of Technology, Kanpur

Lecture – 06
Structure of Digital Communication Receiver, Receive Filter, Signal-to-Noise Power Ratio (SNR)

Hello. Welcome to another module in this massive open online course, in this module we will start looking at a digital communication receiver that is what are different schemes or what is the kind of signal processing the processing that can be employed the optimal processing that can be employed to get the best performance at the receiver in a digital communication system.

(Refer Slide Time: 00:42)



So, let us start looking at the operation and implementation of a digital communication receiver. So, I am here going to look at a digital communication receiver that is basically the receiver in a digital communication system we already looked at the transmitter that is the pulse shaping operation what is the transmit spectrum the power spectral density of the transmitted signal, let us now look at a digital communication receiver. So, when we talk about a digital communication receiver, the schematic of a typical digital communication receiver can be this follows.

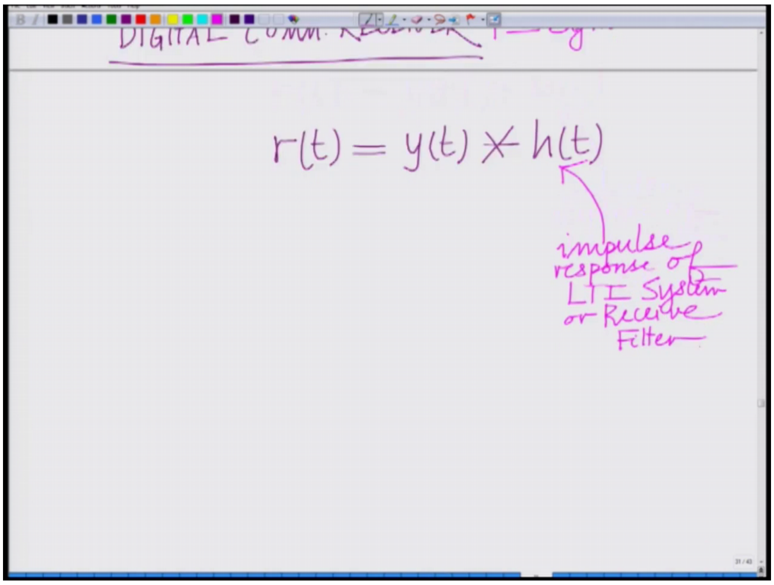
So, let us look at a typical schematic diagram of the digital communication receiver, I have the transmitted signal x t transmitter signal x t, the noise additive noise as we have seen in the previous module. So, that gives the received signal y t this is also poladitive noise channel model and if the noise is white Gaussian then this is our additive white Gaussian channel, this signal y t is subsequently pass through a filter this is an important concept through a filter which is basically an LTI system. LTI stands for linear time invariant system followed by sampling at the symbol time instant and that sample is denoted by.

So, this is basically your sampling operation and this is sampled at the symbol time capital T. T where T equal to symbol duration and this is a schematic diagram schematic diagram of a digital com a digital communication schematic diagram of a digital communication receiver. So, we have the signal x t to which there is additive noise, n t is added we get the receive signal y t. This signal y t is passed at the receiver is passed through a filter which is which basically carried which basically is a nothing but the linear time invariant system its passed through a filter h t; you can also called it as the received filter followed by sampling it is sampled at capital T where T is the symbol duration, and the transmitter digital communication symbol information regarding the transmitted digital communication symbol is extracted from this sample r of t.

So, we can say this is followed by detection; for instance this is followed by you can say detection and what we have to see is, what are the schemes this is followed by detection? So detection; in the sense that we have to extract information regarding the transmitted symbol, so are several components to this one is of course, the filter right and then after you sample how is the information regarding the symbol extractor from the sample. So, all these things remain to be seen and we will look at these concepts one by one.

So, what is the filter; first we will start by addressing the question how can the filter h d b designed, that is receive filter in digital communication system how can that be designed. So, let us take a look at that. So, I have if you look at if I call this of this I call this is filter or basically you receive filter if I call this signal as r of t.

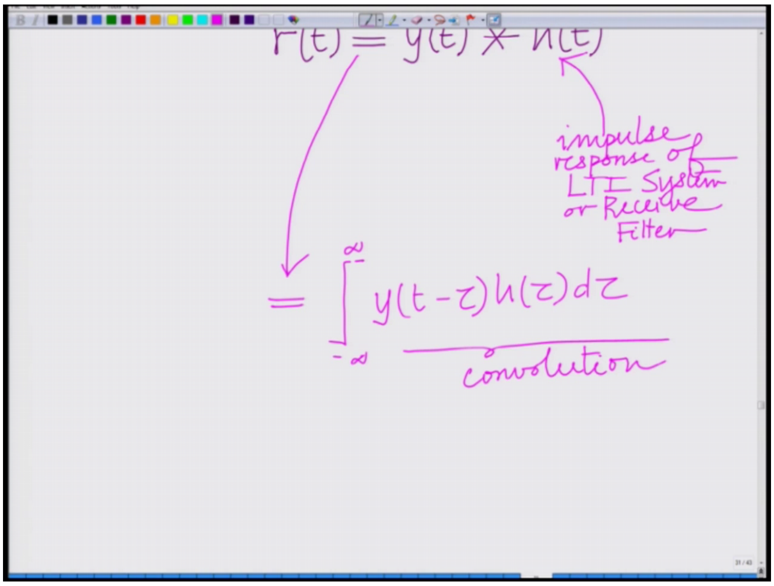
(Refer Slide Time: 05:41)



That is output when y t is passed through h t that is r of t, I can say r of t r of small t that is a continuous time signal is y t passed through h t when it when you pass a signal through a linear time invariant system correct the output is the convolution of the signal with the impulse response. So, h t is basically nothing but the impulse response of the LTI system or the impulse response of the receive filter.

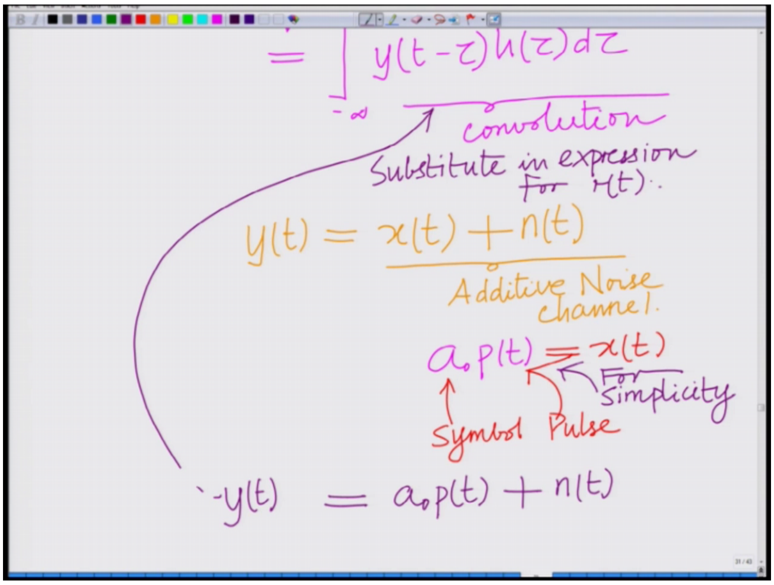
So, this is impulse response of LTI system LTI system or basically the receive filter or you can also termed this as the receive filter.

(Refer Slide Time: 06:36)



Now therefore, I can write this remember the convolution can be expressed as the integral minus infinity to infinity y, t minus tau h of tau this is the convolution operation continuous time convolution or infect linear convolution and now we know let us assume we know the transmitted we know well what do we know.

(Refer Slide Time: 07:02)



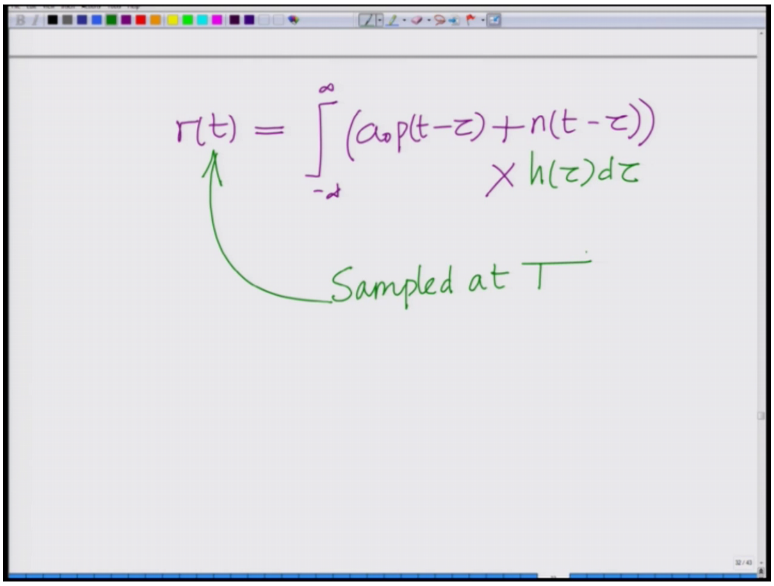
We know y t equals x t plus n of t. So, this is our additive noise model or you can say this is our additive noise channel and further we can assume that this x t equals a naught; let us make the model simple to begin with a naught times P of t where a naught is the symbol P of t is the pulse we have seen this before so ok.

So, a naught is the symbol P of t is the pulse. So, this is our model for x of t. So, this is our model only for x of t. So, let us say this is equal to x of t we have seen this model before the transmitter signal x of t is nothing but a naught p t plus a 1 p t minus start this pulse shifted by t capital T where t is the symbol duration and so on and the sum of all these modulated and shifted pulses, how were to keep things simple here we were are assuming a single pulse that is a single symbol a naught and a pulse and this can be readily extended to the more general scenario. However, let me just be cleared to make things simple to make the following derivations and the following analysis simple I am restricting to a single symbol a naught and pulse p t.

And therefore, what I have here is I get this is equal to a naught P t plus n t again let me mention here this is for simplicity, just to make the initial presentation and then swing analysis simple.

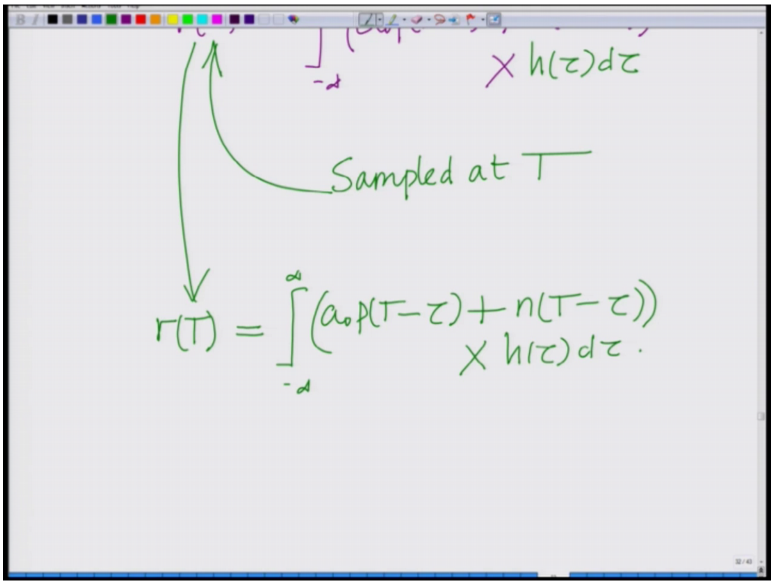
Now, a naught is the symbol p t is the pulse and now, therefore, if I substitute this in the expression for r t now what I am going to do is I am going to take this y t expression for y t, and I am going to substitute this in the expression.

(Refer Slide Time: 10:15)



Substitute this in the expression for r t and what I am going to get is that r t equals integral minus infinity to infinity well we already know it is y t minus tau into h of tau, but y t is a naught p t y t minus tau is a naught p t minus tau plus n t minus tau times or let me write it down times h of tau d tau, this is r t. Now let us consider the sample of r t now these remember this r t is sample at t. So, let us consider the sample at capital T.

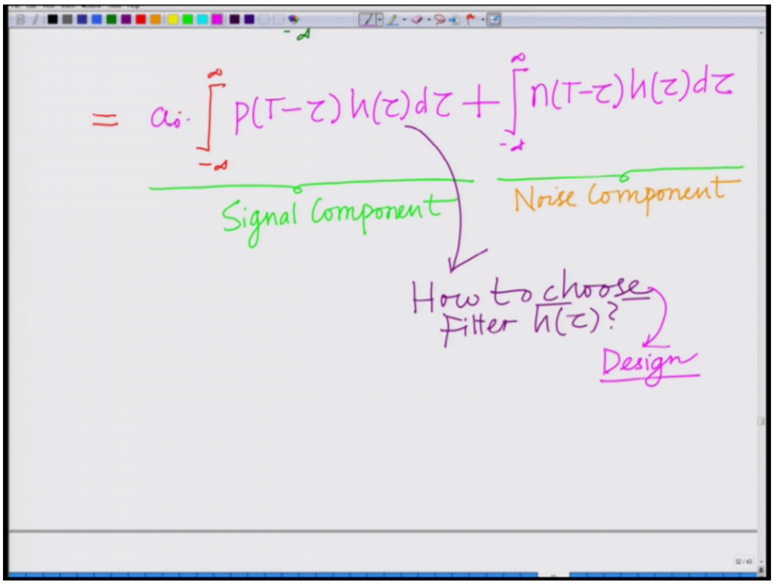
(Refer Slide Time: 11:14)



So, sample at T which means I have to substitute small t equal to capital T that is nothing but minus infinity to infinity a naught P of capital T minus tau plus n of capital T minus tau times h of tau d tau this is the output signal r t output of the linear time invariant system r t sampled at capital T, where capital T is the symbol duration.

Now, let us sub let us separate this into signal and noise components.

(Refer Slide Time: 11:59)

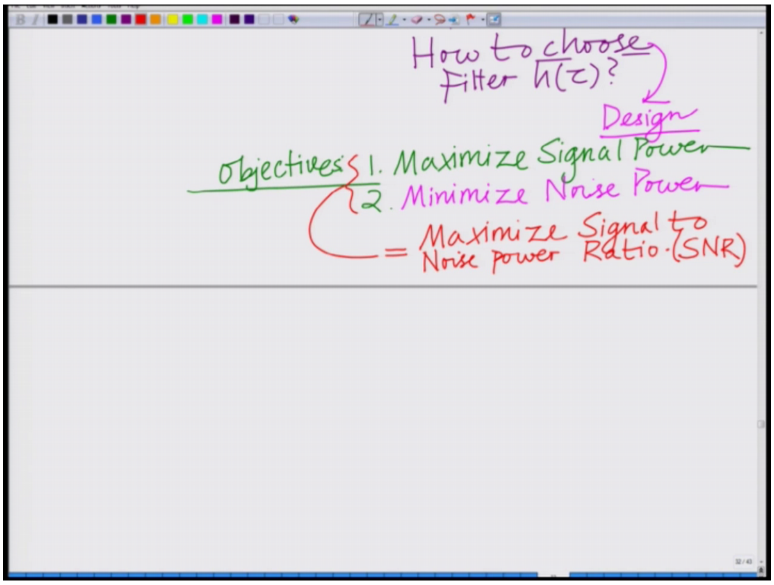


So, now I can separate this into signal and noise components that is this is nothing but integral minus infinity to infinity of course, a naught is constant a naught is the constant symbol, so this will be come outside of the integral it does not depend on the variable of integration d tau. So, this is a naught P t minus tau, h tau, d tau plus integral n capital T minus tau, h tau d tau and now you can see we have separated this into the signal and noise components this is your signal component. So, this is the signal component and this is your noise component signal component. So, we have the signal component we have the noise component.

Now, the point is the question that we would like to address as a already described how do how does one choose how to choose the filter h tau; how to desire or either how to rather than choose I have to say how to design, how does one design this filter h of tau that is how does one optimally design this filter h of tau. Now we have the signal power and we have the noise power, now realize that we would like to maximize the signal power while minimizing the noise power. So, the noise is the undesired signal we would like to minimize right we would like to minimize the noise power while maximizing the signal power or in other words we would like to maximize the signal to noise power ratio that is we have a ratio with the signal power in the numerator, noise power in the denominator we would like to maximize the signal power minimize the noise power.

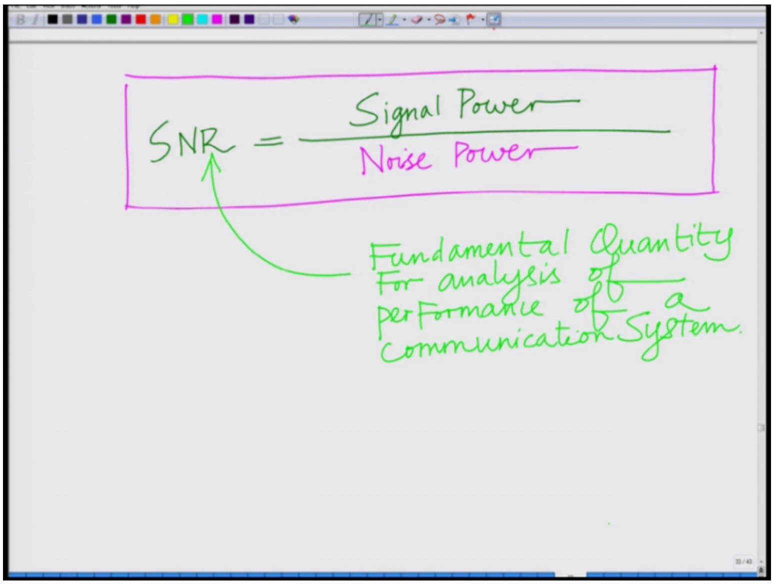
Therefore, we would like to maximize the signal to noise power ratio that is something which is very convenient to consider in a communication system. So, the most typical matric that you will see to characterize the performance of a digital communication receiver is the signal to noise power ratio at the output after processing the received signal.

(Refer Slide Time: 14:46)



So, that is something that is fundamental which you might already seen before. So, the objective is. So, we have dual objectives really cycle call object. So, objective is maximize signal power 2 to minimize the noise power. So, I would like to maximize and both these things together you would like to maximize signal to noise power ratio this is defined as SNR. So, this is a fundamental quantity in the analysis of communication system.

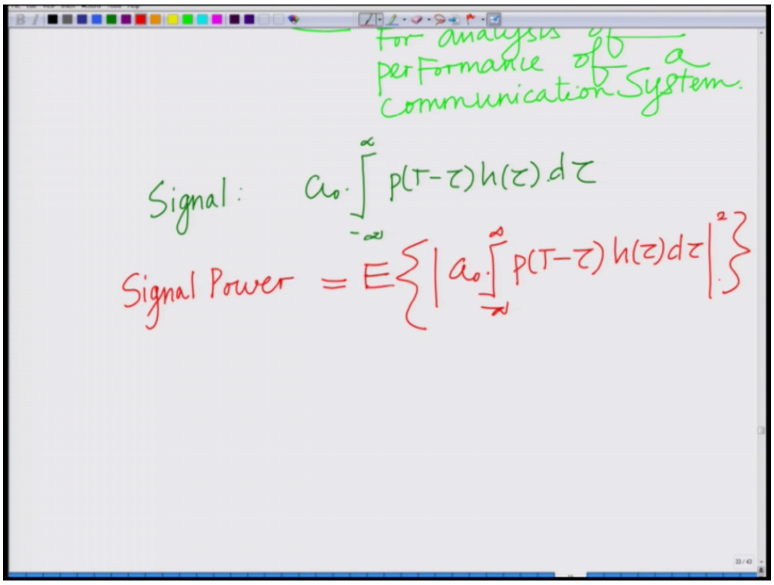
(Refer Slide Time: 15:49)



So, we have the SNR which is the signal to noise power ratio which is equal to your signal power divided by noise power. This is the signal to noise power ratio and this is a fundamental quantity in analysis of a communication system; for analysis of performance not only digital communication system for that matter any communication system actually even analogue communication system because noise is an inevitable part or inevitable component in any communication system.

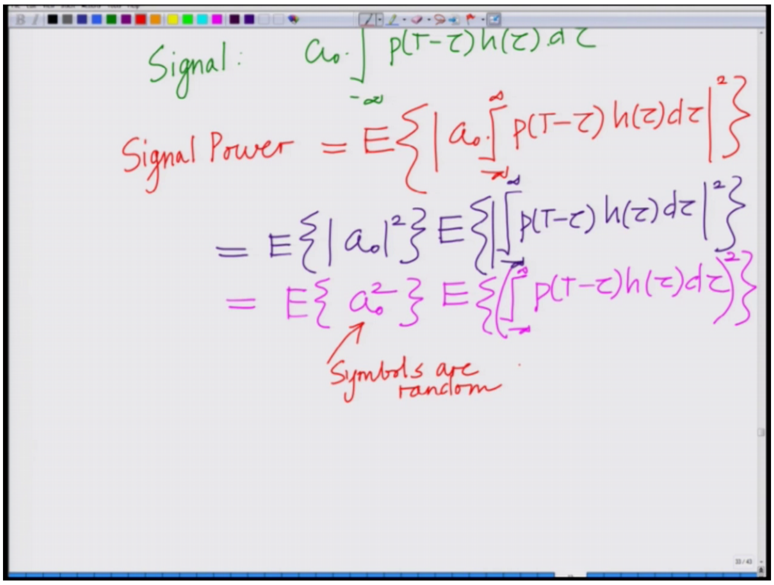
So, the signal to noise power ratio is one of the central metrics which is used to analyze the performance of a communication system, any communication system in particular in this context of the performance of that of a digital communication system.

(Refer Slide Time: 17:30)



So, now let us see; what is the signal to noise power ratio for this digital communication system. So, we have the signal power, now let us look at the signal SNR signal, this is a naught integral minus infinity to infinity well P t minus tau h tau this is your signal. So, signal power would be; so slightly bigger terms. So, let me write this completely expected value of a naught minus infinity to infinity, p t minus tau whole square this is equal to expected value of well.

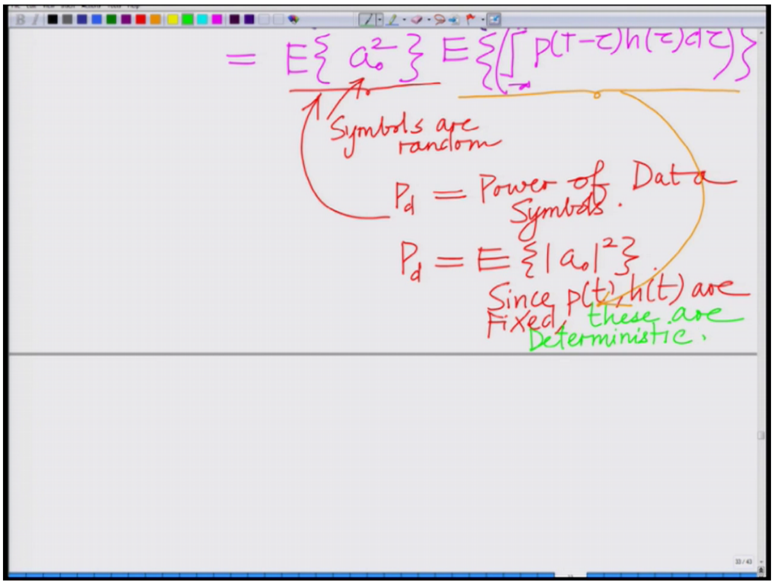
(Refer Slide Time: 18:32)



I can separate this as expected value of magnitude a naught square into expected value of magnitude of minus infinity to infinity, P t minus tau h of tau d tau whole square considering only real signals I can even drop the magnitude, but this is finite is general. So, or let me just drop it considering let say consider only real signals, I can simply write this as expected value of magnitude a naught square minus infinity to infinity, P t minus tau h of tau d tau whole square.

So, this is your signal power; now if you look at this now there are two interesting things in this first we know that the symbols as know symbols are random these are the information bearing symbols. So, these are random and we have characterized the power of the transmitted symbols, we have used the notation to represent the power if you remember correct the power of the transmitted data symbols we have termed that as P d.

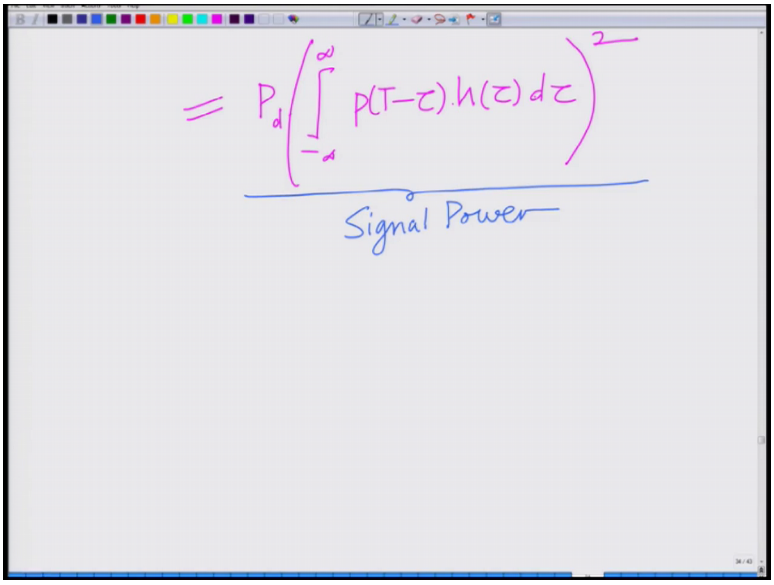
(Refer Slide Time: 20:06)



So, expected a i square we have denoted by naught a naught square, but expected a i square I have denoted it by P d was the power of your data symbol. So, the power of the data symbol is expected. So, P d we have P d is expected magnitude a naught square that is a power of the data symbols.

Further if you can look at this quantity over here, you will realize something important it depends only on the pulse p and the receive filter h d both of them are not random; that is a pulse p t is fixed. Once you design the filter h the impulse response h d is also fix. So, both these quantities are deterministic there is nothing about random about these quantities, so these expected value simply equal to the quantity itself. So, let us note that also since P t and h t are fixed; since p t and h t are fixed these are deterministic quantity or in other words.

(Refer Slide Time: 21:38)



There is nothing random about them therefore, this quantity the signal power will simply be P d now put this things to two things together this will simply be p d minus infinity to infinity, we have I think P t minus tau h tau, d tau whole square where we have got the read of the expectation because this is the deterministic quantity. So, this is basically your signal power and that is the first important thing that we can realize. So, this is the signal power.

So, we have computed the numerator term in the signaled noise power ratio. So, with this let us stop this module here in the next module we are going to look at how to evaluate the noise power, compute the signal to noise power ratio and then derive the optimal from that derive the filter which maximizes the signal to noise power ratio. So, we will stop here and continue in the subsequent module.

Thank you very much.