Indian Institute of Technology Kanpur

National Programme on Technology Enhanced Learning (NPTEL)

Course Title
Applied Electromagnetics for Engineers

Module – 10
Impedance transformation and power flow on a transmission line

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Hello and welcome to the NPTEL MOOC on applied electromagnetics for engineers, so in this module we will first tackle the problem of power calculation very briefly we will have more to say about it later on and then consider the relationship between input impedance and the length of a transmission line and consider few cases that are quite important when dealing with this transmission line circuit.

We begin by looking at the power calculation we know that transmission line carries a forward going voltage and of course because the impedance is usually not matched at the other end there will be a reflected voltage so there is a forward going voltage from the source going to the going to the lode and from the load there will be a backward travelling voltage wave coming towards the source, okay.

The question that normally occurs is that well there is this voltage V_0^+ and the voltage V_0^- and there is a characteristic impedance of the transmission line at 0 even for the lossless case will there be any power that would be dissipated in the transmission line turns out that for a lossless transmission line no power will be dissipated in the transmission line eventually the entire powers from the source will be delivered to the load okay, a more detailed analysis as I said will follow in a different module.

For now let us just look at very briefly what is the power carried by the transmission line what is a power that is carried backwards and what is the relationship between all this, okay. So let us dig righting.

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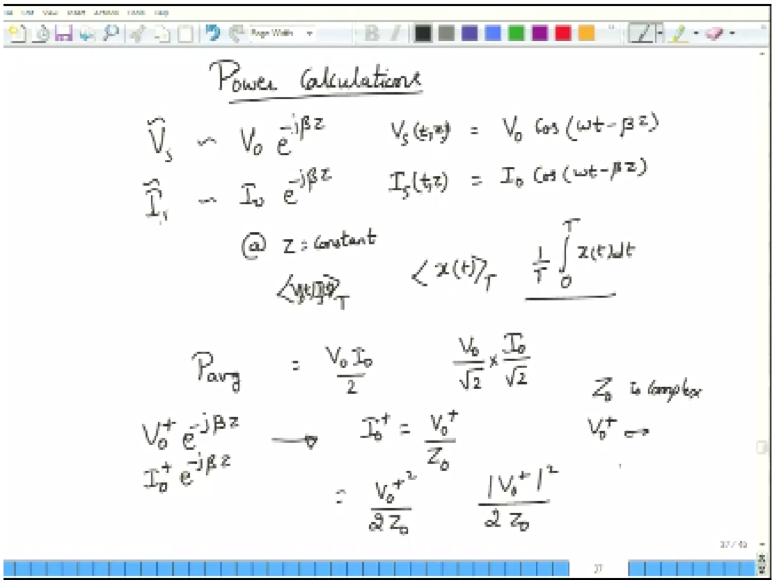

By assuming that the source is a phasor okay, producing the voltage from $V_0e^{-j\beta z}$ and I have a source whose current is again a phasor producing a current of I_0 $e^{-j\beta z}$ okay. Please note that these are phasers the corresponding expressions in terms of time and z is given by after converting this phasor into a proper equation what will you get this would be V_0 cos ωt – βz, similarly the current that you are considering as a function of both time and z will be the current I_0 cosωt–βz.

Suppose I consider some constant z plane okay, it does not matter what plane that we are considering but suppose we consider a constant z plane in which case it can simply represent a source okay, does not even have to represent a transmission line. So for this constant z plane what would be the average power that is dissipated so what is the power that is dissipated instantaneously that is v_st x I_st in a given resistor well I do not need a resistor here, because I know the current I_s here.

And then you average this one over the time, what is the time average suppose x(t) is a function of time then the average of this x(t) over a time t is given by this integral 1/T integration over any time period t x(t)dT okay. If you find what is this average power you can see that the instantaneous power will be cosωt considered z=0 the constant plane, so that this would be 0 I_0 $\cos^2$ωt integrated over one time period of this time variation which will be related to ω what you get is $V_0I_0/2$.

In fact this relationship you already know because the peak value of the voltage is $V_0/\sqrt{2}$ sorry peak value is V_0 the RMS value is $V_0/\sqrt{2}$ the RMS current is $I_0/\sqrt{2}$ the product of these two will give you the average power, okay. Now we have a transmission line whose positive going voltage is given by V_0^+ peak value $e^{-j\beta z}$ similarly the current that would be carried will be $I_0^+ e^{-j\beta z}$ okay, again these are essentially phasers that we are considering.

You can go from the phasor to the average power when we do that there is a small problem that might come u, okay. In general I know that I_0^+ can be written as V_0^+/Z_0 even when you consider these are plus to be a real quantity which sometimes it may not be the Z_0 to be a real quantity then there is no problem. However in general for a lossy line Z_0 is complex or even when the line is lossless V_0^+ might be complex.

This complex only indicates that this source is having a certain phase difference with respect to the current and that could be for a different reason, okay. So what is the power that is carried by this positive going voltage you will have to multiply these phases after converting this phasor into the time domine multiplying and then writing this and it will turn out to be $V_0^{+2}/2.Z_0$. In this case we are considering the characteristic impedance to be Z_0.

In general this can be written as $|V_0^+|^2/2Z_0$ case Z_0 is considered to be real without doing any work if I were to ask you what would be the power.

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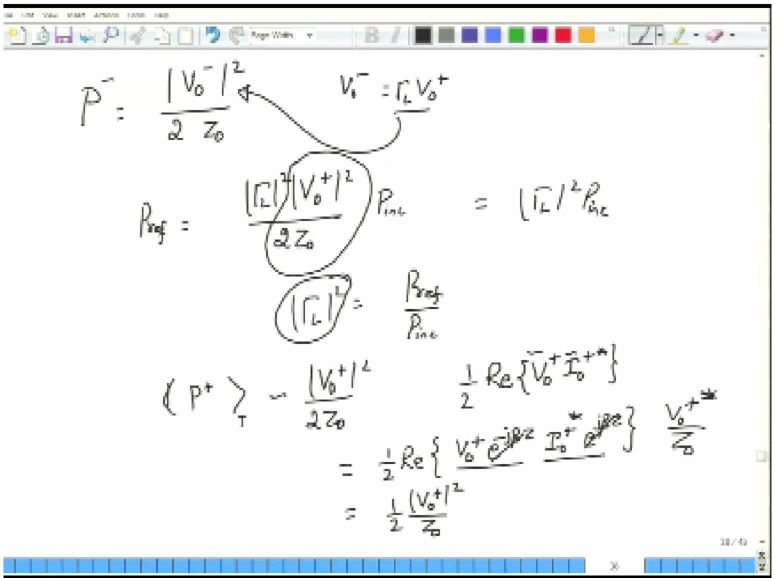

That would be carried by the negative going voltage you will be able to write this as $|V_0^-|^2/2Z_0$ correct, so when you write like this $|V_0^-|^2/2Z_0$ you know that V_0^- can be related to V_0^+ through the load impedance γ_L and this load impedance γ_L will then tell you the amount of power that would be reflected, inside γ_L was always given to the ratio of the reflected voltage V_0^- to the incident voltage V_0^+ this time when we substitute for V_0^- from this expression into this expression for the power carried by the voltage in the backward direction this turns out to be magnitude γ_L^2 times $|V_0^+|^2/2Z_0.$

If you identify this power with the incident power then the power that is reflected will be given by $|\gamma_L|^2$ times t incident, okay. So you see that this is given by $|L|^2$ into P incident and therefore the ratio of the power that is reflected to the power that is incident is simply determined by γ_L or the $|\gamma_L|$ okay. Now you might ask what happened to the lossless condition that we derived well, we only did half the part right, we considered the source to have generated a voltage the voltage came hit the lode that the load does not match the characteristic impedance therefore produces the reflection.

Now as the voltage comes back right, so at the voltage we have some power dissipated in the load, but there is a power that is not dissipated that is coming backwards carried by the negative propagating voltage or negative traveling voltage and this voltage comes in if I have a source which is matched to the characteristic impedance then this reverse power or the reflected power will be completely absorbed.

If that is not absorbed then there will be one more reflection here which will then carry some power and because reflection magnitude is less than 1 the power carried again will be less then there will be power carried backwards and this infinite series will happen such that every time there is a reflection the amount of power carried by that particular voltage will actually go down and down and eventually reach to 0, I did not say that in a lossless transmission line the moment to connect the load I mean the power will be dissipated it actually goes at infinity I mean it can potentially be at infinity.

But if you terminate one of the ends with a characteristics impedance so that there is a proper matching happening then there would not be any further reflection and all the power will be delivered to either the source or the load wherever you have achieved the power match. Usually you want to achieve the power match at the load side, okay. But this is something that you have to remember, so the reflected power is given by $|\gamma_L|^2$ times incident power and the difference between these two will be the one that would be delivered to the load.

There is one additional way of deriving these relationship, I know that I am average power has to be $|V_0^+|^2/2Z_0$ considering this is a time average power right, considering only the lossless transmission line case that I am considering, okay. So this is the power that is actually carried by this is the average power that is carried by forward going voltage. I can obtain this by this particular mathematical relationship I take half real part of $V_0^+ I_0^+$ complex conjugate, right.

Let us go back there this V_0^+ I_0^+ are not just ordinary V_0 and I_0 that you know that these are the phasers that we are considering. So if I know the phasor forms of these so I know this is nothing but half of real part of the phasor for the forward going voltage is amplitude V_0^+ $e^{-j\beta z}$ the phasor for forward going current will be I_0^+ which can be written as V_0^+/Z_0 but more importantly because I am complex conjugating this one this becomes conjugate here the amplitude gets conjugated, okay.

And $e^{-j\beta z}$ becomes $e^{+j\beta z}$ clearly this integrals will be equal to 1 and I_0^+ complex conjugate can be written as V_0^+ complex conjugate divided by Z_0, assuming Z_0 to be real and what I get is half of $|V_0^+|^2/Z_0$, so you have to remember his if you know the fuser forms you can take the voltage phasor, current phasor, conjugate the current phasor. So take the voltage phasor current so that conjugate the current phasor multiply it by both of them okay, so that the term $e^{-j\beta z}$ there and

$e^{+J\beta z}$ get cancel with each other and give you a product of 1 and you get the average power, okay.

You do not have to I mean this is the way to show that you get the average power using the phasor relationship. So this was all about simple power calculation we will not do the exercises related to that but what I want to do in the rest of the module is to give you some interesting inputs.

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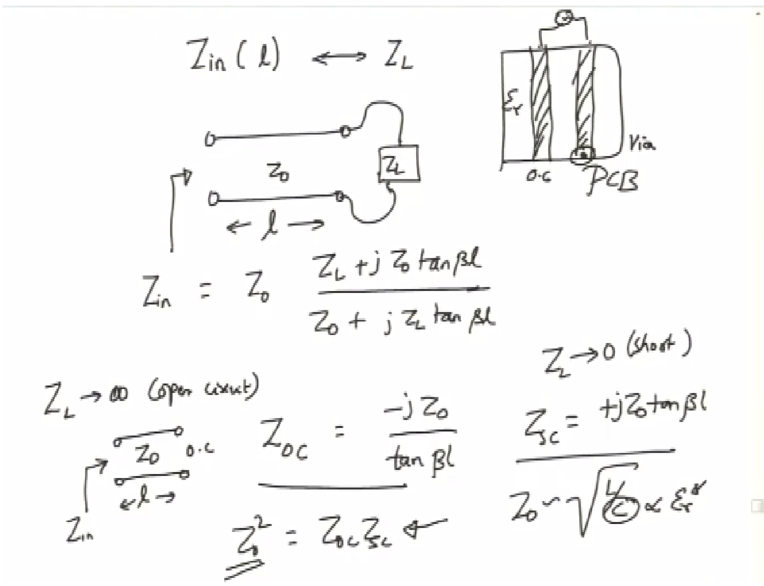

About the value of the input impedance of a transmission line whose length is some L and how is it connected to the load Z_L okay, we consider again the transmission line of characteristic impedance Z_0 this could be terminated in a variety of loads we can denote this load as Z_L please note that this Z_L is not indicating an inductive load, it could be any load that I am considering just the way I am denoting this Z_L in this demotion that L stands for load, not for the inductor.

The transmission line has a length L and what I am interested is to find out what is the input impedance of this transmission line, of course I know what is the input impedance right, so this is nothing but Z_0 Z_L+j$Z_0$and β_L/Z_0+jZ_L and β_L this is the relationship that I know off, okay consider a special case where I take the load Z_L to go off to infinity that is I open circuit the other end.

So when I open circuit let me denote the input impedance seen from the transmission line as Z_{OC} okay, so this is the situation that I am considering an open circuited the transmission line terminals and I am trying to measure what is the input impedance here of a transmission line whose each characteristic impedance is that 0 and a length of L. You know what this is by substituting for $Z_L=\infty$ or Z_L going to ∞ you get this one as - jz0/tanβL.

From a similar analysis I know what will happen when Z_L goes to 0 that is to say a short circuit the load side then the Z_{SC} will be equal to + jZ_0 tanβL look at these two equations carefully if I do not know what is Z_0 okay in a typical practical scenario you do not normally know what is Z_0 you can only estimate what is the value of Z_0 if I do not turned I want to measure Z_0 there is a very simple procedure for us to follow or of course this has its problems.

But looking at the equation Is a simple procedure, I can play the transmission line open circuit the other end ensure that this end is open circuited so there is no current flows in measure the input impedance here what I will be measuring is Z_{OC} then replace the rotor circuit by a short circuit connect a piece of wire out there and then measure the input impedance here, when I take the two measurements then I can multiply this Z_{OC} and $Z_{SC.}$
And take square root of this to obtain Z_O you can see that very easily right so multiplying both sides will give you a multiplying both terms will give you will cancel tan θ on both sides -2n + 2n multiplication will give you one so the numerators Z_O and Z_O will give us Z_O^2 so when you find Z_{OC} and Z_{SC} you can find out what would be the characteristic impedance Z_O , infact if I take a printed circuit both and then I want to measure the ε r, the relative permittivity of this one, okay.

One way to measure the relative permittivity is to actually draw two micro strip lines over here identical micro strip line okay one micro strip line is left open circuited the other one is short-circuited by connecting it to aground plane via you know by actually tell you and then connecting a short circuit by placing a small conductor so that it goes down and connects to the ground plane.

And from the other end connect a voltage source so you can connect a voltage source or you can connect an impedance measuring bridge over here measure the impedances on the two ends from there you find out what is the Z_O and we will let us show that Z_O is given by we already know that for a lossless line it is given by $\sqrt{L/C}$ but the C will be dependent on the permittivity ε_r, so which then allows you to find out what is the permittivity or estimate the permittivity.

This is not a very accurate method but for most applications this method is alright, so this is how you can actually use the knowledge of the measurement of the input impedance okay for various conditions of the load in order to estimate the unknown value of the characteristic impedance itself, in practice this is not exactly how it is done I just told you that it could be one it is not very accurate but okay for some applications or most applications.

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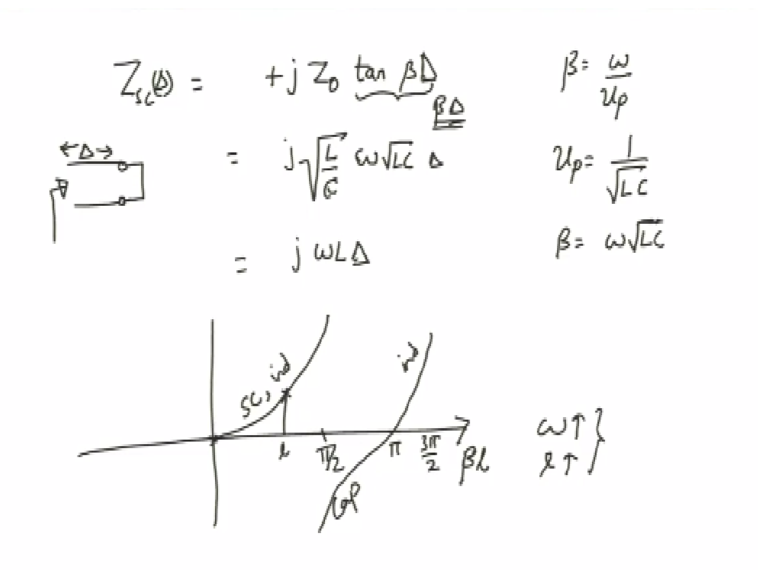

Let us go back to the input impedance expression okay and I want to consider again only the special case of short-circuited load I know that short circuited load is given by or the input impedance of the short circuit regular is given by $+JZ_O$ tan ßl I know what is ß, ß is given by ω/U_P and for this transmission line I know that U_P is given by $1/\sqrt{LC}$ right and therefore I can write ß as $\omega\sqrt{LC}$ where L and C are the distributed inductance and capacitance values of this transmission line.

So I will go ahead and write that one and instead of writing this small L, I will write this as Δ , where Δ is a small distance away from the load so you can think of this transmission line which has been short-circuited and this length of the transmission line is this Δ that I am considering, what is that length I mean what is the input impedance seems here of this short segment length we can obtain that one by going back to this expression so ß will be equal to ω √LC.

So you will get J and I know Z_O, Z_O is nothing but √ L/C and the tan ß Δ can be approximated as ß into Δ because I am considering Δ to be very small so the length of the transmission line is very small compared to the wavelength so I can do a small angle approximation and write this as ß into Δ ß is ω √LC and Δ is of course the length of the transmission line. You can see here that √C can be cancelled and what you get is J ωL Δ.

There is a very short piece of transmission line which has been short-circuited on the other end the input impedance just above about a small distance Δ from the from the short circuited line actually gives you reactive impedance, in fact you can vary the frequency and change the amount of reactants that you are going to see right. Well of course you change the frequency than λ changes so the value of Δ will not be the same it has to also change that you get the idea.

You can actually get in fact if you do not do this approximation just assume that you can change the frequency ω then ω changes F changes λ changes λ and L relationship changes ß x L product changes tan ß L will change giving you any value of reactants that you want. In fact if I plot the reactants as a function of ß x L for this transmission line you will see that when ß L is small and zero the reactance is not zero already and a ß L goes towards Π/2 the reactance rises to ∝ so at Π/2.

The reactants would have become plus ∝ this is when you have short-circuited the load okay and you get a inductive impedance over this side of course what would happen when ß L is negative or you know you just go to the other end where you go from Π/ 2 to 3Π/ 2. You will see that there will be a capacitive reactance that would come up from Π/2 to Π and again from Π to 3Π/2 you will go back to the inductance so the same piece of transmission line when it is terminated with a short-circuit and then you change ß L either by changing ω or by changing L okay.

Or by changing the product of these two it is possible for you to go from an inductive reactance of any value you want the value you want to find the value of this one you will be able to find it by considering the length of the transmission line to be something like this okay. So any length you want if you fabricate a lossless transmission line and terminated with short-circuit you can get inductive reactance we can get capacitive reactance you can get inductive reactance again what would be the behavior for a case of a circuit transmission line that is open circuited at the lower side.

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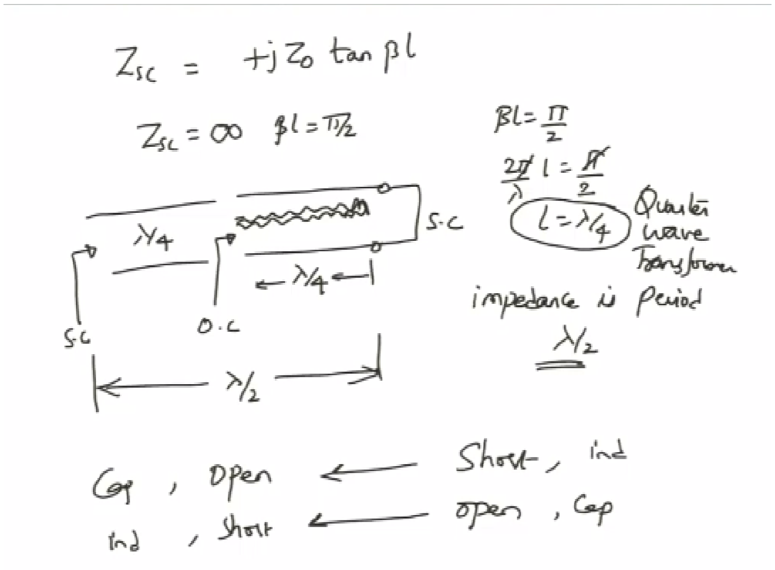

Before answering that question let us look at a very interesting scenario I know that the short circuited transmission line of a length L will have an impedance input impedance of + J Z_0 and ßl right so as I grow from ß = 0 to Π/ 2 this value is always positive giving you a inductive value right but what would have what was the impedance value Z_{SC} at ß L = Π/ 2 at ß = Π/2 tan Π/2 was actually giving you ∝ and the value of Z_{SC} was actually equal to ∝ right. So you actually started off with a transmission line which was short-circuited and then you started exchanging you all started measuring the voltage so measuring the impedance at different points and you move the distance of about λ/4 why λ/ 4 because ß L = Π/ 2 implies that L must be equal to λ/4.

Because you see here ß is 2Π/ λ, Π cancels L = λ/ 4 so as you start moving along the transmission line and come to a distance of λ/ 4 you actually see that this input impedance here I have seen at a distance of λ/4 from the load will look like an open circuit if you again move another λ/4 distance then again it will look like a short circuit thus if you move a total distance of

$\lambda/2$ you are back to the same situation where the impedance will be exactly equal to the impedance of the other side.

Of course this all works for a lossless transmission line but this is a very interesting behavior so on this transmission line lossless transmission line impedances are periodic okay impedance is periodic with a period of $\lambda/2$ a fact that will be very important when we discuss Smith chart okay, in fact what this $L = \lambda/4$ kind of a transmission line this is this is affectionately called as quarter wave transformer I have already alluded to what is quarter wave transformer in the earlier module okay.

It is quarter wave because it is $\lambda/4$ okay so what is $\lambda/4$ transformer does is it turns a short circuit into an open circuit it can also turn open circuit into a short circuit in case you start with an inductance it can turn it into a capacitance you can start with a capacitance and obtain or turn it into an inductor so the same node will show impedance different impedance at different points along the transmission line and this magic can happen because of this particular relationship.

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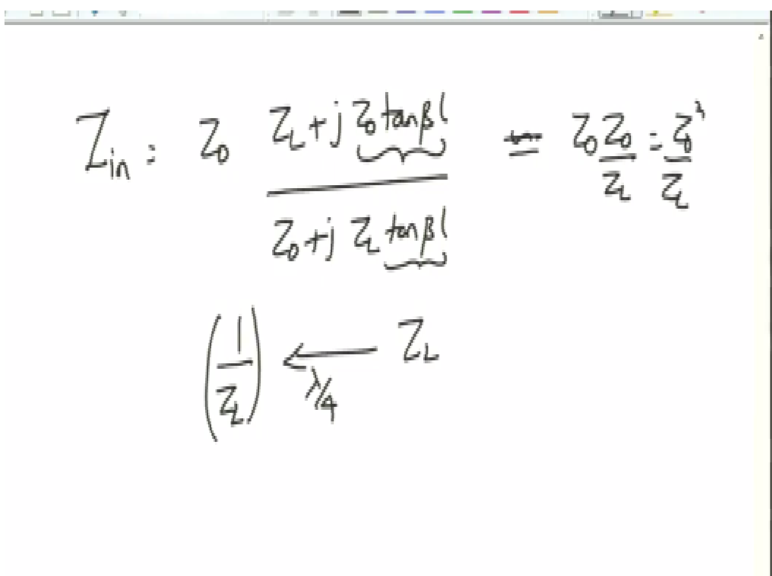

Inside you can easily show that this is true for a general case by going back to the expression for Z_{in} I know Z_{in} is Z_0 x Z_L + JZ_O now tan ß L this would be $\propto$ divided by Z_0 + JZ_L tan ß and so again when tan ß L goes to $\propto$ then this relationship is actually equal to Z_0 x Z_O/Z_L or Z_0^2/Z_L you can see that no matter what Z_L I take after traveling $\lambda/4$ I will obtain $1/Z_L$ so I would have inverted the load impedance inductor turns to a capacitor, capacitor turns into an inductor.

So this is all about the relationship between input impedance and the length of the transmission line that I wanted to talk to you about, in the next module we will see that you know it would not have to use calculators or use this complicated formulas to always transform one impedance to another impedance because in a typical transmission line problem especially when you are constructing something on a printed circuit board you will have lot of components.

And many of these components will be connected by transmission lines if you have to go back to this equations all the time to transform impedances that will be not only not intuitive to you, you would not understand what is happening is just some calculator buttons that you are pressing the second problem is that even pressing the calculator better is very tedious task okay.

So both you do not get an understanding from what is happening and you should also very difficult to keep doing it you can write a program but still it is of you know if you write a program that will use our intuition so because of this people have developed graphical ways of addressing this impedance transformation formula I mean transformation problems and we will see one very famous graphical aid which really illustrates how these impedances are transforming across transmission line.

And helps you solve many, many transmission line problems without in fact using a calculator okay that miracle graphical help is called a Smith chart and the equations that describe the Smith chart is what we are going to take up in the next module, thank you very much.

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