Indian Institute of Technology Kanpur
National Programme on Technology Enhanced Learning (NPTEL)

Course Title
Applied Electromagnetics for Engineers

Module – 12
Smith chart applications
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Hello and welcome to mooke in applied electromagnetic for engineers we have been talking about smith chart we derived smith chart equations in the previous module, we will now understand those equations by plotting them okay here is for your reference the equations that we derived.

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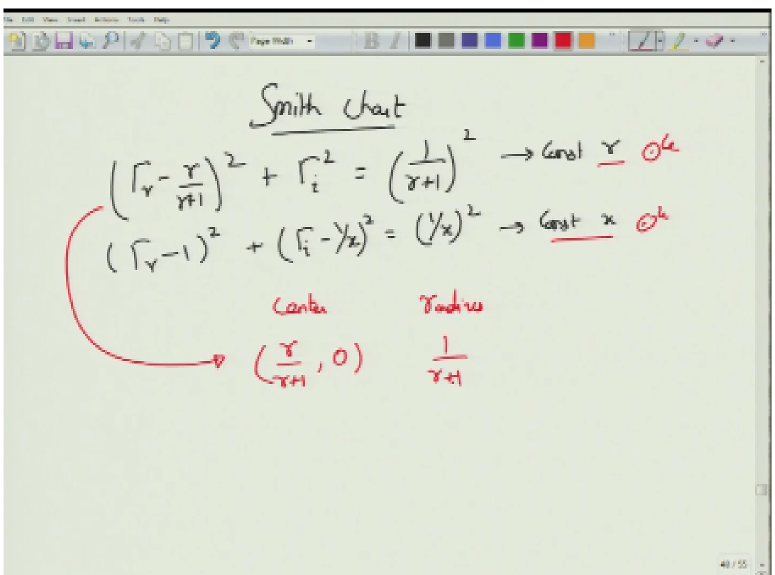

The first equations that we derived was for the constant r values and the second equation is for the constant x values actually they are not constants, but if you fix r to a constant value the this would correspond to a circle right, and similarly if you fix the value of x and this would correspond to another circle the two equations correspond to equations of circles one for r and one for x okay, now let us first look at the constant r circles okay. What is the center of that constant r circles the center is given by r/ r + 1, 0 right, and the radius is given by 1 / r + 1 if you now try to plot this.

We will have to consider the possible values of r first right, you know that r will you know start with value of 0 which corresponds to short circuit and r can go all the way up to ∞ which corresponds to open circuit okay in that case what will happen to the center and.

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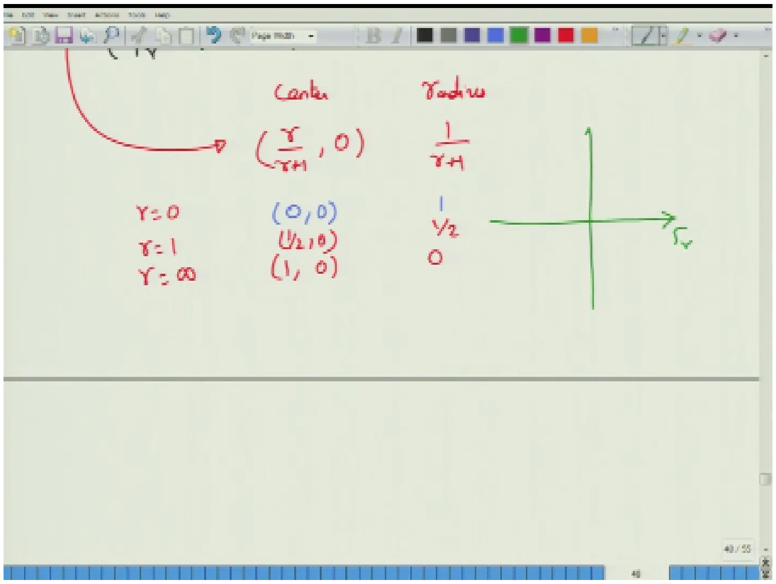

The radius so for r = 0 the center is given by 0 0 right, because the numerator here is 0 and this one is 0 and 0 and the radius is equal to 1 this corresponds to the largest possible radius that we can consider in our smith chart okay, similarly for r is equal to ∞ the center becomes 1,0 okay and the radius becomes 0. So radius of 0 corresponds to a point so what really happens for r = ∞ is that the center moves other to 1, 0 you know on the complex γ play and the radius of that also shrink to 0.

Any value of r should lie in between these values, for example consider r = 1 circle okay this will be very important it would very important for impedance matching okay, r = 1 in this r = 1 circle the center of the circle is actually given by half and 0 right. And the value of the radius is given by ½ so r = radius of this one is given by ½ where are that r = 1 the center is equal to ½ , 0 okay so let us try to plot some of the circles in order to understand what is really going on, to do that first let us start with the complex γ plane this is γ or and this γi so this corresponds to the complex.

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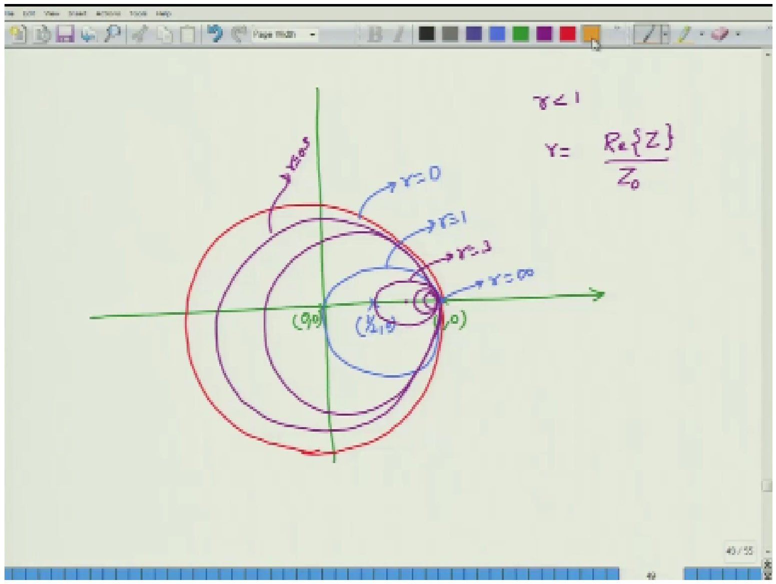

γ plane with center being 0,0 and the maximum 1,0. That we will need we will plot these two points first okay, and then look at what is the radius for r = 0 so when r = 0 the radius is equal to 1 okay so I can simply draw a circle with this center and the radius of 1 and that would pass through this 1,0. Okay and may not be drawing very nice circles but that is because I am not a very good painting person I cannot draw very nice circles I will later show you how the actual smith chart looks you can purchase the smith chart for about 1 rupees or 2 rupees or you can actually find online.

Download and print them okay, so this is the circle that corresponds to r = 0 and really r = 0 corresponds to short circuit because the resistance part has gone down to 0 okay what about r = ∞ well r = ∞ the center is here at 1,0 even the radius is at 1,0 so this r = 1 corresponds to short circuit this point would correspond to kind of an open circuit okay. What about r = 1 r = 1 circle must have a center at ½,0 where is ½ , 0 located it is located in the real γ or axis this is 1/2 , 0 and with this the radius is also ½ therefore with these are the center ½, 0 has a center.

I will now draw one more circle okay this circle is the r = 1 circle this circle is r = 0 circle and this point that we have written corresponds to r = ∞ circle, and r starts to increase the center keeps moving towards this1,0 point right so if you want to write say r = 3 right the center for r = 3 will be 3 /4, 0 and the radius will be only be ¼ ¾ is located at this point okay, ¾ is located here and it also has the radius of ¼ so that circle would look something like this. So this would correspond r = 3, so as the radius increases here the circle will start to shrink.

When r is less than 1okay, so in that case your circles actually would be out of this one all the time within this red radius okay, so this for example could r = 0.3 you might wonder at this point how can r be less than 1 remember what is r? r is nothing but the value of the actual resistance or the real part of the impedance, okay the complex impedance on the transmission line which have been normalized by Z0 so you can actually have the real part be less then Z0 or greater than Z0, correspondingly give you two distinct region one is r less than 1 and r greater than 1, so these are the circles which are known as constant r circles okay how do a constant x circles again we will go back to the equation.

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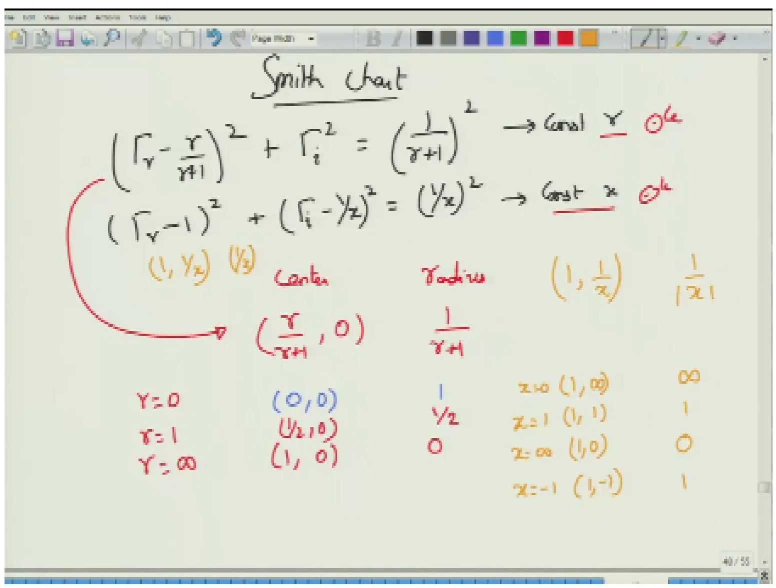

To not that what is a center and the radius okay, this time we know that the center is here must be given by 1, 1/ x and the radius must be given by 1/x so now what I have a center as 1, 1/x and the radius to be 1/x so when x is equal to 0 where be the center, center will be 1,0 remember this is the right most point that we have looked at this also corresponds to r = ∞ by the way and what could be the radius, radius will be ∞, now what is the circle with an infinite radius is just a line so this will correspond to a line on the other extreme when x = so this was for x = 0 so on the other extreme that x = + ∞.

That is an inductive + ∞ the center of the circle will be 1,0 again you will be seeing that the center will not change the centers are x = 0 circle actually 1,∞ with a radius of ∞ and the center for x = ∞, value actually 1,0 with the radius value of 0 okay. So I will show you how it would look and for x = 1 the center will be at 1,1 okay and the radius of this one will also be equal to 1 okay, finally for x = - 1 the center will be at 1, - 1 and the radius will again be of course the radius have to be in magnitude therefore the radius will be equal to 1 again okay except that this circle will be in the other 1/2, so if you not.

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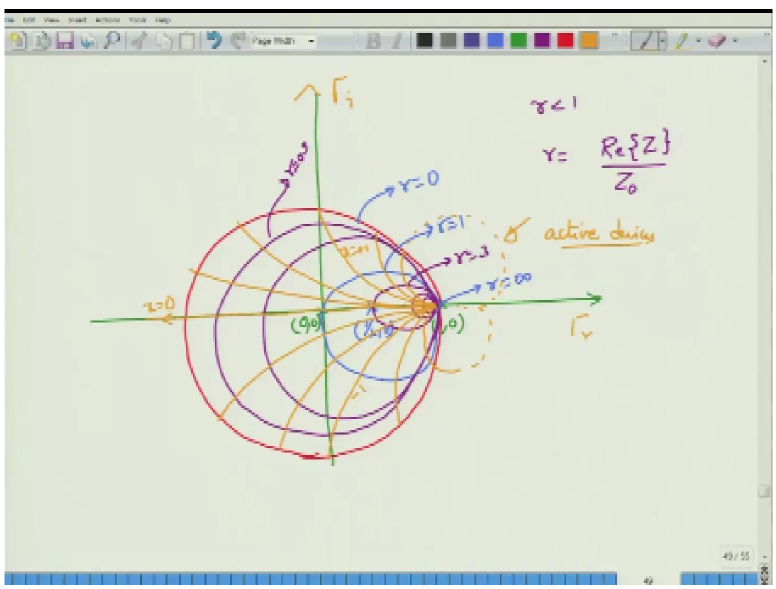

All these values of constant x circles this horizontal line okay with the center at 1,∞ but does not really matter to us it correspond to x = 0 circle okay, similarly x = 1 circle would have the center has x = y circle would have center at 1,1 and has as radius first one and that could be just this quadrant okay, So this corresponds to x = + 1 and this corresponds to x = - 1 as x increases in magnitude, okay your circle radius is start to come down so this for x changing in as x increases and then x is less than 1 they would move away into this particular quadrant so these are the circles.

They do not really look like circles there more like ½ or quadrant circles and that is because the actual circle corresponds to this one okay, so the actual circle must be corresponding to this one we can appropriate center into radius, but unfortunately or fortunately we know that all values of r and x must be in mapped to within this particular minute circle right and because of that the really do not consider in the components or the circle parts which are away from this unit circle okay, these will be considered when you are designing microwave and rf circuits where you will using active devices.

Okay for them you will have to consider the circles outside of the unit circle because those devices can actually give you gain, so this super imposed constant x and constant r circles will give you smith chart and then actual smith chart which I downloaded from the internet would like this.

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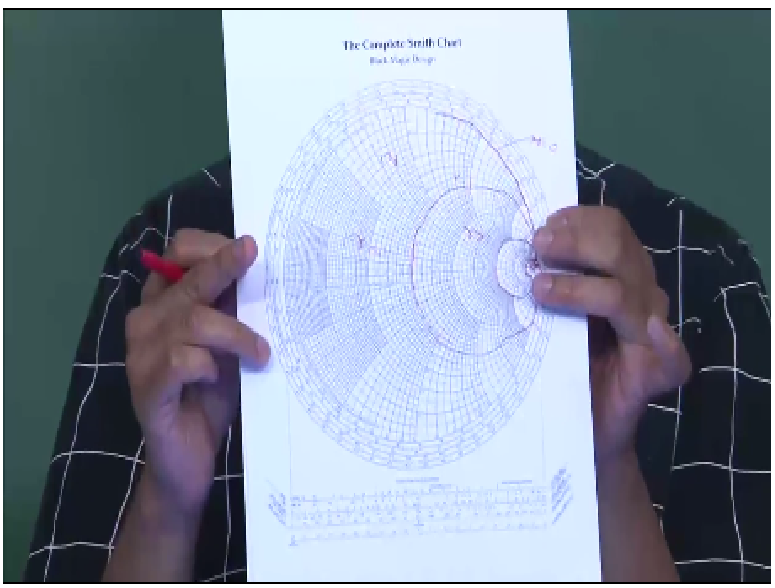

We can see the commercials smith chart which corresponds to super imposed constant r and constant x circles okay, we can see that this x circles so not really circles but they are all arcs this was I will said downloaded from internet you can also download and there are various circles that you can see, so I can try and may be you know give your circle that would correspond to this one. So this is for example a circle that would be plotted on the smith chart commercially available and you can see that the radius on the circles keep on reducing eventually living to a point.

The alter position which corresponds to this one is r = 0 circle where as this one is the important r = 1 circle and this region corresponds to r greater than 1, and this region corresponds to r less than 1, so you can openly see all this constant r circles here similarly we can also look at the x circles you know those x circles are the arcs, that you can see. This hemisphere here corresponds to inductants because for these values the corresponding values of x is positive and below that will be corresponding values of x that will be negative okay, so you can see the corresponding arcs as well as this circles.

There are also other markings on the smith chart which I will explain to you shortly but this is how the actually smith chart would look, now this is my suggestion for this module and for the next few modules please have a smith chart with you and repeat the steps that I actually do it on the in the module in the video, if you do it yourself along with me or maybe after if you have listen to the module then you will learn how to use smith chart this is one of the most important graphical visual tool that is available to microwave and rf engineers okay and also bar would and used by other engineers.

So this is very important have a smith chart with you, which will and the navigate this smith chart making navigation on the smith chart will become easier okay, and verification of how the problems that I would solve would also be easy in case we have smith chart in hand okay let us come back to.

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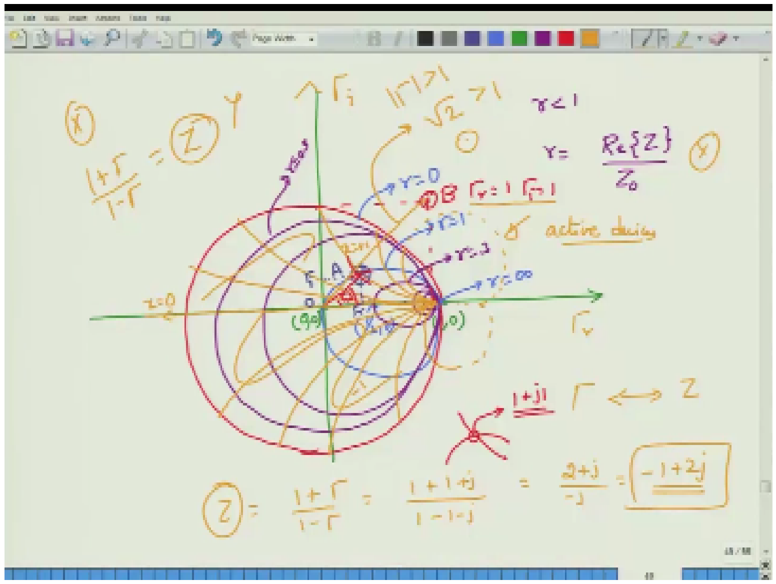

By use of smith chart the basic use of smith chart okay, what how to be use this smith chart in a very simple sense as you can see smith chart is actually giving you or relationship between γ and the complex impedance Z, please note that I am drop the normalization bar over here, okay that is just or notational convenience and also I have dropped this function of Z clearly the quantity on the left and the right hand side of this expression arcs functions of the position on the transmission line but I have just kind of dropped it, so that it does not clatter or equations okay so this value or so this smith chart.

Is actually a relationship between the complex γ and the complex Z values let us pick a particular value over here so may be in a will denote that particular value and call this as the value that I have picked and I am also circling it, with I know I have purposely excite with the circle that this is the point that I am considering what does that point represent let me also denote the center of this one of the smith chart as point o, and the point that I have marked here as a okay, so what is this point a really represent if you look at it the way it this a represents in the rectangular γ chart would be the corresponding values of γr which is the real value of γ at this particular thing and a component are the imaginary part of γi this is in the rectangular form if you draw a line okay connecting o to a then the radius of that line will give you magnitude as γL and the angle will give you or magnitude of γ.

And the angle will give you the phase of the reflection coefficient at that position this is not γL well you know that of course it is also γL because γL magnitude does not change along with the last line transmission line so this gives you the radius gives you why a radius gives you the magnitude of a reflection coefficient which is also equal to magnitude of γ l and the angle will give you the value of the phase of the reflection coefficient but on this chart I have circles okay as well as this arks so I have an r = 1 circle passing through this point I also have an x = 1 circle x = +1 circle passing through this point.

So this is actually inter section of r = 1 and x = 1 point okay so therefore the corresponding impedance at point a will be given by 1 + j 1 and please note that this 1 = j 1 value that we have written clearly the is the normalized value of impedance and this is not the value of this γr and γi please note that this is not γr and γi this values the corresponding impedance. To make that point little more clear let us consider arbitrarily the point here okay which I have marked and let me call this point as point b.

The corresponding value of b is I am going to specify the value of γr and γi γr = 1 and γi = 1 what would be the corresponding impedance here okay, the corresponding impedance would be obtained by going back to the equation that I have because the normalize impedance are the impedance that I have is given by 1 + γ/ 1 – γ the impedance is given by because γr = 1 at point b and γi = 1at point b this would be 1 + j / 1 – 1-j, so clearly 1 + 1 would corresponds to 2 this would be 2 + j / 1-1 is gone so therefore this would be –j down here and what you get is -1 = 2j.

This would be the complex impedance that will be represented by this plus I mean point b, but what exactly this means how can b think of the impedance has -1 + 2j clearly it is not the reason why it is not is because we consider the completely unphysical scenario of γr =1 and γi = 1 what would be the magnitude for point b the magnitude of point b will be equal to √2 and √2 > 1 making the magnitude of reflection becomes greater than one which cannot happen in a passive circle.

Let me re, you know tell you once more. Point a that we consider lied within the unit circle of the smith chart there the value of magnitude of γ l ˂ 1 there was a certain phase within but let us not worry about the phase, the corresponding value of γr and γi can also be rate from the chart. But most importantly the point a represented the inter section of the constant r and the constant xr rx right the constant r circle and the constant x circle.

And in that case correspondingly we obtain the value of impedance as well you can checked that out, so with the value of a given as a point 5 + j. 5 in the for the γ value you can find out what could be the corresponding value of impedances okay. So every point inside the unit circle corresponding to physical value of impedance and the same point will also tell you the reflection coefficient okay it only depends on what coordinates you are considering.

Of you considering the circles in the ark then you are in the impedance coordinates if you consider the straight rectangular axis you know γr and γi then you are in the complex γ and else those points corresponds to complex reflection coefficient and clearly for points which lie above or outside of the unit circles which ahs the point b that we consider a few minutes ago the corresponding value of γ magnitude of γ will be > 1 and that cannot happen for a passive note that we have consider.

A passive load cannot reflect voltage which is higher in magnitude than what it has received so the reflection coefficient will always be less than one for a passive load it is another question that if you actually have an amplifier in the reflection from an amplifier being an active device you know voltage or current which is greater in magnitude than it what it receiving oaky. In that case yes those correspond to a proper point but in that case r will be negative but we have already said that r negative corresponds to a gain rather than loss okay.

Negative resistance corresponds to gain and then presence of an active device, but you know mathematic does not care you can specify any value on this rectangular chart you can specify here you can specify this point you can specify may be this point for each of this points the equation will faithfully give you what is 1 + γ / 1- γ but the result of z you get will be completely non sense except the results that you get then relay within this region okay.

So colorful slide is the summary of our smith chart and basic race of using the smith chart is what I will tell you next okay so we have already seen the smith chart right, let me just draw at basic smith chart for you.

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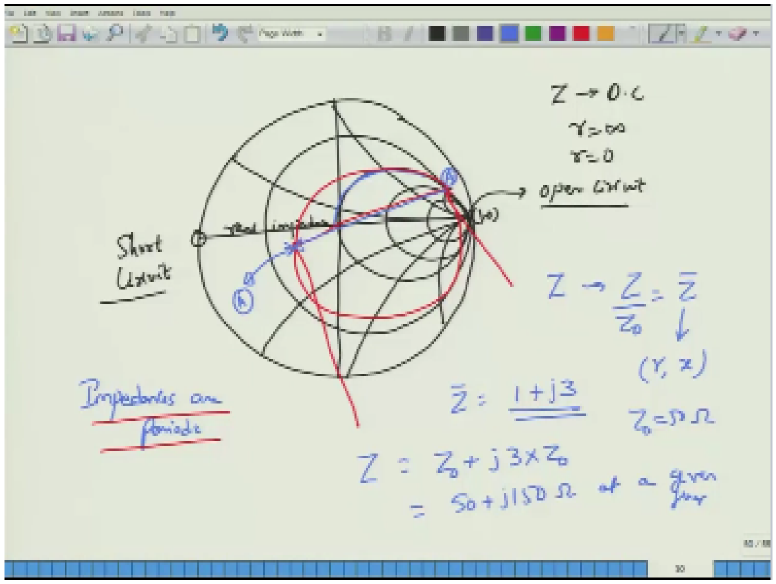

As I said please have the smith chart along with you when you are listening to these videos that will help you reinforce the concepts that we are talking about and you can also notice couple of peculiarities with the smith chart the smith chart all points okay will pass through this 1, 0 point here okay also this point so suppose now consider the complex impedance z to be open circuit for the open circuit and assuming that it is only the real part or that it becoming open circuit this corresponds to the circle and this corresponds to the radius.

The phase angle for an open circuit it also equal to 0 therefore this is really the open circuit point on the smith chart, similarly when you consider r = 0 that corresponds to this circle but when r = 0 if I also look at the phase for areal impedances right. So if I for a real consider real impedances and the x= 0 is this line and r = 0 in this line therefore the intersection of those two points must corresponds to short circuit.

So this is between the short and the open circuit along this line is the value of R which goes from 0 to infinity, where x is simultaneously equal to 0 this all real impedance line okay, so this point here is the open circuit point this point is the short circuit point. If you want to enter the smith chart or if you want to locate any impedance on the smith chart all we have to do is first take the impedance then normalize the impedance by dividing by z0, so what you are going to obtain normalized impedance.

This normalized impedance will be the intersection of r and x circle and rx right, so for example let say z bar the normalized impedance is 1 + j3 okay the normalized impedance is 1+j3 then all we have to do is locate the r = 1 circle which happen to be this one and let see the x = 3 circle which is located at this point okay. So the intersection of these two would correspond to the point, so this is the point A which corresponds to the normalized impedance 1+j3.

Again the meaning of 1+j3 is very clear at this point the impedance if you un normalized is implied to multiply by z0 to un normalize the impedance it actually given by z0 + j 3 x z0 if z0 = 50 Ω of that particular transmission lien the impedance at this point a was actually given by 50+ j150 Ω at a given frequency okay. The other important point about smith chart is that impedances are periodic what do we mean by that fact or the statement that impedances are periodic consider the simple scenario of a short circuit first okay.

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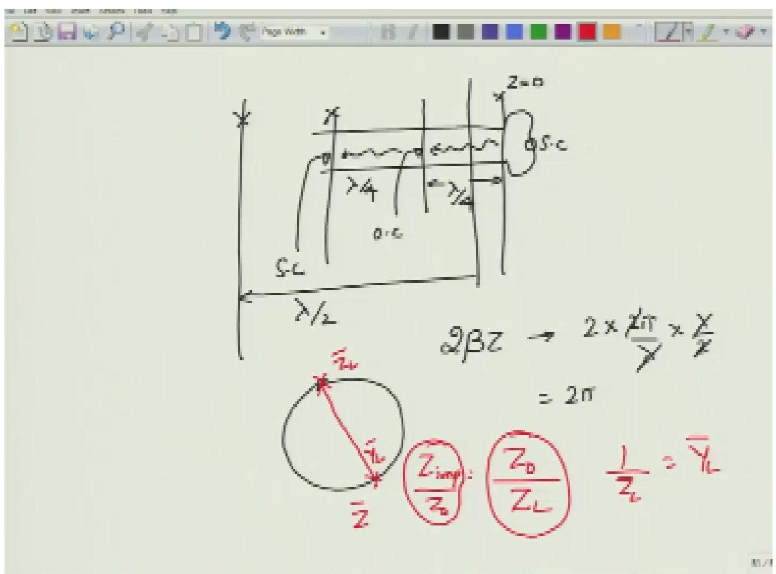

Suppose I consider a transmission line and then short this two terminals okay so I create a short circuit, we know very well in one of the previous modules that as you move along the transmission line the impedance changes and if you move a distance of λ/4 the short circuit will actually more like an open circuit sorry it would look like an open circuit. This is the quarter wave transformer that we have talked about.

And once more if you move a distance of $\lambda/4$ then the open circuit would be transform in to a short circuit, so the impedance at this plane is exactly equal to the impedance at $z = 0$ plane. So whenever you start with any impedance it does not have to be at the load it can at any point z that you start and then you move a total distance of $\lambda/2$ that would corresponds to going back the same value of the impedance this is for lossless cases.

So impedances will repeat every $\lambda/2$, so then every $\lambda/2$ impedances are repeating what it means that the angle $2\beta z$ must be going to 2π rotations right. So you start with $2\beta z$ we know the β is $2\pi/\lambda$ and when you take $z = \lambda/2$ you see that this total phase would be 2π which corresponds to 1 complete rotation on the smith chart. So I can start point A and go along the constant radius oa which corresponds to a constant you know corresponds to a constant γ circle and when I travel completely at one rotation which corresponds to $\lambda/2$ movement away from the load towards the generator I land back on e.

So that is the meaning of impedance being periodic and therefore when you move $\lambda/2$ you come back to the same point that corresponds to a 360^0 movement or 2π movement on the smith chart on the constant γ circle but if you move only $\lambda/4$ then what you actually end up is on the opposite side of this okay diametrically opposite side it is not really ionize as I said the circles but if you know do it on the actual chart then you will be able to get this correctly.

So when move only $\lambda/4$ then you will land up on the other side and here the impedance would be inverted because there is there is $\lambda/4$ transformer length doing it changes the impedance to z_0^2/z, so it start with the load then this would become z_0^2/z_1 and this would be the new impedance that you would see at $\lambda/4$ distance but I know that this z_0^2 can be not distributed equally to z impedance and zl that is I take this z0 derived both sides by z0 and what I get here could be the impedance at $\lambda/4$ it is tense which can be normalized will be equal to the normalized value of 1/zl bar.

And 1/zl bar is the normalized admittance yl bar this is we started with the load yu can start with any value of the impedance that you want okay, so does not really have to be that but the important point is that you started one point on the smith chart and you move $\lambda/4$ distance you end up with diametrically the opposite point on the circle this point would correspond to the normalized admittance corresponding to the normalized load here.

So this would be the normalized load zl bar and this corresponds to the corresponding normalized admittance, in fact this allows to use the smith chart as an admittance chart also why because I can go back to this smith chart if I want to you know add admittances or you know do something about the admittances I only know the impedance to begin with I start with the impedance draw the constant γ circle and then move a distance of λ/4 landing on to the opposite point giving you the admittance that you would have seen on at this particular point.

Form now onwards you can think of this chart as admittances there in market you will also get an admittance chart.

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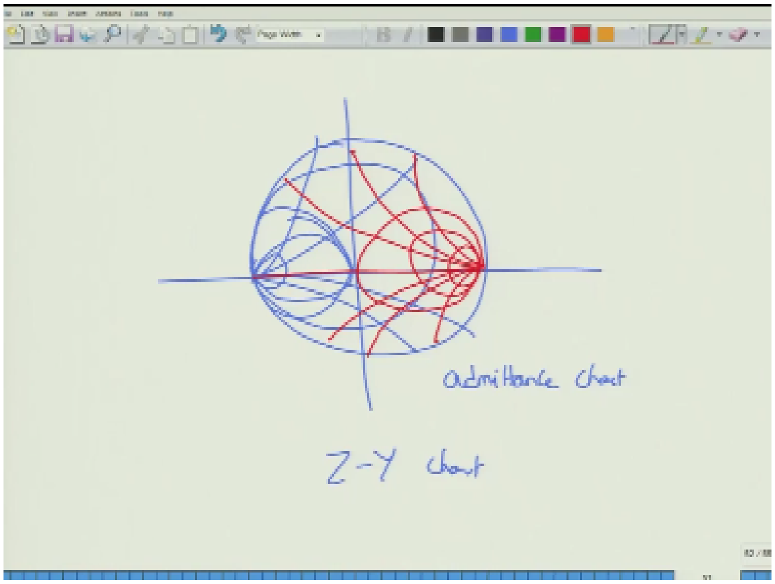

Which looks like the inverted you know you just take this smith chart and then invert it, so wherever you have the constant r = 1 circle that circle would now being along this way and then you start getting this circles so the circles would just be inverted, so every behavior is just inverted okay. You will also get additionally few more larger circles so this is how you actually get an admittance chart sometimes in market you will also get what is called as a zy chart in then zy chart which will be slightly confusing to you I will just what is I think would have both impedance values as well as admittance values super imposed okay. But these zy charts are quite useful for you know impedance transformation problems you can buy or you can download a zy chart and work with it.

For most cases you are satisfied with you know just working with the smith chart starting with the impedance and then converting the impedance in to admittance, so this brings then end to our smith chart basics we have derive the smith chart equation and we have looked at how to basically use the smith chart basic calculation. From the next module onwards we will look at a few applications which require us to use the smith chart and then how to solve the transmission line problems. Thank you very much.

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