Indian Institute of Technology Kanpur

National Programme on Technology Enhanced Learning (NPTEL)

Course Title
Applied Electromagnetics for Engineers

Module – 28
Towards Maxwell's equations-Part 1

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Hello and welcome to NPTEL mook on applied electromagnetics for engineers. In this module we will discuss Maxwell's equations. Maxwell's equation are the basic loss which govern the range of phenomenon associated with electromagnetics from frequencies as low as DC or zero frequency to frequencies as high has about 10^{15}hz that is well beyond the optical range. That is remarkable that only a few set of equations known as Maxwell's equation supplemented with a few additional equations okay, can describe this entire range of electromagnetic phenomenon.

There are various ways in which one can approach Maxwell's equations; one can take a purely mathematical way of understanding Maxwell's equations by looking at the equation and then by knowing the vector field and the calculus of vector field that we have discussed in the previous modules, you might already have a picture of Maxwell's equations, if you just look at them, you would be able to interpret them mathematically and physically as well.

However, historically Maxwell's equation was not developed as a set of equation, I mean Maxwell did not one particular day come up and then give all these equations along with all the other boundary conditions that we are going to discuss. Historically, the phenomenon of electromagnetics was not even to begin with considered as electromagnetic phenomenon people developed notions of electricity first, magnetism, and then developed these two tracks simultaneously or rather parallely.

And then only in the 18^{th} century Faraday managed to bring these two concepts together in order to give you the fact that a changing electric field can induce a change in magnetic field and a change in magnetic field can induce a change in electric field. Based on over 2000 years of development of electromagnetics Maxwell was finally able to unite the various subjects of electricity magnetism.

The work that was started by Faraday was completed by Maxwell, well he introduced the ideas of displacement current. So unfortunately, in this course we will not be able to take that historical route, I will give you a very brief and a very simple overview of how this was developed the actual history is very deep and this is no means a historical lecture or a historical introduction to this subject.

But it is important to understand a few basic milestones in the subject which was the theory of electromagnetics. Then what will we do is we take the math's, you know the mathematical approach, we will look at the equations at once, we would have defined the corresponding quantities, we will look at the equations, and from the equation we will have physical picture formed.

Later we will consider these Maxwell's equations at the starting point and extent that to the notions of static as well as time varying electric field with the key that we are mostly interested in the time varying fields rather than the static fields okay. That is because very interesting phenomenon happened when fields are varying with respect to time, and it is when fields are varying with respect to time, that the true beauty of electromagnetics in that electricity influence in magnetism, magnetism further influencing electricity would come into picture.

So to give you basic introduction as to what is the historical route in which Maxwell's equations was developed, you need to understand that the notion of electricity and notion of magnetism was not something that was not unknown to people. People had developed over 2000 years ago, especially the Greeks had understood the idea that if you take a glass rod and rub it with silk, and then hold it to matter such as small pieces of wood, the small pieces of paper, bits of paper.

Then this glass rod would attract these bits of paper. Then that was essentially the phenomenon, because Greeks were not really interested in experimental observations, but they were mostly interested in philosophizing Greek. So they nevertheless call this phenomenon as electricity okay, and then that was the beginning of electricity in one sense okay. People have observed, so this process of, you know taking the glass rod and rubbing it silk can be extended to many different materials, combinations and people who throughout the centuries are actually observed that this is what happens.

Now you take two different materials not all the materials, but there are few pairs of materials which you could take and then when you rub this process is known as electrification, then you will be able to attract other bits, a small, you know bits of material by the process of this electrification. The fact that electricity you know or the electrification could attracts bit of matter it was not really permute very much until Benjamin Franklin explored this phenomenon in a slightly more detail and he actually came up with the nice you know explanation for this process he said that when you take this glass rod and the rub silk on it, then there is a in balance in the amount of charge so because of this frictional movement the glass rod will move some charges while this silk will gain some charges are it could be vice-versa does not really matter.

He actually define two types of charges he define positive and negative so he said that if you rub them there is one particular component either it could be a glass rod or the silk which I completely forget which one it is but one of those materials will end up having excess of positive charges and the other will have you know excess of negative charges at that time in the concept of a proton or an electron was completely unknown people did not know atomic theory at all because you know it was not developed until about a century later on by that, so nevertheless they understood that there are two kinds of charges.

And they also knew this one because when you rub this materials you know this combinations there was a certain place where you bring two different materials which have been rubbed on two different other materials okay, then sometimes this material would attract and sometimes this materials would result, so this was the idea behind so in that light charges would attract and unlike charges would repelled so in another words you take a positively charge object produced by vigorously rubbing it.

On a silk rod or some other kind of thing and then it will say the glass rod has all the positive charges and then you bring this positively charged the glass so it is positively charge material then it would not attract each other they will repel each other, similarly if I take two silk materials after having rubbed then try to bring them together, then they would essentially try to I mean they would resist that motion so there is a source of attraction and source of repulsion which people I know simplified it.

With saying such as light charges attracted like unlike charges repel okay, this was all about electricity this was the state of affairs in about 1800 or so, or slightly before that and somewhat 1700 also after Franklin did this initial you know given nice pushed to it. The other branch which was developing it was magnetism so people had seen observed a direct there are certain naturally occurring material which was called as load stone and this load stone would attract iron material which was called as load stone.

And this load stone would attract iron material, so you had a rod made out of an iron then that iron material or the iron rod would be attracted by this load stone not only that load stone would also attract load stone in a different load stone in a different load stone and it would also repel different load stones, based on what is the orientation so people us this effect which they called as magnetism use this effects to construct one of the most important instruments that was known to man in navigation called as compass.

Okay so if you look at old compass you would actually see a small needle kind of a thing hanging around okay and this needle would point to always point to the north and the opposite end would always point to the south, so this was actually made by a magnet so this is where the concept of magnetic poles was developed, so every magnet that you would think of would have a north pole and a south pole and way back in around 60 and it is or something I do not remember the exact years okay.

A person postulate that if I take a magnet and then start cutting it into smaller and smaller pieces you know I take this big magnet cut into pieces I do not ever isolate a single pole so I do not ever able to hold a piece of material and say okay, this is a north pole material this is the south material all I always obtained is that if this is the small pole of a magnet that you are this is small magnet that you have actually curved this would be the north pole and there would be a corresponding south pole.

At the other end okay, so in a way the electric charges and the phenomena of electricity was slight was different then the magnetic phenomenon except that in magnetic also or magnetism also like poles would repel and unlike poles would attract, so a pole and charge were similar but they were also different because you could isolate positive charges and negative charges whereas you could never isolate a positive or a north pole and a south pole of a magnet.

So it is very interesting thing that these two phenomenon exhibit similarity but also exhibit differences and this is how these two subjects parallely group one of the most well known classics was theory of magnetism by Gilbert where this person who was physician to the queen of England did lot of experiments on magnetism and condensed all that knowledge into a particularly big book okay. So you can try you read that book it is available on archived or as a free copy okay.

There is no copy right onto that one. So to some up you had a concept of electric charges there were two types of charges and like charges would repel unlike charges would attract, similarly you had magnetism on this side you had two types of poles north pole and a south pole but you could not isolate a north pole from a south pole and like poles would attract unlike poles would repel.

So this was essentially the state of fair until when until more quantitative no measurement where undertaken by first a person called as Coulomb okay, Charles Coulomb and he was a French person what he did was he constructed delicate balances they were known as this force torque balances and then he managed to measure the amount of force that two charges would be you know either attracted to or repelled by.

So he actually did very, very interesting experiments we would not have time to go through of experiments but he did that and then he showed that you can actually not only calculate I mean you can not only observe that there is a repulsion or an attraction okay attraction or an repulsion but he could precisely measure this amount of repulsion or an attraction okay. This was a very important thing and he codified it in terms of what is called as Coulomb law.

Coulombs law actually has two statements you know to complete that particular law it says that two charges would attract or repel and the magnitude of attraction or repulsion false of inverse square as a distance between them thus if I have two objects or two charges separated by 10 meter then the force is actually proportional the force of attraction is they happened to be of opposite charges or the force of repulsion if they happen to be of the same charge type.

Then that force of attraction with actually inversely proportional to the square of the distance so 10 meter separation the force would be proportional to 1/100 meters, okay. So this was 100 meter so this was very interesting because he could you know measure this when an inverse square law was not something that was not unknown to people by then because a few years ago Newton or may be about few 56 years ago Newton had actually formulated his even more famous inverse square law which tells us that there is in a world.

Or in the universe objects masses are always attracting each other and this attraction force is called as the force of gravitation and his gravitational force was also proportional to inverse square so inverse square idea was there okay and in fact in much of a sense the inverse square law of coulomb kind of resembles the inverse square law of Newton, okay. The second component to coulomb law was that.

The line of the force mean which way would the force be attracted to so if I keep these two charges you know imagine if I have kept these two charges here would these be attracted like this or would they be going like that, right. I mean so that question was answered by a second component of coulomb law which most people do not really you know look at that more detail what coulomb said that the line of force of attraction or repulsion would always be along the line that joins these two charges, okay.

So this is my charges and this is the line of the force okay, in our modern vector day notation we write coulombs law in this way, okay.

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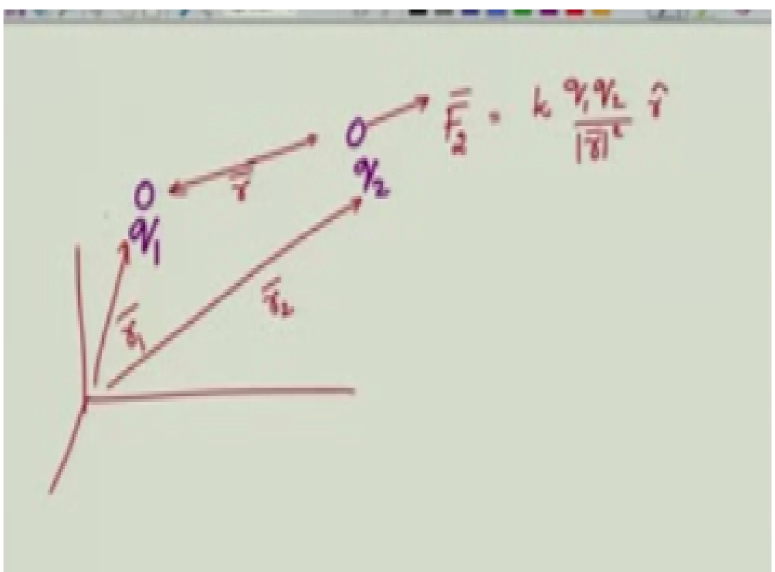

Suppose I have two charges called this charges as Q1 and a charge Q2 so the force between these two of attraction or repulsion can be more specified as the force on charge 2 because of the presence of charge 1 okay. So you imagine that charge 1 was already present it was fixed to the position and then you brought in this Q2 and the separation between these two was actually some distance R, okay.

But I know that this point can be given you know with respect to some coordinate system you can actually specify a position vector R1 we can similarly specify another position vector R2 okay an then the distance between these two can be specified by another position vector R, okay. So this force that you actually observe on charge Q2 because of charge Q1 and of course is a mutual course.

Just as Q2 would experience a force of Q1, Q1would also experience a force 1 because of Q2 and this was given by some constant k, Q1 Q2/the magnitude of this vector Rand it would be in the direction of the unit vector that joints the two charge okay, if I could be along this blue line which is where the force is acting on so along this particular line is where the force is acting on and if you find out what is the unit vector of this one, of course what is the vector r we already know how to obtain the vector r that is given by r_2-r_1 correct, so r=r_2-r_1 therefore you could go back and rewrite this as constant k $|q_1q_2/r_2-r_1|^3$ times r2-r1 vector okay.

Because r_2-r_1 vector divided by $|r_2-r_1|$ will give you the unit vector, so this is how you would write in the modern way the force experienced by the charge q_2 which is kept at position vector r_2 in the space as this particular thing. Now this constant k can be different in different systems of units in the modern NKS system or the SI systems this constant k is given by 1/4πε0 where 4π is of course what you know these are the constant and this is called as rationationalized MKS.

Because the rationalization with respect to 1/4π is happening in Colum's law otherwise this factor of 4π would show up in Gauss's law and in all the other different laws, okay. SO Colum's law because we are putting this 4π here this factor 4π does not show up in Maxwell's equation when you write those equations in SI units. So this is what your force law is and this is what the constant is where ε_0 because I am assuming that the medium here is vacuum ε_0 is called as the permittivity of the vacuum.

If I were to replace or if I were to do this same experiment inside water then the amount of force that you would observe on the charge q_2 because of charge of q_1 would actually been lesser. The force that we experience or the force that the charge q_2 experiences in a material is less than the force that is experienced in vacuum and if you take the ratio of the forces you end up with another constant called as dielectric or relation dielectric constant or relative permittivity of the two mediums, relative is always with respect to vacuum, okay.

And this permittivity because it is a relative measure is always greater than 1 okay, and the value of permittivity of air or the relative permittivity of air is about 1.0008 which is practically closed to 1 as is the case of a vacuum, vacuum of course has a relative permittivity equal to 1, okay and in SI units this ε0 is given by 8.85×10^{-12} F/m because this permittivity is an important aspect for defining capacitances and capacitances are measured in faradays the SI unit is a very practical unit it will allow you to measure ε_0 and specify them in terms of F/m okay. So this is all that was there know to us through Colum and that was the kind of observation which was available at the time of Colum which was a very important step in electricity.

After a few years, people actually develop the notion of current as well okay, and then they calculated they found that you know if I take a piece of current okay, and then take one more piece of current in which the current I_1 is flowing and then there is another current I_2 is flowing then there will be a force of attraction or there will be a force of repletion, so just like this you charges are either attracting themselves or repelling them source seems to be the case currents attracting or repelling, okay.

But this discovery did not come in this way, it actually came about in a slightly different round about manner people as I told you already had this idea of magnetism so there was this person called as Ampere who was performing you know experiments and or rather Orsded who was performing this experiments and then he actually observed a very interesting thing, he was demonstrating in front of a class, you know in a classroom and then he observed that whenever he brought a current carrying wire close to a magnetic compass right.

The moment there was a current the magnetic field or the magnetic compass got deflected, so there was some kind of a relationship between current and magnetism otherwise the current carrying wire would not have deflected this compass, okay. So there was some kind of an interesting observation that this Orstred made and based on this observation later on okay, and almost kind of independent already based on that but on the another continent or another country in France where this humpier started to develop his theory okay and then he came up with law or with relationship that tells you force of attraction because of the two current carrying conductor.

And under go in right that one because of the little complicated expression and I do not want to write that one without introducing due to another effect which people kind of try to understand this okay so it just keep this idea but current can influence magnetic compares and then leave it at that particular point okay.

Now if you go back to this you know you know force on a charge right we have seen that this force is given by some Q_1 Q_2/r^2*r^ right so that is the in model the vector notation that how the force can be return but people were not really happy with this lecture just has whether they are not happy with the picture of Newton you know when the Newton said that there are two masses one may be sun and one may be earth.

And if I move earth here by coming that I think of then immediately the sun would also you know carry a force which will move sun as well so this phenomenon in which one could change you know at one particular point mass M_1 and then inflict on force on mass M_2 was called so you have mass M_2 and mass M_1 and when it change M_1 position your M_2 position also change because of the force.

So this phenomenon in which you could distantly controlled the movement or the behavior was called as action at a distance and people not does not very happy with this action or distance sow what I said was may be does not work that way we of course know today that it does not work that way that if I just take earth and suddenly change earth position sun will not really change immediately there will be a some amount of delay right.

And that is what unscented but now at that time unscented does not born so people what they did was that they came up with a very interesting notation which is now become very common place in physics they said that whether the second mass M_2 is present or not we do not know but the mass M_1as a field of influence around it.

So it is like this I have this pen okay this pen I have may be or I have this holder here whether I have a pen here or not does not matter the holder as a certain field of influence and if I slowly bring this particular mass here then these two masses will attract each other so since coulomb law was essentially same as Newton's law people started I have applying the idea of field of influence okay to charges as well.

They said that when we have a positive charge over here and there is a certain field of influence and then you bring in this other charge right when this other charge would experience a field which would impart a force controverts so it is a intermediary step that was created which would have taken the effects from the charge and transfer it to a other charge.

This intermediate field of influence is what we now call as field okay and we have a first electromagnetic quantity from Coulomb's law in that you have a charge and there is a field of influence around a charge or simply the field around this charge and because which is electric phenomena that we are considering there is an electric field around this charge okay.

And this electric field will take the force or will act on the other charge that is being brought into a field of influence of this first charge and impart of force on to it how much force is impart, imparted and how is the electric field related to the force we define electric field as a force per unit charge okay.

So if you go back to this coulomb's law over here you see that you have f2=1/4πε_0 q1 Q_2/magnitude of r_2 right so if I take this force and then divide this one by Q_2 okay on both the side if I divide this one by Q_2 then Q_2 will vanished from the right hand side over here for this Q_2 will not be present and whatever I obtain on the right hand side is what I define as the electric field of the charge Q_1 so let us very interesting way of looking at it.

So this kind of eliminated the problems of action at a distance because you no longer had this idea that a charge here shaking up would immediately shake the other charge what would happen instead is that charge shaking up here not too bigger okay but if I take this charge a very slowly change it up there is a field of influence or a field electric field around this charge which would change and this changing electric field would act like a wave kind of motion and then it would impact of force on to the other charge so force experienced by any test charge qt would be = the electric field at that particular point in the absence of the charge multiplied by the charge magnitude itself okay.

This is the very interesting phenomena because the positive charges must have a field of influences which would repeal you know the negative charges or rather repeal the positive charges where as the negative charge must have a field of influence which would attract so you could in fact think of because the force of attraction in the force of repulsion you could think of force going away from or the field moving from the positive charge and a field coming towards a negative charge this is just a conversion you could reverse the convention but the convection tell you that the field around a positive charge is always diverging away and a field surrounding a negative charge is always converging into it okay.

This is just a single charge that we are talking about so we introduced our first electromagnetic quantity called as electric field we call it field because it is depended both on the position right the field will be different at points and it is a vector field so the field at this point would be directed here the field here would be directed along this wave for point charge right and if the point charge happens to be a negative charge then field at this point would be directed towards the charge in a different way, so the electric filed is a vector quantity which further depends on a vector therefore this is a field vector.

And when I change the charge the with respect to time then the field also will change with respective\time right so therefore this is a space or rather in general it is a time varying electric field okays similarly we can defined magnetic field and the we will do that one in the next module when we talk of you know defining the magnetic field and then bring the concept of electric field and magnetic field together in the form of Maxwell's equation so until next module thank you very much.

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