Indian Institute of Technology Kanpur

National Programme on Technology Enhanced Learning (NPTEL)

Course Title
Applied Electromagnetic for Engineers

Module – 03
Sinusoidal waves on Transmission lines

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Hello and welcome to the NPTEL mooke on applied electromagnetics for engineers. In the previous module we derived equation for voltage as well as an equation for current which I gave you as an exercise on the transmission line.

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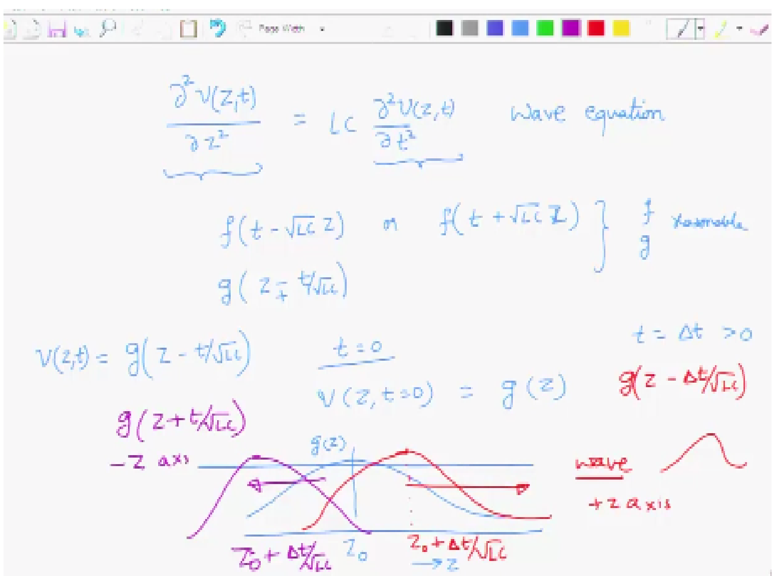

If you recall what those equations are that equation tells you on the left hand side how the voltage would be changing, voltage being a function of both Z and T on the transmission line. So on the left hand side of the equation you have the voltage changing as a function of Z remember Z is the axis along which the voltage or the axis of the transmission line that we have considered.

The right hand side of this equation will have $\delta C \times \delta^2 V/\delta t^2$. Again V is the function of both Z and T right.

The left hand side is telling you how the voltage is changing along Z and the right hand side is telling you how the voltage is changing a long time. The voltage is actually a function of both Z and time. Now how do we solve this equation. Now mathematically these equations are called as partial differential equations of the second order and mathematicians have come up with various ways to solve this equation.

We will now go through the solution of this equation at this point, but instead I will tell you the solution and later in the exercise you can demonstrate that the form of the solution that I give you is actually satisfying this particular equation. Incidentally this equation is called as a wave equation for the voltage V, a similar equation will be there for current I. Now this particular wave equation will admit solutions of the form $F(T- \sqrt{LC}xZ)$ or $F(T+ \sqrt{LC}xZ)$.

It will also admit solutions of the form Z-T/square root LC and you can very clearly see that I have just interchanged the positions of Z and T from the function F in this function G. It will also satisfy or it will also admit the solution of the form GZ+T/square root LC you can actually substitute these functions into them, we only demand that these functions F and G be reasonably you know mathematically reasonable, physically reasonable that they do not have infinities anywhere or they are not physically unrealizable.

These functions are assumed to be physically well behaved mathematically well behaved okay. Let us consider what is the implication or what is the meaning of this type of solution. So let us consider for example that the voltage V at any point on Z and T of the transmission line will be of the form G(Z-T/square root LC) okay. We do not really care about what is the shape of the wave form here or shape of the function G here, they are only interested in looking at what would happen if this is the kind of solution that will be satisfied by the voltage and what is the implication of this one.

Let us say I somehow freeze time right by freezing time I mean I take a particular value of time, I can take any value of time for this function. So but let me assume that I am taking T=0. And then I will look at the entire transmission line, so I have a transmission line here and I have frozen time at T=0 right. Now what will happen to the function then, then V at Z at time T=0 will be some function G(Z).

So if you actually have a transmission line in this way and Z-axis is one that is travelling in the direction that I have shown, then you can see that this function g(z) would be present along the transmission line let us say that the function g(z) is peaking at some arbitrary position set 0 at some arbitrary position set 0 the function g(z) is peaking now I will increase prime from 0 to additional time Δt such that Δt is greater than 0.

The what will be the shape of this one now you look at what will happen to the function gz gz-$\Delta t/\sqrt{LC}$ will be the new function correct and from your earlier significant systems you would that if g(z) would represent a function of z in the particular manner which is peaking at z0 g(z) – $\Delta/\sqrt{LC}$ will be the same function g(z) it as got delayed or separated. So now earlier this was peaking of z(0) now this function would peak at z0+ $\Delta t/\sqrt{LC}$ right so it would actually peaking at this point and the shape of the function would remain the same the essential point is that if earlier the peak was situation at z0 now the peak is situation at z0+ $\Delta t/\sqrt{LC}$.

So this is the shift with respect to Δz or with respect to z axis the amount of shift is Δz that needs to happen such that the peak get shifted by a cretin amount of which is given by Δt LC. Right and to which is this shift happen now suppose I go from t= $\Delta t = 2 \Delta t$ that is why I increase the time from Δt to $2\Delta t$ and what will happen to this position where the peak would be two present.

It would further shift towards the z axis so as Δt increases the shift of the way from also happens in the positive z direction therefore this kind of a thing would respect a wave behavior with a wave moving along the positive z axis okay so the shift is happing along the z axis and the voltage if actually a wave right with a particular wave shape that wave shape is not important for us but it is a wave which is moving along positive z axis. Okay so this is a wave that we are moving along positive z axis now instead if you had started off with the function let us say g(z) + $\Delta t/\sqrt{LC}$.

At t= 0 you would still have the original function g(z) but if I now increase my time the peak should now occur earlier in set correct so the peak would now occur but – z0 + $\Delta t\sqrt{LC}$. Rights so this is a wave which is actually moving in the negative z axis anther for this particular voltage would correspond to a wave which is moving along – z axis.

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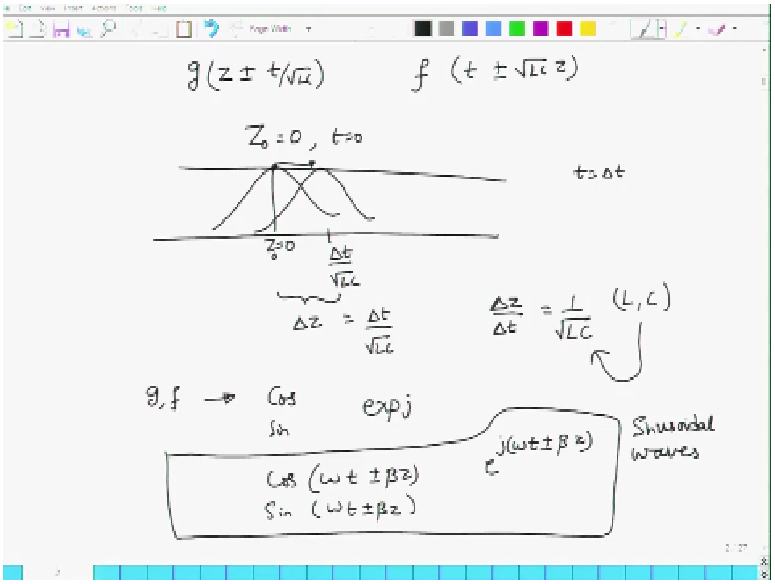

So to summaries transmission line equations for the voltage admit the class of solutions which are given by g z+- Δ t√ LC or equivalently ft +- √ LC into z okay with the + standing for waves moving in the – z axis – standing for wave moving in the positive z axis by the way what would be the velocity of this wave it is kind of very easy to understand that velocity right or derive that velocity v concreteness we will assume that z0 = 0 is the position at which the original function gz would be peaking right so originally g(z) would peak at z= 0 along the transmission line okay now at t= Δ t.

So this was the situation at t=0 and g of c was speaking at z = 0 or said not equal to 0 now as I increased t from 0 to Δt while function would has shifted to Δt/ √ nc this is the change that has happened in Δz the location of the P has changed were distance Δz which is given by Δt/ √nc and if you take the ratio of Δc/Δt which will be give you the velocity at which this speak is moving this particular point is moving towards or along the positive z axis will be the velocity of the wave and that velocity is given by 1/√ nc it is kind of quite surprising that.

The two parameters L and C of the transmission line L and c of the transmission line determining the velocity of the voltage and you can also show that because the current satisfies the same set of equations write it instead of V of let we replace V of set e the equation would be same therefore the velocity of the current would also be the same so whatever the voltage that is moving as the function of z and t a similar their form would be available for the current as well so current could also be.

Moving along either plus said axis are along – z axis at the same velocity of 1/√nc so we have seen that voltages and current so the transmission line for no longer just have z independent quantities where actually waves which are moving either along positive axis or along negative axis okay so that is the idea of a voltage or a current wave on the transmission line okay now let us actually specialize these functions g or f into very specific type of functions this functions that we would want to specialize are trigonometric functions Cos or Sin or equivalently the complex exponentials.

So I would like as two consider functions which are of the form Cos ωt + or – βz your Cos can be also be Sin with the argument of ωt + or – βz or we will consider the corresponding exponential complex exponentials of ejωt + or – βz this class of solutions in which the function g or f of the particular form of a trigonometric Cosine Sin or a complex exponentials or called as Sinusoidal waves okay not out something that would be very familiar to you from your earlier wave theory classes.

So sinusoidal wave is the one that would ne varying as a functions of Cosine or a Sin or equivalently as a complex exponential again you can see that the + sin would present a wave that is propagating along the – said axis and the – sign would correspond you wave that is moving along the + z axis okay there are couple of additional constants that we have seen to be introducing in an arbitrary manner okay it is not very arbitrary I will tell you what this ω and βr this ω.

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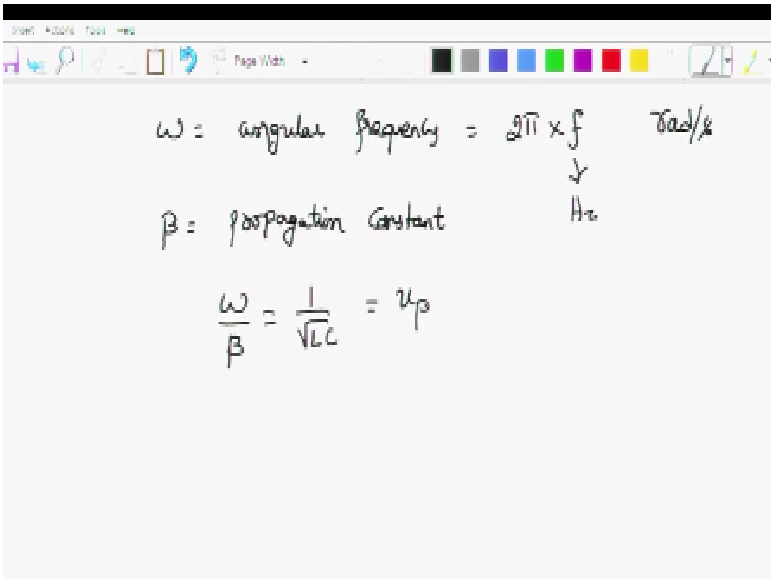

Is what is called as the angular velocity and this angular sorry it is called as a angular frequency and this angular frequency is given by 2π x f where f is the sequence measured in Hats angular frequency is one that is actually measured in radiant's per sec okay β is called as the propagation constant of the wave in this particular case propagation constant β happens to be a constant itself in general β made need not be a constant it can be a function of frequency when β is the function of frequency.

Then in non trigger wave is a function of frequency when it give phase to phenomena called as dispersion as we will see later on this propagation is constant and the angular velocity are sorry angular frequency are related to the constants of the transmission line L and C in the fashion that ω/ß is given by 1/√ LC okay where 1/√ LC is also called as the phase velocity. If why would it be called as a phase velocity.

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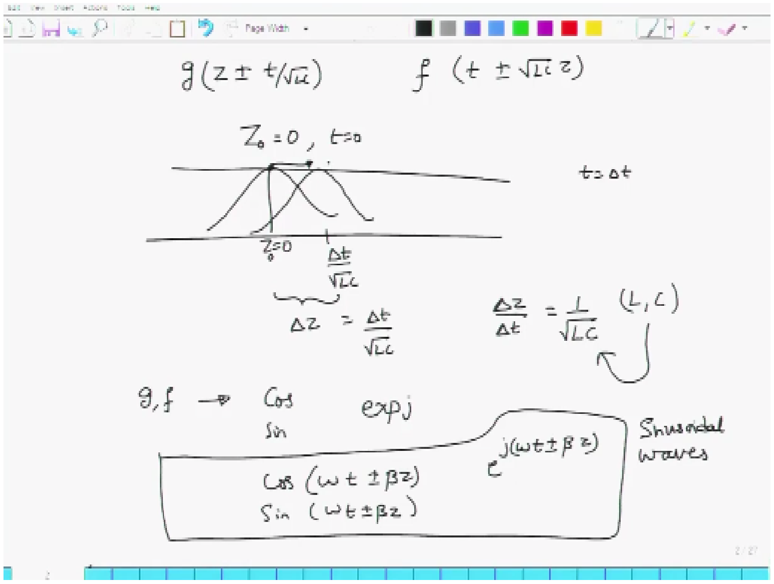

Go back to this one as Δ T changes Δ Z also changes in a proportionate manner so as to keep the peak of this function moving along so if you actually imagine that you are a little person who is able to sit on this particular peak and travel along the wave you would see that the velocity with which this particular point or velocity with which you will be moving along the wave peak right along the wave peak will be given by 1/√ LC and that is the phase velocity, if you have ever served in an ocean then this phenomenon could be quite familiar to you, you know you are moving alone the wave and you have positioned yourself at the top of the you know of the wave.

And the wave is actually carrying you, while the wave is carrying you towards the shore or towards the sea does not matter but if you are on the peak the velocity with which you move along that particular peak will be given by this 1/√ LC okay I mean in a transmission line if you imagine you are self on that one monist seashore it that velocity will be quite different.

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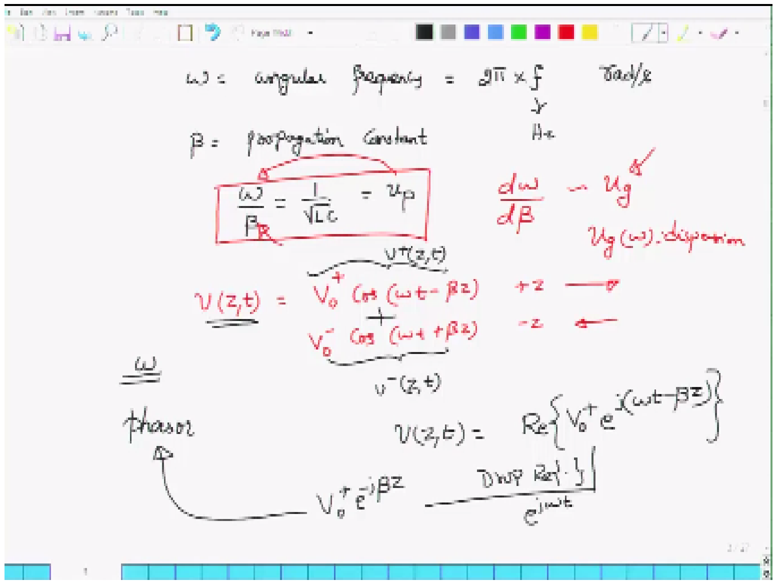

So this is the meaning of what we call as the phase velocity and it is the ratio of ω to ß that determines what is the phase velocity and in this particular case it so happens that this ration omega/ß is given by 1/√ LC, okay. So this is something that you should remember we will later on see that the ration D ω/D ß which tells you how the velocity itself is changing will be the one that will decide what is call as the group velocity.

And it is this group velocity that will tell you how a particular packet or the when you have the modulated wave the peak of the modulated wave how quickly or how fast it is moving is determined by this group velocity, it also tells out that group velocity itself might be a function of frequency in which case this change of group velocity with respect to the frequency omega, ω angular frequency or the regular frequency F.

When the group velocity itself depends on this frequency ω this gives rise to what is called as dispersion, we will see all this things later on, okay. Now coming back our voltage V as a function of z and t is now given by some constant V0 Cos ω T – ß z if the wave is propagating along +z- axis it will be given by Cos (ω T) + ß z if the wave is propagating along –z axis.

To denote a wave which is propagating along + z -axis let me introduce a super script to the amplitude V0 call this as V0 + and to denote a wave amplitude which is propagating along –z direction let me denote it by V0 – okay, since the wave equation happens to be linear our mathematician friends will tell us the total solution will actually be a linear combination of these two, that is a wave equation can support both positive z axis wave as well as a wave which is propagating along –z axis.

So this wave we can call this as v+ (z, t) indicating that this is a positive z travelling wave and this would be a v- (z, t) that wou8ld be propagating in the –z axis or in the –z direction, there is one additional thing which I want to introduce at this point, we will now consider from now onwards we will consider all ways to be at the same frequency ω, okay everything in the transmission length problem that we will consider from now onwards we will consider all waves to be at the same frequency ω.

Okay, everything in the transmission line problem that we will consider from now on at least until the point where we stop considering this one the frequency domain behavior will have the same frequency ω, okay and it is kind of painful for us to carry out this or carry along this ωt factor, okay and more over the donation of complex exponential is much better suited or it is much suited for discussing the problems mathematically so it kind of simplifies our mathematical expressions.

Therefore instead of considering of cos and sin I would like to move on to consider the complex exponentials, okay. in terms of a complex exponential I can write my voltage v(z,t) as real part of $v_0^+ e^{j\omega t}$-βz assuming that I consider only wave propagating along +z axis, correct this is true because if you go back to this expression way of z,t here you see that this expression would be the same because e^{jx} the real part of that is given by cos x, here x=ωt-βz being the argument of the complex exponential.

If I want to represent in the complex exponential form for a wave which is propagating along –z axis simply replace ωt-βz/ωt+βz, okay, $e^{j\omega t}$ will be a constant for us I mean it will be a constant factor for us to include everywhere what people do in order to simplify the notation further is that they drop this real denotation that is they would not denote it by real, okay they also drop this factor $e^{j\omega t}$, okay. When I drop real as well as I drop this factor $e^{j\omega t}$ what I get is simply this v_0^+ e-$^{j\beta z}$ which would be a complex number for every value of z.

If I say z is a constant at some particular point on the transmission line then this $v_0^+ e^{-j\beta z}$ will be a complex number and this complex number is called as a phaser, okay.

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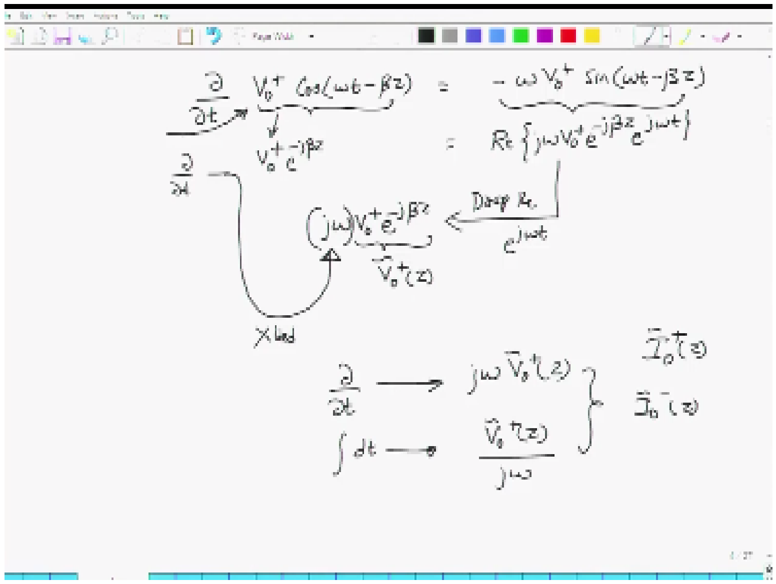

What is the advantage of using a phaser consider what happens to the situation where I am trying to find out what is a partial derivative of the voltage with respect to time, okay so my voltage was of the form v_0^+ cosωt-βz but when I differentiate this one with respect to time what I get will be $-\omega v_0^+$ sinωt-βz I hope we know that we agree to this one, right. Now this right hand side can be written in a complex notation by calling this as real part of $j\omega v_0^+ e^{-j\beta z} e^{j\omega t}$ I just removed $e^{j\omega t-\beta z}$ into two factors right, e^{ab} so $e^a.e^b$ is e^{a+b}, correct.

To obtain the phaser form of this I will drop real and I will drop this $e^{j\omega t}$ factor from this expression then I drop this factors the result will be $j\omega v_0^+ e^{-j\beta z}$ where $v_0^+ e^{-j\beta z}$ is the original phaser let me denote all my phaser by drawing this small arc on the values and this phaser will be dependent on z because of the factor $e^{-j\beta z}$ therefore I write this as $v_0^+(z)$, okay and there is a term which is getting multiplied to this $v_0^+(z)$ phaser and that term is jω.

So if I represent this one in terms of its phaser this would be V0 + $e^{-j\beta z}$ and when I differentiate this one with respect o time this is as though I have simply multiplied it the phaser by j ω, so differentiating this ∇t of the original cosign way form this equaling of multiplying the phaser by j ω.

So mathematically ∇ ∇t in the phaser doming becomes multiplication by factor j ω in the phaser doming as I said okay, similarly you can show I would not show it here I live it an exercise the act of taking an integration with respect to time will be equivalent in the phase doming of dividing the phaser by a factor j ω okay. This simple rule will actually help us in writing the equations in a slightly better and mathematically simpler form for us okay.

So this is the phaser notation which I wanted to introduce you to there is nothing specific about a voltage phaser I can as well consider the current phase I will have I 0 + of z or I will have I 0 – of z both be in the current phaser.

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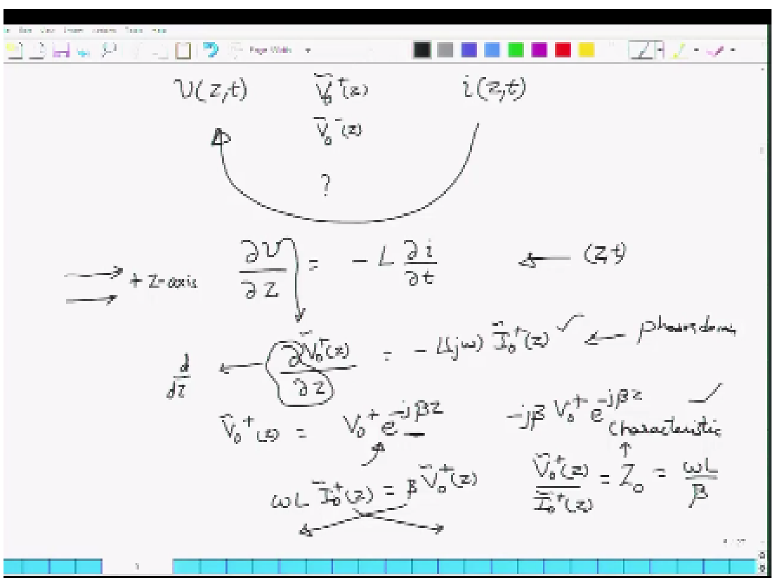

Now let us get back to an interesting thing we know how to obtain the voltage as a function of Z and T on the transmission line we also know how to obtain the corresponding you know phaser's V + (Z) or V- (Z) or V0 + (Z) or V0 - (Z) can I actually obtain what is I (Zt) from the voltage it should be possible for us to do so right, I should be able to obtain what is the current from the voltage in order to do that I simply need go back to the original equation for the voltage remember when we were applying the Gibbs of voltage law in order to relate voltage and current we said that the total voltage change ∇V/ ∇Z = -L ∇i/ ∇t.

Now instead of considering this voltage v and I if I go to the phaser doming then ∇ V0 +(Z) would be the phaser representing the voltage here that change with respect to z will be given by ∇V0 + (z) /∇z this is the partial derivative of the phaser with respect to z, this would be equal to – l x j ω and the current phaser let us call this as I 0 + (Z) okay.

Oh I need to denote my phaser by this bar that I putting please note that I have gone from the z and t kind of a notation to the phaser doming okay, so I have gone to the phaser doming by changing this voltage and current in to their appropriate phaser okay. However I do know that V+ z phaser ser is given by the amplitude V_0 times the factor $e^{-j\beta z}$, if I take that derivative of this one what would I get after I get derivative.

It of this one what would I get after operating the derivative one I do not even need to have the derivative these partial derivative become total derivative because the depends on p have been taken away.

Right if I differentiate this voltage phaser with respective to z what do I get – jxβ. Right and then what ever the original v0+ -jβz that would still remain. Now equate this one with this expression for the right hand side and take out all the – sign out there and j will also cancel o0n both side.

So what do you get is ωl i0 + z given by v+(z) x β. Okay I know that the ratio of the voltage to the current must be sometime of a impedance. This impedance in this particular cases has been define has ratio of positive of going voltage to the positive going current. That is t he both voltage as well as current are v going along the =z direction.

Both of them are travelling around the z direction and this ratio will be denoted by the quantity z0. This z0 is called as a characteristics impedance of transmission line. Okay and this characteristics impedance can be of complex but in most piratical cases this characteristics impedance will be real quantity. And what is the characteristics impedance use this equation to bring i0 to down here β on this side is given by ω l/β.

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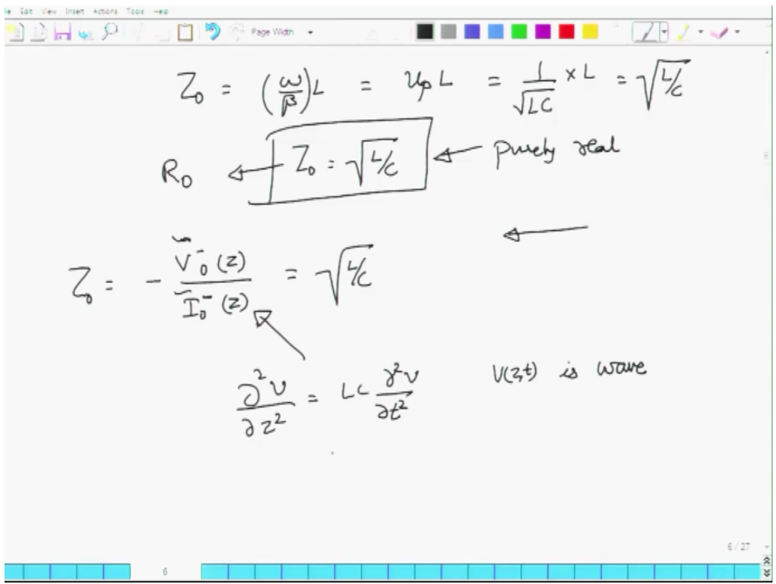

What is ω l/β I can simplify this further because I know the relationship between ω and β. So I have ω /β x l is nothing but where the phase velocity x l up the phase velocity given by 1/le for this particular loss vision vertical line this one l/c. So the Characteristics Impedance of this transmission is purely assuming that both l and c are real and r and c are positive.

In some cases it might not be so but in our case l and c to be real quantities and positive quantities and therefore z0 will be a purely quantity. Purely real number and real quantity will be very interesting completely independent of the frequency. Okay completely frequency the only quantities depends on the characteristics impedance and the capacities of the transmission of line itself.

That is the reason why it is called as the characteristics impendence it is the impedance is the characteristics of given transmission like it has nothing to do with the frequency the approximately result in the practice that defense because we hare cultured the lusted line it is the lusted transmission line character line that will be purely real quality quantity since purely in some time instead the detonating time z0 is detonated it by r0 okay the definition.

This is a character impediment is also be contained considering the negative going voltage and the negative going current this is v0 - this the negative going voltage okay this bar this still indicates the this is the facer that considering the character impediment that actually defined as a - over here so because the - sign appears will because current well be going in the other direction so because this reason the character impediments will be define by the writing - v0 –of z/i0-of wok this character impediments in the value of LC this is appropriate .

When you have a negative going voltage and a negative going current okay so to summaries what we are doing so far with the equation to for the voltage and then we showed the voltage should be z t engine along the transmission line is actually a way a propagating with a long said direction or - z direction mathematically is more reason why you cannot both + and – direction in case you know physically one direction that is proffered or both that depending on situation that will later we consider on okay.

That also be written in terms of the fazer in all trhe solution the form go z _or + Δ t/ √ lc are admitted as a solution finallly we define to quantity is called as a charactetrstic this given as zols also define the face well velcoity that is ω / β okay we will say more about these quantity in the next module we are consider terminating a transprent line thank you very much.

<u>Acknowledgement</u>
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