Indian Institute of Technology Kanpur

National Programme on Technology Enhanced Learning (NPTEL)

Course Title
Applied Electromagnetics for Engineers

Module – 32
Boundary conditions for Electromagnetic fields

by
Prof. Pradeep Kumar K
Dept. of Electrical Engineering
Indian Institute of Technology Kanpur

Hello and welcome to NPTEL mook on applied electromagnetics for engineers. In this module we will discuss the boundary conditions for the electromagnetics field quantities A, B, D and H. what are these boundary conditions and why should one consider or one should study that.

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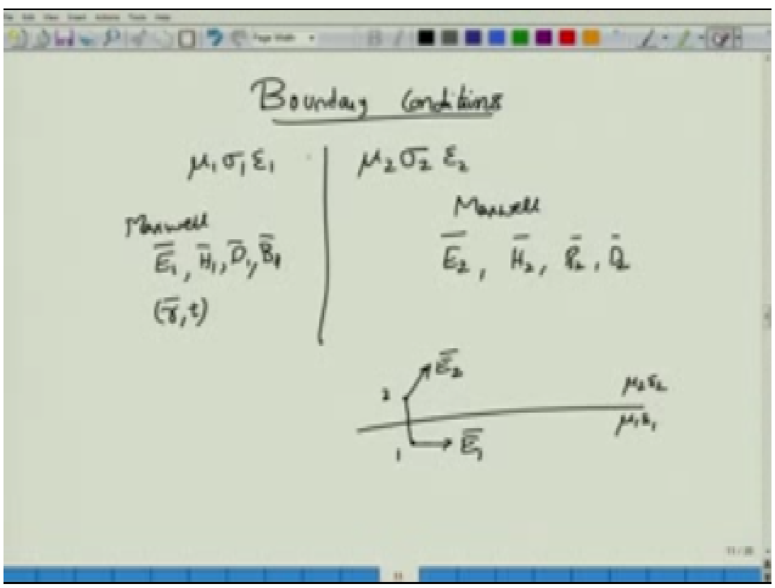

Now let us assume that this paper that I have which separates two regions. This region, you know where this pen for example would be present okay, is what I call as region 1, here the material properties are described by a certain set of values for μ and ε. So let us say the region in this region here to this particular side of my hand, this is μ1 and ε1 where μ1 is the relative or the μ1 is the permeability and ε1 is the permittivity.

That could also be conductivity, so let us call this as σ1 in this region, separating on this side okay, on this side where my hand is showing, so I have a second medium which is μ2, ε2, and σ2, so these are described by the parameter values of the material μ2, ε2, and σ2. This boundary between the two mediums is actually very thin, although you can see that there is a certain amount of small thickness of the paper okay, which is actually existed in a real medium.

In a real medium this thickness of these papers that I have considered as the boundary required two regions will actually correspond to the few layers, few atomic layers or few molecular layers where the properties change continuously from μ1, ε1, σ1 to μ2, ε2, σ2, no change can be abrupt like boundary. But if you are looking, you know like from me using the, you know telescope or whatever that is or rather if you are looking at it from a distance, a few layers of molecules which might be about a few 10s or 100s of times chomps does not really bother you.

Because if you actually look at for example, you take a lens okay, a lens will have a certain glass property that could be different from the surrounding air medium. However, the thickness of the lens is so much larger compared to the few atomic layers 10s of atomic layers or 10s of molecular layers thickness that you can consider the thickness of the boundary that separates the medium to be almost equal to 0.

So we call this as jumped boundaries and we call these boundary conditions as jumped boundary conditions, because the material property jumps suddenly from one value to another value okay. And when such a thing happens is then a guarantee that the electric fields, magnetic fields, D fields, the fields, whatever that we calculate in region 1 and we calculate them in region 2 are related to each other or not first of all shroud they related if they are related how are they related that is precisely the subject of this module to set the stage okay.

Let me draw a line here and call this line as the dividing boundary between two regions as we said this is μ_1, σ_1 and ε_1. Although for now let me just focus on μ1 and ε1 we will talk of σ_1 later because only one equations requires σ_1 and suddenly on the other side of this boundary you μ_2 σ_2 ε_2 okay we have solved Maxwell here okay and Maxwell tells us that the fields here are given by, the vector quantities E_1 and H_1, D_1 you came also have a J_1 in case you wish to talk of the continuity condition for J okay.

But for now we will not worry about J okay so we solved Maxwell equations along with whatever the initial conditions that I am necessary and we used the Maxwell's equations to come up to the corresponding values E_1, H_1, D_1 and B_1okay so these are the fields and we solved Maxwell in region 2 as well where we obtain E_2, H_2, B_2 and D_2 these are all your vector quantities which vary both as function of r okay which vary both as a function r as well as a function of time.

So all the field quantities are functions of space as well as time so these are the varying and space changing field quantities electric field magnetic field H or the magnetic field intensity H electric flux density D and magnetic flux density D now that we have these equations with us what is the relationship between E_1 and E_2 we are not interested only the relationship between E_1 and E_2 here we are also interested in the relation between the boundary okay.

So how does you know having electric field E_1 in region μ_1, ε_1 and electric flied E_2 integer μ_2, ε_2 and having a boundary separate the two regions relate the values of the electric fields E_1 to E_2 in order to understand that one let us fix two points okay so let us fix two points let us call this as point 1 and point 2 and I will also consider a unit vector in the direction from 1 to 2 okay you know that between any two points there will be a line right.

So can always draw a line so this is the line between any two points and then between these two points there will be a bisection for simplicity let us actually take these points 1 and 2 in such a way that bisector happens to be just the boundary that you have considered okay so between any two points you have boundary okay and in fact if you think of this as a 3 dimensional extension right.

We can see that this is actually defining the plane as well right so this is actually designing a plane as well right, so this is actually designing a plane as well so you can imagine that I have two point here and then there is a line here and then there is essentially a plane that connects between these two points so this is a situation that I am considering so the point 1 is located here and point 2 is locate on this particular side okay.

Now by some means we do not really know what those means are we have calculated the electric field that is called this as even, and we have figured out there it is direction at this particular time is given by this particular black arrow that I have different similarly E2 we have calculated and that also is seen to be having in this direction for maybe you know we can just we erase this one and then get back to a simple thing that I would actually use to calculate the boundary conditions, so I will orient my boundary in such a way that this is the boundary because it is makes my task little easier.

Okay so these are two points, point 1 and 2 and we know that these two points essentially are bisected by the plane, above this plane you have $\mu2$ and $\varepsilon2$ below this plane you have $\mu1$ and $\varepsilon1$ this is the same situation I am just you know made the boundary horizontal rather than vertical so that I can write the equations on this side okay, so I have considered the electric field to be okay let me consider this electric field over here to be calculated and pointing in this direction even and the electric field here to be pointing in this direction.

And having a certain value E2 here, now what is the relationship between E1 and E2 in order to answer that we need to go from what we call as the point form, of Maxwell's equations into integral forms of Maxwell's equations where no more difficult than the point form and for our benefit here okay we will just write down the ϕ forms okay this ϕ forms were the or more basic there more fundamental they will always hold, even when the boundary is actually moves it okay.

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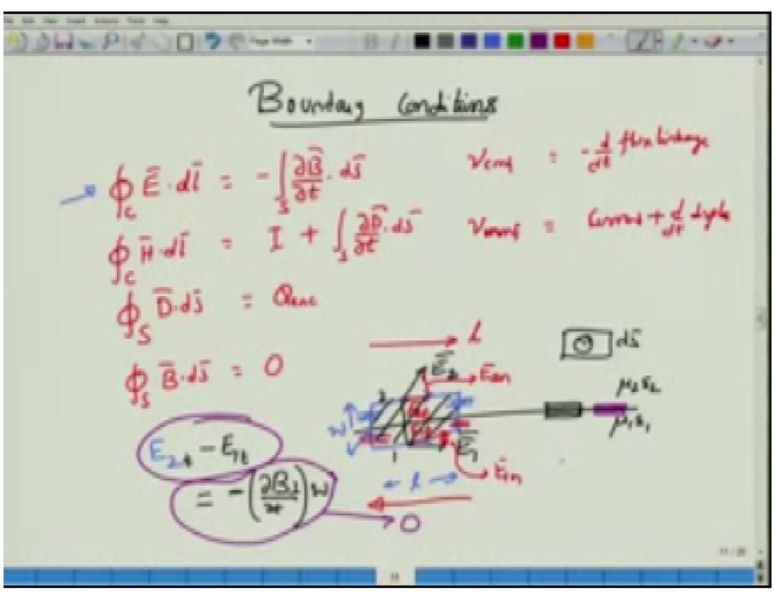

It does not matter when the boundary is present or not even when the boundary is moving this integral forms are always valid so what are the integral values we know that ∂ x e you know if you just go back and write down the curl expression in terms of the line integral over the closed path is going to move nothing but E.dl right and that should be equal to – integral of ∂b / ∂t.dS, so this equation is Faraday’s law which is simply telling you that emf over that closed loop okay, must be equal to – d/ dt of flux linkage right.

So the rate of change of flux in case will be driving the emf around this a closed for there is what this equation tells you them ampere Maxwell’s law will tell you that the magneto motive force over a closed loop which is given by integral of H. dl must = the total current I surrounding a particular surface the open surface through which this contour is being defined so there will be a current I + ∫ ∂D/ ∂t . ds which is how the displacement current is actually changing right, and then you have two additional equations over the closed surface so these where the closed loop okay and these where the open surfaces that we had considered.

Over the closed surface the way in which in D. ds or the total electric flux should be simply equal to the amount of charge that is enclosed and similarly integral over the closed surface of B. ds must be equal to 0 okay because no there are no magnetic charges out there to enclose, so these are the integral versions of the equations these are fairly the same so let me just write down this as the magneto motive force, okay.

Must be equal to the current + whatever the time rate of displacement in other words this is the displacement current that I have, so now I have all this equations what I do is very simple, I imagine that you know I extend a path here which includes paths or points 1 and 2 and then I take this particular path, okay. So I take this path okay and the you know this is the way in which I will take the path and I will also assume that over this path the electric field E2 or the electric field E1in the appropriate regions are not varying much.

So let us call the path as having a length L and having the width W okay, clearly half the width lies above and half the width lies below this particular boundary okay. We will also assume that the value of L is not so large I mean you are not taking a kilometer a long paths out there, you are just taking a path which is very, very short it is just a few infinity symbol 11 paths that we are taking.

Now over this path I will apply faradays law, okay. So what do I get when I apply faradays law? Well I in order to apply the faradays law let us also see that if I go from left to right and call that as the positive direction in which this is increasing then I need to find out the corresponding tangential component of E2 that is obtained by projecting this E2 onto the path which goes from left to right, right?

So when I do that one I see that I can write this as E2 x L or rather E2 T L where E2 T will be the tangential component, so this electric field here will have two components right, so one component is a tangential component which is what is important because you know this tangential component multiplies to the path length out there and there will also be a normal component which we will call as E2 L.

This normal component will be perpendicular to the boundary, so therefore this is not really the path that you are looking at so imagine again that this is my boundary, okay. And on this path so let me keep this particular point here and this is how my electric field is present okay so this is my electric field, now this electric field on this you know if I take the path along this paper, okay I need to first decompose this electric field into corresponding normal components.

So you can see that the normal component is coming perpendicular to this paper and because of this there will also be a tangential component out there, okay. So there is a tangential component which is along the path that you are looking at and there is a normal component which is coming perpendicular to the boundary sand because lien integral always demands the tangential component I write this as or I obtain I am interested only in the tangential component in region to the tangential component is E2 T and that is what I write.

You know multiplied by the path length which is L right and on this side I have the normal component okay of corresponding to E2and on this side I have the same normal component but these two are in the opposite directions, therefore even if I include them because the science here will be different and the magnitude of the normal component is the same on these two paths okay, then normal contribution to this closed curve will be equal to 0. So the only thing which I am interested now is what is the contribution of this path and what is the contribution of this one.

Again if you decompose here electric field E1 into corresponding normal and tangential components so this is the tangential and the normal components so you have E1t and you have

E1n as the tangential normal components so you can clearly see that only the tangential component will contribute because on this path the normal component will have whatever the contribution that will cancel out the contribution of E1 normal component of electric field on this path.

So it is only the tangential components that contribute then there is also another catch, on this side we consider the traversing of the path as positive, therefore if I move along this direction then I need to consider the direction of dl to be opposite to the direction of the path along which I have taken in region 2. The net effect is that what I obtain here for the next move for the electric field in the region 1 will have a negative value multiplied by the tangential component E1t times l.

So this is what I obtain for Faraday's law on the left hand side, but what is on the right hand side on the right hand side I should have no, I have a minus sign here that is no problem. But then the total flux that is linking this closed loop you know has to come because of the magnetic field that is coming out of this particular surface, so now I have actually taken this surface and I need to find out what is a normal component of this surface okay, and integrate $\partial\bar{D}/\partial t$ on to that normal component.

The total area that I have is lxw and it is the normal or that is the $\bar{B}$ component that is coming perpendicular to this loop that I must include over here, so let me call this as some $\bar{B}$ perpendicular component times lw or rather $\partial B/\partial t$ component times lw is what I'm looking at okay, and this is the equation that I now have, what I do is I divide by l on all sides okay, so when I divide by l on all the sides what I obtain is that E2 tangential- E1 tangential should be equal to $-\partial B1/\partial t$ times w.

Now what I do is I shrink this path going to 0 so I shrink this path eventually I actually obtain okay, or in the limit of w going to 0 I obtain above and just below the path okay, so which means that unless my magnetic field component B perpendicular blows up to infinity the product here on the right hand side will tend to 0 and clearly the magnetic fields cannot blow up to infinity, because infinity magnetic field does not really make sense, okay.

We assume that all fields are finite and therefore the right hand side completely goes up to 0, so giving you the first boundary condition that the tangential electric fields must be continuous okay, so the tangential electric fields must be continuous across the two boundaries that you have talked about. So let me just write that one the first law or the Faraday's law tells you that.

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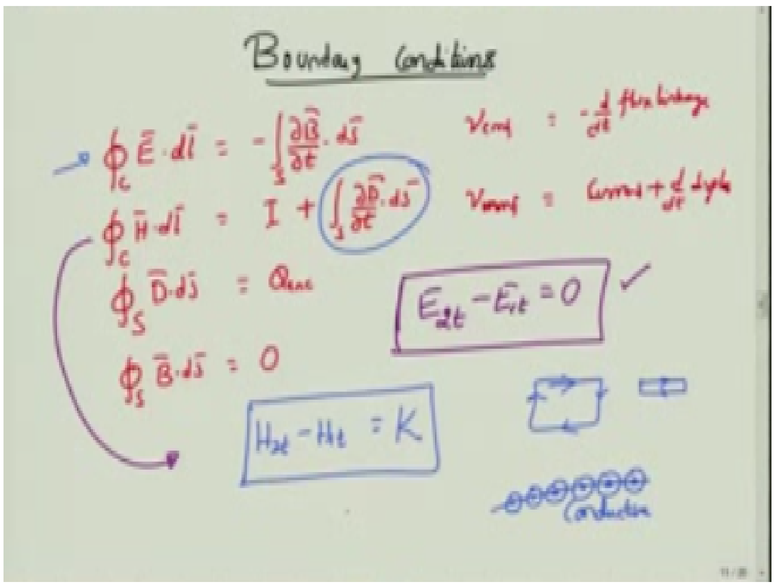

E2t-E1t must be equal to 0, so the tangential continuity of the electric fields is the first boundary condition that you are looking for, by the same logic this second Ampere Maxwell law will tell me that $\int\bar{H}.d\bar{l}$ can be rewritten by following the same path that we actually followed so I can write this one as H2t-H1t that must be equal to whatever the current that is coming out of this path, so this was the boundary.

So and this is the magnetic field that I am writing out there okay, so I get H2t-H1t and then whatever the current that must be coming out of this particular plain plus whatever this current out of this plain plus ∂d perpendicular / ∂t time lw clearly if I divided every, so this is H2t x l H1t x l = the current the plus this fellow. Now when I take or I when I remove l from all this equations that is divide both sides by l okay, I do not consider or I will see that this ∂D1/∂t becomes or the l component goes away and then you have ∂d1/∂t x w okay.

Then if I now take the limit of w tending to 0 this quantity goes off to 0 simply because we do not allow for infinity fields d. so the contribution as the loop shrinks down to a point or loop shrinks down in such a way that you actually end up just having this kind of a loop here of infinitesimally small thickness the contribution from the right hand side or the displacement current actually goes up to 0.

Now should this current I also go to 0, well it does not have to go to 0 for example if the medium here is a conductive medium okay then I can actually have currents on this surface so all these arrows or rather circles that I am drawing also putting this one over here, so this is simply the case that the current is actually flowing on the sheet. So you can again go back to this boundary out here imagine that below everything is a conductor and this these you know I am keeping these pens over here and the direction of these pens you imagine that this is you know just on the surface and these are the direction over which the magnetic fields is coming.

And since my loop is going up in this region going down and then going back up in this region and coming again to the first region to complete the path, the current that I am interested is the one that is coming out like this so if one of the medium is the conducive medium then I can actually have the current component flowing along the plain or along the boundary and we call this current as what is called as the sheet current okay.

And we use a different symbols for that one we use symbol of K to denote that this is actually the sheet current and what the conclusion that you have to draw from this by writing the same path and then trying to apply that boundary condition is that the tangential magnetic field components H will be discontinues by the amount of this sheet current K and you cannot rule out this sheet current unless the medium both medium happen to be only dielectrics, so if the medium happens to be a conductor then there is a good chance that there will be a sheet current and you have to consider that sheet current as the discontinuity component for the magnetic field.

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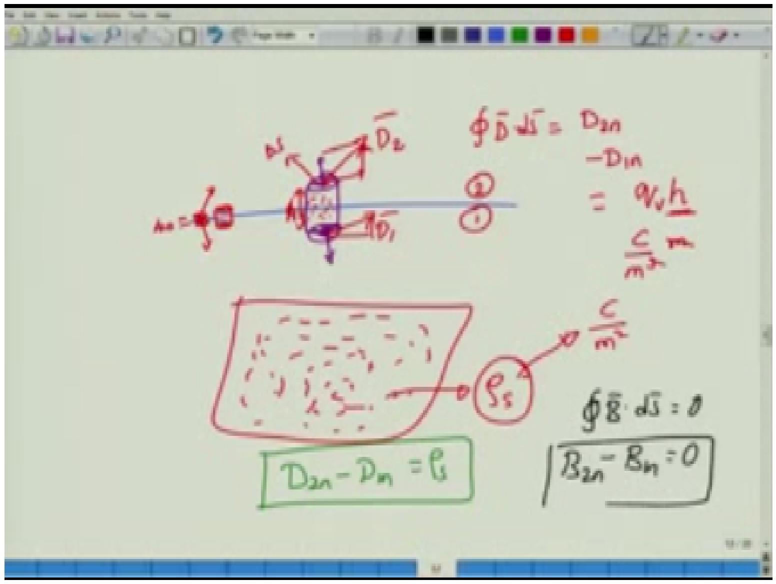

Now let us move on to talk about that conditions for d field and the d field imagine that I am actually keeping this what is called as a pill box okay so I keep this pill box okay there is a certain area on top there is a certain area below and you have to remember that these areas are always pointing the opposite directions because the surface area is actually governed by the right hand side road so if go like this then the surface on this plain is pointing up and if you go in the bottom side the surface will be pointing in the below direction.

So the open surface components are up and down out there and what you are interested now is the magnetic field component so let us say at this point this is d2 and at point this is D1 okay you again split the magnetic field in to normal and tangential components and for this case you are not interested in the tangential component yu are only interested in the normal component because that component is the one that would be perpendicular to the surface area okay.

And there is a certain length or the height of this one let us call this height of the pill box as H okay. So integral of d.ds over the close surface when you apply to this two regions so this is medium 2 this is medium1 what you obtain is the that the normal component of the D field in the second medium which is D_{2n} multiplied by Δs where Δs is the area of this particular closed surface this surface on top as area of Δs.

And if you go into the region there will be magnetic field but then the directions or the surface element has become negative and if you now take that into an account is that $-D_{1m}$ Δs and this should be equal to whatever the charge that is enclosed again there will be a case were the charges or a enclosed because in right hand side what is that you get the charge enclosed for other than charge density multiplied by so the charge density the volume charge density multiplied by Δs multiplied by H okay.

Now if I take Δs out I know divide on sight by Δs and then let Δs go to 0 then what I obtain is Qv*H or rather not Δs if I now et H go to 0 Δs is anyway gone on all this sights so if I let H go to 0 this volume density is measured in Colum's per meter cube and this H is measured in meters therefore the quantity that I obtain will be Coulomb per meter square and that Coulomb per meter square will be charge that you have taken and sprinkled on the surface.

So if this is my boundary look some were top so if I know put some charges on top of this one we deposit some charges by some were whatever means that I can then this charge would correspond to the surface charge density which will able has ρs and we measure this one as Coulomb per meter square and as a pill boxion in height eventually you just give me two kinds of surfaces of height H is equal to 0.

You see that the displacement vector or the D vector the normal component of D vectors actually become discontinuous by the amount of surface charge density so the equation that describes the normal components D_{2n} and D1n that discontinuity or the difference between these two must be equal to the surface charge density.

This is intuitively kind of placing to us because what does this mean D2n is the flux density that is this coming out of the second you know pill box in the second region and D1n is the flux density that is coming in to it right so D1n you can because there is –D1n sign that we are considered D1n is incoming D2n is coming out the difference in this two flux density must be equal to the flux that is contained within that volume.

And that flux is nothing but the charge that is contained again that if the two medium happen to be dialectics then if I sending some D_{1n} here D_{2n} will be exactly equal to D_{1n} because that cannot be any left over D values otherwise there will be some charges and for two perfectly dialectic medium there are no charges but if one of the medium happens to be conductor then that conductor can have charge inside on its surface constituting.

The surface charge density, in that case the ion flux and the out flux might be different and the difference between the influx and the out flux will be exactly equal to the surface charge density, that the surface of the conductor can hold. Finally we have ∫B.ds = 0 and you can very easily see that, this simply leads to the condition that the normal component $B_n – B_{1n} = 0$. So these are the 4 major boundary conditions that we have described and we have discussed.

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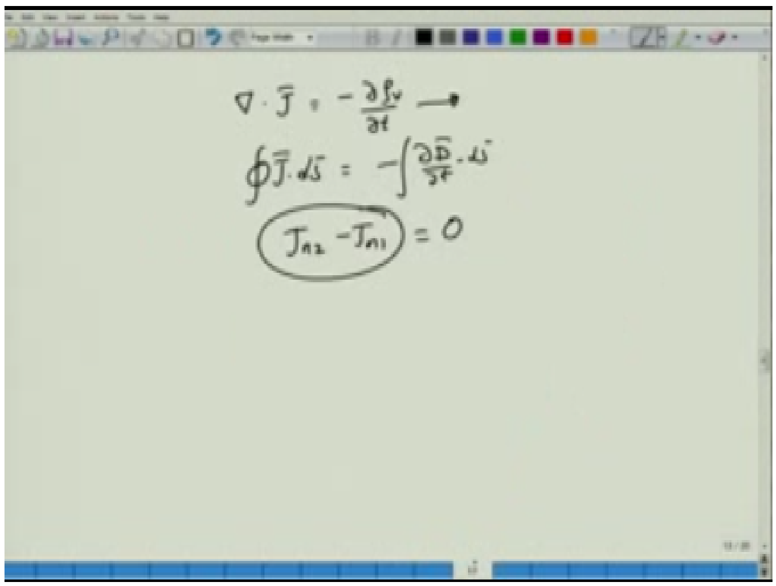

The other boundary condition that involves ∂.J = -∂ρv/ ∂t I will leave this one to you to figure it out and this can happen in the medium in these both are conductors but have different conductivity or one of the medium is the conductor and we can conductor for other case. So you will again have to write the ∫ form of this that would be .J = -∂D/ ∂f ds it is the continuation equation and you can very easily show that, the normal component $J_{n2} – J_{n1}$ when you evaluate on the two boundaries.

That must be = 0 or $J_{n2} = J_{n1}$ okay. The conduction current as to be = that particular thing, so you can study this one at you leisure, I would not describe this one because we will be using this condition for the current density vector J as much as the boundary condition for the other field components. The real value of boundary condition is that once I calculate the electric field in a particular region by using Maxwell equations, usually the materials properties, I can calculate immediately what should be the field on the other side.

That will allow us to know for example if you know I have a mirror or if I have a glass and the light is coming in, and incident on this particular glass, these boundary condition allow me to determine not only the fields over here, which neither I have determined, but it will also allow me to determine the fields in the other region okay. Because I can relate e1 e2, d1 d2, b1 b2, h1 h2, I can find out what are the fields in the other region.

Then I can answer questions like okay, if I incident I waltz of power on to the glass, how much power will actually be reflected? What is the fraction of the power reflected? What is the fraction of the power transmitted? So instead of having one boundary what happens if I have multiple boundaries and if I incident 1 Walt power here, 10 boundary layers what would be the power that come out okay. So boundary conditions are very important when they deal with problems saucy as incidence, reflection of a wave, understand concepts like total internal reflections.

And this boundary condition that you have derived are fairly general okay, there are no usual things going on except the surface current density and there is the surface charge density, both are little idealization, those idealization will serve very well for us in order to understand how does the field behave from one point to another point, thank you very much.

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