**Indian Institute of Technology Kanpur**

**National Programme on Technology Enhanced Learning (NPTEL)**

**Course Title**
**Applied Electromagnetics for Engineers**

# Module – 39
# Inductance calculations

**by**
**Prof. Pradeep Kumar K**
**Dept. of Electrical Engineering**
**Indian Institute of Technology Kanpur**

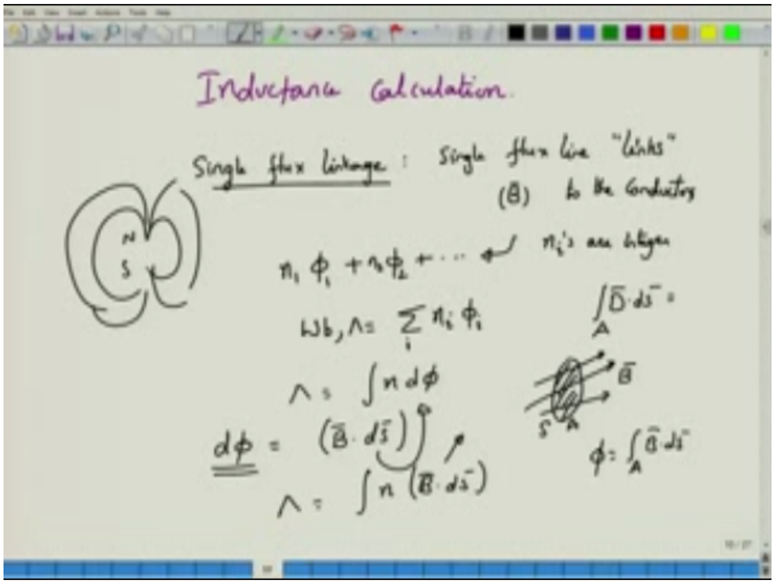
Hello and welcome to NPTEL mook on applied electromagnetics for engineers. In this module we will calculate inductance one of the important circuit parameter for a couple of cases, while doing so we will also be calculating the magnetic fields for them. So let us begin by first understanding what is inductance? You have to understand that inductance is a concept associated with magnetic flux that links to a particular conductor, this is one way of defining inductance.

Another way of defining inductance is to actually calculate what is the amount of magnetic energy stored in a group of conductors or in a conductor, and then based on that define inductance. We will not take that magnetic energy root, because it is little more complicated and does not release server purpose. What we do instead is that we define inductance in terms of the flux linkage associated with a particular circuit.

You notice that I am keeping the word links the very vague word, because I do not want to specify mathematically or rigorously, because it is actually very difficult to do so. But most of the times it should not cause us confusion, because intuition tells us what is this flux linkage and how to define that flux linkage, how much flux linkage, the amount of flux linkage to a particular conductor usually gathered by the induction okay.

Sometimes it so happens that a particular conductor might have more than now one flux linkage associated with it, or sometimes you might have flux linkage coming from one source link and one times, another source linking and two times, and another linking three times to a given inductor.

(Refer Slide Time: 02:07)




So we say that if we consider, you know if we consider a conductor we say that one single flux linkage exists okay, one single flux linkage exists when a single flux line, what is flux? Flux is the magnetic flux density lines that we are talking about, so called B lines, you know for a simple magnet you have seen those flux lines, so say N and S if you see these are all the field lines or the flux lines that you can think off.

So we are talking about how much of these flux links to a particular conductor okay. So a single flux linkage is said to exist in a particular conductor when a single flux line links as I said this word is little vague at this point, but I will soon clarify that one. So when this single flux line links to the conductor. As I said sometimes you might have a flux which we will denote this as $\phi$, so you might have a certain circuit being linked with the flux $\phi_1$.

And it might link at times $n_1$, flux $\phi_2$ might link the same circuit $n_2$ times and so on. So you can actually think of the total flux linkage, and this total flux linkage is usually measured or it is measured in webers okay. And this total flux linkage we denote it by this $\Lambda$ it will be given by $\Sigma n_i \phi_i$ okay over all possible values of i. The assumption that we have made here when we wrote $n_1 \phi_1 + n_2 \phi_2$ seems to be that all of these nis are integer okay.

It need not be so; in fact you might have a flux linking not completely to a circuit, but only partially okay. This concept of partial flux linkage is very important and it will come up when we calculate the inductance of in a coaxial cable for example. Therefore, one has to generalize and allow for fractional flux linkages, therefore, one should not assume that these ni numbers are always going to be some whole numbers, you know 10, 5, 7, 8 and so on, you might also have about 8.3253 kind of a number for these nis okay.

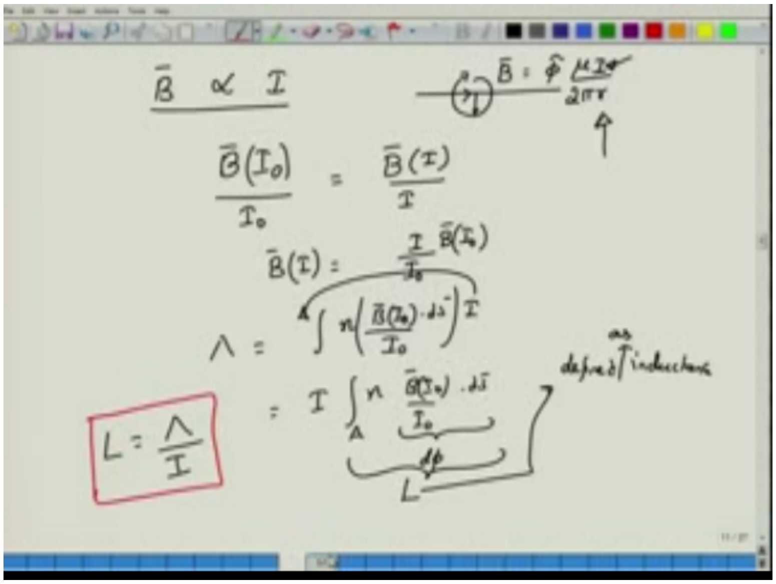
Whenever, this type of a fractional flux exist then the total flux is usually defined as the integral over the differential amount of flux that gets need to a particular circuit. However, the given circuit the amount of magnetic flux linking will be given by the integral, the open surface integral of the magnetic flux density correct, this comes of because, you know you have the total integral of D.ds where d was the electric flux density.

When you integrate this one over an open surface okay, you would obtain whatever the charge that was present okay, or the flux lines that was crossing that particular surface, when you close it, you would find the source of that D lines as the charges, but if you keep this open surface, this is the open surface that I am looking at and if the D fields are going around or crossing this particular surface, then the amount of flux crossing this hatched surface area will be obtained by the open surface integral of D.

In a manner that is very similar to that you might think of this has B lines and this as the cross section or the open surface over which I am actually trying to integrate. So if I denote the area integration by subscripting this integral sign with A, this would be the amount of flux that would be linked okay. of course, this is the flux that is actually D.ds is the flux that is linked partially dϕ is not the correct thing would be to actually say that the partial flux linkage is D.ds where we are assuming that these kind of constant over that particular surface.

So pushing this back into the expression for the total flux linkage Λ will be equal to integral of n B.ds okay. This integral still make sense, because B.ds is the differential amount of flux linkage, of course if you integrate over the complete area you will get the total flux that is linking that particular cross section okay. Since, I am only interested in the differential amount of the flux that is linked or the infinite decimal flux linkage, I write this as B.ds okay.

(Refer Slide Time: 06:36)



In a linear medium the magnetic flux density is proportional to current. We have seen that for example, in the case of a long infinitely long wire, the magnetic flux density B will be given by along the ϕ direction we have placed the wire along the Z direction let us. So this would be given by μI the wire is carrying a current of ϕ then this would be given by μI/2Лr where r is the radial distance from the wire at which we are evaluating this magnetic flux or magnetic flux density okay.

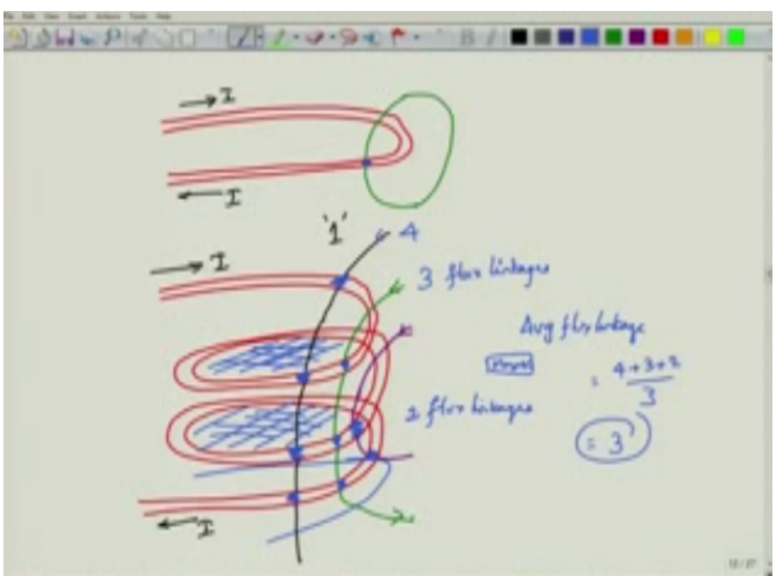
So in a linear medium B is function of I, how much current is being carried by that particular current and it is also directly proportional to this I. If I evaluate B at some reference current I0, so if I evaluate this B at some reference current, then I obtain some magnetic flux density which we will denote it as B(I0). If I divide this one by I0 which is the reference current, this should then be equal to whatever the flux that I calculate with the actual current I that is propagating or being carried by the wire divided by the current in the wire okay.

For a linear medium this of course holds true, because right hand side quality simply becomes the constant independent of I right. So right hand side of this equation is a constant independent of I. Now I can substitute therefore, what is the magnetic flux density when the wire of the conductor is carrying the current of I and that could be equal to I/I0 B(I0) and I substitute this into the expression for the flux linkage.

So I obtain n B(I0).ds/I0 and there is also another I along with this okay. Since I is not dependent on what surface area we are evaluating, so I can push this I outside of this integral okay. So I obtain I integral of n BI0 which is the magnetic flux density given a reference current I0.ds which will tell you the amount of the differential magnetic flux that is crossing from the surface area. And this quantity I can now call this entire right hand side quantity okay, once I go through the complete area A, I can call this right hand side quantity by some symbol called L, and this L is now defined as inductance associated with that particular conductor okay.

So this is defined as inductance. So in simple words okay, inductance is defined as what is the total flux that is linking to a particular conductor divided by what is the current that is being carried by the conductor okay. And this equation is important, so if you remember this then you can actually solve and calculate inductances.

(Refer Slide Time: 09:35)




As an example, we will consider, before we consider the example I should point out what do we mean by flux linkage okay. Now imagine that I have a wire, so I have this wire which I have bound it up in the form of a coil. So this is the wire which is carrying some current I okay, and of course the same current is returning back from this. Now you imagine that I actually have a metal ring okay, what I do with that metal ring is, I take the metal ring okay.

And then pass it through, if this is an imaginary metal ring of course okay. So I pass this metal ring through this coil okay, such that the coil actually cuts this particular region okay. So the coil is essentially cutting this region, of course it is not cutting in the real sense, it is only cutting in the imaginary sense okay. So I can show you by an example, suppose this is the wire that is carrying a current, so let us say the current being going around this particular direction.

Now you imagine that my hand, you know this is making the symbol okay, this is the loop that is having. So if I now simply go through this loop and then link through this wire okay, you can imagine that this part of the wire is actually coming back. So this part of the wire is coming back, but my thumb and the index fingers are not crossing that other path okay. So there is this written path is there, but my loop which is made by this fingers are not crossing that one.

So this we say as a single flux linkage okay, so this we say as a single flux linkage. So here if we calculate what is the amount of flux linkage, that if you say what is the numbers of flux linkages, the number of flux linkages is 1. Now we consider a slightly different example, this time it will be little complicated okay. So we do not just consider the current carrying wire, but we actually wind it up in the form of a coil okay.

So we wind it up in the form of a coil, so let me also draw the other part of this. So the current I is entering here and that same current I will be coming back to the coil okay. Now let me show you one line okay, and you guess what would be the amount of flux linkages that happens to this particular line okay. So let me go through this line okay, so this is the line that in green which I have written and I have marked.

And I know that wherever I show that, you know it is the line is not continuous, it means that the corresponding wire or the loop is actually passing through this, this imaginary loop is passing through this or cutting through that one. So how many cuts we have now here, we have a total of three cuts correct. So this is an example where we have three flux linkages okay.

We have to imagine that there is a sheet that you can think off and we can imagine that you can take a metal wire or something and then just that wire passes through these planes, this is one plane that I shown with the hatched area, this is another plane that I have shown in the hatched area. And there is one more cut coming through this one, because you can imagine that this itself is one plane, and we are cutting this plane exactly once. So there is total number of cuts that we have made or the flux lines cutting these coil is about three times and therefore, this is a three flux linkage example.

On the same graph I would like to show you a different kind of a cut okay. This time let me show you a cut where we begin with this flux line okay, and then we go through here, go through here okay, rather we go all the way over here. And then we come to this, and then we come back over here. So let us see how many cuts we actually made okay. So here we made 1, 2 and 3 okay, so we only have made 3, so let me actually erase this line, and then copy it in a slightly different way okay.

So let me cut at this point as well. So how many times I am actually cutting this coil, I am cutting this coil four times. So here is one time, this is the second time, so this was the first time, second time, third time, and fourth time. So the number of flux linkage here is actually four okay. I will show you one last example, so that it becomes clear to you. Suppose this is the flux line that is coming here okay.

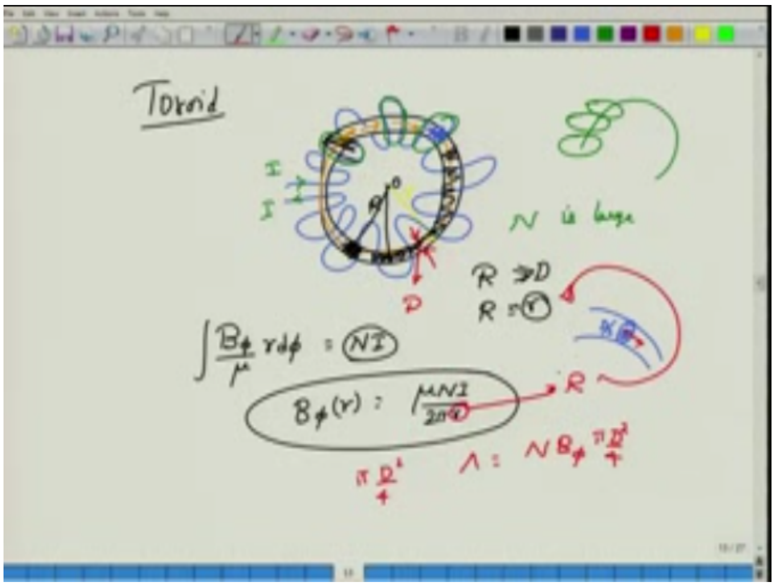
And then this comes all the way here, and flux only this particular surface okay. So this particular flux line that is coming in comes all the way over here, and then cuts through this particular coil. So 1, 2 and 3, so maybe this is the third coil from the top okay. How many flux linkages do I have now, I have about 2 flux linkages. If I remove this second line okay, and instead assume that the cut that I am going to make will actually pass through this coil okay.

So it passes through this coil, then how many cuts do I have, this time I have about 2 flux linkages okay. So as I said, the word links is little weak okay, you are not most of the time interested in calculating individually these flux lines okay. What you will be interested is to calculate what is the average flux linkage. In this case how many lines were there, there were three lines and each line cut or linked to that particular coil in a different amount of time.

So the first one was the four, second one was three, the third one was two, so 4+3=7, 7+2=9, 9/3 on an average you have three flux linkages to this particular coil okay. of course, these flux lines have come from a certain magnet, or some kind of a magnetic field around this. This magnetic field if it is lying externally or supplied externally to the coil, then this associates to the external inductance.

However, we know that if there is a current flowing through the coil, it itself will generate a magnetic field right. So if that magnetic field links, then you have an internal flux linkages okay. so let us look at two examples.

(Refer Slide Time: 16:07)




The first example that we are going to look at is what is called as a toroid. Toroids are very important; they are used widely, especially in PCDs where you require a good amount of inductance for chokes and other cases. So toroids are very important way of obtaining inductances okay. And what are the toroid consist of, the toroid is simply a magnetic material or a long solenoid okay that has been completely bound in the form of a circle.

And then you have a wire that goes through this okay, so you have a wire that is linked, let me just show it with the way like that okay. So the current I will be entering this toroid and the wires, and then the current, same current will be coming back okay. And in violet end circles here each time you can simplify this movement by assuming that, there is actually a closed loop okay, and there is one more closed loop at this point and you said there are about N such closed loops through which this is linking the solenoids or through these loops are actually going into the solenoid.

So the simpler version is to imagine that there are loops instead of this kind of a spiral kind of a winding okay. This assumption is true as long as the number of windings N will be very large, so N is large and the windings are very tight okay. Also if I call the radius of this solenoid as R, so this is the radius, so at this point is the radius placing on that one, this is from the origin to the point where I have indicated here is the radius.

And the coil itself is assumed to have a certain amount of thickness okay, the solenoid is assumed to have a certain amount of thickness which we will denote by D okay. Now how do I find the field okay, the first step would be to find the field, magnetic field and from the magnetic field I will then be able to find out what is the amount of flux, because amount of magnetic flux linkage and from that I will be able to calculate the inductance.

So how do I calculate the field of this toroid. Well, we have seen in the last two modules that whenever there is a current flow in a given path, the magnetic field will tell to end circle that. So if there is a wire, magnetic field is circling here, and if the current itself is going through the circles in this way to the circle that is shown in this green color, the magnetic field has to come out as perpendicular to this.

So it has to come out perpendicular here, it has to come out perpendicular here perpendicular and what you will see is that the magnetic field is going to form circles along the $\phi$ direction. So if I consider this as a $\phi$ direction this would be the circle that I am going to obtain. We will also make couple of additional assumptions saying that R is much larger than D, so we are going to neglect the thickness of the toroid material.

And because of this if I want to find the magnetic field inside with this between these regions, I know that value or that particular radial distance let us denote it as some value of r, in our approximation that R is much larger than D, R is approximately equal to r okay. Of course, if I consider a particular radius R such that it would be in this region. So in these hatched regions if I consider their radius R to lie okay, so this is the radius R.

Then the magnetic field or the magnetic flux density integrated around that particular or at a constant radius R will be given by $B\phi$, because there is a direction in which you are magnetic field is. So this $B\phi$ divided by $\mu$ integrated over the corresponding loop here which is $rd\phi$ that must be equal to the total current that is being enclosed, and the total current enclosed is N times

I, because through each green loop you will have one current coming out or whether the each D field encircles one current in this loop and there are N such loops.

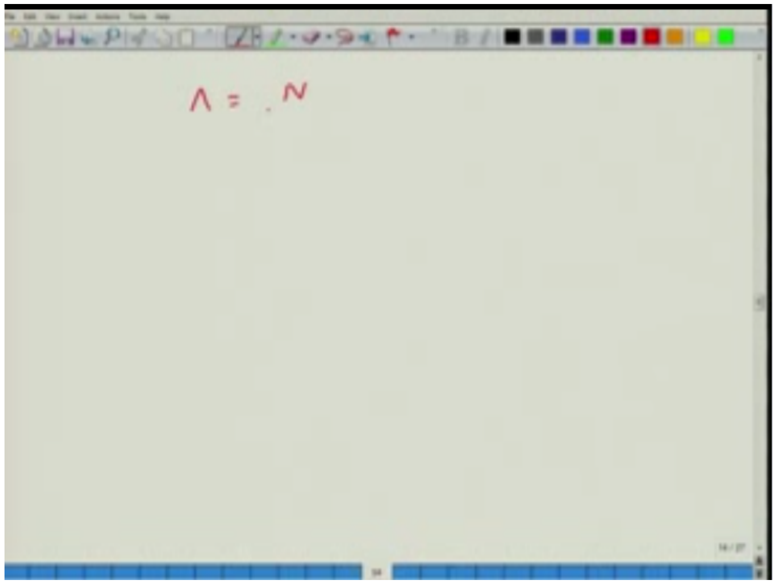
So the total current enclosed will be equal to NI from which you can calculate what is the value of the magnetic field at a given radial distance R, and this would be given by μNI/2Лr okay, as long as R is in this hatched area okay. So we are calculating the magnetic field, and the magnetic field is confined into this thickness okay, so the magnetic field does not come out of the toroid, because it cannot come out of the toroid in a tightly bound toroid, the magnetic field down side will have to be equal to 0.

So this is how the magnetic flux would be, and I want to find out how much of magnetic flux is linking this particular current loops okay. So imagine that I know, I have, I am looking at this particular circle, although I am, you know it is a little exaggerated showing that these wires have come out in actuality the wires are only coming or tightly bound on the thickness itself right. And the magnetic field is now crossing this perpendicular to this surface area, and what is the surface area of that one, that is actually the surface area of a circle that you would draw on the toroid outside which has a radius of d/2.

Therefore, the surface area of this one through which the magnetic field will be coming out and perpendicular to that one, and therefore, corresponds to the total flux associated with that one. That surface area is $\phi d^2/4$ okay. So I can multiply the amount of or I can find out what is the total magnetic flux, the total magnetic flux will be $NB\phi(ЛD^2/4)$.

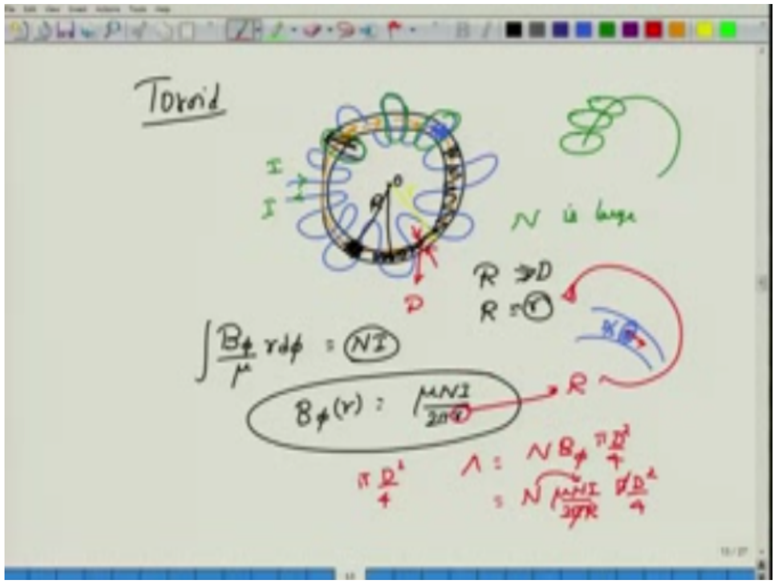
And in this expression I will replace this small r by the R, because that is what the assumption that we have made, the thickness of these toroid is actually very small. Therefore, I can replace this small r by R.

(Refer Slide Time: 22:10)



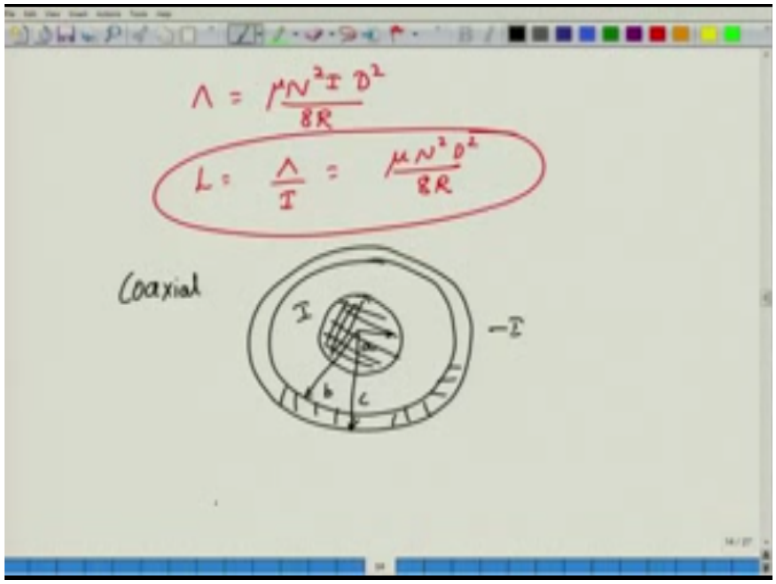

And then, I can write the total flux linkage as N.

(Refer Slide Time: 22:16)




You remember this, the total flux linkage will be given by NBϕ(ЛD$^2$/4) so we can write it here itself, Bϕ is given by μNI/2ЛR after making this approximation times ЛD$^2$/4, so clearly Л cancels out N increases by N$^2$.

(Refer Slide Time: 22:36)

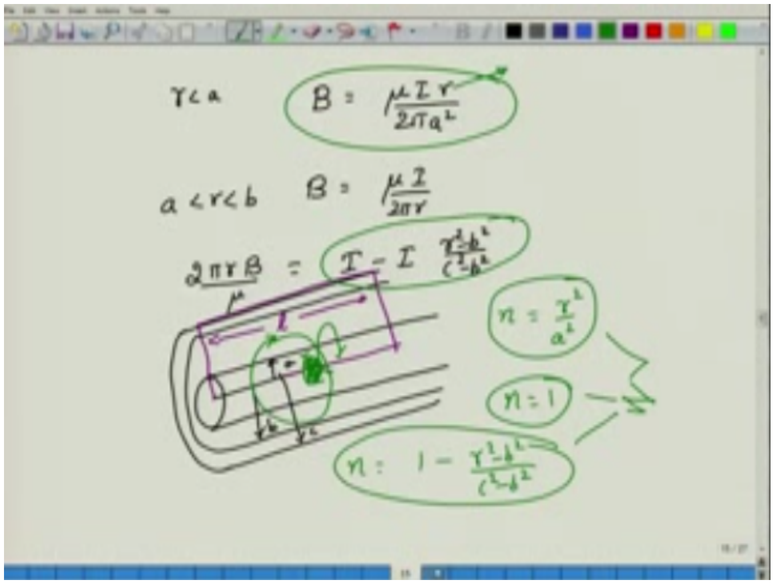



So the total flux linkage that we are going to get will be $\mu N^2 ID^2/8R$ okay, and the inductance L is given by Λ/I which in this case becomes $\mu N^2 D^2/8R$, so this is the inductance of a toroid coil. As the next example we will consider the coaxial cable okay. You remember from the previous module that the coaxial cable had an inner conductor of A and an outer conductor of thickness C-B, because we assume that the outer conductor, you know you had a two concentric circles one of radius b and the other one of radius c.

So this is one conductor which actually carries the written current of –I, this is the forward conductor which carries a forward current of I or the inner conductor which carries a current I okay. What is the inductance of this cable, we have to remember or recall what is the field that we actually calculated okay. And once we know the magnetic fields, then we will have to find a particular surface through which we will have to calculate the flux linkage.

In this example we will see that the flux linkage will not be complete in at least two sections of the coaxial cable okay. Flux linkage will be complete only in one section, in the other two sections would not be complete okay. First let us write down what is the fields that we calculate.

(Refer Slide Time: 24:10)



For this one we calculated fields in three different regions, the first region when r<a was the inner conductor case, where the magnetic field B is given by μTr/2Л$a^2$ and for the region where you are in between the inner and the outer conductor the magnetic field B was given by μI/2Лr. And for the region between b and c that is in the outer conductor region, the magnetic field was given by 2ЛrB/μ so this 2ЛrH, B/μ is actually H correct, this would be equal to the current I-I $r^2$-$b^2$/$c^2$-$b^2$ okay.

So we saw this yesterday and from which of course we can find out what could be the magnetic field B. Now notice something over here, in this region the magnetic flux okay, depends on the value of R at which we are calculating the magnetic flux. Therefore, the corresponding flux linkage well we have to tell you where the surface that you are going to look at, the surface for calculating the amount of flux linkage can be considered to be by looking at the coaxial cable longitudinal cross sections.

So this is the longitudinal cross section, this is a, this is b, and this one is c right. So we have to first find a surface through which each flux line cuts only once, and because the magnetic field is circulating right, so this is how the magnetic field is circulating right. So the surface must be chosen in such a way that the magnetic field actually comes out of it. So if I have this wire the magnetic field has to, because it is circulating it is coming out in this way.

So my surface has to be kept here, so that this four lines which I am showing are the magnetic field lines or the flux lines that are coming out. So this is the direction of the inner conductor, this is the magnetic flux, and this surface if I now pick will have a magnetic flux line coming out. So the surface that I am going to pick will be in the constant ϕ plane, because B will be in the ϕ plane and therefore that will be perpendicular to the ϕ plane.

So if I draw that constant surface, let me take that surface as all the way from ar=0 to this one, and the length of this surface element will be denoted by L, and let us see we are not interested in the inductance section, but we are interested in the inductance per unit length okay. So this is the surface that we are going to consider, and if you consider this surface as long as you are in the inner conductor, you see in the inner conductor the magnetic flux lines would actually be complete in this way.

So the magnetic field lines at a given R would be circling the current through this one right. So if I consider this small value of R it would be circling through here, but only this portion is getting linked right. So it is only one portion or the fractional portion that actually gets linked to the surface that we have drawn. So because of that the fraction through which the flux links is given by $r^2/a^2$.

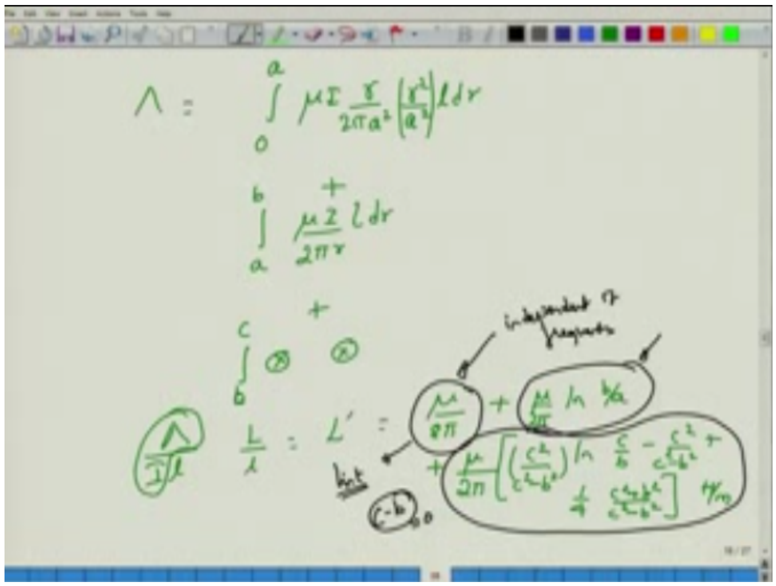
If you are not convinced with this, you can just think of how much current is actually carried by this particular hatched surface area, the green surface area. It will not carry the complete current, it will only carry a current which is the fraction of the total current carried by the conductor, the inner conductor has a current density of $I/Лa^2$. And only a fraction that density times $Лr^2$ is being carried by this loop.

And therefore, that is only a fraction of the conductor that the magnetic flux links. On the other hand if you go outside, the magnetic flux lines links completely okay, it encloses the complete current. So in the outside region the fraction N will be equal to 1, because the current density is $I/Лa^2$ and the fraction that is carried by any R outside of this one will not be, the density might be $I/Лa^2$, but when is your outside this loop right, outside the inner conductor then the entire flux gets linked.

So essentially in the region outside the inner conductor or in between the inner and the outer conductor, the fraction will be N=1. And by looking at this equation you can imagine what would be the fraction that would be carried in the outer conducting region and that would be equal to 1-

$r^2$-$d^2$/$c^2$-$d^2$. So these values of $r^2$/$a^2$ and 1 and1-$r^2$-$d^2$/$c^2$-$d^2$ are the partial fractions that are linking to this inner conductor and the outer conductor in this way.

(Refer Slide Time: 29:05)



Now to obtain the total flux linkage I simply have to integrate over the three regions, actually I should have shown you that there are three surface regions over here, one will go from 0 to a, the other one will go from 0 to b, the other one will go from a to b, the other one will go from b to c, so there are three regions of integration depending on the value of R. And if you carry out those integrations with the appropriate value of the magnetic fields and sign.

For example, for the inner conductor you are there from 0 to a and μIr/2Л$a^2$ $r^2$/$a^2$ ldr I have actually taken the liberty of multiplying this by L. So this would be the contribution from the first integral or the inner integral or the inner conductor, and for the region between the conductors you will have to integrate from a to b, that magnetic flux density is μI/2Лr there L times dr, the fraction does not enter, the fraction in this case was $r^2$/$a^2$ for the inner conductor, whereas from the outer conductor it does not.

So this is what you get, this is from integral region, and for the outer conductor you are going from b to c, and you will have to write down the corresponding magnetic field as well as the fraction through which this is linking, both I have indicated in the previous slide, so I will ask you to put that one in. And when you carry out the integration, and once you obtain the integration this will all give proportional to the current I.

And if you divide this flux linkage by IL what you obtain is Λ/IL, but Λ/I is the inductance L divided by L will give you the inductance for unit length okay. So inductance per unit length let us denote it as some L’ which is the quantity that we were very interested in the case of transmission lines when we were discussing. And this L’ will be equal to μ/8Л+μ/2Л log(b/a)+ this is a very complicated term you can actually sit and do the integration, and then show that this is correct okay.

So it is given by this value, this is very important practical, therefore I am actually writing this entire thing okay. $-c^2/c^2-b^2+1/4\ c^2+b^2/c^2-b^2$ times H/m. So it might look complicated, but this actually has three contributions okay, one is what is called as the internal contribution, and this internal contribution will be independent of frequency okay. We have calculated inductance only for the case of VC frequencies, but this calculation when you extend it for the AC frequencies or for higher frequencies we will see that the other terms will actually start to disappear.

However, this term μ/8Л being completely independent of the frequency will always survive okay. In fact this we will show to be contributing as to what is called as the internal inductance of a given transmission line, and in this case it is actually μ/8Л. The second term will be present okay, more or less always except in very, very high frequencies, but this term will be present, because the region between the conductors will always enclose the current I.

If I remove the outer conductor, then this term will not be present, but if I include the outer conductor this term would always be present. This term can actually be considered to be negligible when you consider the thickness c-b would be approximately 0. So in other words if I consider the outer conductor thickness to be 0, then this term can be eliminated okay. So this is what I wanted to talk to you about the inductance calculation.

With this we will stop with magnetic electric field calculations we have performed, magnetic field calculations, capacitance or inductance we have looked at. And we have looked at a few transmission line structures for which we have calculated the capacitance and inductances. Thank you, very much.

**Acknowledgement**
**Ministry of Human Resource & Development**

**Prof. Satyaki Roy**

**Co-ordinator, NPTEL IIT Kanpur**

**NPTEL Team**
**Sanjay Pal**
**Ashish Singh**
**Badal Pradhan**
**Tapobrata Das**
**Ram Chandra**
**Dilip Tripathi**
**Manoj Shrivastava**
**Padam Shukla**
**Sanjay Mishra**
**Shubham Rawat**
**Shikha Gupta**
**K. K. Mishra**
**Aradhana Singh**
**Sweta**
**Ashutosh Gairola**
**Dilip Katiyar**
**Sharwan**
**Hari Ram**
**Bhadra Rao**
**Puneet Kumar Bajpai**
**Lalty Dutta**
**Ajay Kanaujia**
**Shivendra Kumar Tiwari**

**an IIT Kanpur Production**