Indian Institute of Technology Kanpur

National Programme on Technology Enhanced Learning (NPTEL.)

Course Title
Applied Electromagnetics for Engineers

Module – 46
Finite Difference Method

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Hello and welcome to NPTEL mook on applied electromagnetic for engineers. In this module we will first finish an example of finite difference method that we discussed in the last module. And then discuss another numerical technique called as method of moments. So I hope you remember what we discussed in the finite difference method.

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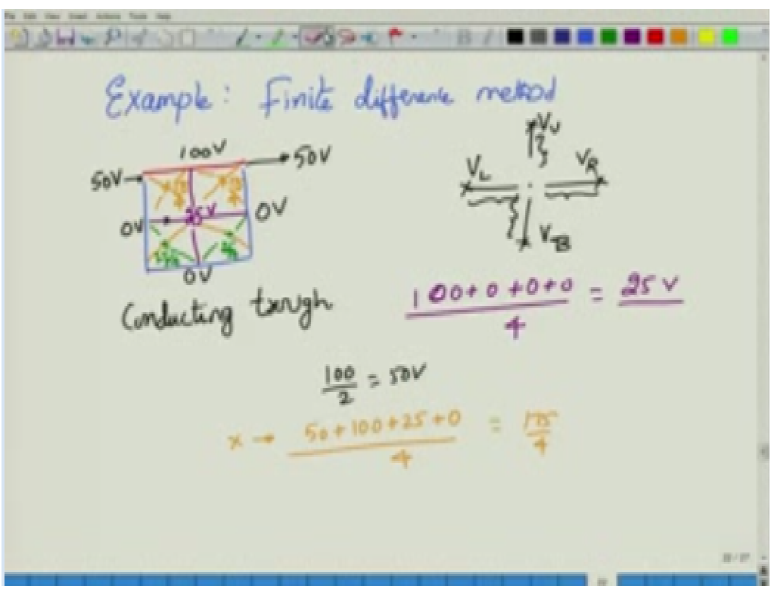

If you choose your step sizes appropriately, then the potential at any point okay, will be the average of the potential of the neighboring four points. So for example, the point I need to find out the potential is given by this dot, then the potential to the left, potential to the right, potential to the top and potential to the bottom are the ones that we need to use and then average them, provided the sizes or the step sizes from the point where you are calculating the potential to the point where the potentials are known okay. That step size will remain same.

So the step size thing will actually become independent when you actually calculate using the finite difference method. So here is an example, which is the conducting trough okay, we have already looked at the analytical solution of this one in the form of a strip line with the finite strip that we had kept for actually a two parallel plate wave and for the parallel plates that we had kept two plates in parallel we had kept.

And then, we have shorted the two plates by an appropriate conducting strip. And then we had calculated what would be the corresponding equipotential surfaces or equipotential contours as well as what would be the electric field. So I would ask you to go back in case you have to follow the next few steps, I would ask you to review whatever we have discussed in terms of the earlier example, so that you can see how the analytical solution can be obtained from this numerical solution okay.

So we start by the problem that is shown here, so all these colors of the lines that I have made, they are all suppose to represent ideal conductors, the top conductor is kept at 100volt and the other three conductors which are all joined together are kept at 0volt. Of course, you need to put a small gap otherwise everything will be on the same potential. So we put a small insulating gap so that the top plate is at 100volt potential and all the other plates are at 0volt potential.

Now what we want is to solve this Laplace’s equation, inside the empty region that we have, and then determine what are the equipotential lines okay, once we find what is the equipotential lines, then either by graphical method or by a computer method you can easily find what would be the electric field, because electric field will be perpendicular to the equipotential lines. So let us dig into this problem.

So you have 0volt, 0volt and 0volt, so in the first iteration, I actually want to determine the potential at the center point here. To do that I need to look at what is the potential onto the left which happens to be 0, potential onto the right which is 0, potential on the top which is 100 and potential at the bottom which is 0. So the first potential that I will be obtaining will be given by 100+0+0+0/4 and that you are going to obtain as about 25volt okay.

So this is the value the first time that you are going to write will be 25volt okay. So this seems to be very simple. Now what we do is, we need to find out the potential at other point, suppose I try to find the potential at this point, I know what is the potential to the left, I know what is the potential to the right, but I do not know what is the potential onto the top, because remember this step size was equal to this step size, however the step size is now reduced to half.

So if I want to find the potential at this cross point, I need to know what is the potential at this 0 point or this particular point which I do not know. So what instead I can do is that, I know that the potential at the gap is going to be 100 or the top conductor will be about 100volts. But I can find out what would be the potential at this gap okay. So this gap potential will be the average of these two neighboring point.

So if I try to find out what is the potential at the gap it could be of the neighboring points, this time I have only two points here okay. So I have one point here at 0volt potential and the other one is at 100volt potential. So the gap can be written as 100/2=50volt. So we make an initial guess that this would be 50volt, of course this is just an estimate of the potential that would be present.

In fact we would say that there would be lot of, I know the potential has to go all the way from 100volt on the red line to 0volt on the blue line, therefore there will be many, many such potentials that need to be calculated. But for now we can assume that the gap potential that we estimated is 50volt. Similarly, we removed this gap over here and then say this is also 50volt.

So now we have slightly more information about the potentials so we might want to calculate the potential at other points okay now let us take this particular axis okay so essentially what I mean is that we have turned around the x and y axis by about 45^0 and I seek the potential at the center over here now to this potential if now imagine yourself turning around or turn the paper around.

And in this way if you are calculating this one the piece of paper you see that the potential to the left is 50v potential to this one is about 25v potential to the top here and the bottom here will be 0 and 100v respectively therefore the potential at the cross point that I have made which would also be the same potential here because it would also be by symmetry this same potential so if the potential at the cross point where which I have made will be an average of 50 + 100 + 25 + 0 / 4 so this would be about 175 / 4.

I do not really know what is the value but you can actually calculate it by a calculator and then but that one so that would be the same value over here as well so this would be 175 / 4 this would also be 175 /4 I will rewrite this one in a neat manner so that you can see this correctly later on but for now look at what is the potential at the green color over here again I have a potential to the left which is 0 potential to this one is 0 potential here is also 0 but the potential on the center of the trough is 25 v therefore the average here or the potential at this point will be 25/4 by the same symmetry this potential will also be 25/ 4.

So after 2 iterations okay so after 2 iterations the potential would look something like this the grid potentials would look something like.

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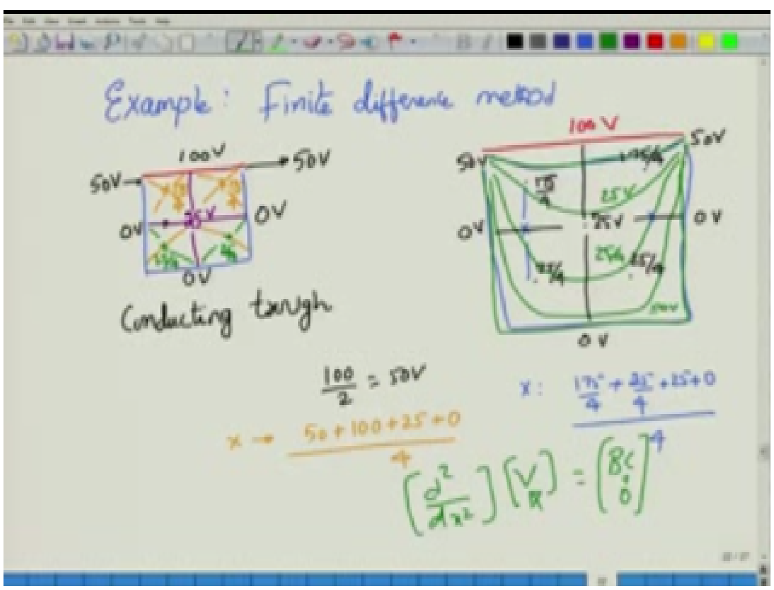

So let me redraw this one so this is slightly bigger picture that I am going to draw okay and then I have the top conductor which is kept at 100v and I have 50 v at the gab 50v at the gab at the center I have 25 v and no so this is at the center that I have these potentials are 0v, 0v and 0v, we have also calculated the potential at this point which is 175/4 the potential at the center of this square is 175/4.

The potential here is 25/4 the potential here is 25/4 now I have you know reasonably more number of points I can now find out what is the potential at this cross points okay which are essentially at the center of these 2 points now that would be the potential at this cross point would be 175/4 + 25/4 because these are on the no if you think of this is top bottom to the right and to the left to the right will be 25 to the left will be 0 this entire thing divide by 4.

And again by symmetry this would be the same value over here so you get additionally 2 more points so by looking at this one you are able to get additional points over here, so if you actually continue this okay this of course will be different but if you continue the entire grid what you would see is that the potentials will actually be you know larger over here this would be say 50v contour so this would be a 50v then the potential actually starts to draw up this would be a 25v contour.

Because it is actually passing through the center with a 25 this potential will be whatever 25/4 point and this would essentially be very small so that this approximately 0v of course the actual 0v potential is done I mean it is given by the walls of the conductor so this is the method by which you can actually calculate the potential in a complicated scenario I want to add one important point the potential that we have calculated in this manner is okay for a paper and pencil calculation.

This is strictly speaking nit the way in on a computer that we are going to implement it one a computer we would write the d^2/dx^2 whatever the equations that we obtained after appreciating the derivative in terms of you know those 4 values that we obtain and the right hand side is either 0 or at the boundary we will have to take care little bit of a problem there but we have to take care of the boundary and convert this entire thing in terms of a matrix equation so you will have the matrix corresponding to d^2v/ dx^2on the left hand side okay.

And then this would be v actually this would be the matrix corresponding to the operator d^2/dx^2 times v that must be equal to some boundary condition + most of these value it will be = 0 and the you proceed to solve for a un potential values v which you have tried you know which you have marked out there by there are matrix solution okay, we will follow a very, very similar method in the next numerical techniques that I am going to discuss and that technique known as method of moments to introduce methods of movements let us begin with the actual problem I mean problem in a slightly.

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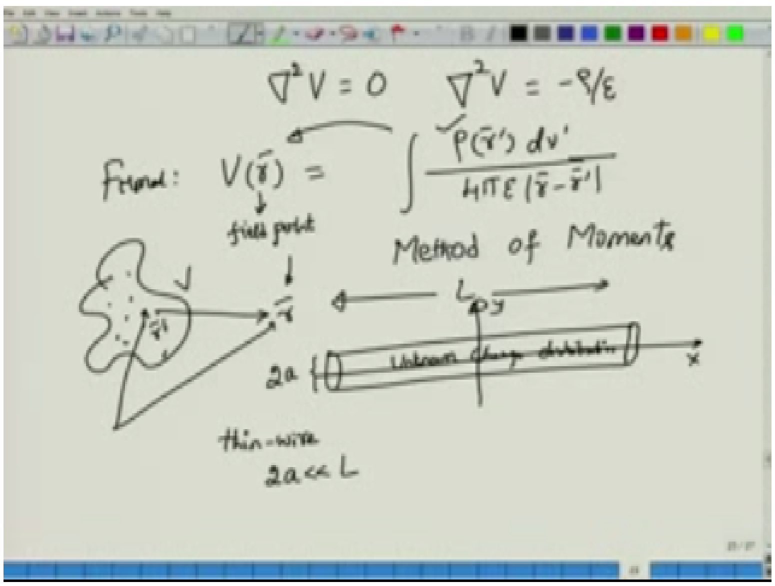

Simpler context we have seen that Laplace equation tells you the $\nabla^2 V = 0$ but Poisson's equation tells you that if there is a charge distribution in the region then this ∇2V will b = - δ ε and this ∇2V = - δ ε the row in general can be a volume charge density always have a solution at any source sorry at any field point r okay so this is your field point this is the position vector of the point at which we are considering the potential this always has a formal solution okay, so this formal solution if an integral over the charge distribution.

The charge distribution in a given the region so for example this is the region where the charge is distributed okay so any point at which I am actually evaluating the potential is called the field point and the corresponding position vector for the source point will be denoted by a prime on the letter r so you have δr r prime which is the scalar field of course and it is function of a vector r prime but it the result is actually a scalar over the volume d prime of the occupied by that particular region over there.

That we are considering derived it by 4π ε r – r` magnitude okay this r – r` will be the line that joints the source point the point where the chargers are located to the point vary or calculating the field okay, this can be you know line integral volume integral or a surface integral depending on what type of geometry that you have, so given δ of r` everywhere in the region it is a reasonably okay matter I am not showing it is very simple because someone has to carry out this integration.

And this can be even done numerically but if this source is given then it is rather trival matter to determine what is the potential at entire space of course the problem is that your only allow to specify potential can most of the cases we can specify the voltages at different point in the conductor in most of the cases you do not know what is the charge distribution so if the charge distribution is unknown how do we calculate the you know I mean you know the potential at certain points only.

Then we need to calculate the charge distribution once the charge distribution is known in the potential at all the other points where we have not specify the potential also have to be calculated the method that we are going to adopt is what is called as method of moments okay a full discussion of course we will take you into a different codes numerical methods which are running in NPTEL you can referred to that but a brief idea of method of movements is like this, in order to understand this method.

We will look at a simpler problem okay we consider a very thin wire the wire has a diameter of about 2a okay and certain length which is much, much larger so this let us make it has a L so this length L is take to be much larger than the diameters so essentially what we are considering is what is known as a thin wire okay, the radius a or the diameter 2a is very small compared to the length L okay let say that is some unknown charge distribution you do not know what the charge distribution here is they revolved you are trying to calculate okay.

We also let this wire lie on the x axis okay so the wire is lying on the x axis this other axis is the y axis for now this is the two dimensional picture that I can show you, what we do here is we reconsider.

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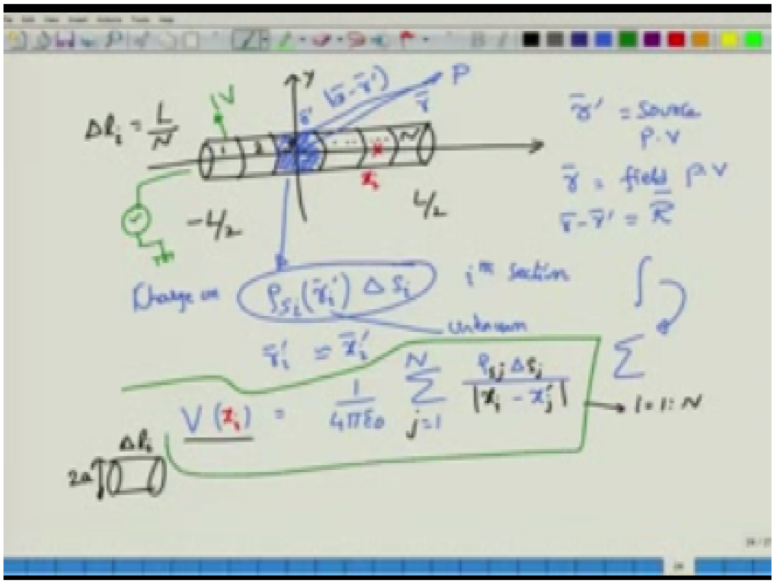

That wire okay so you gad this wire which was extending over say minus n / 2 to + n / 2 what we do is the section this wire into appropriate sections okay, we cut this wire not actual wire and then cutting it we imagine that the wire has been cut and there are different sections section 1 section 2 section 3 and so on and so forth up to the last section n okay this is the y axis at any point for example if I want to find out the potential at the point p marked here there is a certain position vector to this.

Which is r and if I consider the charge on a particular section right so there will be n number of section over here if I consider the charge in a particular section but charge itself we will have a certain potential vector r’ and from the charge to the point p will be the joining vector r – r’ okay so please note that r’ is what is called as the source position vector r is called as the field position vector okay what we are interested is not r or r’ themselves but the difference between these two which we will designate by some capital R okay a vector r if necessary okay.

Now what we assume further is that within this particular section here it is a section three but it can in general be any one particular section j okay in this or eighth section in this section we assume that the charge distribution is going to be constant okay for now neglecting the this is actually a volume because this is the conductor most of the chargers are actually residing on the surface so what you are actually calculating is one the eighth section what is the unknown charge ρ s I and this ρ si will have a certain position vector ri’ or you know because I am having a x axis and a y axis defined and I am assuming that the thickness of the wire is very small ri’ is approximately xi’ that is I am measuring the position along the x axis okay as I go through that.

This times whatever the surface charge density that I have Δ si will actually give the charge total charge on the eighth section okay, which we will assume to be constant so charge on the eighth section assume to be constant of course this ρ si which you know given at this particular eighth section is completely unknown to us but it is essentially consider to be a constant. So the potential at the point r which is the point p can be written as 1 / 4 π ε_0 this is actually just a constant so you can pull this constant out you can even set this constant equal to one and re introduce the constant wherever you want but let us not do that.

And now how do I go from a continuous to a discrete well if you remember the definition of an integration okay integration is actually made out of small pieces you know multiplying the values of the particular function and multiplying it by the width and this summing them over so you have one infinite number of such summations that will result in a summation going in to a integration but on a computer I cannot do an integration directly so I will always have to do a discrete version of that which essentially is a summation.

So the summed value the potential at this particular point r is given by sum over all the sections that I have so I have made n section of the wire so it would be I = 1 to n ρ si Δ si / x I – so we run in to a small problem here because I have to specify not the potential at this point r okay as I thought I would want to but I would like to determine what is the potential at this particular point which is say xi okay.

So if I want so do that one then instead of considering a general point r which would be in the xy plane I am trying to find out the potential on the wire okay I will tell you in the moment why I am trying to do this one the potential at the point xi because I have used the you know index I for this one I have to change the index over here, so let me change the index so instead of talking about the eighth section I will be talking about j^{th} section okay so I have j = 1 to n ρ sj Δ sj Δs is the surface of the cylinder okay whose diameter is about 2 and whose length is Δ li.

Where Δ li is the length of each section they of course have made n number of sections therefore Δ li = L / N that is the length of the wire divided by n. so I have Δ ρ sj Δ sj / x j xi okay I do not need to put prime over here at least or may be if I want I can always put a prime over there of course if I put the prime here I should put a prime Δ sj' and ρ sj' but I will not putt those primes just to confuse all of us.

But it is very important to note that this particular expression that we have written will be evaluate at a constant xi that is on a particular I the section of course this I itself has to go from one to n, so let us actually write it on completely oh before we do that one I want to specify why it correspondently I mean I why I wrote the potential at the center point xi on the wire because if you see the conductor I actually set the potential of the conductor okay set the potential of the conductor to some value. Let us say I set the potential entirely to one volt. So how do I do that? I of course I have to take the battery or something and correspondingly connect the 1 volt battery to this particular wire.

When I do that that the entire wire is assume to be uniformly under the same potential and the value is taken to be 1 volt. Just for our simplification it could be any other potential, in fact the potential could also vary that does not really change what we have written in this equation okay. so that is the beauty of the numerical method, you can make v(xi) different sections okay. Now that we have written here j goes from 1 to n. I also goes 1 to n let us completely rewrite for all the values π.

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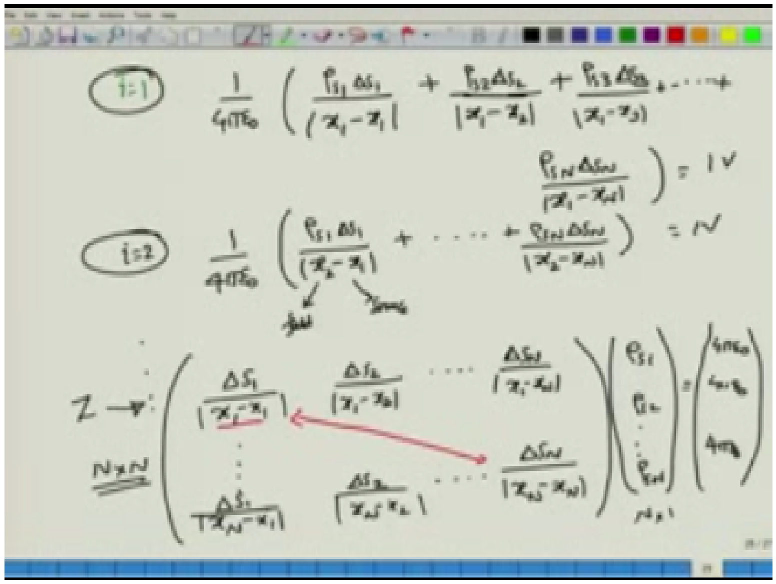

What would be the corresponding expression? And you can very easily see that for i = 1 the Σ has to be expanded and I what I get will be 1/ 4π ε which is the constant, (ρ_{s1}) it is at the section 1and I am looking at the potential 1 itself because of x_1 + charge on the section 2 the point where I am evaluating the potential is still x_1 but the source is located at x_2 that is on the x axis it is located at x_2.

Similarly you get ρ_{s3} Δ_{s3}/ $x_1 - x_3$ all the way let us go up nto ρ_{fn} Δ_{fn}, let me just rewrite below here, so this would be going all the way up to ρ_{fn} Δ_{fn} / x_1 still that point where I am calculating the potential – x_n magnitude. For i = 2 it would essentially would be the same constant ¼ ε_0 then you have ρs1 Δs1 which is the charge on the 1st section/ $x_2 - x_1$. Why? Because this is the field point at which I am evaluating the potential and this where the charge or the source is located.

So if I carry out for the last section I get ρ_{fn} Δ_{fn} charge on the last or the end section / $x_2 - x_n$ and so on. I can put everything together in terms of matrix okay that would be very interesting to look at in matrix location this would be Δ_{s1} $x_1 - x_1$ magnitude, Δ_{s2}/ $x_1 - x_2$ magnitude so on upto Δ_{fn}/ $x_1 - x_n$ magnitude go to the last section which would be Δ_{s1}/ x N – x_1 which is the section 1.

Then Δ_{s2}/ $x_n - x_2$ okay go all the way up to the last section you get Δ_{sn}/ $x_n - x_n$ magnitude okay, so this is the matrix which will be n cross n matrix okay. You can put a short hand notation as some z matrix, so z matrix is this one. What is the unknown vector, ρ_{s1} ρ_{s2} all the way up to ρ_n. These are the values which I do not know so I put them as a n x 1unknown vector. Finally on this side I have all these constant which is 4 ε_0, 4π ε_0 why is this?

Because I know that the potential at i=0.1 will actually be = 1 volt. Similarly the potential at the 2nd section will also be = 1 volt because that is the voltage that we have set on the wire, we have set the voltages and voltages is set to be uniform, if it is not uniform then you can know write the appropriate values of the potential whether it is 1 volt, 2 volt, how you have define the potential in this case, the potential at all the section = 1 volt.

Therefore this 1/4 ε_0 when you take it to the right hand side simply becomes 4π ε_0, so now I have a very interesting matrix out there okay. There are two things that would you immediately recognize, if you look at the diagonal elements, the diagonal elements are going to the ∝ why because there $x_1 - x_1$ which is obviously will be = 0. So this diagonal element will shoot up to ∝. Similarly there will be $x_2 - x_2$ in the second row, 2nd column, that will also shoot upto ∝ the last row and the last column will be having $x_n - x_n$ which is again 0 and this will also shoot upto ∝, that is the 1st observation that you are going to make.

The second observation is the value δ x2/x1-x2 will be you know similar to the value that you are going to obtain in the other case if you be the matrix of you have written so you will have δs1/x2-x1 so apart from δs2 part 1/x1-x2 magnitude will be the same as 1/x2-x1 magnitude okay the top will also be the same because we are going to assume it everything to we having the same surface area therefore.

The matrix essentially will be symmetric it could be symmetric such that upper diagonal element will be equal to the lower diagonal elements okay so you have an matrix which is symmetric which means if I take the transpose of the z then I obtain z transform will be equal to z itself and further the problem that all these diagonal elements seem to go to ∞ so how do we overcome this problem of ∞ I will not completely you know give you the way in which it has done.

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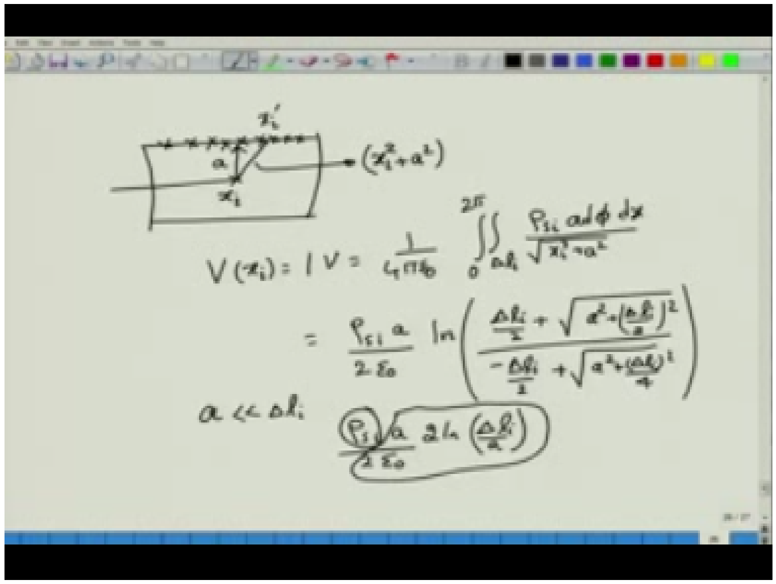

The idea is that if I blow up one session of you know what ever the session I am looking at then I know that I am calculating the potential at this point xi right on this same section I assume that the charges are kept on the surface right so the charges are off course kept on the surface because if there is a conductor and if I take the charge direction so any point on this one that I am going to measure will be xi prime and the radius of this tube or the wire is actually equal to A.

Therefore the vector which joints the source point xi prime and the field point xi is actually given by xi^2+a^2 okay so because of this right I can actually go back to that potential calculation potential at any height session v(xi)=1v is given by 1/4πe0 instead of using the equation I will actually I mean now I will actually use the formal equation where I am assuming that the charge will be constant.

And we will have some adϕdx okay which is the way in which the cylindrical surface area will be defined divided by $\sqrt{xi^2+a^2}$ if you evaluate this integral over the range 0 to 2π and over δ li okay we will see that you can show this as an exercise that this is given by δsia/2 log δli/2+ so this is the little complicated quantity but this is very straight forward integration that I has you do this one as an exercise so $a^2+\delta li^2/4$ so this is essentially the quantity that I have and if I further assume that a is much smaller than δli that is if I am assuming the thin wire appreciate then this complicated expression can be rewritten as si a/2log δli/a so this would be a finite number that you can calculate off course you do not know what is si but that is okay because the coefficient for that height session will be this a/2logδli/a and please remember all these session will be the same.

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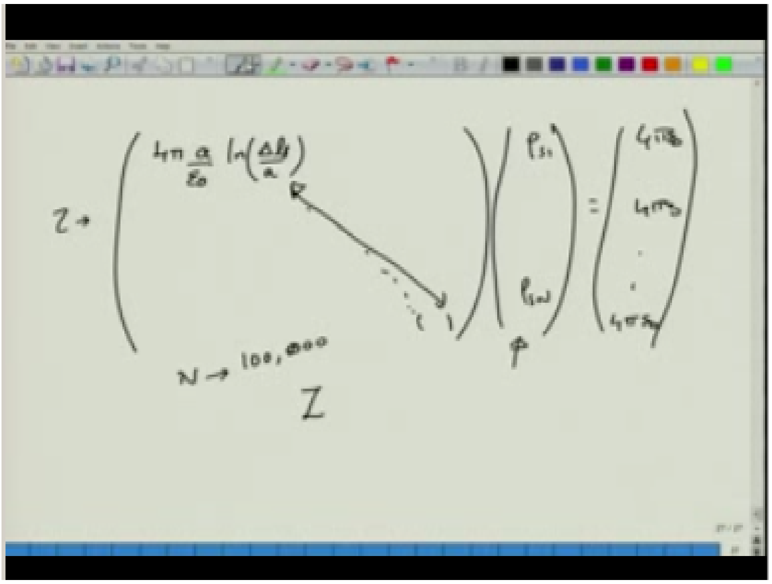

So you are modifying coefficient matrix z matrix will instead consist of 4πa/e0 because there is a 1/4πe0 you have to multiple this π 4π/e0 over here log δli/a and so on for all the other cases okay so this would be the modified z matrix which will not have infinities on the diagonal elements because we have removed the infinity on this part okay the unknown vector ps1 to pn remains as it is the right hand side vector 4πe0 will also remain as it is and you can now apply any matrix in order to solve this matrix okay and to obtain the unknown parameter before we conclude this method of moments I just like to point out.

The that in these scenarios the value of n can be very large you know sometimes it can be about 100,1000 and this case no one actually computes the inverse of the matrix z inverse okay there are powerful methods of solving matrix equation none of them involves determines or actual calculation of the inverse okay so we do not calculate the inverse you do not find out the determines because it is very difficult and completitionally intense to do that one common methods are there to decompose the matrix in terms of its lower and upper diagonal matrix using a elimination process or you apply an alternative solution such as some successive over relaxation or successive relaxation methods okay.

So coming back we now have a matrix and a vector equal to the known vector so this is say the known potential we prime or just for that case okay and then we can use powerful matrix it is all your to determine the unknown row okay all these elements are known to you because everything else δl1 is known δl2 is known everything else is known in this particular matrix and you can be able to find what is the corresponding unknown vector unknown vector with that thank you very much.

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