Indian Institute of Technology Kanpur

National Programme on Technology Enhanced Learning (NPTEL.)

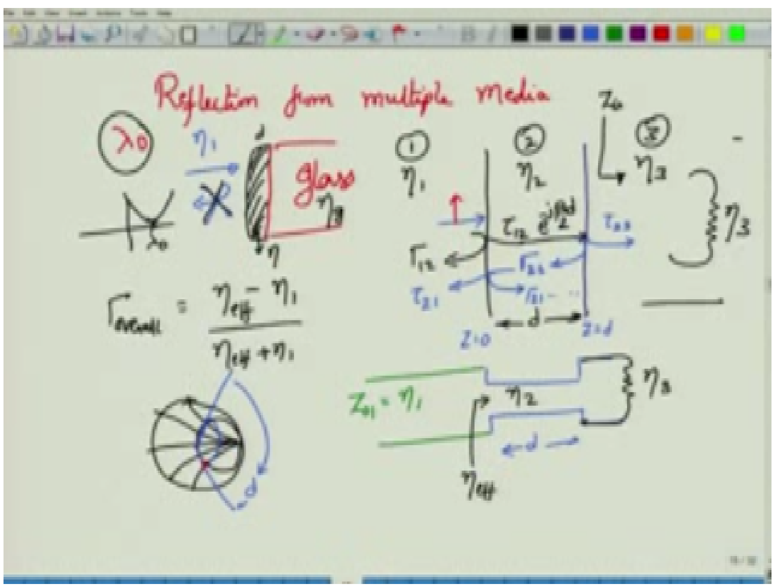
Course Title
Applied Electromagnetics for Engineers

Module – 48
Application: Reflection from multiple media
and anti-reflection coating.

by
Prof. Pradeep Kumar K
Dept. of Electrical Engineering
Indian Institute of Technology Kanpur

Hello and welcome to the NPTEL mook on applied electromagnetics for engineers. In this module we will first briefly look at reflection of uniform plane waves from multiple boundaries and see how you can use the concepts from transmission line theory in order to solve this problem. And after that we look at energy flow from a uniform plane wave, we discuss the concept of pointing vector. This is the problem that we want to solve.

(Refer Slide Time: 00:44)



So you have three regions let us say of space each angle impedance which is distinct from each other. So you have impedance of the medium 1 to be η1 impedance of the medium 2 which is of a thickness D meters, micrometers, nanometers whatever the appropriate widths of meters that you have chosen. And units of distance that you have chosen and the third region having an impedance of η3.

So if you were to go with the pure wave based approach to solve this problem, you would have had an incident wave normally that is the angle of incidence and normal to this interface would be equal to 90°. So we consider wave incident first travelling in the medium 1 and incident on the boundary between medium 1 and 2 okay. So if you want you can label this boundary at say Z=0 and the other boundary between 2 and 3 can be labeled as Z=D.

So as I was saying if you go purely from the wave perspective, you have an incident electric field right. And because, η1 is not equal to η2 there will be a reflection at the first boundary itself. Of course, there will be partial transmission into the second boundary, we have calculated what is the reflection coefficients from the first medium to the second medium which in this case because there are multiple boundaries to consider, I have just indicated with the subscript 1 and 2, the corresponding reflection coefficient.

Similarly, there will be a corresponding transmission coefficient from medium 1 onto the medium 2 and this is how the new wave which is still uniform namely of the same frequency will begin the travel. As it travels it picks up a phase of $e^{-j\beta d}$ here β is the propagation constant for the second medium, therefore, I should write this as β2D okay. At this point what happens you will have one more reflection, because the medium impendence η2 is in general not equal to η3.

So there is one reflection over here, and the subsequent transmission into the second medium, the transmission coefficient will be tou t to 3 and the reflection coefficient here between medium 2 and 3 will be γ23 okay. While the wave is travelling back it picks up another phase factor of $e^{-j\beta 2d}$ thus if you are looking from the boundary between 1 and 2, the wave that actually completed around trip propagation with the total phase change of $e^{-j2\beta 2d}$ and rather the phase change of 2β2d.

So there will be another phase factor, but at this point our work is not complete, because you will have a reflection into this second medium back, and a transmission into the first medium. The transmission into the first medium from the second medium will be denoted by the coefficient tou 21 and the reflection coefficient here is γ21, 2 indicates the first medium, 1 indicates the second medium.

And this cycle goes on infinitely right, because there are an infinite number of times that this wave has to travel from the medium 1 medium boundary to the other medium boundary. And if you want to find out what is the overall field that is reflected in the first region, then you have to sum all the partial waves that you get from the first reflection, from the second reflection, from the third reflected and transmitted and so on.

So you have to sum all the partial waves similarly if you want to find out what is the total transmitted field we have to sum all the partial waves that go into the 3rd medium now you can do that and you can develop what is called as a power series approximately I mean power series approach for this problem but we no remembering the ideas from transmission line theory can solve this problem with much simplicity okay.

So our idea would be to employee the fact that linear isotropy and homogenous medium of impendence η1 can be considered as a transmission line of characteristic impedance η so this is the characteristic impendence zo1 or here Z0 is the characteristic impedance one corresponds to the 1st medium and the characteristic impedance of this medium is obviously = η1 then you have a 2nd transmission line which is not infinity in extent but it is a finite length transmission line right.

So this finite length is of length d in meters as I told you earlier where as the characteristic impedance of this one is given by η2 okay the boundary between 2 and 3 will need one more transmission line to be written but luckily for us this 3rd medium extend all the way to ∞ which means if I look at what is the input impendence you know from the edge of the 2nd to the 3rd boundary condition if I just look at into the 3rd medium this entire thing would appear to be a lumped impedance of value η3.

So I actually have instead of a transmission medium I actually have a lumped impedance of η3 I am assuming all loss less media therefore η1, η2, η3 are all real quantities now how do I obtain what is the amount of reflected power or how do I obtain what is the overall reflection coefficient this problem of transmission line we have already solved many times we have solved you can in fact employee smith chart approach in order to solve this problem what you will be doing is to first transform this impedance η3 through the transmission line of characteristic impedance η2.

And a distance of d okay so in order to get an effective impendence which we will call as effective impendence here effective input impendence which acts as a load to the 1st transmission line so now the overall reflection coefficient which I will denote of γ over all will be given by η effective – η1/ η effective _ η1 is not it very simple in case it turns out that your η3 is not real if it is complex there is no problem the procedure remains the same and if you are not going to use any equation to convert η3 to ηf you can always use smith chart right to convert this particular impedance from η3 to η whatever that value.

So you first find out or normalize η3 to the transmission line characteristic impedance of η2 may be that is located at this point you know what is the length d of course you should know what is the wave length of operation that should be specified as part of the problem so on the SWR you move an appropriate distance land at a new point and then from that new point so this is the distance d that you have moved and the coordinates at this point marked in red will tell you want is the effective impedance once you found that one you can find what is the overall transmission or over reflection coefficient.

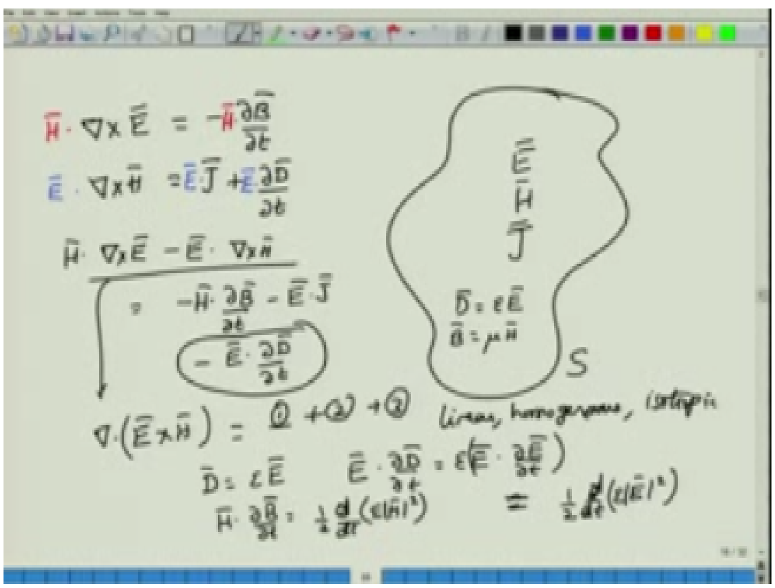
Now these type of problems arises quite naturally especially in optics community when you are trying to match to media okay you have medium and the you have lt us say a mirror okay and you want these to be matched in such a way that there should be no reflected light or not a mirror or some other transmission in a optical element where they should not be any reflection at that particular wave length okay so there are what is called as anti- layer or anti reflection coatings which you will sometimes see on the mirrors or on the optical elements okay such that.

They blocked all reflection into the first medium and therefore there is complete transmission into the second medium, but we have to coat the material with a certain coat in which is called as the air coating so let say this is the medium that you have so let us say this is glass and at that particular wavelength some λ0 you want no reflections into the first medium so you simply take a certain amount of thickness okay and then coat this one this will have a certain impedance η the impedance of the glass.

Let say is ηg and then you shine light from the first medium of impedance η1 then there will b no reflections so reflections are possible because you have this anti reflection coating created of course that drawback of this one is that it works only at a particular wave length and therefore does not really you know eliminate reflections if you happen use different wavelength okay, so in fact as you change the wavelength you know the amount of reflection magnitude will have actually start to increase.

We will have more to say about this type of reflection and how to you know design anti reflection coatings or how to design filters called as fabric filters in a special case study module that we will take about it later, by gone and now would be discuss the power flow right and I want to do that one by the help of what is called as pointing vector the idea of pointing vector is very simple we assume that.

(Refer Slide Time: 09:58)



There is a certain region okay and in this region there are some electric field there are magnetic fields okay and if possible the material also has some surface current density j indicating that the material could be conductive or have some amount of conductance associated with that one, so this region can be assumed to be closed okay having the closed surface yes and the volume that is quite large okay, now we have couple of equations right so we have the curl equation which tells curl equation for the electric field.

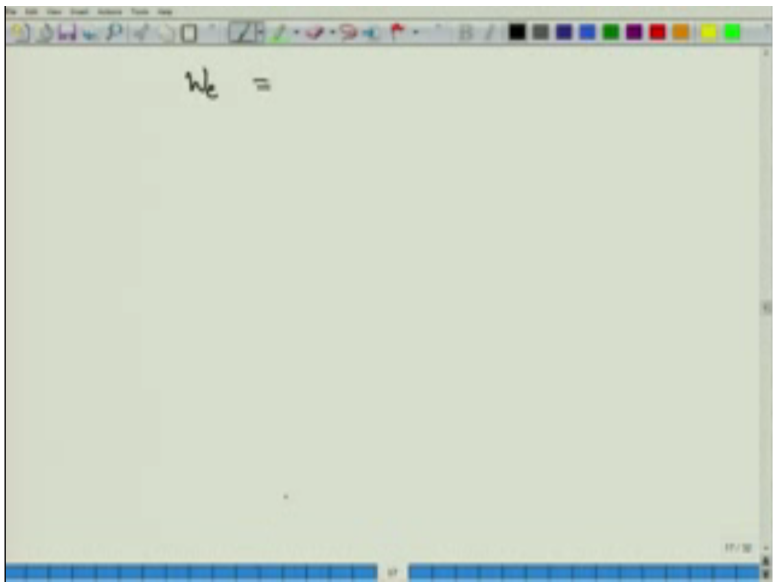
Which is $\nabla \times E = - \partial B / \partial t$ and then you have curl of H = j I want to eliminate j because I just send that you can have some amount of conducting material out there so $j + \partial D / \partial t$ further I will consider a material medium such that $D = \varepsilon E$ and $B = \mu$ times H where ε is the permeability sorry permittivity and μ is the permeability I am assuming that the medium is linear homogenous and isotropic okay, so linear homogenous and isotropic media okay so we have $\nabla \times E$ and $\nabla \times H$ what we do.

Is an interesting thing we take the electric field quantity E and dot on both sides to this $\nabla \times H$ equation okay, so we take electric field E and dot on both sides so this is what happens after you take E and then dot it on both sides then you have H okay which I will do on both sides so I have $- H. \partial D / \partial t$ on the right hand side now what I doing is simply subtract these two equations okay so I subtract this second equation from the first equation and therefore I obtained $H. \nabla \times E - E. \nabla \times H$ to be equal to $-\bar{H}. \partial B/\partial t - \bar{E}.\bar{J}$ and then I have $-\bar{E}. \partial d/\partial t$ okay, so I have three terms on the right hand side, I have two terms on the left hand side but I can actually employ relationship okay, which allows me to simplify this equation into $\nabla.\bar{E}x\bar{H}$ okay, so this equation that I have is $\nabla.\bar{E}x\bar{H}$ so you can simply the left hand side equation by using a vector identity which we would not prove of course.

But $\bar{H}. \partial x\bar{E}-\bar{E}. \partial\bar{H}$ is actually $\partial.\bar{E}x\bar{H}$ okay, so these are the point relationships on the right hand side you have the three terms okay, so term 1+term 2+term 3 the first term is $\bar{H}. \partial\bar{B}. \partial dt$, the second term is $\bar{E}.\bar{J}$, the third term is $\bar{E}. \partial D/\partial t$. Let us look at this term little more carefully the $\bar{E}. \partial\bar{D}/\partial t$ I know the $\bar{D}=\varepsilon\bar{E}$ therefore $\bar{E}$. product with $\partial\bar{D}/\partial t$ can be written as $\bar{E}. \partial\bar{E}/\partial t$ and ε that actually comes out because I am assuming ε to be a constant.

Now this $\bar{E}. \partial\bar{E}/\partial t$ from differential equations or from the differentiation rules will be or you know you can show that this is actually equal to $\partial/\partial t|\bar{E}|^2$ okay, so you can show that this is equal to $|\bar{E}|^2$ because $|\bar{E}|^2$ is $\bar{E}.E$ itself, so you simply take this one and of course it is because it would be twice so you have to put a half factor to this and there is an ε term here, so instead of retaining the ε term as a constant outside I can even put the ε term inside so eventually obtaining for this entire term on the right hand side to write this one as, $1/2\partial/\partial t\ \varepsilon|\bar{E}|^2$ okay. Similarly you can show that $\bar{H}. \partial\bar{B}/\partial t$ can be written as ½ rather this is not $\partial/\partial dt$ this is d/dt or d/dt $\partial/\partial t$ does not matter in this particular case so I have d/dt of $\varepsilon|\bar{H}|^2$ okay.

(Refer Slide Time: 14:45)

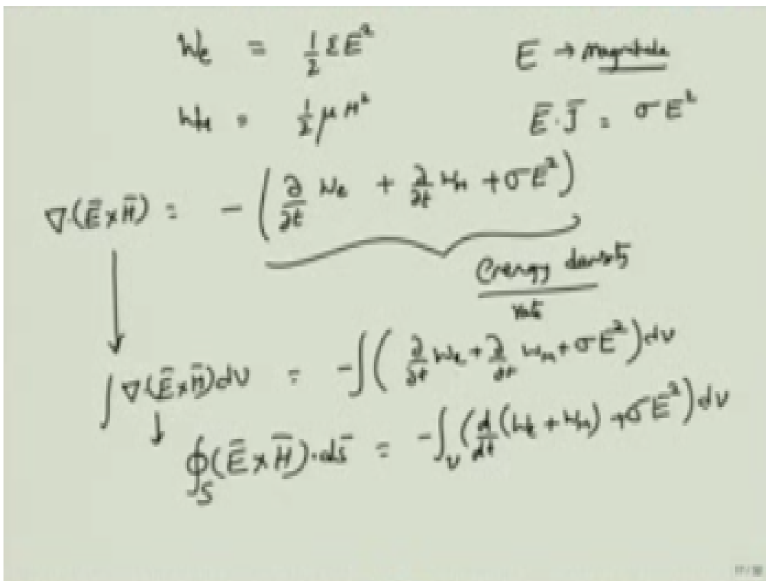


We have not really talked about it but there is a quantity called as electric energy density okay, which tells you how the electric magnetic energy is actually store in the form of an electric field okay, I am strictly speaking applicable only for the static electric field but luckily these concepts are also applicable for the time varying case that we are considering here, okay. In the static case you do know that the electric field you know electric energy density can be written as $1/2\varepsilon E^2$ okay, this $1/2\varepsilon E^2$ you know the intuition to that is the $1/2cv^2$ term and if you know that capacitance is primarily and electro static concept then this $1/2\varepsilon E^2$ kind of make sense.

Similarly an inductor stores an energy of $1/2LI^2$ okay, and the corresponding energy in the magnetic field it stores it in the form of a magnetic field because inductance is primarily a concept associated with the magnetic field or magneto static field and that would be $1/2\mu H^2$, so what you are looking at is the rate of change of electric energy density, energy density because these are defined over a very small volume that we have taken.

And the rate of electric energy density change or the rate of magnetic field density that is the rate of change of electric energy density or the magnetic energy density should tell you about the power that is increasing right so in a closed surface if you say that there is an energy or the rate at which the energy density and the magnetic energy density that is present inside is increasing or decreasing depending on the side for increasing we keep the sine as plus for decreasing we keep the sine as minus okay. So essentially there is some power change or energy density change with respect to time that is happening.

(Refer Slide Time: 16:36)



Now I write w_e as ½ ε e 2 so I have remove that magnitude term I am assuming that when I write e without any vector sine this actually points to magnitude itself okay. So you have ½ ε E2 and similarly the magnetic energy density is ½ μ H2 and the terms that you had d / dt or ∂/ ∂t terms you had your ∂ / ∂t w e + ∂ / ∂t w m of course there is an overall – sine because this is how they were in the right hand side of the equation well we have not forgot in the other term we have e.j but since j is related to electric field in terms of the conductivity so this can be written as σ E2 right.

Because j = σ x E and e is the magnitude on this particular notation therefore e. j will be equal to σ E2 to that again or we will add this term also to the right hand side so you have a +σE2 and these are all the energy density quantities okay these are all the energy density or the rate of change of this energy density vectors what is the left hand side here left hand side is ∇. E x h now I can apply stocks theorem okay to the entire closed surface that we had considered and integrate this out over the volume.

So if I integrate this one I have ∇. E x h okay dv that is the first integration that I have to do this should be equal to - ∇/ ∇t we + ∇ / ∇t w m + σ E2 this entire thing integrated over the volume because the quantity inside is all energy density term multiplied by the volume will give you the total energy so energy density in measured in terms of volume multiplied by the volume itself will give you the total energy I am not yet done as I said I apply stocks theorem to this one and convert integral of ∇ . e x h to dv as integral over the closed surface of the quantity e x h. ds where we consider the surface right this was a surface that we consider and this would be the normal to the surface and this itself had a volume of v over which we are actually integrating on the right hand side.

So this quantity let me remove the minus sin over here or maybe I can keep minus n it does not matter so this should be equal to integral over the minus sin or volume okay then d / dt or ∇/ ∇ t does not matter + w m which tells you how the total electromagnetic energy is changing plus the energy that is actually getting dissipated because of the presence of non perfect conducting material integrated over the volume.

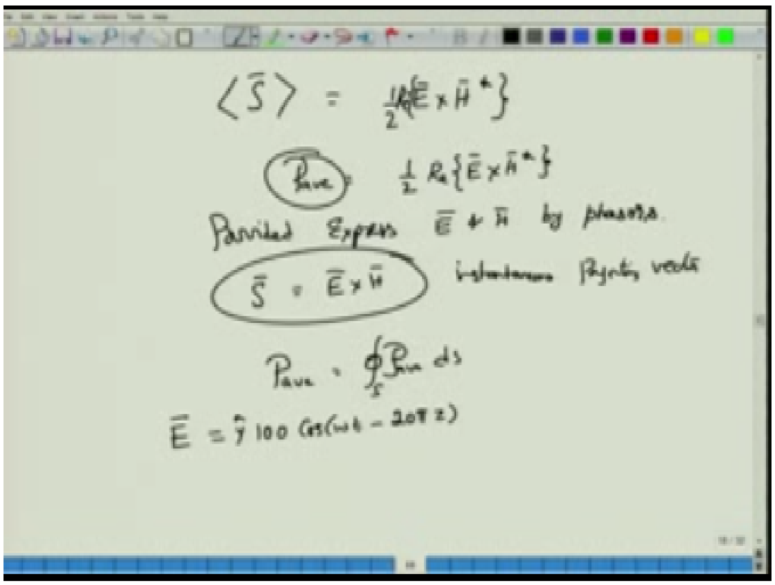
Now look at this right hand side it tells you that in a closed surface in the volume that is enclosed by the close surface the electromagnetic energy is decreasing the rate of electromagnetic energy is negative which means an electromagnetic energy is decreasing with time and there is also you know dissipation in the form of the omit you know loose by this σ E2 by heating of the material so even that power is actually reducing. So if in a close surface if the power or if the energy density is decreasing that obviously something must be coming into it right, some other agency must send that amount of energy, so that the energy actually drains out.

This was like you know inside closed surface charge is melting way but the charge is melting way someone as to put the charges in order to maintain the continuity and pitting the charge means the current was a flowing into it.

So that is the meaning out here, if you change the – sign on to the left and to the right then the – sign on to the left will tell you that this energy is actually coming out okay. so in essence this term E x H is actually describing the way the electromagnetic energy not always but most cases yes, the way the electromagnetic energy is changing okay. You are not at content with having E x H because you do not want that instant energy or E x H will always tell you the instant energy that is flowing.

But you want a time average energy right or you want the time average power, so if you assume that the electric field sinusoidal magnetic field sinusoidal then you can show that.

(Refer Slide Time: 21:22)



The time average pointing vector S defined as E x H, E x H tells you that there is the power out flow right, so this s pointing vector if you take the time average value okay. This will be given by ½ E x H the real part of I want prove this relationship to you, this can be proved easily by actually by calculating the electric field and calculating the magnetic field. So what you actually have is that the time average power will be given by ½ of real part of E x H provides you express.

Electric field and magnetic field by their appropriate phasors, so it is important that you know this is the phasors the actually pointing density that you have E x H when you do not express E and H in terms of the phasors this one is called as the instantaneous okay, pointing vector and this is again at the density only because you go back to this one, the total power flow is given by density times the closed surface area.

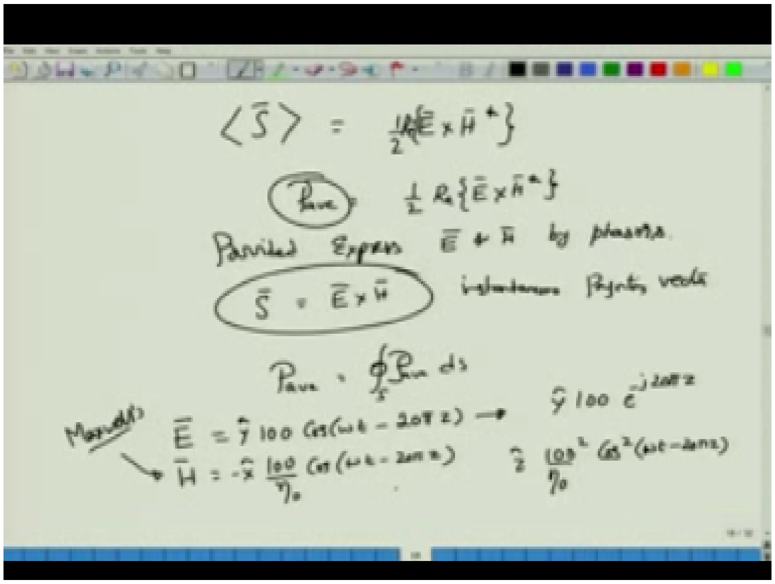
So this is actually power/ m^2 so these are all the power density or pointing power density and this is the power density term okay and average power density are the average power flowing will be given by half of real part of this, if there is you want to x this one by the surface area then you will get the time average. So this is the density that actual average power will be given by integrating this power density over the appropriate closed surface.

So if you apply this pointing vector to the cases that we have already seen, suppose if I have an electric field of the uniform which is propagating along the z axis but it is oriented along the y axis let us say 100, 100 is the aptitude of this one cos ωt – sum 20 πz okay so this is an example where off course I am not specifying the frequency ω but the propagation constant is specified as 20ϕz okay what is the corresponding phase for this the corresponding vector phase will be y hat $100e^{-j20\pi z}$ what will be the corresponding magmatic filed.

For this well you know that e/h should point along this z although we have not talked about that one from Maxwell equations you will know that so by applying Maxwell equation you can show that the magnetic field must be oriented along –x direction. And should have an amplitude of 100/e0 impendence of the medium and this will be in face so it will be cosωt-20πz if you just take e/h you will get a term which is going along the z direction or oriented along the z direction and then you get 100^2/e0cosωt-20πz.

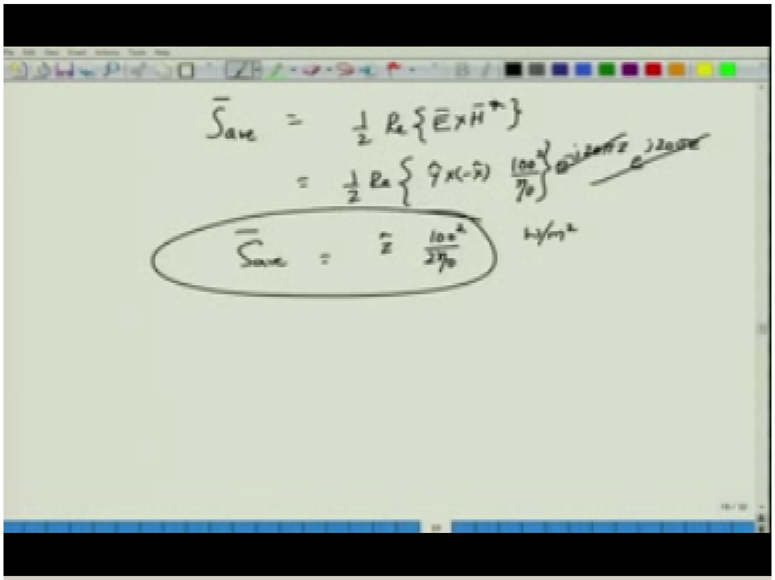
But as I said you are not interested in the instantaneous energy density because you can never measure the instance energy density the optic frequent are changing to fast no detector is able to measure variation of the electric field at that particular time scale of evolution so you are only always measuring in a average power integrated over many cycles and that can be obtained rather easily.

(Refer Slide Time: 25:26)



We express them into the phase form you have already expressed the electric field magnetic field will be along –x direction having an amplitude of 100/e and you will have $e^{-j20\pi z}$ okay.

(Refer Slide Time: 25:43)



Now what is the average pointing vector density is average that is ½ real part of e/h complex conjugate right so this is very easy to find out right so there will be an x sorry there must be a y parser coming because of the electric filed okay cross –x phase coming because of the magnetic field the amplitude gets multiplied so you have 100^2/e0 but what happens to the face electric field will have $e^{-j20\pi z}$ magnetic field will have $e^{-j20\pi z}$ but when you conjugate that one it becomes $e+^{j20\pi z}$ real part is not really I mean no point about that one.

So this would be z 100^2/2e0 w/per2 so this is the average time average pointing vector and this is when we integrate it over the close surface will give you the total power so this completes our discussion of pointing vectors in case of the power calculations we just have to follow the you know equations that I have given here we will have little bit more to say about this is case study that we will consider thank you very much.

Acknowledgement

Ministry of Human Resources & Development

Prof. Satyaki Roy
Co – ordinator, NPTEL IIT Kanpur

NPTEL Team
Sanjay Pal
Ashish Singh
Badal Pradhan
Tapobrata Das

Ram Chandra
Dilip Tripathi
Manoj Shrivastava
Padam Shukla
Sanjay Mishra
Shubham Rawat
Shikha Gupta
K.K Mishra
Aradhana Singh
Sweta
Ashutosh Gairola
Dilip Katiyar
Sharwan
Hari Ram
Bhadra Rao
Puneet Kumar Bajpai
Lalty Dutta
Ajay Kanaujia
Shivendra Kumar Tiwari

an IIT Kanpur Production