

Indian Institute of Technology Kanpur

National Programme on Technology Enhanced Learning (NPTEL)

Course Title

Applied Electromagnetics for Engineers

Module-53

Introduction to Waveguides

by

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Hello and welcome to NPTEL move on applied Electromagnetics for engineers in this module we begin the discussion of wave cuts we have already seen one way of conveying information that it was the transmission line so a transmission line we have seen that also support transverse electromagnetic wave or if we have not seen that we will see that when shortly but a transmission line structure is conventionally different from a waveguide structure because a transmission line requires a pair of conductors whether it was a two wire conductor.

That we talked about or the micro strip that we have not talked about but showed you that or the coaxial cable that we talked about you know any of the transmission line structures are differentiated from the waveguide structures by having two conductors atleast a pair of two protectors you can actually have multiple conductors so that becomes what is called as a multi conductor transmission line something that we won't be talking about it here so why do we need wave guides if you already have transmission lines unfortunately if we have transmission lines the structure of a transmission line is such that if you take a two wire line.

There is a nice separation between the two wires and any of the external electromagnetic fields can actually interfere with that okay moreover the geometry of the transmission line that we consider determines the inductance and the capacitance of these structures as we have already seen right so we have evaluated capacitance of a few transmission line structures we have evaluated inductance of a few transmission line structure and because these inductance and capacitance is eventually you know show a frequency dependent behavior the bandwidth of these structures as the frequency of operation starts.

To increase the response of this drops out and hence they have a bandwidth which is defined depending on what transmission line structure that you are considering the bandwidth could be a few hundreds of megahertz or it could be up to about say three gigahertz or a very designed coaxial cables can work up to 5 to 10 Gigahertz but beyond that if you try to increase the frequency of operation these transmission line structure simply cannot you know convey information because of what happens you know.

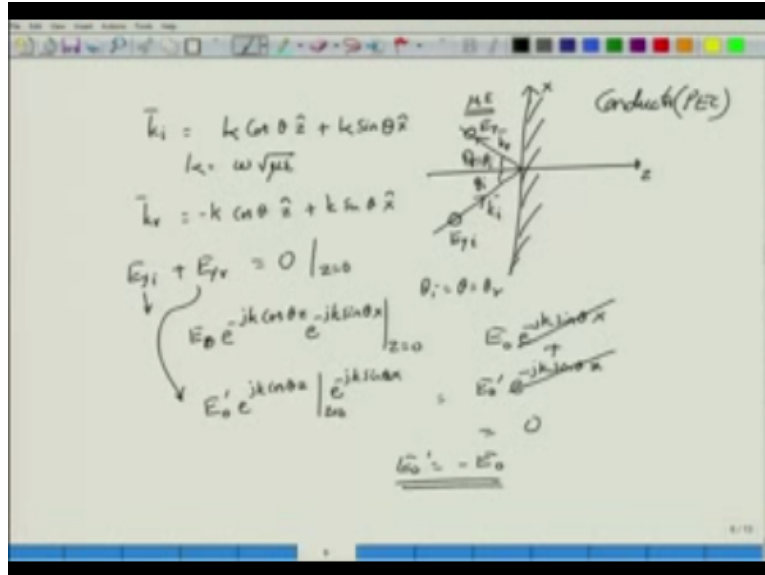
Because of the technical thing is what is called as a higher-order mode propagation that is there will be additional modes which are not present when the frequency is low that these frequencies these additional modes rather start to propagate and they will corrupt the signal or rather bring down the bandwidth okay so you have to redesign this transmission line structure such that you either emphasize those higher-order modes which anyway start to propagate at very high frequencies or you try to eliminate them because eliminating is not possible.

You try to make the best scenario out of this you actually design structures such that for a certain frequency range you know up to a certain frequency range there won't be any propagation but once you start propagating but once you exceed the frequency minimum frequency or the cutoff frequency then multiple modes can actually propagate the advantage here is that you are actually pushing the range of operation anywhere from 10 gigahertz all the way up to hundred gigahertz or slightly more than that in some cases okay.

So what you have is a very interesting you know structure which will be different from the transmission line structure because this structure will have only a single conductor and this single conductor structure as we will see will not be able to support transverse electromagnetic waves okay ,but they will support higher order modes and the advantage is that they go much beyond than what is possible with the simple transmission line structures okay in terms of the frequency of operation so we will look at in this model will just give you a brief idea of what these wave guides are and give you couple of ideas as to how to relate at least one of the wave guides to a transmission line okay and show you what should be the procedure in order to analyze.

This wave guide structures and from the next module we will actually analyze the common wave guide structures okay before we even talk about wave guides here is a problem that we have okay.

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So I have an interface so let us say on the right hand side of this interface is a nice conductor for the moment I will assume it to be a perfect electric conductor which means there is no tangential electric field on the boundary let us also call this as the x-axis and the incident wave propagating along the z-axis I do not want to choose the normal incidence I want to choose a oblique incidents in this case so let me choose an oblique incidence at an angle of θ I okay because the right-hand side is a conductor obviously no electromagnetic wave propagates into the second medium and whatever the incident EMA wave.

That you have would simply be reflected back at the same angle of you know incidence the angle of reflection is the same angle at the angle of refraction the moment you have a obliquely incident light you have a choice between TE and TM mode and I will choose in this particular case to work with TE modes because they are slightly simple to understand the concept without getting too much into mathematical details so for the TEK the electric field will be along the y direction so the incident light will be our incident wave will be along the y direction.

So let us call this as V by I and this K vector you know which is at an angle θ I already know how to decompose this KI vector or the incident KI vector into the magnitude of the vector $K I$ times some $\cos \theta I$ along z axis so there is a third component $+ K I \sin \theta I$ along the X component and obviously this magnitude of k_i is given by whatever the frequency that we have here times

square root of mu epsilon where mu and epsilon are the permeability and permittivity of the medium of incidence over here so this first medium.

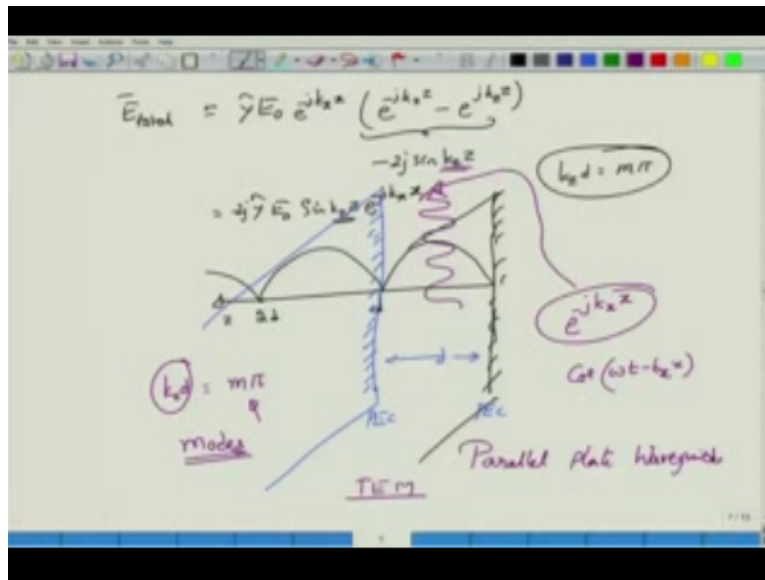
That we are considering since light is reflected or wave is reflected so the reflected wave also is assumed to be in the is polarized along the Y direction itself and the reflected wave vector K_r will have the same magnitude as the incident wave vector obviously because it is in the same medium only thing is its sign along Z direction will change because the wave is now propagating away from the Z direction.

So there will be a minus sign to this one and I have used the same angle θ I here so let me eliminate all these angles and let me also eliminate any you know denoting this one by I so I have to rewrite this one so this would be $K \cos \theta Z$ where $\theta = \theta$ which is also $=\theta$ R and this would be $K \sin \theta Z$ where the magnitude of the K vector is $\Omega \sqrt{\mu \epsilon}$ the reflected one will have the magnitude of K but it would be at a - sign with respect to Z + K sine theta along X okay.

What is the boundary condition well the boundary condition is that the total tangential electric field in the first medium must be +0 that means Y by incident+ EYR reflected must be = 0 there are no tangential electric fields on the second region okay, and what is VY I it will have some magnitude so let us say EY or E0 is the magnitude of this electric field incident electric field the phase factor will be of course this condition will be the boundary condition is actually kept at Z = 0 or actually have to be done at Z =0.

So you have $E_0 e^{i(\omega t - kx \cos \theta - kz)}$ - $E_0 \sin \theta X$ correct so this is the incident one and when you evaluate this one at Z = 0 you will have $E_0 e^{i(\omega t - kx \cos \theta)}$ for the incident wave for the reflected wave the amplitude would be so let us say a_0' is the amplitude of the reflected wave so this would be $a_0' e^{i(\omega t - kx \cos \theta - kz)}$ this of course at Z = 0 there is also a power- $E_0 \sin \theta X$ okay, so this will give you $V_0' e^{-kx \sin \theta}$ the sum of these two should be =0 obviously if this has to be valid for every point in X .This should be = I mean there should not be existing and a_0' must be =- E_0 right.

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So the total electric field in the region one can be written as a field which is y directed because that is the field that we have considered $E_0 e^{j\omega t - jk_x x}$ and instead of writing $k \sin \theta$ I will write this one as k_x where it is understood that k_x is magnitude $k \cos \theta$ and what about the phase factor for the z well you have for the incident field $E_0 e^{j\omega t - jk_z z}$ and for the reflected field $E_0 e^{j\omega t + jk_z z}$ so all the other things are the same except that we have a sign over here now this is nothing but $2j \sin k_z z$ of course.

You see that said you know less than in the in the sense that away from the interface okay so away from the interface the total electric field actually has a very interesting scenario so it will come in interesting expression you have $2j \sin k_z z e^{jk_x x}$ so look at this if you want to sketch the magnitude of the total electric field I would like to sketch that one as a function of z over there at $z=0$ obviously this would be $=0$ and as the value of z you know as you move away from the interface at some distance $k_z z$ into z equal to some distance D .

So if I am sketching this one as a function of $k_z z$ times said at case it or rather if I am sketching this one as a function of said at some distance z such that z equal to D again you might have a situation where $k_z z$ into D will be equal to some multiple of π correct so when this happens when the wave vector along said times the distance D from the interface is equal to $m\pi$ the magnitude of the electric field again goes to 0 or rather the electric field there goes to 0.

So I am sketching the magnitude but the total electric field there goes to 0 it does not stop at this point you can actually continue this one so this 0 happened at D from the interface $Z = 0$ or $-D$ in case you are you know in case you want to be very specific I mean at another distance which let us say is $2D$ which is the multiple of this one the electric field goes to 0 again.

Okay now imagine that I actually have kept one more perfect electric conductor at this point okay so I already had a perfect electric conductor here at $Z = 0$ which was my original interface now I have placed another perfect electric conductor at this point where the distance between the two perfect electric conductors is now D right, nowhere is a very interesting question does the electric field realize that there is been a second perfect electric conductor the electric field does not even realize that there is a second perfect electric conductor.

Because any rate would have gone down to 0 there the electric field would have gone to 0 there just because we placed a perfect electric conductor means that we have not changed the boundary condition at all and it does not have to be that you have to place this perfect electric conductor at D we can place it at $2d$, $3d$, $4d$ whatever so wherever those integral multiples of D that you place provided that you keep all the other things constant that is the angle θ to be constant then the electric field and the corresponding magnetic fields automatically satisfy the boundary condition okay, and in case you also make instead of these two wires.

If you make them 2 plates okay, so if you make these 2 as 2 plates which extend all the way along the x direction and along the Y direction but then they are separate by a distance D along the z axis then you will realize it even more interesting scenario or even more interesting thing that the wave actually shows oscillations along the X direction there is there is a phase factor $e^{-jk_x x}$ which when you go back and write on the real I know $\cos(k_x x - \omega t)$ kind of a thing you will see that this would be $\cos(\omega t - k_x x)$ right so is not it amazing that the wave although was incident.

And then got reflected the resulting wave is actually transmitted or is it guy it is guided or it is propagating along the x axis that is along the interface direction this is actually you know propagating so this structure supports propagation of the electric fields and of course the magnetic fields which we have not written here but it does support the magnetic field as well when you analyze it in the TE case or TM case you will see that just by having two parallel plates which are separate there will be wave propagation inside.

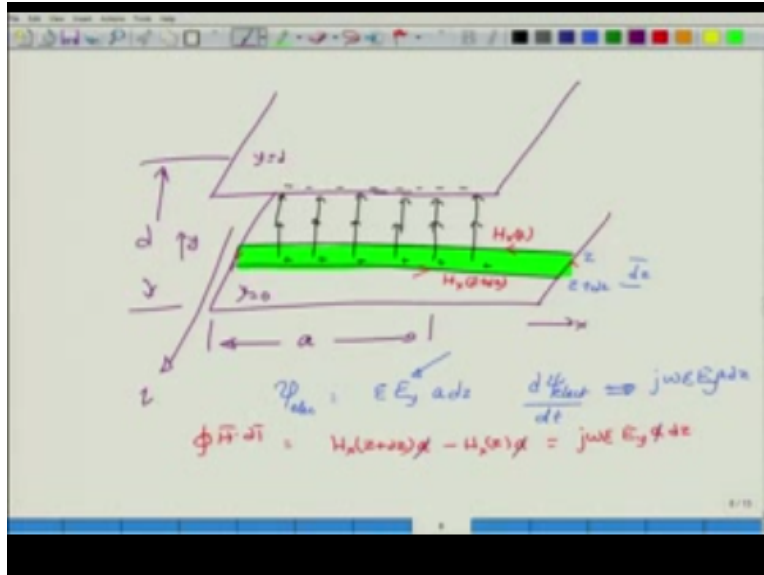
And this wave propagation of course depends on the angle of incidence and as the angle of incidence changes there will be some situations where the electric field automatically goes to 0 but there are other situations where it does not go to 0 then it does not go to 0 the field would definitely see a different you know try to it will be different so sine kz said will be the way the field is actually going back but then if the second perfect relative conduct is not placed to the appropriate position then it won't be able to satisfy the boundary condition and the condition that there is a PC on the same on that particular distance.

So because of that for those angles θ there won't be any propagation but when you have the angle θ adjusted such that there is a boundary condition satisfied at the position where you keep the second perfect electric conductor then that particular angle of incident slight incident or the electromagnetic wave incident at that angle of incidence actually propagates along the interface or it is guided by the two parallel plates okay this is an example, of what is called as a parallel plate they have guide okay because it allows light of different modes and these modes are the ones which satisfy this angle.

So all those angles kz said times B or you know the kz said times D must satisfy the condition that it must be equal to some odd multiple or even multiple of ω right so that the sine function the sine of kz automatically satisfies the boundary conditions from the said perfect electric conductor those angles will be pasted at discrete values there won't be continuous angles so those angles which you know satisfy this condition will also allow propagation of electromagnetic waves incident at those angles and those angles are those electromagnetic waves at those angles are called as modes okay.

So we say that parallel plane waveguide supports multiple modes the lowest order mode turns out to be what is called as P E M which is what I would like to talk about in the remainder of this module okay.

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This gives you a nice different point of view let us go back to the parallel plate waveguide but this time I will change the coordinate conditions because it is common to consider propagation along Z direction rather than propagation along X direction so this time I considered two plates which go all the way to infinity or extend all the way to infinity as well as infinity on two sides one along the x direction along the Y direction okay and one along the X direction and along the Z direction they are separated along the Y direction so you actually have a separation along the Y direction by an amount of D okay.

So let us just put an amount of D as a separation and these are the two parallel plane wave guide that I am considering okay so these are the two waveguides that we have the separation between the two is D along the Y plane so this might be equal to $y = 0$ $y = d$ do not worry about Y whether it is $y = 0$ here $y = D$ there or y equal to 0 on the top plate and $y = 0$ on the bottom plate minus t on the bottom plate okay so this axis is along the y axis that we have considered this is the x axis okay because along x axis this would be infinite.

It would rather be difficult for us to treat this infinite concepts okay, so what we do is we only pick a portion a along the x-direction and work with that finite length portion okay, we do not really lose anything because of the fact that this is infinite I can just pickoff a particular region and see what is the electric magnetic fields and everything per unit length and then I can work back.

To that okay along said I have so this let us say is there is a direction that I consider so along Z is the direction in which the waves are supposed to be propagating well what kind of waves are supposed to be propagating first of all look at this thing so because these two plates are kept at two you know like they are kept at a distance and if there is a wave which direction should the wave be present well if I pick a small strip here okay ,so I pick a strip over here the electric field because of the fact that this is a perfect conductor.

The $y = 0$ is a perfect conductor the electric field must be directed from one conductor to another conductor if I arbitrarily choose that this conductor from which the electric field originates as the bottom conductor then I have the electric field to be uniform and originating from the bottom conductor and reaching all the way to the top conductor so equivalently what I you know have kind of assumed is that there are these positive charges in this strip down here and there are these negative charges on the top plate.

So essentially we have a positive charge here negative charge here so that the electric field lines are along the Y direction and they would be going into going from one plate to the other plate okay now if I consider the same strip here okay so if I consider this strip and ask you what is the total flux that is enclosed by this strip which actually is at Z and Z +DZ that is the strip has a width of DZ and a length of a what is the total amount of electric flux that is enclosed by this the amount of electric flux let us call this as ϕ Electric is = epsilon E Y the strip width is a into DZ okay why should it be epsilon Y.

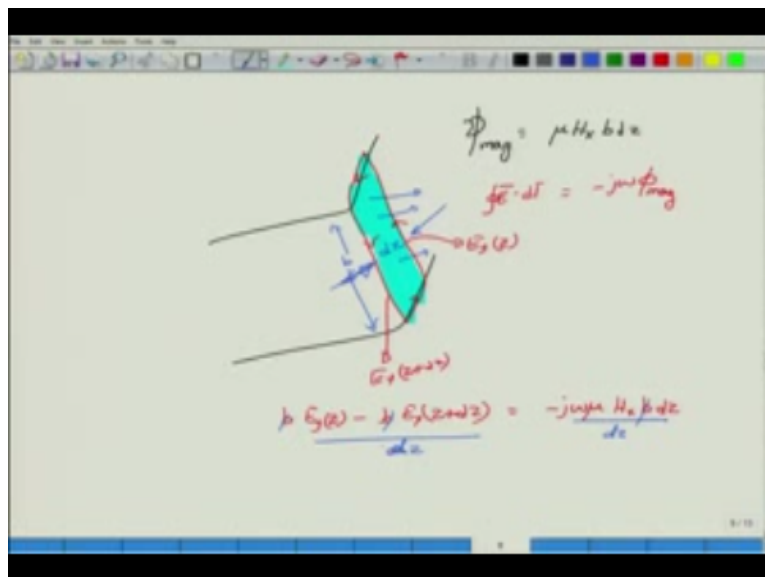
Why because the electric field lines are uniform and originating along I mean and oriented along the Y direction epsilon times electric field will be their flux density flux density times the area of this green strip will be A into d and this is the total electric flux density of which if I now assume EY to be time varying then the displacement current which is dying d 5 electric flux by DT must be in the phasor form be equal to $J \omega \epsilon E Y A$ D said okay so this is the total displacement current that is because of these arrows the black arrows that we have written these arrows are varying with respect to time only in magnitude the orientation is always along y direction okay now on the same strip if I apply amperes law here.

Okay so by going through this particular part okay so what do I get along this path the magnetic field is H and let us say it is oriented along the X direction so this is H X of Z + DZ and this would be HX of Z itself so you can look at the way the fields and other things are oriented and

you can show that when you apply the integral $\oint \mathbf{H} \cdot d\mathbf{l}$ right which must then be equal to the displacement current as we will show I mean as we know that one so if you apply this integral of $\mathbf{H} \cdot d\mathbf{l}$ and you will get H_x at $Z+DZ$ times A that is because there is the length of the strip.

The perpendicular direction there is a long said there is no H component okay we have assumed that electric field is along Y and magnetic field is only along X component so you have H_x set plus DZ a- H_x of Z times A this must be $= J \omega \epsilon \int \mathbf{E} \cdot d\mathbf{S}$ it obviously a cancels out on all on the right-hand side and the left-hand side and it must because we have as we said we are going to consider a strip of a but our results will be independent of this a here okay so at this point the results will be independent of a now let me consider a different kind of a strip.

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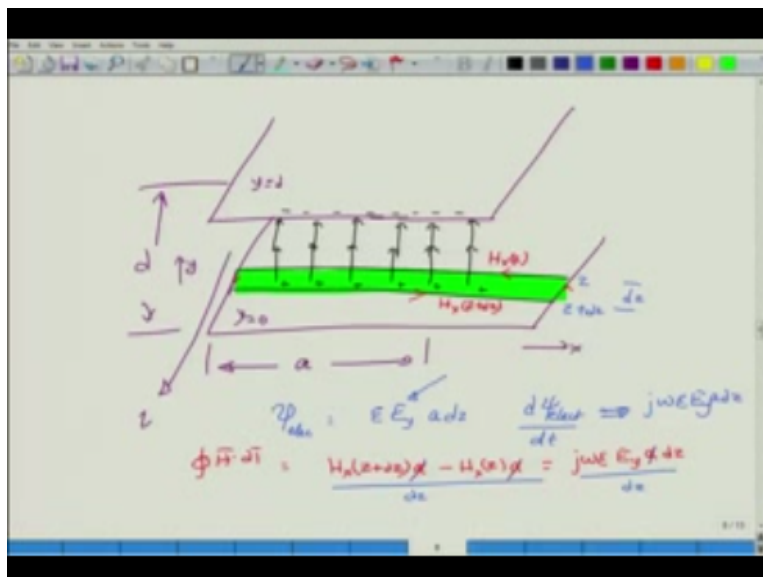
Okay I have a parallel plate waveguide okay so I have a parallel plane waveguide coming off like this but this time I consider a strip so let me consider the strip here and this time the strip that I consider will be like this okay so this is the strip that I am going to consider the width of this strip will be again it should be in this direction so the width of this strip is DZ it but the height of this trip will be b okay so what is the total magnetic flux coming out of this so H is along the X direction.

So what are the total magnetic flux coming around so let us call this as ϕ_{mag} and this magnetic flux will be μH_x which is the magnetic flux density B vector times b DZ will be the width of this strip so it will be B times DZ okay and now if I apply appropriately oriented loop here so if I

apply the appropriately oriented loop around this particular loop if I apply integral of $\mathbf{E} \cdot d\mathbf{L}$ that must be $= - \frac{d\phi}{dt}$ magnet ϕ mac right so this would be the total you know the magnetic or the magnetic flux and then the rate of change of magnetic flux would be the EMF induced over this closed Count to E of course E on this path will be evaluated at $Z + \Delta Z$ whereas on.

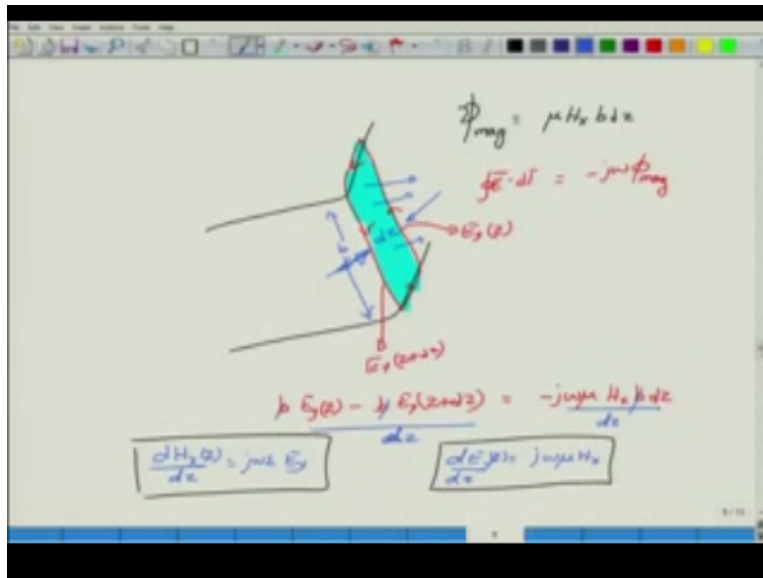
This path must be evaluated as Z itself okay so when you apply this one so you can see that this would be B times e Y of $Z - B$ times e V of $Z + \Delta Z$ this must be $= - \frac{d\phi}{dt}$ okay again the equation will be independent of B here okay so you can remove B from both sides and now if you divide both sides by ΔZ or rather DZ and let this DZ go to I know whatever the 0 there the left hand side will be a differential of any Y with respect to z and.

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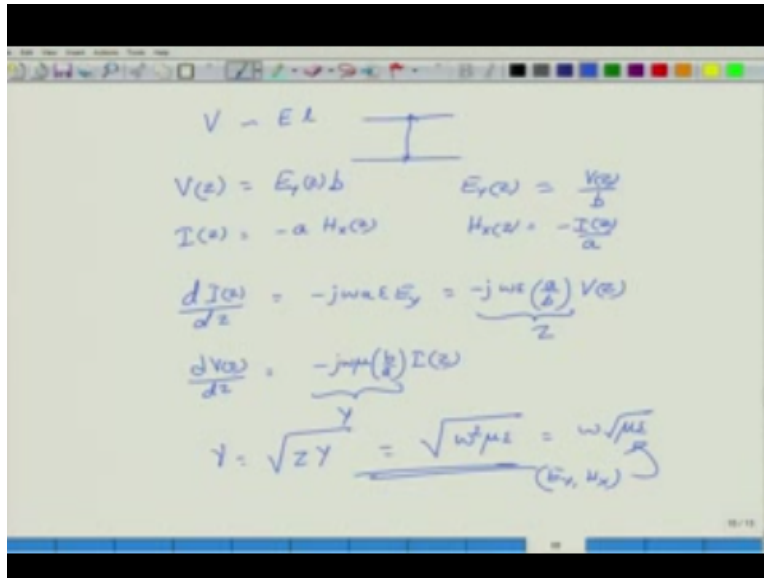
Similarly if you differentiate on both sides by or divide both sides by DZ and then let DZ go to zero the left hand side will be $\frac{dE}{dz}$ checks off Z by DZ right.

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So if I do this one I obtain two equations so one will be $D H X$ offset divided by $DZ = J \omega$ epsilon DY and similarly I obtained $D DY$ by DZ okay to be $= J \omega \mu H X$ so these equations are the equations that govern how the electric field and the magnetic field along this particular parallel plate wave guide would change okay.

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$$V = EL$$

$$V(z) = E_y(z)b \quad E_y(z) = \frac{V(z)}{b}$$

$$I(z) = -a H_x(z) \quad H_x(z) = -\frac{I(z)}{a}$$

$$\frac{dI(z)}{dz} = -j\omega\epsilon E_y = -j\omega\epsilon \left(\frac{a}{b}\right) V(z)$$

$$\frac{dV(z)}{dz} = -j\omega\mu \left(\frac{b}{a}\right) I(z)$$

$$\gamma = \sqrt{ZY} = \sqrt{j\omega\mu\epsilon} = \omega\sqrt{\mu\epsilon}$$

Just one last thing we know that voltage is electric field times the length along which you are actually calculating the voltage so if you pick the lower and the upper four plates and then pick a particular value of Z then the corresponding potential difference at the point said between the lower and the upper plates will actually be equal to the electric field that would exist at that particular point times the difference between the two plates so this is volt per meter into meter therefore that would essentially be the voltage difference similarly the current from amperes law earlier.

We have seen that the current here at a particular Z where desert can be thought of vanishingly going to 0 be = - A H X offset okay ,so because I know voltage, I know current, I can go and replace in terms of V Y I mean replace if I said in terms of V that would be the offset by B similarly I replace HX of Z as- I of Z by a when I do that one I obtain two equations so one equation will be D offset by DZ which tells you how the current is changing okay and this is = - J ω a epsilon times DY but that is actually = - J ω epsilon a by B D of Z.

And then you have DV of Z by DZ = - J ω new D by a times I have said okay, so these equations remind you of the factor the mind you of the form that this is some Y and this is some Z you know the impedance series impedance it and this is the series said or the shunt admittance Y therefore the propagation constant of these two voltages and the currents will be = square root of Z into Y.

And that would be equal to in this case because a by B cancels out with B by a - J and - J would I mean if I do not even have to consider the - J and - J let me just consider the absolute value over here so $\omega^2 \mu \epsilon$ is what you will get so this would be square root of $\omega^2 \mu \epsilon$ which is ω square root $\mu \epsilon$ so you see that your field components V_Y and H_X both propagate with a velocity given by $1/\sqrt{\mu \epsilon}$ and they have a lossless propagation of propagation constant ω square root $\mu \epsilon$ okay, in fact I would also ask you to find out what is the characteristic impedance.

There you will see that the characteristic impedance also depends on the ratio of B by a okay, I mean in it won't happen in γ but it does happen in the propagation for a characteristic impedance of this particular thing what I would like to conclude is that a parallel plate waveguide is a very special type of a waveguide okay it can support transverse electromagnetic waves where the electric field is along Y depends only on Z the magnetic field is along X depends only on Z okay and this TEM wave propagates with a velocity that is independent of the frequency.

But as the frequency is increased okay then it is possible that the same transmission I means the same parallel plane wave guide actually supports higher order modes okay and these higher order modes are called as the TE higher order modes or TM higher order modes depending on this so-called oblique wave incidence analysis that we have done granted in both viewpoints we have not really looked at how the fields for the Highmore's actually exist and in order to do that when we have to solve Maxwell's equations and then apply appropriate boundary conditions.

Which is what we are going to do in the next module but the point is that the transmission line structure and the equations that we wrote are actually relatable to the electromagnetic wave propagation that we have discussed and that is the key point okay but when the frequency increases the parallel plane waveguide structure or the panel plate structure cannot just support TEM but higher order modes in fact that is the point where we say that the parallel wire or the parallel plate wave grant has lived to its utility and we do not really consider it to be useful when higher order modes start to propagate.

Okay because there will be excessive loss out there okay to overcome this is what we go for a different type of a waveguide a proper waveguide and the example that we will be looking at in the next class is that of the rectangular waveguide thank you very much.

Acknowledgement

Ministry of Human Resources & Development

Prof. Satyaki Roy
Co – ordinator, NPTEL IIT Kanpur

NPTEL Team
Sanjay Pal
Ashish Singh
Badal Pradhan
Tapobrata Das
Ram Chandra
Dilip Tripathi
Manoj Shrivastava
Padam Shukla
Sanjay Mishra
Shubham Rawat
Shikha Gupta
K.K Mishra
Aradhana Singh
Sweta
Ashutosh Gairola
Dilip Katiyar
Sharwan
Hari Ram
Bhadra Rao
Puneet Kumar Bajpai
Lalty Dutta
Ajay Kanaujia
Shivendra Kumar Tiwari

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