Indian Institute of Technology Kanpur

National Programme on Technology Enhanced Learning (NPTEL)

Course Title
Applied Electromagnetics for Engineers

Module –54
Rectangular waveguides

by
Prof. Pradeep Kumar K
Dept of Electrical Engineering
Indian Institute of Technology Kanpur

Hello and welcome to NPTEL mook on applied electromagnetic for engineers. In this module we will study what is called as rectangular waveguides as we have already discussed wave guides offer to work at a much higher frequency and also have a larger bandwidth than a simple two wire like transmission line structures. The difference between a transmission line and a waveguide also we have emphasized so if you want to imagine how a rectangular wave guide would look.

You can actually look at this you know duster, so you see here that this is actually a cubicle or rather this is actually having a cross section which is rectangular okay, it may not here it is a rectangular cross section this is the height of the structure and this is the width of the structure in our usual rectangular waveguides we would find that the width is actually larger than the height of the structures and what you have to also notice is that there is a uniform cross section along the z axis.

This is the direction where we assume that the waves are propagating okay and this is actually a very good model because you see this entire material the so called waveguide is actually made out of a thing this is not a waveguide is a duster but anyway so the entire thing is actually made out of a single material except for this no dusting portion, if you just remove this one then everything is just you know in just a wood material.

This is filled with the same wood material but in practice you would see these wave guides to be just air filled air field is another fancy word for saying that they would be Hollow okay. So you will have an x axis you will, so you will have an x axis you will have a y axis and then you will have a z axis along which the wave is supposed to propagate but please remember this entire thing is actually made out of metal okay.

So this is this is the model of a waveguide that we are going to analyze and I have already pointed out if I were to cut this duster at various places which is, now like cutting a waveguide at various places and examine the cross section the cross section actually remains the same. So that is the reason why we call this as uniform waveguides okay, so how do we go about analyzing such rectangular waveguides or for the matter. If I do not consider the rectangular waveguide I considered a circular cross-section then I will be considering what is called a circular waveguides right.

So how do I analyze a circular waveguide or a metallic rectangular waveguide for whatever you know to find out the fields and find out when, what is the operating range and all the other things right. There is in general a nice systematic way of doing this the systematic way of doing this is to begin by separating the fields into two parts one part would be what is called as the exile part which is the way in which the which is the field component along the direction of propagation.

So in this case either and head said for a rectangular waveguide would correspond to the exile or the longitudinal components. Then you have the other components which are transverse to the direction of propagation direction of propagation is Z, so transverse to the Z would mean X and y direction. So the fields ex e / hx and hy are called as the transverse components and usually it turns out that these metallic wave guides can be analyzed by first expressing all the transverse components in terms of the longitudinal components.

Solve for the longitudinal component apply boundary condition and from there once you have the longitudinal components known to you as a function of x, y, z and time okay then you can go back and find out the other components either by Maxwell's equations or by a relationship of transverse -longitudinal components okay. Unfortunately we will be not having enough time to develop this method of prone analyzing wave guides in complete detail.

So I will be skipping over a few details leaving them as exercises for you they are not very hard they just take up a little bit of time because they involve lot of algebraic manipulations okay. But the four part methodology is what we are going to follow okay, so we will first begin by looking at the Maxwell's equations themselves that is the zero part you might say and then the first part would be to express the transverse components in terms of the longitudinal components okay and then you up to solve the longitudinal component expressions for the longitudinal components must be obtained.

Appropriate boundary conditions will be applied and once you have done that solution it is easy to go back and find out the full solution because transverse components are already known in terms of the longitudinal components okay. So we begin by recapitulating Maxwell is equations or maybe even before that I can first give you the cross section that I am going to consider.

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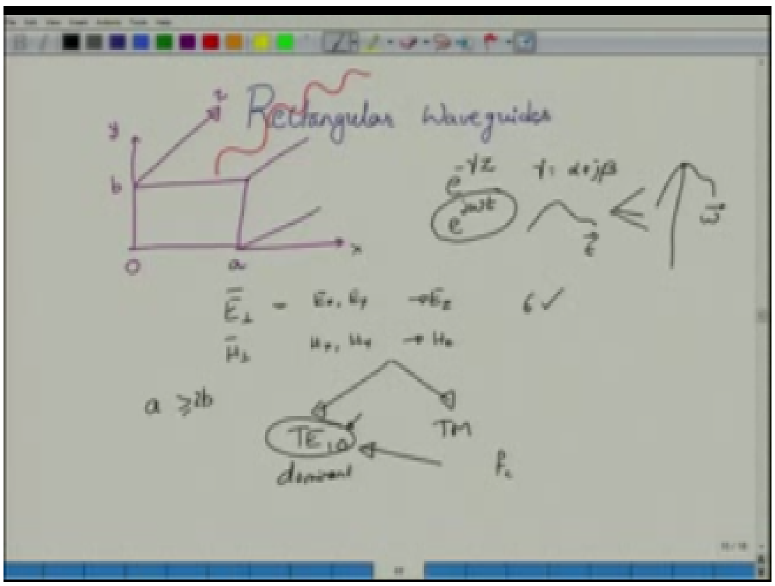

This is the cross section of the waveguide that I am going to consider okay, so here I have not drawn the cross section I am trying to draw the 3d picture of this, so I have 1 axis let us say axis x which along which we have the width of the waveguide, so it is bounded by x = zero and x = a and y along this axis is the y axis which is from 0 to B and this one would be the direction of propagation okay. So it could be either this direction or because of the right hand rule then this would be the actual direction I mean this would be the conventional direction.

But the wave is assumed to propagate along the waveguide itself, so in this case the wave is propagating along - z direction or you can just redefine sent and it does not really matter what you do alternatively you can interchange the x axis, so you can consider x = 0and x = - a and then you will obtain the conventional z direction of propagation okay. So do not worry about what that z axis is just look at the fact that this entire thing is actually made out of a single conductor and inside this waveguide there is actually nothing it is an air-filled waveguide as we would call it okay.

First step would be to recapitalize Maxwell's equations and I am going to do that one okay with little bit of changes in the sense that I know that the waves have to propagate along the z axis therefore along z as a function of z we would like all the field components to have a form which is exponential of - γ z where γ is the complex propagation constant in general okay. If we consider loss less waveguide then γ will be pure imaginary being = J β in general it will also have some amount of losses.

I mean the waveguide has some amount of losses because of the imperfect dielectric or imperfect conducting material, so there will be some non zero value of α. Whatever that is all the field components are assumed to have e to the power - γz dependence on the z coordinate. We will also assume that all field components are being evaluated at a particular frequency ω okay, the reason for that one is that any given you know function of time which is reasonable function can always be split into its Fourier components or different frequency components.

In other words you can actually find out the corresponding spectrum and if you know how each spectral component propagates through the waveguide then it is just an easy matter to actually put propagate each of these frequencies and then put them back together in order to obtain the way in which the originally pulse like function or some general function of time would propagate okay. So this is the reason why we initially started with the phase assumption that we will be looking at only one frequency and then expressing all the quantities as phases and that is what we are going to do okay.

So from now onwards we will not really carry over the C J ω T because all field components are phasers okay all field components in terms of z will have $e^{-\gamma Z}$ okay. For the rectangular waveguide that we have assumed, so let me remove this one so let me just remove this axis for otherwise it might be a bit confusing okay. So for this rectangular waveguide the perpendicular or the transverse components will be the components E_x and E_y whereas the longitudinal component will be the z component longitudinal or exile component for the electric field of course.

For the magnetic field you will have Hx Hy and Hz in general in a waveguide all six components are nonzero okay all 6 E_x E_y E_z Hx Hy and Hz are non zero. It is possible for us to separate out the kind of fields that you are going to see inside the waveguide into two groups, one group is called as the TE group or the TE modes we will define what a mode is shortly and then the other mode or the other group is the TM mode solutions okay.

This TE and TM breakup actually comes just by the simple thing of you know an oblique incident light can be broken up in terms of its TE and TM components individually you know you know how to you know reflect off a TE component TM component. So if I have an initial polarization which is a combination of TE and TM break it up see how there you know reflect from a given interface and then add them up together.

So that is essentially what we have been doing and that is true for the rectangular waveguides as well. So you can break it up into TE and TM and it turns out that as long as the width A is larger than the height B, usually about 2 B or something then the first group of waves that would propagate inside the waveguide turns out to be the TE modes. So practically speaking the TE modes are the ones which are called as the fundamental or dominant modes not every TE but there is a dominant mode called TE 1, 0 okay.

Where we will also discuss what we mean by TM know the 1 and 0 subscripts that we have put in but this is called as the dominant mode because as the frequency starts to increase and reaches beyond a certain critical frequency this mode TE 1 0 mode is the first one to begin propagation okay. So we will focus our attention onto the TE modes I will not really look at the TM modes because the analysis is kind of very similar you will do that as part of the exercises okay.

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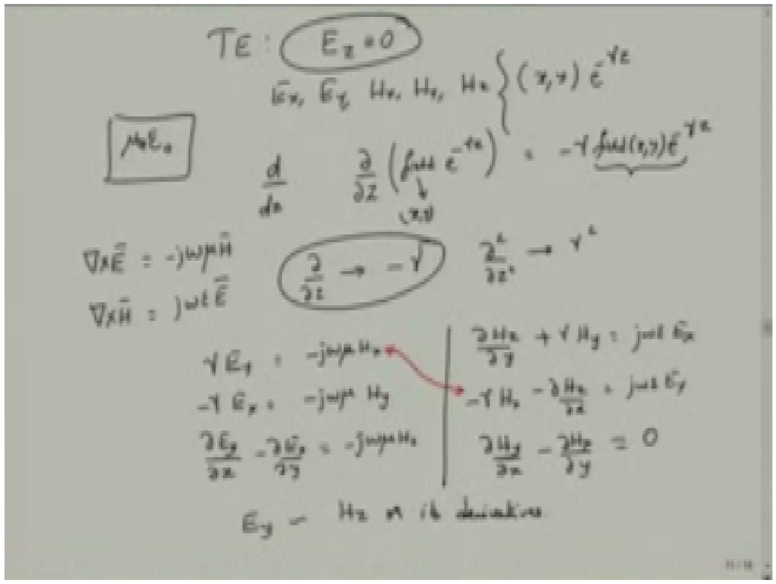

So what do we do with the TE modes well in the TE modes the characterization is that the electric field component Ez would be = 0 okay. So that is the reason why we would call as TE mores it is all transfers the electric field is completely transfers to the direction of propagation transfers meaning it is having components only in x and y that is it has components only of x and y. So Ez = 0 anyway so that is the transverse electric mode, so what components are remaining now?

So you have you have Ex, Ey you have H_X Hy and H_Z all these quantities will be functions of x and y but all of them will be functions of z in just a single manner that is of e power - γ said okay. Now you can actually look at an additional you know result because of the dependence of e power - γ said what would be d by dz or let us say the partial derivative del by ∇ Z of any field quantity whose z dependence will be in the form of $e^{-\gamma Z}$ and the field dependencies of x and y.

Well this is a function of x and y therefore this would not change anyway but because ∇ / ∇ z of $e^{-\gamma Z}$ would simply pull out - γ leave the other things as it is field of x,y $e^{-\gamma Z}$. So this is the original field component that we looked at as a function of x y & z but because that by ∇ said is there it would pull out - γ and you get - γ field of this quantity okay. So I can replace wherever in the curl operations that I get or in any of the other operations ∇/∇z with – γ.

I can also replace $\nabla^2 / \nabla z^2$ with γ^2 obviously because γ will be pulled out twice from the differential so that would be - γ into – γ, so that would become γ^2 okay. With this setting let us write down the curl equation, so I have ∇ cross e is = - J ω μ x H okay. So this is the point form of Faraday's law that we have written and then the second curl equation is Del cross H will be = J ω ε e okay.

Why because there is no current inside the hollow medium there is no wire which is actually carrying a current or there is nothing like a conduction current present inside whatever current that is there inside the rectangular waveguide that has to be displacement current and displacement current is ε ∇ e /∇ t ∇/ ∇ T is in phaser or notation J ω. So that is the reason why you have J ω ε times.

So the waveguide is filled with mu and ε initially we have assumed that these you know can be air or can be anything else but whatever it is essentially something that would not support any conduction current ideal and dielectrics is what we have assumed. Usually as I said it would be μ_0 and ε_0 but let us be general and then say it could be any μ and ε it could be filled with some class or something else but as long as they have a constant value of mu and ε and these do not have any value of σ this theory that we are going to develop will be all right okay.
Otherwise you will have to make a little bit of modifications okay write down the curl equations for E and curl equations for head separately all the time realizing that ∂ / ∂ Z can be written as –γ. So if you do that and also realize that ez is equal to zero you get two groups of equations so you get γ ey when I write e Y I obviously mean that it is a function of both x and y as well it is not a function of said because well the set dependency is - γ we have already taken it out right.

So this would be equal to minus J ω μ HX okay then you have –γ e x - J ω μ H Y then you have ∂ e Y / ∂ X well unfortunately I cannot simplify this because I do not know how ey behaves with respect to X nor I know how e X behaves with respect to Y therefore this I cannot remove. So I will just keep it as it is I have a second group of equations coming from ∂ cross H that would be ∂ HZ / ∂ Y+ γH Y = J ω ε e X then you have -γ H X - ∂ HZ / ∂ x = j ω ε EY finally I have ∂ H Y / ∂ X I do not know how they behave with respect to X or with respect to Y.

So I will just keep them as it is but luckily this is the equation now these equations are all mixed in terms of EY EX HX HY and HZ what I would like to do is to express EY purely in terms of HZ or it is derivatives and I can do that in order to do that when I just have to combine a couple of equations. I know how to express a y in terms of HX from this expression correct and this expression here has HX on the right hand side it has a component HX there is an e by here if I write down e Y in terms of head or other HX in terms of e Y substitute into this second equation that two equations are linked kind of a thing right so substitute into one then I will obtain YY in terms of HX.

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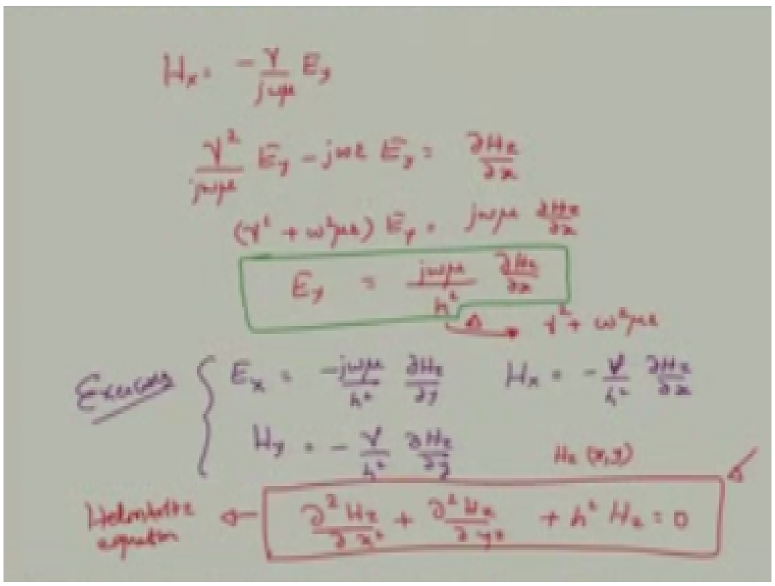

So if I do that what do I get I already know how to express HX so I will have minus γ / j $\omega\mu$ so HX is actually this so if you are not convinced you can look at this one HX = - γ / J ω μ x correct I go and substitute that into the other expression so substituting this into the other expression which I have marked will give you γ^2 / J ω μ ey – j ω ε times ey = ∂ hz / ∂x I can take a as a common factor out so I will get γ^2 + ω^2 $\mu\varepsilon$ x ey = j $\omega\mu$ ∂hz / ∂x or finally ey = J $\omega\mu$/ h^2 ∂hz / ∂x where H square is what we have defined this quantity γ^2 + ω2 as okay.

So I have defined this as H^2 so this is by definition so what I have done is to start with the Maxwell's equations and somehow be able to express all the or at least I have shown you how to do it for one case I will leave the other expressions for you to find out so how do I represent the transverse components in terms of the longitudinal components okay you can then show I will leave these as exercises for you can show that e X can be written as- J ω μ / h^2 ∂ hz / ∂y please note that electric field ey will have ∂hz / ∂x electric field e X will be ∂ hz / ∂y.

Similarly H X will be equal to – γ / h^2 ∂hz / ∂x / and then finally you have h y – γ / H^2 ∂ hz / ∂ Y okay so I will leave these as exercises for you to show that again you just have to combine a few equations and you will be able to find this out okay. So at least our problem is simple right I just need to solve for headset so if I only need to solve for hz or what I can do is to do so equation 3 you know on this one I can actually differentiate this one with respect to X right so after substituting for the equation e Y and substituting the equation for e X from the ones that we already have I can differentiate this one with respect to X and differentiate this one with respect to Y and put the two solutions together then what do I obtain I obtain ∂^2 Hz / ∂ X^2 + ∂^2 Hz / ∂ Y^2+ H^2 hz = 0 okay.

So all I have to do is go back to this equation and then substitute for e Y I know ey is equal to J Ω μ by ∂ X I mean H square I do not have differentiate I just have to substitute for e Y and then substitute for e X into these expressions because ey will be J Ω μ by H^{2x} ∂ hz / ∂ X there is already a∂ by ∂ X over here so this fellow will become J Ω μ /H^2 ∂^2 hz / ∂ X^2 okay. Similarly you can show what will happen to this one combine them pull them together and you will be able to find this okay leave this also as an exercise incidentally this equation is called as Helmholtz equation okay.

So what we have done is to derive an equation for headset in these equations headset is actually a function of x and y okay so we do not really put the function of Z because we already know how it would go as a function of Z now how do I solve this equation having met this equation earlier yes we have met this equation earlier if you remember in the Laplace's equation solution right we had solved equations which were similar to this so you had ∂^2 e / or ∂^2 V / ∂ X^2 + ∂^2 V / ∂ Y^2 equal to some term in the poisons equation that was equal to some right hand side equal term which was a source term.

And we applied a particular method called as variable separable method we in fact did this for that infinite squared rough problem right so we did this problem of variable separable where we showed if you have a single component whether it is the voltage V or HZ which satisfies this particular you know type of an equation partial differential equation then you can write that component as product of two functions one of which is function of X alone and the other one is a function of Y alone okay.

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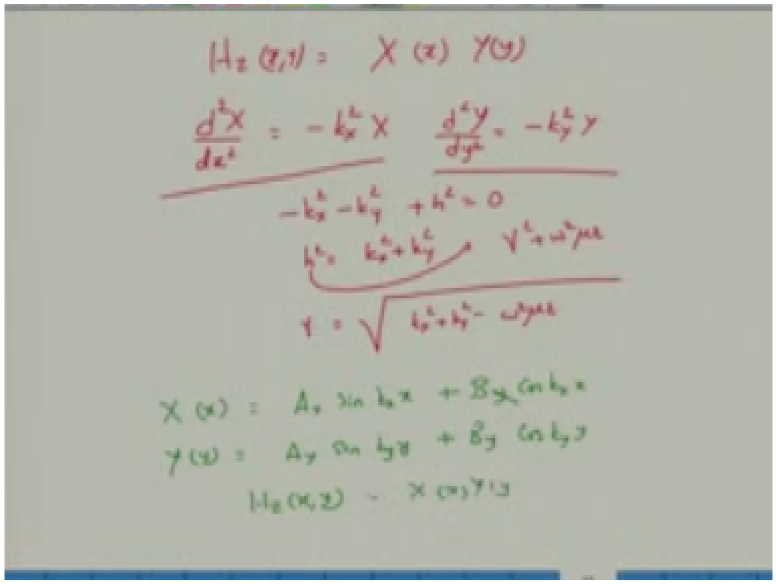

So what we mean there is that HZ as a function of x and y can be written as X of x and y of Y and you can substitute these expressions into the Helmholtz equation and then obtain the resulting differential equation second order differential equations ok when you do that you will get d ^{2} X / D X ^{2} equals some minus KX ^{2} X and d ^{2} Y / dy2 equals some minus KY^2 Y okay such that the constraints - KX^2 – KY^2 +h ^{2} should be equal to 0 or H ^{2} = KX ^{2} + KY^2 since H ^{2} is nothing but γ^2 + Ω^2 μ∂ right so I can write this as γ^2 equals or γ equals the complex propagation γ is equal to √ KX^2 + KY^2 - Ω ^{2} and of course I still do not know what is KX and KY.

But I will be able to find that one out by solving these equations individually now these are simple second order differential equations whose solutions I know X of X will be some a X sine KX of X plus some B Y or rather be X cos KX of X because of the second-order solution which is this one similarly Y of Y will be a Y here it is the same thing except that KX is replaced by KY and it X replaced by Y plus you have dy cos K Y into y the full solution for head side as a function of x and y is given by X of X into y of Y okay.

Now this we have completed is in this second part now what do we do well we need to apply boundary conditions so what sort of boundary conditions we should apply we have boundary conditions for the magnetic field but the magnetic field tangential component boundary condition means that I have to know what is the surface current since I don't know the surface current and surface currents can exist no current can exist in the hollow region but on the surface currents can exist because of system metal right but I do not know what is that surface current so I cannot really apply the conditions for the tangential component of the magnetic field.

But on a metallic perfect electric conductor the electric field tangential component must go to zero so I know that equation and in this example that we are considering the TE mode there are only two components ex and ey and there are four walls that I need to consider right left wall right walled bottom one and top one and I have to find out on these walls which are the components which are tangential and apply the boundary condition that that tangential component at that particular wall must go to zero right. So let us go back and write down the cross section.

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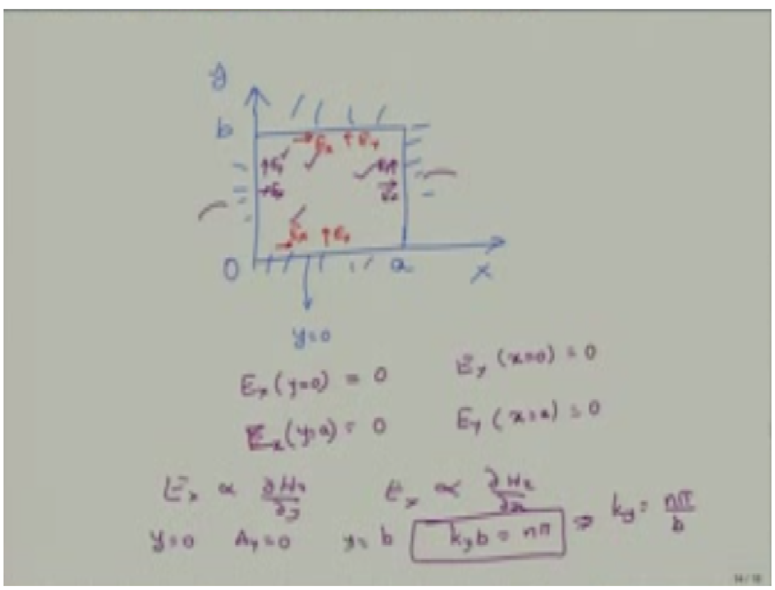

So this is the cross section that I have so this is the this is y equal to this is the Y then this is y equal to B so this is zero and this is along X this is equal to a so I have this wall here bottom right top and left wall okay so on this one what is the change that is happening the X is actually changing but Y is equal to constant right so on this one y is equal to 0 how is how are the electric field e ex and ey oriented e X is along this way but ey will be oriented this way right it would be going from bottom to the top so it would be vertical out there so clearly this is not the tangential component this is the tangential component so I will put a tick mark against that okay.

So my boundary condition is that e X at y = 0 µst be equal to 0 okay similarly if I look at what is the tangential component here again the x directed component is tangential the Y directed component would still be vertical whether it is up or pointing up or pointing down does not matter it is still vertical out there so the other boundary condition that I am going to obtain will again depend only on this eX and I have e X at y equal to a equal to zero okay.

Similarly for the right wall and for the left wall you can see that this is e X whereas this one is e Y so obviously this is the tangential component e X is normal here ey is the tangential component I can put it right, so I will have EY at X equal to 0 being equal to 0 ey at X equal to a is also equal to 0 as my problem solved well not really I need to know what is ex and ey but I do not know ex and ey except that I know e ex and ey through this relationship I know ey as J Ω µ / H^2 ∂hz / ∂ X I know what is HZ, HZ is X of X into y of Y where X of X is this Y of why is this so if I differentiate this one with respect to X then I will get e Y component and multiply it with some J Ω µ by H^2.

Similarly if I multiply by some constant and then differentiate this expression X of X Y of X I mean H set of X Y by Y then I will get another I mean I will get a component for EX component right so if I do that what I obtain since e X is proportional to H Z /∂ Y and ey is proportional to ∂hz / ∂ X okay and substituting for y = 0 so what will happen is there is a component here sine KX of X so differentiating sign will give you cos pulls K X out but then differentiating cos will give you sine and pulls minus KX out right.

So similarly it will be for Y as well so there will be a minus sign in the differential x' of x and y' of Y you can actually show that one right and then apply the boundary condition now here I am going to leave this as an exercise for you so when you apply the boundary condition at y = 0 you will see that even a very interesting thing when you apply y = 0o you will see that a Y is zero okay but when you apply the boundary condition at y equal to B when you apply the boundary condition this means that KY x B must be equal to some integer multiple of π because there will be some sign of component right. So there will be K Y x B = n π which actually gives you the value of K Y given by n π / B okay

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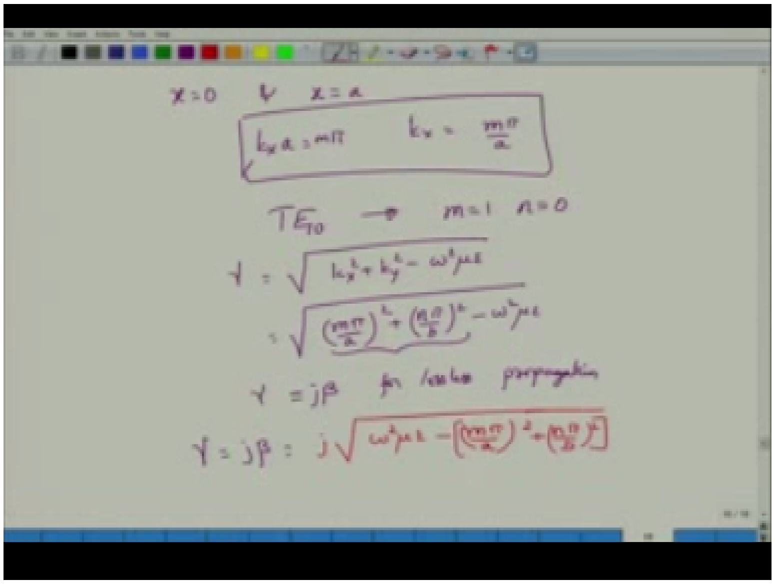

Similarly if you apply the boundary condition at X = 0 and X = 8 you will find K X a equal to some integer multiple of π, so that K X itself is equal to M π/ a and these values are now known because a is known π is known m and n are in your control the mode that I mentioned te10 is actually obtained by putting m=1 and n= 0 okay and then what happened to $\gamma=\sqrt{}\ kx^2+ky^2-\omega^2\mu$ but this is $kx^2$and ky^2 square are nothing but $\sqrt{}m\pi/a^2+n\pi/b2-\omega2\mu$ what kind of a γ do you want you want γ to be pure imaginary right γ to be pure imaginary for a lossless propagation right.

When will this square root thing become imaginary right or when will this fellow become imaginary when the quantity here will be greater than ω2μ or sorry less than Ω square mu epsilon correct so that when that happens you can rearrange the equation and say γ equals J $\beta=j\sqrt{}\omega\mu-m\pi/a^2+n\pi/b^2$ let me put this inside the root inside the bracket so this right hand side of this expression $j\sqrt{}\omega\mu-m\pi/a^2+n\pi/b^2$ right will give you the propagation coefficient or the propagation constant of the modes.

Okay and this has actually happened under the condition that $\omega^2\mu>m\pi/a^2+n\pi/b^2$ okay if I define this quantity in what is this $n\pi/a^2$ and $n\pi/b^2$ if I define this as sum $\omega c^2\mu$ because it is just a constant I can redefine it as $\omega c^2\mu$ and please remember ωc will depend on M and N okay so ωC actually depends on M and N on mu and epsilon of course this does not depend on that one and since ωc is nothing but 2 π FC right this FC is what we call as cut off frequency so only when the applied frequency or the operating frequency actually exceeds the cutoff frequency then γ will become pure imaginary which means there will be lossless propagation inside a waveguide as long as the frequency is less than the frequency cutoff frequency for that value of m and n.

Because this actually changes with M and N right so if you if your operating frequency is less than the cutoff frequency for that given pair of numbers m and n then that particular mode will simply be attenuating it will never propagate okay so that is the reason why sometimes you know these wave weights are called as high-pass filters or exhibit a high-pass filter characteristic because when the frequency is less than the cut off frequency.

For the given mode number m and n there would not be any propagation only when the frequency exceeds the cutoff frequency then that particular mode actually begins to propagate and the TE 1 0 mode is the mode which will propagate but the lowest frequency possible.

So given a rectangular waveguide of certain cross sections the lowest frequency cutoff frequency occurs for the so-called TE 1 0 mode where m is equal to 1 and n equal to 0 we will see what is that cutoff frequency well cutoff frequency.

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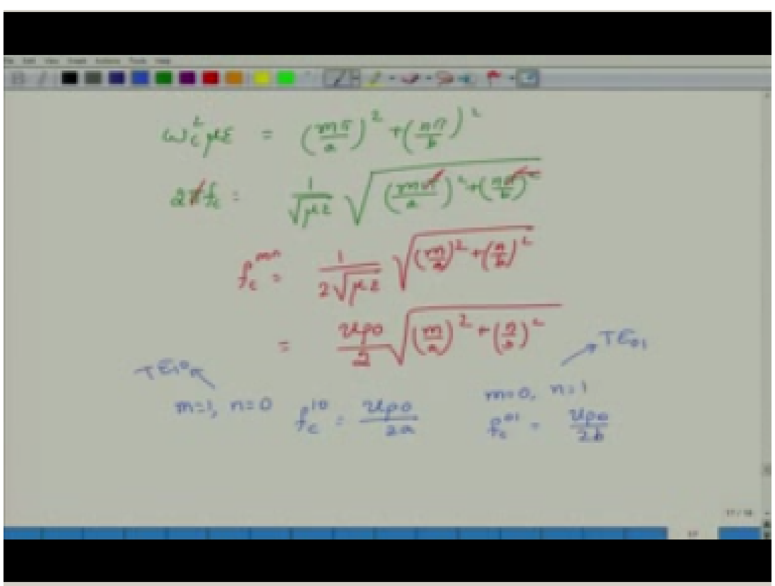

Is $\omega c^2\mu = m\pi/a^2 + n\pi/b^2$ right so this is in general now I can write down what is ω C or equivalently I can write down what is 2 π FC this is $1\sqrt{\mu}\sqrt{m\pi/a^2+n\pi/}$ in these equations this Π and π can be π removed outside the square root so when you remove them outside the square root it becomes π that π can be cancelled off with this π okay so essentially I can cancel off this so FC the cutoff frequency which is actually dependent on MN and sometimes I will use the upper script to just denote that this is a quantity that depends on the more number m and then this is given by $1/2\sqrt{\mu}\sqrt{m/a^2+n/b^2}$.

If you had instead of a waveguide if you just had the medium as it is and just you know imagine that the waveguide has a width a going off to infinity and width or the height B going off to infinity then it is just a medium in between with μ and epsilon and if you launch a plane wave then that plane wave would propagate with a certain phase velocity given by 1/√μEpsilon and if μ is equal to μ naught epsilon is equal to epsilon then that velocity will be the velocity of light in free space right that would be seat speed of light in general.

Let me call that as up0 where u p0 denotes the phase velocity of the medium with the waveguide walls moved to infinity and the medium is essentially you know consisting or characterized by μ and epsilon values itself so that could be up0 so this is u p0 by 2 m by K whole square n by B whole square under root okay I promised you that t10 is the fundamental mode I will show you that one okay when M = 1 and M = 0 what will happen to this expression here this expression will be u p0 divided by 2 because n= 0 so that will be gone and this is equal to this.

So the cutoff frequency FC1 0 will be u p0 by 2 a okay now suppose you try M= 0 and n=1this would correspond to the mode te 0 1whereas this corresponds to the mode te1 0 right so what would be the cut off frequency here FC of 0 1 is equal to up 0/ 2 B well we have already said that a is greater than B because a is greater than B and a and B appear in the denominator rather than in the numerator the cutoff frequency of FC 1 0will be less than the cutoff frequency of FC 0 1 okay.

So this is true for at y π cal waveguide that we consider the cutoff frequency for the 1 0 mode will always be lower than the 0 1 mode you might question whether M = 0 n = 0 condition is possible well let us go back and look at the expressions X of x and y of Y ok so you had H of Z to be equal to X of X into y of Y so in this we have also seen after applying the boundary condition where we had applied a Y is 0 and similarly I had shown you I think a X is also 0 okay so yeah I is also 0 that would have come from this expression I think somewhere over here okay I have not mentioned but please note that 100k X is also zero.

So if I go back to these expressions a X is zero ay is zero so the equations are not containing the sign terms they will contain the cosine terms okay but our electric field components are actually do you know proportional to the differential of these quantities right Eva is proportional to Delta set by Del X which means they will be of the form sine KX into X sine K Y into y and when M=0 and n =0 then what will happen KX will be zero K Y will be zero so electric field components will be completely zero so the condition that we had here M equal to 0 n equal to zero can never occur in practice.

Because this condition is the most trivial condition which tells you that there is no field at all okay you cannot have just a magnetic field right you don't just have h xh h way and head said because it is a time varying scenario and eye and e^x because they are proportional to sine KXx and sine k YY if they are not present if m is equal to 0 and n equal to 0 both are 0 then the total field will actually be equal to 0 okay so please keep that in mind and therefore this F c10 the frequency t10 whose cutoff frequency is FC 1 0 is called as the fundamental or the dominant mode now I have used the word mode a lot of time so what is the mode that I'm talking about mode is just a pattern of the electric field or the magnetic field depending on what you are you know what you would like to use it or sometimes the pattern of the power itself the pointing vector itself.

But more or less it is taken as the way in which the electric field pattern looks like as a function of x and y so in the cross section of the waveguide that I have what is the way in which the electric field whether it is how is it distributed please note that this distribution is governed by two things one is by Maxwell's equations right because that will tell you how the electric fields are actually you know propagating inside the waveguide and second the way this particular fields are you know range is determined also by the boundary condition okay so you will see that the mode shapes for the TE 1 0 mode will be slightly different for T 0 1 t20 and what not other modes.

And if you replace the metallic wave grades with dielectric waveguide such as optical fibers in the modes will be different ok because it is not just the Maxwell's equations which are determining the modes but also the boundary of the waveguide or the boundary conditions that you need to impose okay.

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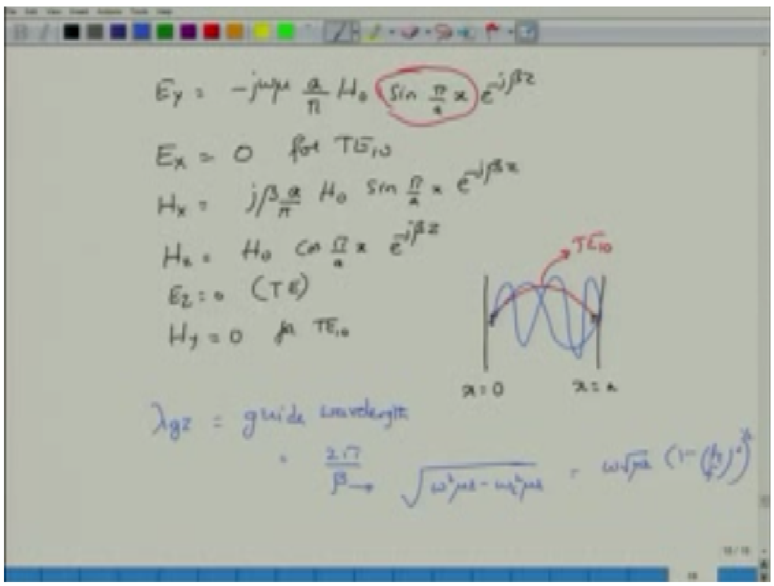

Let us go back we are not completely done with the wave guide solution out there so far we had considered the general te modes but we know that TE 1 0 is the dominant mode whose cutoff frequency FC 1 0 is given by u p 0 divided by 2 a correct so we have already seen this one with that and with the fact that HZ of x and y will be some constant right cos of KX x okay with n equal to 0 that cos of K Y into y will be equal to 1 okay so this will be cos of KX and what is KX here we are looking at em equal to 1 therefore this must be equal to Π by a for the TE 1 0mode KX is equal to Π by a therefore this would be caused by by a into X and in terms of Z it would be a power minus β into Z where beta we have already determined from that expression.

And this would be some constant which we will call as constant H 0 right if you are not happy with the constant well you know this is the constant that would be BX into or rather be X into B

Y something like that right so BX into B Y that would be the constant that would be left out we simply put all of them into a single constant h0 okay now I know that is proportional to Delta Z by Del X and E X is proportional to Delta Z by Del Y right it is just proportional because there are also factors of J Ω mu by γ square or not γ square it is J µ mu by H square right H is the quantity I think so let me go back and correct that one for you so that is J µ it is the proportionality constant is J Ω mu by H square but now H square is actually equal to γ square plus µ square µ epsilon right but γ is actually pure imaginary so that would be minus beta square plus Ω square mu epsilon so this is actually µ square µ epsilon minus beta square okay so you can actually put that one down out here.

And instead of this you can look at this also you know that β square is equal to Ω square mu epsilon minus KX square plus KY square in the TE 1 0 mode KX KY square is actually equal to 0right because KY n equal to 0 so µ square minus beta square is actually equal to KX square KX square is nothing but Π by a whole square so you can actually put all these constants and then show that the corresponding you know the corresponding component a Y will be equal to minus Jµ minus because there would be some term that would be coming out in the negative sign there in the in the solution you can show that times H 0 find Π by a into X e power minus J β Z luckily or unluckily you do not have X component in this case.

So X is actually equal to 0then you have H X component given by J β a by Π H 0 sine Π by a X E power minus J β times Z okay then finally Hz although I have written already Hz what it is, Hz is H0 cosπ/a x $e^{-j\beta z}$ okay, what about the other components that we have so Ex0, Ey0, Ez anyway is equal to 0 because this is th TE mode and then turns out that Hy you will also be equal to 0 for TE_{10} mode okay, so these are the conditions for TE_{10} mode as you can you know derive and then show that these expressions are correct. Now I would like to take a look at this expressions okay, especially look at this Ey and observe how it is actually changing with respect to x.

The corresponding you know way it which is changing with respect to x the function is actually is a sin function that it make sense well it does, because remember Ey is tangential to two walls which are those walls they are the side walls right, so you have the side wall over there which was at x=0 and another side wall at x=a, correct. And of course you had the other two walls along y=0 and y=b.

But on x=0 and x=a Ey have to go to 0 so essentially its amplitude would be 0 at this point, similarly amplitude of Ey at x=a must also be equal to 0. What sort of functions trigonometric function that can I fit into this one. Well, I can fir in a nice half a side right, such that sin of x=0, sin of x at π will also be equal to 0, right. So I can fit in this way and I can also fit in a different one, I cannot of course make this 1 because it is on 0, so the other way I can fit this function would be to fit this way one complete sinusoidal right.

And what is stopping means from fitting other type of functions, well I can fit this one as well. So I can fit any number of integral half, any number of half multiples of the sin wave that is or the multiples of half sin wave okay, the most fundamental mode will be the one that will actually fit with only a single half cycle of the sinusoidal signal and that happens to be the TE_{10} mode, so this component will actually be for TE_{10} mode and these are the higher order components, okay so these are the higher order modes are nor components where the higher order modes, okay.

There are a few definitions that go with wave guides that we should be familiar with, we define λg okay, as the guide wave length and this guide wave length is actually something that is measured along the z axis which is the direction of propagating modes, right so the modes are actually propagating along the z direction and this is given by 2π/β, β is actually the propagation constant which tells you how it is, how the phase factor is changing with respect to z, but I know what is β in terms of ω right, so β is actually given by $\sqrt{\omega^2\mu\varepsilon-\omega c^2\mu\varepsilon}$ because I just wrote that kx^2+ky^2 in this fashion, I can pull this $\omega^2\mu\varepsilon$ and as a common factor so I will actually have $\omega\sqrt{\mu\varepsilon}$ for β I am writing inside it would be $\sqrt{(1-fc/f)^2}$ okay.

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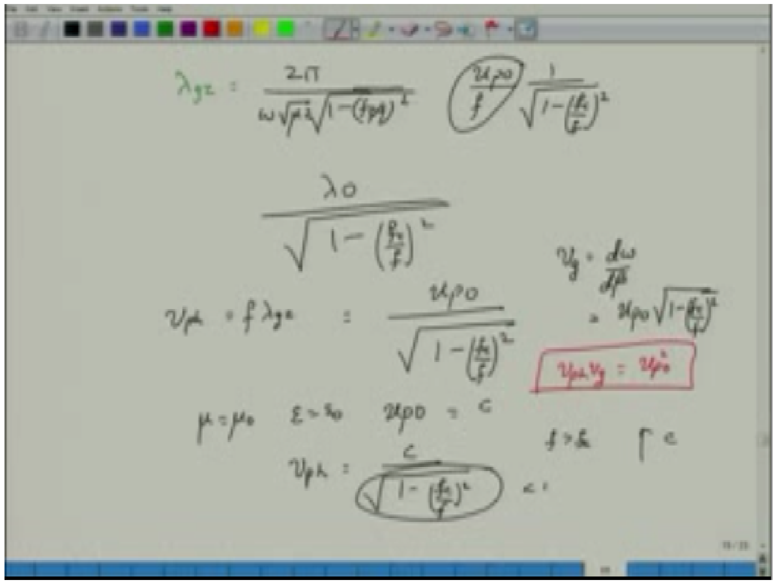

So now what is guide wave length $\lambda gz=2\pi/\omega\sqrt{\mu\varepsilon}\ \sqrt{(1-fc/f)^2}$ okay, what is $2\pi\omega\sqrt{\mu\varepsilon}$ now, ω is $2\pi.f$ correct and $1/\sqrt{\mu\varepsilon}$ is nothing but the phase velocity of the free space media that we consider, so up0/f into this factor $1/\sqrt{(1-fc/f)^2}$ okay, what is up0/f it is actually the you know wave length of a wave which is propagating in the medium characterized by μ and ε correct, this is the velocity, velocity by frequency is the wave length, so this is actually operating wave length $\lambda 0/\sqrt{(1-fc/f)^2}$ okay.

What would be the phase velocity, well phase velocity will be related to the guide wave length because the phase velocity is the velocity with which the wave is travelling along the z direction that is the direction in which the wave is propagating so you will have to calculate the wave guide length along the z direction and multiply by the frequency, okay. It is not calculated in the direction normal to it I will come back to that in a moment but the phase velocity that we mean is the velocity.

With which the wave is propagating along the z direction and for that one you have to just multiply the frequency and λgz and if you do that you will see this is given by $up0/\sqrt{(1-fc/f)^2}$ it is a problem well what will be μ when $\mu=\mu 0$ and $\varepsilon=\varepsilon 0$ in that case up0 which is the phase velocity of the medium will b equal to speed of light so what we say is that the phase velocity is equal $c/\sqrt{(1-fc/f)^2}$ right.

What is f and fc, how is f and fc related, f is actually greater than fc so which means the denominator here will be a quantity less than 1 if this is less than 1 then the phase velocity is actually greater than c, does it violate relativity Einstein's relativity well it does not really violate Einstein's relativity because the wave which is propagating with this phase velocity is carrying 0 information.

The information is actually carried at a different velocity called as group velocity which we will meet in the next class okay, or in the next lecture but for now vg the group velocity defined as $d\omega/d\beta$ that is not the ratio of ω to β as it could be but this is $d\omega$ to $d\beta$ and it terms out to be $u_{p0}\sqrt{(1-f_c/f)^2}$ okay, so luckily we have a relationship which tells you that the phase velocity and the group velocity, groups velocity is the you know velocity with which the wave is actually propagating on a carrying information so this is given by u_{p0}^2, so this relationship always holds phase velocity by itself does not mean anything, okay.

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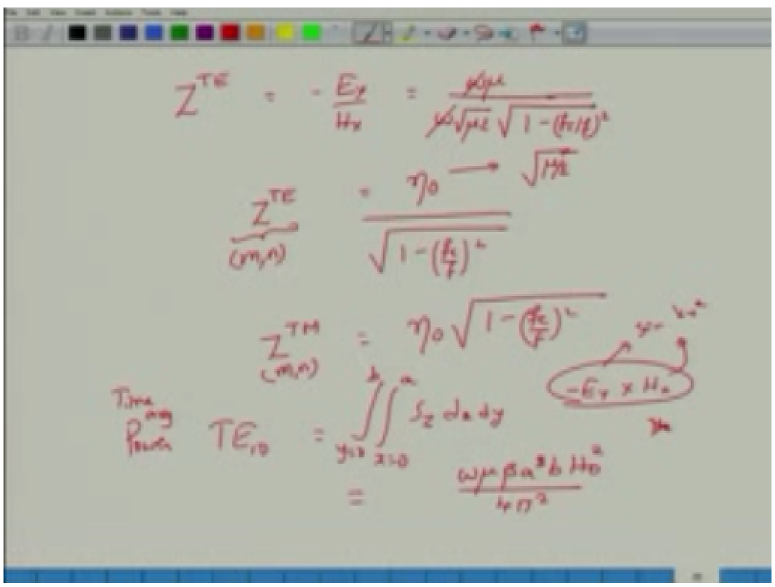

Well, we have waves and we know that we can actually form the ratio of this wave components or the field components that are there and when we form the ratios of electric field to the magnetic field we end up with impedances right, so we have a TE impedance which is defined as $-E_y/H_x$ okay, this $-E_y/H_x$ is simply because the wave is suppose to propagate along z direction so the impedance also a student such the rate kind of points into the z direction.

So the impedance Z^{TE} is $-E_y/H_x$ and you can substitute from the previous value of pervious expressions for Ey and Hx and show that this can be written as $\omega\mu/\omega\sqrt{\mu\varepsilon}$ and this factor $\sqrt{(1-f_c/f)^2}$ factor okay, ω and another cancel out $\mu/\sqrt{\mu\varepsilon}$ is nothing but the medium impedance η_0 okay, so η_0

is $\sqrt{\mu/\varepsilon}/\sqrt{(1-fc/f)^2}$ okay, so this is how the TE mode would actually be present and if I, I have only shown you the expression with fc I am nothing but the values of m and n but you have to putting the values of m and n to figure out the appropriate frequency cut off frequency and for a given operating frequency form this $\sqrt{(1-fc/f)^2}$ factor and then multiply or divide η0 by that particular factor okay.

So this impedance is actually dependent both on m and n of course I would not show you this but the impedance for TM case you will actually be $\eta 0\sqrt{(1-fc/f)^2}$ so in fact if you measure the TE impedance and TM impedance you can actually measure what is the free space mode η0, okay. Now w have talked about wave guides, wave guides carrying information but the wave guides as they carry fields they also carry some amount of power, right.

The power carried along the z direction is obtained the time average power that is carried along the direction is actually dependent on what mode you are propagating, for the TE_{10} mode so average power the time average power that is being carried so let me write this out, time average power that is carried by the TE_{10} mode is given by integral of the z component of the pointing vector over the cross section of the wave guide, what is the cross section of the wave guide x=0 to a, y=0 to b.

And you can show that one what should be the fz component, fz component is given by Ey x Hz correct, or –Ey x Hz because that is the one that would correspond to the z there is another Hz component but if you take Ey x Hz that would be along the z direction so that would not give you the z directed pointing vector, so you do not want this when, we just want to –Ey x Hx and because Ey is proportional to sinkx, x where kx is π/a, Hx is also proportional to the same thing and these two are independent of the y you know co-ordinate.

You can put in the expressions for Ey and Hx from the previous you know slides that we have put in or may I have shown you that you can show that this power carried the time average power carried will be given by $\omega\mu\beta a^3 b$ that is dependent as a^3b and the constant $H0^2/4\pi^2$ okay. Now in fact, this is used this expression is used to fix the value of constant H0, okay why because you know normally how much power you put into the wave guide and once you know the power you can actually find out what is H0 the field strength and then you go back and plot the values of H0 everywhere.

So I would like to finish with the TE mode here, and I would like to also while I am not discussed TM mode but I just like to give you the intuition of what form of the fields you can actually obtain if you just know the cross section and know the boundary conditions, okay. So it go back to the cross section over here.

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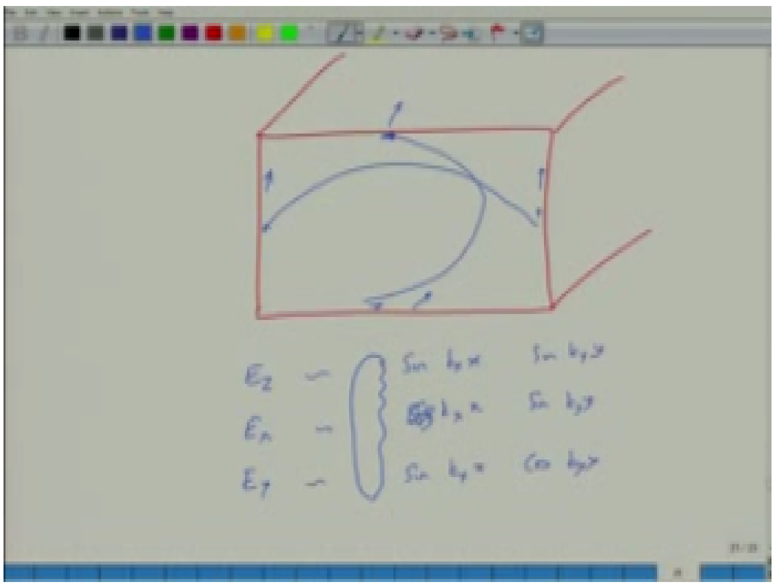

So this is the cross section of a wave guide right, if I am looking at Ey component where I want both Ey to be function of x and y and Ex component to be function of x and y, as well as Ez component for example if I am looking at the TM mode I need to know what is the Ez component as well, right. How do I write down the form of the solutions without actually solving Maxwell’s equation.

Well boundary conditions, how does E so let us also not just show the cross section is also extended because I want to show you EZ as well. We have already seen what will happen to Ey at x=0 and t x=a right, so you need to have the form of a solution in the form of sin some kx into x or ky into y right.

So this is at X= 0 and x=a so you need to have that kind of a thing right. now how about Ex component Ex component is actually tangential onto the y=0 and y=b walls that is for the lower and upper pulse so therefore they must also have a sin type of a function right, what about Ez well is it is tangential to the components so because easy is you know along this one it is actually tangential to the component along the x=0 as well as to the component I mean as well as to the plane or the wall at x=a moreover is it is also tangential at the top and the bottom walls okay, so Ez component will be sin some KX x sin kyy okay, whereas Ex component being tangential only at y=0 and y=b will exhibit a sine nature okay.

Further why so I will write down here so sin nature for the y but it would actually exhibit a cosine nature for the x component I have not shown it for the other modes you know even in the TE other more T21 for example if you value it you will see that this is the form okay, same for Ey as well so Ey will be tangential along the x direction that is for the x=0 wall and x=a wall therefore it would be sine kx x whereas it would be cos ky y okay.

Of course the real reason why you solve Maxwell's equation and go to all that lengthy procedure is because you do not know all these constants and without these constants you really cannot find out all the other values of impedance and other things right, so you still need to learn how to solve the general waveguide with the steps that we have talked about but intuition should tell you where your whether you are getting the right solution or the wrong solution with this, thank you very much.

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