Indian Institute of Technology Kanpur

National Programme on Technology Enhanced Learning (NPTEL)

Course Title
Applied Electromagnetic for Engineers

Module – 55
Attenuation and Dispersion in rectangular waveguides

by
Prof. Pradeeep Kumar K
Dept. of Electronic Engineering
Indian Institute of Technology Kanpur

Hello and welcome to NPTEL mook on applied electromagnetic for engineers in this module we will wrap up the discussion of rectangular waveguides by looking at two topics in rather brief discussion one is attenuation and the other one is dispersion, now attenuation in a waveguide as the electromagnetic wave in the form of a more begins to propagate will be of two sources that is it is because of two sources one is that the material that is used to fill the waveguide if it is a dielectric material then you do not expect the dielectric to be perfect in the sense that there would not be any shunt current.

So this is the same thing that we have already encountered in the case of a imperfect dielectric and we modeled this imperfect dielectric as some kind of a conductance component in a transmission line so in a manner that is very similar but slightly different we will you know we can consider the attenuation because of the imperfect dielectric that we fill the material however impact is most microwave waveguides operating in the region of frequencies are 3 and beyond 3 gigahertz are usually air-filled.

So there is no question of you know a separate dielectric material being filled whenever such materials are filled then we need to consider the imperfect dielectric but when it is air then we can more or less neglect the attenuation caused by the imperfect air in the sense that we can neglect the conducting current from one plate to the other plate inside a waveguide because of the imperfect air.

So attenuation because of the dielectric is present but it can be considered to be small or negligible in compared to the attenuation introduced by the imperfect metals, now this is actually much more in the order of magnitude because metals we assumed when we derive those equations were to be of perfect electric conductor which meant the tangential electric field component there on the surfaces or the boundaries went completely to 0 there right however the conductivity of most metals is very large but they are not as large as one would like to make an approximation towards ∞.

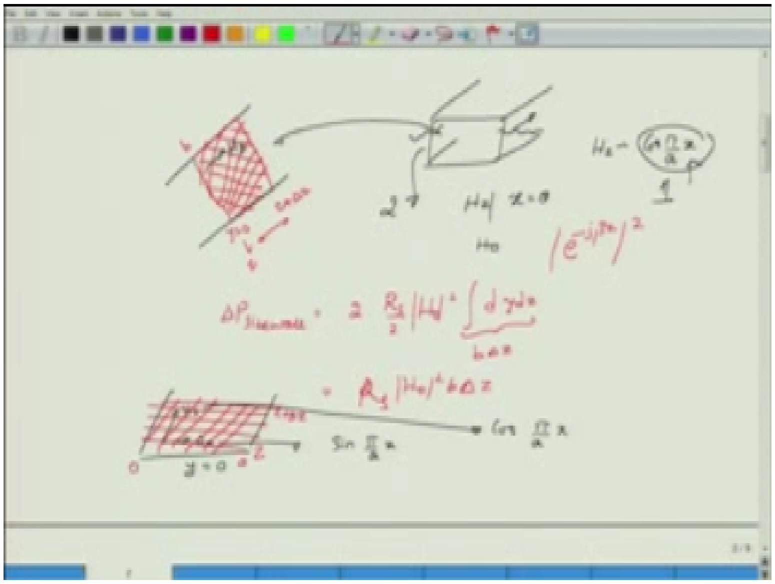
So most for example if you take a wave guide made out of copper then the conductivity is in the range of 10 to the power Siemens per meter you would definitely want it to be much more so that the approximation that you made in terms of perfect electric conductor and the metal to be as better as possible because it is not possible you will have to consider the effect of an imperfect metal and then see what happens to the wave as it propagates inside the waveguide whether there would be any power lost because of the walls that are not perfect anymore insert we have seen a sensation that is very similar to this we have in fact calculated all these things when we discussed the concept of a skin effect right.

So there we had a wire which we considered a round wire or even a planer conducting surface that we considered and waves were all no incident on that one so you would expect on a perfect conductor that no way would propagate or penetrate inside but we found that inside a waveguide there would be a small thickness of an effective thickness what is called as the skin effect or the skin depth through which the fields actually propagate there they do attenuate inside but to a finite extent determined by the conductivity in the form of the skin depth the fields are not completely zero but extend down into the imperfect metallic surface.

Now a very similar thing would happen over here you have a waveguide made out of four walls so you have two side walls and then you have two walls one at the top and the bottom and these walls are all made out of imperfect metals and in our intuitive picture of a mode we have seen that the propagating mode can be considered as something that would be making partner reflections and eventually propagating along the axis of the waveguide.

So this is a wall then the electromagnetic wave at an angle θ would know would be incident on this one then it would be reflected again incident I mean again reflected and so and so it would happen both in the top and the bottom one as well as on the side wall so in that intuitive picture it is clear that you are actually looking at the skin effect kind of a problem because of the light or the electromagnetic wave incident on the top wall where the wall is now not perfect metal okay.

(Refer Slide Time: 04:44)



And we have seen that the average power that is dissipated when you consider the imperfect metal and they have derived the surface impedance of an imperfect metal out there and we call that imperfect metal as RS and you can show that this particular average time average power that is dissipated is actually given by ½ R_s where R_s is the skin resistance or the skin effect resistance that we have calculated right.

So it was related to Δ in our earlier skin effect discussion and then you need to have the current component right, so we have also seen that in the case of a planar conducting surface the current component was actually given by or is proportional to the magnetic field the tangential magnetic field on that particular surface right, so in the example that we considered you had EX and then you had a check or HX and then the current per width along this direction that we have looked at is actually related to tangential magnetic field HX for the case of a plane wave incidence that we have considered.

So in any wall if you are able to find out what is tangential magnetic field on that particular wall then that would correspond to the current per unit width of the on that particular wall so IW is what we used in the previous thing, so if I can substitute IW = H_T where H_T is the tangential magnetic field then the average power that is lost because of the skin effect would be proportional to so since the tangential magnetic field in general could be a vector it would be H_T you know with a vector sign.

So this is given by ½ R_S H_T^2 okay this is the power density you need to integrate over appropriate region in order to obtain the total average power that is lost because of the skin effect okay so we have seen this example and for the TE10 case if I go back and look at what are the walls of the waveguide that we have we have four walls as we said there are two sidewalls 1 at x = 0.

And the other at x = a and then you have two walls one at the bottom at y = 0 and at y = b as the top one on this TE10 mode what field components have not do exist the field companies that exist are the Z component had said with an amplitude of H_0 and a dependence in terms of X as cos Π by a times x. Whereas for the corresponding E_Y component the E_Y component was some constant you know there were all these constants of Jω μ a whatever that is but you are not really interested in the E_Y component you are interested in the other component which is the magnetic field HX and HX is given by Jβ H_0 a / Π this is the constant that would multiply and in terms of the dependence on x it would be sin Π/ a x x

Of course all these components go in terms of z as e^{-j}β times z where β is the propagation constant for the TE10 mode okay now to determine attenuation the procedure that we follow is that we recognize that the power carried by the wave or the mode as it propagates along the wave guide you know what any point of said is given by the power in at z = 0 which is the incident power we are coupling into the waveguide and if this is power is actually getting attenuated okay power is attenuated let us say the attenuation coefficient for the field is $e^{-\alpha}$ z or the attenuation coefficient eα the power goes as e^2
We have seen this in the case of a transmission line with losses as well right so you have p0 e^{-2} αz as the power that would attenuate as the electromagnetic wave part you know propagates through the waveguide okay, so if you differentiate this one so if you differentiate this one with respect to z axis what you get on the left dP/ dz which you know is the power per unit length along the propagation direction or the rate at which the power is actually changing along the propagation direction.

So this would be =- 2α p0$e^{2\alpha\ z}$ but this is nothing but power at that particular z right so power at any point z is this one times 2α and if you are only interested in the attenuation coefficient you do not really bother about the sin you can forget about it by taking the magnitude of all of these quantities and then obtain α as 1 /2P at any particular z that you are considering and then dp/ dz okay.

So if you find out what is the change in the power over that you know along the distance or the propagation direction which we will call as ΔP so if you find out what is the power per unit length along that direction then multiply this one by the power that you are using at z itself this would give you some indication of α okay please note that this procedure although seems to be intuitively correct is not really correct because originally we derived the fields assuming that the boundary condition would tell you that the fields must go to zero at all of the four walls right.

But here we know that the fields are not going to zero so the fields exchanged slightly brief beyond the walls so strictly speaking whatever the modes the expressions that we have obtained is not correct right it is not correct because we have not applied the correct boundary condition the correct boundary condition would have been just a continuity boundary condition because now this is not a perfect electric conductor.

However the procedure if you were to try and you know implement that one would involve applying boundary conditions on an imperfect conductor and then recalculating all of the fields which you know is very tedious, so we adopt this procedure by finding out what is the total power per unit length and then normalizing that one with twice the power that is incident our power twice the power present at that particular plane assuming that the attenuation is there but it is not so high that you need to re-evaluate the fields okay.

So that is the philosophical thing that you should keep in mind that this method intuitively is fine but technically may not be fine but the results that you obtain with this method or more or less okay as long as attenuation is quite small okay so having said that.

Now let us try and evaluate what is this Δ P /Δ Z well on the Left wall that you have what is the tangential H component that you are going to obtain that would be H set component right on the left as well as on the right walls but if you take this waveguide again so you see that this is the left wall so on which the tangential component would be along Hz.

And it would be the same tangential component Hz along the right wall as well so if there is a power getting lost in the left wall power must also be getting lost in the right one and because of symmetry the total power lost in the left and the right side walls will be twice the power lost in the left wall let us say or you can look at the power loss in the right wall as well.

What is Hz at the left wall well Hz has to be evaluated on the left wall at X= 0 and we already know that HZ in terms of X will be going as cos Π / a times X so clearly putting x= 0 will cause this term =1 and the value of Hz at x= 0 will simply be the constant H_0 and then you pick off a particular unit length okay so this headset is existing or is this headset extends all the way from y = y = B, so this is aside wall on this the headset should exist so this is from the bottom portion right.

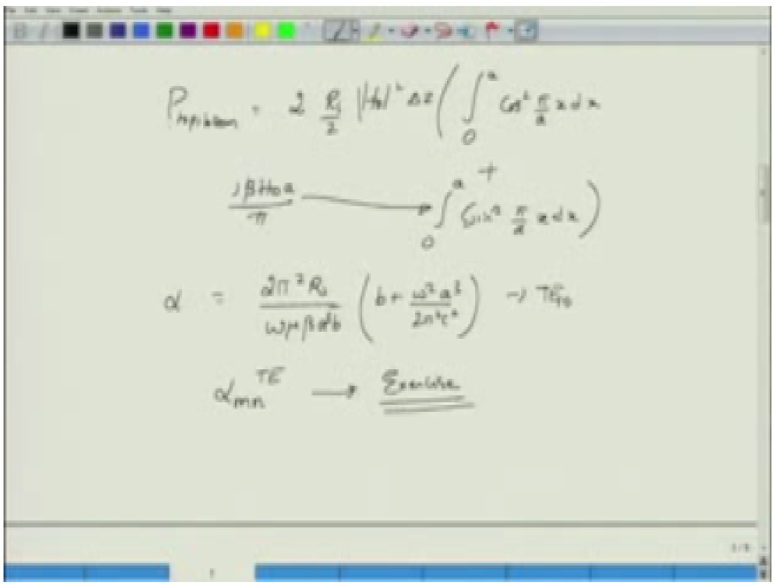
So this is the bottom wall so from the bottom portion at y = 0 all the way up to y = B but luckily hence there is not a function of B therefore when you pick this particular you know claim to evaluate the power being lost right, so the integration from y = 0 to y = B will simply give you a constant multiple of B okay along Z again it would because Hz is not a function of z in this case remember although there is an $e^{-J\beta z}$ term because you are taking the magnitude square that term would be =1.

So if you find out what is the power that is being you know lost because of this side wall over here which extends from y = 0 to B and some z to z + Δ z along this particular axis along the wall then the total power lost because of the side wall will be 2 ½ R_S /2 is present and this 2 is again telling you that the power is not twice the power that is lost and because this is constant this would be $H0^2$ integration of dy dz over the appropriate limits that we have indicated will simply provide you with B x Δ z.

So the power lost because of the side wall is R_S H_0^2 B x Δz okay for the bottom wall what would be the situation let us go back and look at what is the situation for the bottom wall right or the top wall does not matter whichever that is again by symmetry you can see that they will be equal so you can just evaluate the bottom plate as said so this axis is along z this is along the x-axis.

So what tangential component you have here on the bottom wall you have a Hx component right which is tangential you have a Hz component which is also tangential because the bottom wall is actually at y = 0 and both quantities are not functions of Y right HX as a function of X will be in the form of sin Π / ax whereas Hz as a function of x is actually in the form of cos Π / a x x and neither of these quantities reduce to some kind of a constant therefore you need to carry out the full integration if you were to consider the power being lost in this hatched area okay which would extend from y = 0 all the way to it would be from x =0 to a and z=0 to some Δ z or z to z+ Δz okay so x =0 to x=a.

(Refer Slide Time: 15:25)



So if you find out the power lost because of the top and bottom walls so let us call this as the top and bottom walls that would be twice right this is a total power that is being lost so 2 R_S /2 any way exists the constant $H_0{}^2$would come out right and then integration over z will simply pull out Δz but whatever the integration that you have you know in terms of this one that would still be present.

So you have 0 to a cos ^{2} Π / ax dx and this entire thing ∫ 0 to a $\sin^2$ Π / ax dx, so the constant as I wrote was something like Jβ H_0 a / Π so actually you should put this one into HX so this one goes to here the magnitude square of this of course so this goes into this one and then for this H_0 it would still be H_0 as such right so if you do this and then find out what would happen to this overall integral well you can show that after you found this and then divide it by the power carried by the T10 wave which we actually solved it in the previous module.

So the attenuation coefficient α is given by 2 Π^2 Rs / ωμ βe^3 b times b + $\omega^2 e^3$ /2 Π^2 x C ^{2}so this is a expression for the attenuation in fact this expression that we have obtained is only for the TE1

0 mode you can find out the expression for any other mode because you know what is the corresponding TE mode solutions or the expressions for the electric and magnetic fields and then you actually have to find out only the tangential magnetic field on the walls and then substitute them integrate it okay.

I leave this as an exercise for you to find out what happens to the attenuation coefficient of TEmn modes okay and leave this as an exercise it is not difficult it is simply tedious because the walls will not have no nice behavior in terms of the magnetic fields being function of only 1 variable or being a constant as we have found in this particular case, so this is the situation for attenuation but we are also interested in asking what happens if I take a pulse okay or a pulse like waveform and then modulate this pulse like waveform on to a more that is being carried by the waveguide.

And then receive this mode at the output end of the waveguide right and if I receive that mode the main question that I am looking at is whether the mode has attenuated well if the mode has attenuated I might compensate it by either putting it through a gain and it meant you know an RF gain element or by increasing the power at the input side itself but I am actually getting I mean I will be very worried if the pulse that I obtained at the output is not the same shape as the pulse that I hope that I send at the input right.

So the waveguide might not be very long but even if this wave guideis about you know 10 centimeters or about 9 centimeters long and if the pulse input is not maintained in shape right in terms of the duration in terms of the amplitude variation that you would see with respect to time if it is not maintained at the output end then this waveguide is pretty much useless because it is distorting the pulse right.

Any distortion would not be present for the TEM kind of a mode okay that is something that you can show I will leave it as an exercise for you because for the TEM mode the only effect because the phase velocity will be completely independent of the frequency right because phase velocity for a TEM like mode inside a medium plane wave for example will be just $1/\sqrt{\mu \in}$ right there μ and $\in$ are the Constituent parameters.

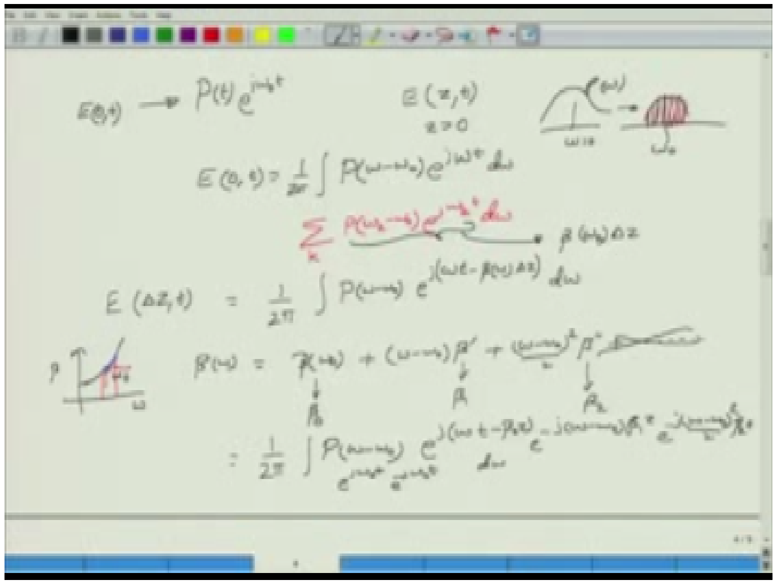
So for a TEM situation or a for a TEM mode if you consider a pulse and the pulse has on like an infinite number of frequencies because Fourier transform tells us so each frequency component will travel through the mode through the wave guide at the same phase velocity and when you reconstruct them back then there would not be any change in the pulse shape so TEM modes do not distort pulses okay there is another way of looking at it while phase velocity is the ratio of ω to β, β is directly proportional to ω $\beta = \omega$ times $\sqrt{\mu \in}$.

And that is the reason why ω in the numerator and ω in the denominator cancel with each other making the phase velocity independent of frequency okay, so this linear dependence of β with respect to ω that is if I double ω β will also double right so in order to maintain the same ratio given μ and $\in$ values so this linear relationship can be shown to be equivalent of a pure phase dela.

So every component gets delayed and every component gets played by the identical amount therefore there is no problem when you put the pulse components back together in order to have the original pulse shapes there is no distortion because of this in a waveguide though the situation is not correct because β is proportional to or β is a nonlinear function of the frequency F why because β is = or β at least as a factor which is $\sqrt{(1 - F / FC)^2}$ or $(FC / F)^2$ right.

So because of this factor $\sqrt{(1 - F / FC)^2}$ the value of β with respect to frequency F is not linearly related it is non linearly related and what means what it means is that different components actually travel at different you know velocities leading to interference among the components at the output and distortion in the pulse okay let me look at that scenario you know in a very kind of a brief manner I will leave most of these statements to you the big picture is what I am planning to talk to you about okay.

(Refer Slide Time: 21:32)



Suppose I have at z=0 okay that would correspond to the input of a waveguide if I have the electric field of a particular mode let us say it is just a team 1 0 mode that I am considering if that electric field would be some pulse like with respect to time and has been modulated in the time domain as well why should I model it because the modes have a cutoff frequency.

So you have to actually operate at the nonzero carrier frequency in order for this pulse to propagate I am assuming that he pulse has it is frequency response centered right Z I mean ω = 0 making this pulse to be a base band pulse okay, so if this with some constant e0 in case you want if this is the electric field of the mode at z = 0 I am concerned to find what is the expression for the electric field or what will happen to this X you know electric field but sum z where z is greater than 0 okay.

I can do so by first looking at E(0 x t) as a inverse Fourier transform right so I can actually think of this fellow as the inverse Fourier transform which would be some P(ω) or you can think of this as P (ω - ω 0) if ω^T dω right so I can think of this fellow as having so if this is P(ω) at ω = 0this is the pulse spectrum so this P(ω - ω0) will be situated at the carrier frequency ω0 the same shape is just has been shifted infrequency from one frequency to the carrier frequency right.
And intuitively Fourier transform can be thought of as many spectral lines that is each of these spectral lines have a different amplitude as determined by this particular graph and then multiplying that one with the carrier $e^{J\omega T}$ or simply putting them back together so intuitively this is many frequencies each having an amplitude so this instead of considering this other function now consider this P(ω) K - ω0 as amplitude of the k^{th} frequency right dω.

So if you just think of this way what you are doing is putting many sinusoids of different amplitudes together now that is essentially what Fourier analysis is telling you okay so once you put this one back you can think of this as simple sinusoidal plane waves which would begin to propagate right, so if I want to find out what is the electric field at some distance Δz from z =0 propagating to z =Δz this will be given by 1 / 2 Π each of these terms here will undergo a phase shift right.

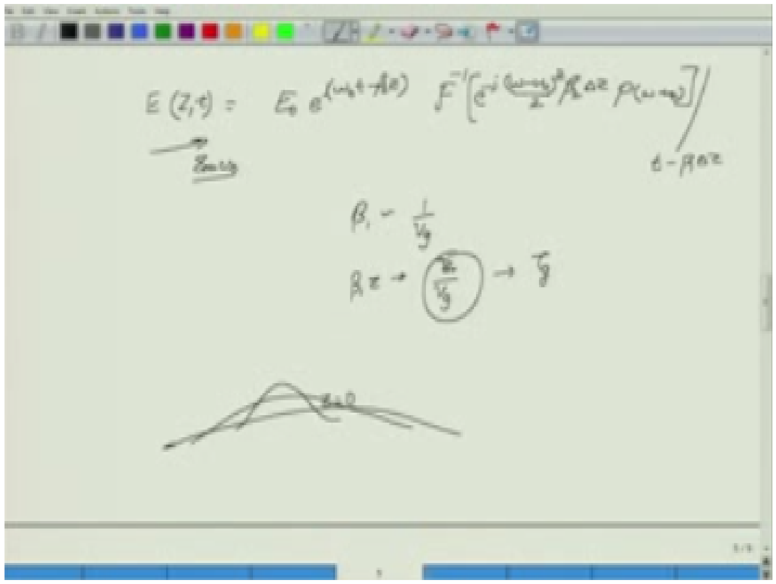
And the phase shift will be proportional to or it will be - βΩ (ωk x Δz) right previously this β(ωk) ω k x √ μ ∈ for the TEM case but for this case it is not it depends on ωk so putting back together is very simple for me if I can rewrite this one as P of ω - ω0 e $^{J\omega t - \beta(\omega) \times \Delta z}$ so this is how the field would look like after you propagated over a distance of Δz if you want to proceed any further you need to know what is β(ω) okay turns out that if β(ω) is this way so let us say this is at ω and this is with respect to β if this is the way in which β and ω look like okay or maybe I can shift ω and β over here.

Do not worry about what I am actually looking at so I am just interested in trying to show you what we normally do okay so normally what we do is when we are given β and ω in this particular manner, so you look for the carrier frequency ω_0 and then you have a carrier or sorry you have a pulse around that particular carrier frequency which means that the total frequency range would be this one so the curve that you are looking at is actually this nonlinear curve given in blue line which represents the interesting range for ω and β relationship okay.

In that range with ω0 as the center frequency I can expand this β (ω) in Taylor series as β(ω0) + ω - ω0 that is at any frequency ω around ω0 I can expand using Taylor series times β` evaluated at ω 0 of course +(ω - ω) ^{2} / 2β``these terms would continue to exist but we will terminate the expansion over here okay if I terminate the expansion over here and then denote this β` as β1, β`` as β2 and β at ω0 as β_0 I can put this back into the expression for the electric field at Δz, so I obtain the pulse spectrum e (ω -ω0) e $^{J\omega t - \beta z}$ e$^{- J \omega - \omega 0 x \beta 1 z}$ e $^{- J\omega - \omega 0) 2}$/ 2 x β 2 x z right this entire thing integrated over dω.

Nnow you have e $^{J\omega t - \beta 0 z J\omega 0T}$ if you multiply and divide this one by e $^{J\omega 0T}$ that is you multiplied by this identity right inside the integral and you can do that because ω_0 is a constant in this integral so you can pair off the e $^{J\omega 0T}$ e $^{J\omega 0T}$ or you can pair it off with e $^{-J\omega 0T}$ and then you have e $^{J\omega 0T}$ and β_0 z which you can pull it out okay.

(Refer Slide Time: 27:27)



Long story short you have e at Z (T) as the carrier e $^{J\omega 0\ T-\beta 0z}$ and inverse Fourier transform of e $^{-J\omega - \omega 02\ /\ 2\beta 2\Delta Z}$ with the original pulse that we have looked at which is ω - ω_0 okay this entire thing evaluated at T - $\beta 1\Delta z$ I leave this as an exercise for you to show is just combining that terms that we have talked about okay let me make a few remarks over here the carrier that we had e $^{J\ \omega 0T}$ would propagate and then give rise to a pure phase delay right so that delay was given by β_0 x z.

Since it is a single frequency e $^{J\ \omega 0T}$ this is what you expect a single frequency would undergo a simple pure phase delay ok but when your pulse you have multiple different frequencies then the correct expression that you are going to obtain at the output or the after the propagation will involve two things one is time has been replaced by t - β 1 x Δ z okay this β 1 is the ratio of dβ / dω or rather it is a differentiation of β with respect to ω since ω / β is the velocity dω / dβ is what we call as the group velocity.

So β_1x Δ z or β_1 x Z in this particular case β_1 x z will be z / vg okay where vg is the group velocity so you have β_1 as 1/vg and then you have β_1 x z as z / vg which is actually denoted by τg and we call this as a group delay okay so you have a pulse which undergoes a group delay which means the peak of the pulse travels more slowly than the carrier and this would be alright if β2 were to be 0 that is all that you would have obtained so the pulse is propagating with the group velocity experiences a group delay but because of this e $^{- J \omega - \omega 02 / 2 \times \beta 2z}$ the actual pulse that you are going to obtain will be the inverse Fourier transform of this nonlinear phase factor times the original pulse.

And depending on what pulse you have given as an input you will see that when β2 is not I mean β2 is present there will always be some kind of a distortion for a simple Gaussian pulse as you know you start at Z = 0 and as you begin propagation the corresponding pulse drops both in amplitude as well as you know expands out you know this is called as a dispersion induced pulse broadening okay, so this is the effect of dispersion and dispersion by itself means the nonlinear dependence of β with respect to ω, so these results are easy to evaluate I will give this as an exercise to you can look at that one thank you very much.

Acknowledgement

Ministry of Human Resources & Development

Prof. Satyaki Roy
Co – ordinator, NPTEL IIT Kanpur

NPTEL Team
Sanjay Pal
Ashish Singh
Badal Pradhan
Tapobrata Das
Ram Chandra
Dilip Tripathi
Manoj Shrivastava
Padam Shukla
Sanjay Mishra
Shubham Rawat
Shikha Gupta
K.K Mishra
Aradhana Singh
Sweta
Ashutosh Gairola
Dilip Katiyar
Sharwan
Hari Ram
Bhadra Rao
Puneet Kumar Bajpai
Lalty Dutta
Ajay Kanaujia

Shivendra Kumar Tiwari

an IIT Kanpur Production